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Discrete-Time Non-fragile Dynamic Output Feedback H_∞ Controller Design

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Abstract— **The non-fragile dynamic output feedback H_∞ controller design problem affected by finite word length (FWL) for linear discrete-time systems is investigated. The controller to be designed is assumed to be with additive gain variations, which reflect the FWL effects in controller implementation. A notion of structured vertex separator is proposed to approach this problem, and exploited to develop sufficient conditions for the non-fragile H_∞ controller design via a two-step procedure. The resulting designs guarantee the asymptotical stability and the H_∞ attenuation level of the closed-loop system. A comparison between our proposed method and the existing method for non-fragile H_∞ controller design is provided, and a numerical example is carried out to support the theoretical findings.**

I. INTRODUCTION

In the course of controller implementation based on different design algorithms, it turns out that the controllers can be sensitive with respect to errors in the controller coefficients ([1], [13]). By means of several examples, it is demonstrated in the control design formalism [8] that relatively small perturbations in controller parameters could even destabilize the closed-loop system. This brings a new issue at the stage of designing controllers: how to design a controller for a given plant such that the controller is insensitive to some amount of error with respect to its gains, i.e., the controller is non-fragile.

This issue has received some attention from the control systems community, and some relevant results have appeared in the last decade to tackle the problem of designing controllers that are capable of tolerating some level of controller gain variations ([1],[3], [6]). Recently, for additive norm-bounded controller gain variations, results have been obtained for both state feedback [13] and output feedback [14] by using the Riccati inequality approach. Multiplicative controller gain variations were addressed in [15].

All the above mentioned works are concerned with the non-fragile problem with the norm-bounded type of controller uncertainty. However, this kind of uncertainty cannot describe the uncertain information due to the FWL effects exactly. Correspondingly, the interval type of parameter uncertainty [9] can describe the uncertain information due to the FWL effects more exactly than the former type. But up to present, there is no work on the non-fragile controller design problem with taking interval gain uncertainty into account. Moreover, similar to the case that the problem of designing a globally optimal full-order output-feedback controller for polytopic uncertain systems is known to be a non-convex NP-hard optimization problem [7], the problem of designing full-order non-fragile dynamic output feedback H_∞ controllers with interval type of gain uncertainty is also a non-convex NP-hard one. On the other hand, the vertices of the set of uncertain parameters grow exponentially with the number of uncertain parameters, which may result in numerical problem for systems with high dimensions. These problems motivate the work in this paper.

To overcome the above mentioned difficulties, this paper is concerned with the problem of non-fragile dynamic output feedback H_∞ controller design for linear discrete-time systems with FWL consideration. The controller to be designed is assumed to be with additive gain variations of the interval type, which are due to the FWL effects when the controller is implemented. And a two-step procedure is adopted to solve this non-convex problem. In Step 1, we give a design method of an initial controller gain C_k. In Step 2, with the controller gain C_k designed in Step 1, an LMI-based sufficient condition is given for the solvability of the non-fragile H_∞ control problem, which requires checking all of the vertices of the set of uncertain parameters that grows exponentially with the number of uncertain parameters. It will be very difficult to apply the result to systems with high dimensions. To overcome the difficulty, a notion of structured vertex separator is proposed to approach the problem, and exploited to develop sufficient conditions for the non-fragile H_∞ controller design in terms of solutions to a set of LMIs. The structured vertex separator-based method can significantly reduce the number of the LMI constraints involved in the design conditions. The designs guarantee that the closed-loop system is asymptotically stable and the H_∞ performance of the system from the exogenous signals to the regulated output is less than a prescribed level.

Notation: For a column-rank deficient matrix H, N_H denotes a matrix whose columns form a basis for the null space of H. I denotes the identity matrix with an appropriate dimension. $\mathbf{0}_{i\times j}$ represents zero matrix of i rows and j columns. The symbol $*$ within a matrix represents the symmetric entries.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. Problem Statement

Consider a linear time-invariant (LTI) discrete-time system as

$$\begin{aligned} x(k+1) &= Ax(k) + B_1\omega(k) + B_2u(k) \\ z(k) &= C_1x(k) + D_{12}u(k) \\ y(k) &= C_2x(k) + D_{21}\omega(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state, $u(k) \in R^q$ is the control input, $\omega(k) \in R^r$ is the disturbance input, $y(k) \in R^p$ is the measured output and $z(k) \in R^m$ is the regulated output, respectively, and $A, B_1, B_2, C_1, C_2, D_{12}$ and D_{21} are known constant matrices of appropriate dimensions.

To formulate the control problem, we consider a controller with gain variations of the following form:

$$\begin{aligned} \xi(k+1) &= (A_k + \Delta A_k)\xi(k) + (B_k + \Delta B_k)y(k) \\ u(k) &= (C_k + \Delta C_k)\xi(k). \end{aligned} \tag{2}$$

where $\xi(k) \in R^n$ is the controller state, A_k, B_k, and C_k are controller gain matrices of appropriate dimensions to be designed.

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$\Delta A_k, \Delta B_k$ and ΔC_k represent the additive gain variations of the following interval type:

$$\begin{aligned}
&\Delta A_k = [\delta_{aij}]_{n\times n}, |\delta_{aij}| \le \delta_a, i,j = 1,\cdots,n \\
&\Delta B_k = [\delta_{bij}]_{n\times p}, |\delta_{bij}| \le \delta_a, i = 1,\cdots,n, j = 1,\cdots,p \\
&\Delta C_k = [\delta_{cij}]_{q\times n}, |\delta_{cij}| \le \delta_a, i = 1,\cdots q, j = 1,\cdots,n. \quad (3)
\end{aligned}$$

Let $e_k \in R^n, h_k \in R^p$ and $g_k \in R^q$, denote the column vectors in which the kth element equals 1 and the others equal 0. Then the gain variations of the form (3) can be described as :

$$\Delta A_k = \sum_{i=1}^{n}\sum_{j=1}^{n} \delta_{aij} e_i e_j^T, \ \Delta B_k = \sum_{i=1}^{n}\sum_{j=1}^{p} \delta_{bij} e_i h_j^T,$$
$$\Delta C_k = \sum_{i=1}^{q}\sum_{j=1}^{n} \delta_{cij} g_i e_j^T.$$

Applying controller (2) to system (1), this yields:

$$\begin{aligned}
x_e(k+1) &= A_e x_e(k) + B_e \omega(k) \\
z(k) &= C_e x_e(k)
\end{aligned} \quad (4)$$

where $x_e(k) = [x(k)^T, \xi(k)^t]^T$, and

$$A_e = \begin{bmatrix} A & B_2(C_k + \Delta C_k) \\ (B_k + \Delta B_k)C_2 & A_k + \Delta A_k \end{bmatrix},$$
$$B_e = \begin{bmatrix} B_1 \\ (B_k + \Delta B_k)D_{21} \end{bmatrix}, C_e = \begin{bmatrix} C_1 & D_{12}(C_k + \Delta C_k) \end{bmatrix}.$$

Denoting the transfer function from the disturbance ω to the controlled output z, corresponding to the state-space model (4), as $G_{z\omega}(z) = C_e(zI - A_e)^{-1}B_e$.
This paper addresses the following problem:
Non-fragile H_∞ control problem with controller gain variations: Given a positive constant γ, find a dynamic output feedback controller of the form (2) with the gain variations (3) such that the resulting closed-loop system (4) is asymptotically stable and $\|G_{z\omega}(z)\| < \gamma$.

B. Useful lemmas

Lemma 1: [12] Let matrices $Q = Q^T$, G, and a compact subset of real matrices $\mathbf{H}$ be given. Then the following statements are equivalent:
(i) for each $H \in \mathbf{H}$

$$\xi^T Q \xi < 0 \text{ for all } \xi \ne 0 \text{ such that } HG\xi = 0;$$

(ii) there exists $\Theta = \Theta^T$ such that

$$Q + G^T\Theta G < 0, \mathbf{N}_H^T \Theta \mathbf{N}_H \ge 0 \text{ for all } H \in \mathbf{H}.$$

Lemma 2: [2] Let $G_{az\omega}(z) = C_a(zI - A_a)^{-1}B_a$, then A_a is Shur stable and $\|G_{az\omega}(z)\| < \gamma$ for some constant $\gamma > 0$ if and only if there exists a symmetric matrix $X > 0$, such that

$$\begin{bmatrix} -X & 0 & XA_a & XB_a \\ * & -I & C_a & 0 \\ * & * & -X & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (5)$$

Denote

$$G_{0z\omega}(z) = C_{e0}(zI - A_{e0})^{-1}B_{e0}, \quad (6)$$

where

$$A_{e0} = \begin{bmatrix} A & B_2C_k \\ B_kC_2 & A_k \end{bmatrix}, \ B_{e0} = \begin{bmatrix} B_1 \\ B_kD_{21} \end{bmatrix}, \quad (7)$$
$$C_{e0} = \begin{bmatrix} C_1 & D_{12}C_k \end{bmatrix},$$

Then, we have the following lemma.

Lemma 3: Let $\gamma > 0$ be a given constant. Then the following statements are equivalent:

(i) A_{e0} is Shur stable, and $\|G_{0z\omega}(z)\| < \gamma$;
(ii) there exists a symmetric positive matrix $X > 0$ such that

$$\Lambda_1(X) = \begin{bmatrix} -X & 0 & XA_{e0} & XB_{e0} \\ * & -I & C_{e0} & 0 \\ * & * & -X & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (8)$$

(iii) there exist a symmetric positive matrix $X > 0$ and a matrix G such that

$$\begin{bmatrix} X - G - G^T & 0 & G^TA_{e0} & G^TB_{e0} \\ * & -I & C_{e0} & 0 \\ * & * & -X & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (9)$$

(iv) there exist matrices A_{ka}, B_{ka} and C_{ka}, and a symmetric matrix $P > 0$ with

$$P = \begin{bmatrix} Y & N \\ N & -N \end{bmatrix}, \quad (10)$$

such that

$$\Lambda_2(P) = \begin{bmatrix} -P & 0 & PA_{ea} & PB_{ea} \\ * & -I & C_{ea} & 0 \\ * & * & -P & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (11)$$

where

$$A_{ea} = \begin{bmatrix} A & 0 \\ B_{ka}C_2 & A_{ka} \end{bmatrix}, \ B_{ea} = \begin{bmatrix} B_1 \\ B_{ka}D_{21} \end{bmatrix}, \quad (12)$$
$$C_{ea} = \begin{bmatrix} C_1 & D_{12}C_{ka} \end{bmatrix}.$$

and

$$A_{ka} = T^{-1}A_kT, \quad B_{ka} = T^{-1}B_k, \quad C_{ka} = C_kT. \quad (13)$$

(v) there exist a symmetric matrix $X > 0$ and a matrix G with structure

$$G = \begin{bmatrix} Y & N \\ N & -N \end{bmatrix}, \quad (14)$$

such that

$$\begin{bmatrix} X - G - G^T & 0 & G^TA_{ea} & G^TB_{ea} \\ * & -I & C_{ea} & 0 \\ * & * & -X & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (15)$$

holds, where A_{ea}, B_{ea} and C_{ea} are defined by (12).

Proof: Due to the limit of the space, it is omitted here. ■

Lemma 4: [16] Let matrices Q, F_1 and F_2 be constant matrices with appropriate dimensions. Then the following statements are equivalent:
(i)

$$Q + F_1\Delta F_2 + (F_1\Delta F_2)^T < 0, \text{ for } |\delta_i| \le \delta_a, \ i = 1,\cdots,s,$$

where $\Delta = diag[\delta_1, \ \cdots, \delta_s]$.
(ii)

$$Q + F_1\Delta F_2 + (F_1\Delta F_2)^T < 0, \text{ for } \Delta \in \Delta_v,$$

where $\Delta_v = \{\Delta : \ \delta_i \in \{-\delta_a, \ \delta_a\}, i = 1,\cdots,s\}$.
(iii) there exists a symmetric matrix $\Theta \in R^{2s\times 2s}$ such that

$$\begin{bmatrix} Q & F_1 \\ F_1^T & 0 \end{bmatrix} + \begin{bmatrix} F_2 & 0 \\ 0 & I \end{bmatrix}^T \Theta \begin{bmatrix} F_2 & 0 \\ 0 & I \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} I \\ \Delta \end{bmatrix}^T \Theta \begin{bmatrix} I \\ \Delta \end{bmatrix} \ge 0, \text{ for all } \ \Delta \in \Delta_v. \quad (17)$$

Lemma 5: ([11]) For any real matrices Y, M, F and E with compatible dimensions and $F^TF \le \delta^2 I$, where $\delta > 0$ is a scalar, then

$$Y + MFE + (MFE)^T < 0$$

holds if and only if there exists a scalar $\varepsilon > 0$, such that

$$Y + \frac{1}{\varepsilon}MM^T + \varepsilon\delta^2 E^TE < 0.$$

III. NON-FRAGILE H_∞ CONTROLLER DESIGN

In this section we will present a two-step procedure which can be used for solving the non-fragile H_∞ control problem, and a comparison is made between the new proposed method and the existing method.

A. Non-fragile H_∞ controller designs with known gain C_k

In this subsection, we will give non-fragile H_∞ controller design methods under the assumption that the controller gain C_k is known, where the gain C_k will be designed in the next subsection.
To facilitate the presentation, we denote

$$M_0(\Delta A_k, \Delta B_k, \Delta C_k) = \begin{bmatrix} \Xi_1 & \Xi_2 & 0 & \Xi_4 & S^TA & S^TB_1 \\ * & \Xi_3 & 0 & \Xi_5 & \Xi_6 & \Xi_7 \\ * & * & -I & \Xi_8 & C_1 & 0 \\ * & * & * & -\bar{P}_{11} & -\bar{P}_{12} & 0 \\ * & * & * & * & -\bar{P}_{22} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} \quad (18)$$

where

$$\begin{aligned} \Xi_1 &= \bar{P}_{11} - S - S^T, \Xi_2 = \bar{P}_{12} - S - S^T, \\ \Xi_3 &= \bar{P}_{22} - S - S^T + N + N^T, \\ \Xi_4 &= S^TA + S^TB_2(C_k + \Delta C_k) \\ \Xi_5 &= (S-N)^TA + F_BC_2 + N^T\Delta B_kC_2 + F_A \\ &\quad + N^T\Delta A_k + (S-N)^TB_2(C_k + \Delta C_k), \\ \Xi_6 &= (S-N)^TA + F_BC_2 + N^T\Delta B_kC_2, \\ \Xi_7 &= (S-N)^TB_1 + F_BD_{21} + N^T\Delta B_kD_{21}, \\ \Xi_8 &= C_1 + D_{12}(C_k + \Delta C_k). \end{aligned} \quad (19)$$

Then the following theorem presents a sufficient condition for the solvability of the non-fragile H_∞ control problem with additive uncertainty.

Theorem 1: Consider system (1). Let scalars $\gamma > 0, \delta_a > 0$ and gain matrix C_k be given. If there exist matrices $F_A, F_B, S, N, \bar{P}_{12}$ and $\bar{P}_{11} > 0, \bar{P}_{22} > 0$, such that the following LMIs hold:

$$\begin{aligned} &M_0(\Delta A_k, \Delta B_k, \Delta C_k) < 0, \quad \delta_{aij},\ \delta_{bik},\ \delta_{clj} \in \{-\delta_a, \delta_a\}, \\ &\quad i,j = 1,\cdots,n;\ k = 1,\cdots,p;\ l = 1,\cdots,q, \end{aligned} \quad (20)$$

then controller (2) with additive uncertainty (3), C_k and

$$A_k = (N^T)^{-1}F_A, \quad B_k = (N^T)^{-1}F_B, \quad (21)$$

solves the non-fragile H_∞ control problem for system (1).

Proof: Due to the limit of the space, it is omitted here. Contact the authors for the detailed proof. ■

Remark 1: Theorem 1 presents a sufficient condition in terms of solutions to a set of LMIs for the solvability of the non-fragile H_∞ control problem. By the proofs of Theorem 1 and Lemma 3, Theorem 1 also shows that the non-fragile H_∞ control problem becomes a convex one when the gain matrix C_k is known and the state-space realizations of the designed controllers with gain variations admit the slack variable matrix G with structure (14).
However, for the non-fragile H_∞ controller design method, it should be noted that the number of LMIs involved in (20) is $2^{n^2+np+nq}$, which results in the difficulty of implementing the LMI constraints in computation. For example, when $n = 6$ and $p = q = 1$, the number of LMIs involved in (20) is 2^{48}, which already exceeds the capacity of the current LMI solver in Matlab. To overcome the difficulty raising from implementing the design condition given in Theorem 1, the following method is developed. Denote

$$F_{a1} = [f_{a11}\ f_{a12}\ \cdots\ f_{a1l_a}], F_{a2} = [f_{a21}^T\ f_{a22}^T\ \cdots\ f_{a2l_a}^T]^T, \quad (22)$$

where $l_a = n^2 + np + nq$, and

$$\begin{aligned} f_{ak1} &= \begin{bmatrix} \mathbf{0}_{1\times n} & (N^Te_i)^T & \mathbf{0}_{1\times q} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times r} \end{bmatrix}^T, \\ f_{ak2} &= \begin{bmatrix} \mathbf{0}_{1\times n} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times q} & e_j^T & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times r} \end{bmatrix}, \\ \text{for } k &= (i-1)n + j,\ i,j = 1,\cdots,n. \\ f_{a1k} &= \begin{bmatrix} \mathbf{0}_{1\times n} & (N^Te_i)^T & \mathbf{0}_{1\times q} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times r} \end{bmatrix}^T, \\ f_{a2k} &= \begin{bmatrix} \mathbf{0}_{1\times n} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times q} & h_j^TC_2 & h_j^TC_2 & h_j^TD_{21} \end{bmatrix}, \\ \text{for } k &= n^2 + (i-1)p + j,\ i = 1,\cdots,n, j = 1,\cdots,p. \\ f_{a1k} &= \begin{bmatrix} \Omega_1 & \Omega_2 & (D_{12}g_i)^T & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times r} \end{bmatrix}^T, \\ f_{a2k} &= \begin{bmatrix} \mathbf{0}_{1\times n} & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times q} & e_j^T & \mathbf{0}_{1\times n} & \mathbf{0}_{1\times r} \end{bmatrix}, \\ \text{for } k &= n^2 + np + (i-1)n + j,\ i = 1,\cdots,q, j = 1,\cdots,n, \\ \text{where } &\quad \Omega_1 = (S^TB_2g_i)^T, \Omega_2 = [(S-N)^TB_2g_i]^T. \end{aligned}$$

Let $k_0, k_1, \cdots, k_{s_a}$ be integers satisfying $k_0 = 0 < k_1 < \cdots < k_{s_a} = l_a$ and matrix Θ have the following structure

$$\Theta = \begin{bmatrix} diag[\theta_{11}^1 \cdots \theta_{11}^{s_a}] & diag[\theta_{12}^1 \cdots \theta_{12}^{s_a}] \\ diag[\theta_{12}^1 \cdots \theta_{12}^{s_a}]^T & diag[\theta_{22}^1 \cdots \theta_{22}^{s_a}] \end{bmatrix}, \quad (23)$$

where $\theta_{11}^i, \theta_{12}^i$, and $\theta_{22}^i \in R^{(k_i - k_{i-1})\times(k_i - k_{i-1})}, i = 1,\cdots,s_a$. Then, we have

Theorem 2: Consider system (1). Let scalars $\gamma > 0, \delta_a > 0$ and gain matrix C_k be given. If there exist matrices $F_A, F_B, S, N, \bar{P}_{12}, \bar{P}_{11} > 0, \bar{P}_{22} > 0$, and symmetric matrix Θ with the structure described by (23) such that the following LMIs hold:

$$\begin{bmatrix} Q & F_{a1} \\ F_{a1}^T & 0 \end{bmatrix} + \begin{bmatrix} F_{a2} & 0 \\ 0 & I \end{bmatrix}^T \Theta \begin{bmatrix} F_{a2} & 0 \\ 0 & I \end{bmatrix} < 0, \quad (24)$$

$$\begin{aligned} &\begin{bmatrix} I \\ diag[\delta_{k_{i-1}+j} \cdots \delta_{k_i}] \end{bmatrix}^T \begin{bmatrix} \theta_{11}^i & \theta_{12}^i \\ (\theta_{12}^i)^T & \theta_{22}^i \end{bmatrix} \\ &\times \begin{bmatrix} I \\ diag[\delta_{k_{i-1}+j} \cdots \delta_{k_i}] \end{bmatrix} \geq 0, \text{ for all } \delta_{k_{i-1}+j} \in \{-\delta_a, \delta_a\}, \\ &j = 1,\cdots,k_i - k_{i-1}, i = 1,\cdots,s_a, \end{aligned} \quad (25)$$

where

$$Q = \begin{bmatrix} \Xi_1 & \Xi_2 & 0 & \Psi_1 & S^TA & S^TB_1 \\ * & \Xi_3 & 0 & \Psi_2 & \Psi_3 & \Psi_4 \\ * & * & -I & C_1 + D_{12}C_k & C_1 & 0 \\ * & * & * & -\bar{P}_{11} & -\bar{P}_{12} & 0 \\ * & * & * & * & -\bar{P}_{22} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} \quad (26)$$

with Ξ_1, Ξ_2, Ξ_3 defined by (19) and

$$\begin{aligned} \Psi_1 &= S^TA + S^TB_2C_k \\ \Psi_2 &= (S-N)^TA + F_BC_2 + F_A + (S-N)^TB_2C_k, \\ \Psi_3 &= (S-N)^TA + F_BC_2, \\ \Psi_4 &= (S-N)^TB_1 + F_BD_{21}. \end{aligned}$$

Then controller (2) with additive uncertainty (3) and the controller gain parameters given by (21) solves the non-fragile H_∞ control problem for system (1).

Proof: Due to the limit of the space, it is omitted here. By (18), we have

$$M_0 = Q + \Delta Q + \Delta Q^T < 0, \quad (27)$$

where

$$\Delta Q = \begin{bmatrix} 0 & 0 & 0 & \Delta Q_1 & 0 & 0 \\ 0 & 0 & 0 & \Delta Q_2 & \Delta Q_3 & \Delta Q_4 \\ 0 & 0 & 0 & \Delta Q_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with

$$\begin{aligned} \Delta Q_1 &= S^TB_2 \textstyle\sum_{i=1}^{q} \sum_{j=1}^{n} \delta_{cij} g_i e_j^T, \\ \Delta Q_3 &= \textstyle\sum_{i=1}^{n} \sum_{j=1}^{p} \delta_{bij} N^T e_i h_j^T C_2, \end{aligned}$$

$$\begin{aligned}\Delta Q_2 &= \textstyle\sum_{i,j=1}^{n}\delta_{aij}N^Te_ie_j^T+\sum_{i=1}^{n}\sum_{j=1}^{p}\delta_{bij}N^Te_ih_j^TC_2\\ &\quad +(S-N)^TB_2\textstyle\sum_{i=1}^{q}\sum_{j=1}^{n}\delta_{cij}g_ie_j^T,\\ \Delta Q_4 &= \textstyle\sum_{i=1}^{n}\sum_{j=1}^{p}\delta_{bij}N^Te_ih_j^TD_{21},\\ \Delta Q_5 &= D_{12}\textstyle\sum_{i=1}^{q}\sum_{j=1}^{n}\delta_{cij}g_je_j^T.\end{aligned}$$

By (22) and (27), it follows that (20) is equivalent to

$$\begin{aligned}M_0 &= Q+\textstyle\sum_{i=1}^{l_a}\delta_if_{a1i}f_{a2i}+(\sum_{i=1}^{l_a}\delta_if_{a1i}f_{a2i})^T\\ &= Q+F_{a1}\tilde{\Delta}_aF_{a2}+(F_{a1}\tilde{\Delta}_aF_{a2})^T<0\end{aligned}\tag{28}$$

holds for all $|\delta_i|\le\delta_a$, where $\tilde{\Delta}_a=diag[\delta_1,\cdots,\delta_{l_a}]$. By Lemma 4, it follows that (28) is further equivalent to that there exists a symmetric matrix $\Theta\in R^{l_a\times l_a}$ such that (24) and

$$\begin{bmatrix}I\\ \tilde{\Delta}_a\end{bmatrix}^T\Theta\begin{bmatrix}I\\ \tilde{\Delta}_a\end{bmatrix}\ge 0 \tag{29}$$

hold for all $\delta_i\in\{-\delta_a,\delta_a\},\ i=1,\cdots,l_a$. Notice that the set of Θ satisfying (23) is a subset of the set of Θ satisfying (29), hence the conclusion follows. ■

Remark 2: From the proof of Theorem 2, it follows that when $s_a=1$, the set of Θ satisfying (23) is equal to the set of Θ satisfying (29), and the design conditions given in Theorem 2 and Theorem 1 are equivalent. Θ satisfying (24) and (29) (or (25) with $s_a=1$) is said to be a *vertex separator* [5]. Notice that the number of LMIs involved in (29) or (25) with $s_a=1$ still is $2^{n^2+np+nq}$, so that the difficulty of implementing the LMI constraints remains. To overcome the difficulty, Theorem 2 presents a sufficient condition for the non-fragile H_∞ controller design in terms of separator Θ with the structure described by (23), where the number of LMIs involved in (25) is $\sum_{i=1}^{s_a}2^{k_i-k_{i-1}}$, which can be controlled not to grow exponentially by reducing the value of $\max\ k_i-k_{i-1}:\ i=1,\cdots,s_a$. Compared with the Θ being of full block in (24) and (29), Θ with the structure described by (23) satisfying (24) and (25) is said to be a *structured vertex separator*. However, it should be noted that the design condition given in Theorem 2 may be more conservative than that given in Theorem 1 because of the structure constraint on Θ. But the smaller value of s_a is, the less conservativeness is introduced.

B. Comparison with the existing design method

In this part, the result of a non-fragile H_∞ controller design with norm-bounded gain variations is introduced, and the comparison with our result is made.

Similar to [13] and [10] for non-fragile problem with norm-bounded uncertainty, the norm-bounded type of gain variations $\Delta A_k,\Delta B_k$ and ΔC_k can be overbounded [11] by the following norm-bounded uncertainty:

$$\begin{aligned}\Delta A_k= &\quad M_aF_1(t)E_a,\Delta B_k=M_bF_2(t)E_b,\\ &\quad \Delta C_k=M_cF_3(t)E_c,\end{aligned}\tag{30}$$

where

$$M_a=[M_{a1}\cdots M_{an^2}],E_a=[E_{a1}^T\cdots E_{an^2}^T]^T,$$

$$M_b=[M_{b1}\cdots M_{bnp}],E_b=[E_{b1}^T\cdots E_{bnp}^T]^T,$$

$$M_c=[M_{c1}\cdots M_{cnq}],E_c=[E_{c1}^T\cdots E_{cnq}^T]^T,$$

with

$M_{ak}=e_i,E_{ak}=e_j^T$
for $k=(i-1)n+j,\ i,j=1,\cdots,n,$
$M_{bk}=e_i,E_{bk}=h_j^T$
for $k=n^2+(i-1)p+j,\ i=1,\cdots,n,\ j=1,\cdots,p,$
$M_{ck}=g_i,E_{ck}=e_j^T$
for $k=n^2+np+(i-1)n+j,\ i=1,\cdots,q,\ j=1,\cdots,n.$

and $F_i^T(t)F_i(t)\le\delta_a^2I$ for $i=1,2,3$, represent the uncertain parameters, here δ_a is the same as before.

Noting that the problem of non-fragile dynamic output feedback H_∞ controller design with norm-bounded gain variations is also a non-convex problem, and similar to Theorem 2, when the controller gain C_k is known, it can be converted to a convex one.

To facilitate the presentation, we denote

$$\bar{F}_A=\bar{N}A_k,\bar{F}_B=\bar{N}B_k,$$

$$M_{a1}=\begin{bmatrix}0 & \bar{S}B_2M_c & 0\\ \bar{N}M_a & (\bar{S}-\bar{N})B_2M_c & \bar{N}M_b\\ 0 & D_{12}M_c & 0\\ 0&0&0\\ 0&0&0\\ 0&0&0\end{bmatrix},$$

$$M_{a2}=\begin{bmatrix}0&0&0&E_a&0&0\\ 0&0&0&E_c&0&0\\ 0&0&0&E_bC_2&E_bC_2&E_bD_{21}\end{bmatrix}.$$

Assume that C_k is known, by using the method in [13] and [10], the non-fragile H_∞ controller design with norm-bounded gain variations is reduced to solve the following LMI:

$$\begin{bmatrix}\bar{Q} & M_{a1} & \delta_a\varepsilon M_{a2}^T\\ * & -\varepsilon I & 0\\ * & * & -\varepsilon I\end{bmatrix}<0, \tag{31}$$

with matrix variables $\bar{S}>0,\bar{N}<0$ and scalar $\varepsilon>0$, where

$$\bar{Q}=\begin{bmatrix}-\bar{S} & -\bar{S} & 0 & \bar{S}(A+B_2C_k) & \bar{S}A & \bar{S}B_1\\ * & -\bar{S}+\bar{N} & 0 & Q_1 & Q_2 & Q_3\\ * & * & -I & C_1+D_{12}C_k & C_1 & 0\\ * & * & * & -\bar{S} & -\bar{S} & 0\\ * & * & * & * & -\bar{S}+\bar{N} & 0\\ * & * & * & * & * & -\gamma^2 I\end{bmatrix},$$

with $Q_1=(\bar{S}-\bar{N})(A+B_2C_k)+\bar{F}_A+\bar{F}_BC_2,\ Q_2=(\bar{S}-\bar{N})A+\bar{F}_BC_2,Q_3=(\bar{S}-\bar{N})B_1+\bar{F}_BD_{21}$.

The following Lemma will show the relationship between condition (31) and the condition given in Theorem 2.

Lemma 6: Consider system (1), if condition (31) is feasible, then the controllers design condition given in Theorem 2 is feasible.

Proof: Due to the limit of the space, it is omitted here. ■

Remark 3: From the proof of Lemma 6, it follows that condition (31) is more conservative than the non-fragile H_∞ controller existence condition in Theorem 2 with $s_a=l_a$. However, as indicated in Remark 3, the case of $s_a=l_a$ is the worst case of the new proposed method. So the existing non-fragile H_∞ controller design method with the norm-bounded gain variations is more conservative than the one given by Theorem 2.

C. Design an initial controller gain C_k

In this subsection, we focus on the problem of finding an initial feasible solution C_k to the non-fragile H_∞ control problem.

Consider the controller (2) with $\Delta A_k=0$ and $\Delta B_k=0$, which is described by

$$\begin{aligned}\dot{\xi}(k)&=A_k\xi(k)+B_ky(k),\\ u(k)&=(C_k+\Delta C_k)\xi(k).\end{aligned}\tag{32}$$

where ΔC_k is the same as in (30).

Combining controller (32) with system (1), we obtain the following closed loop system :

$$\begin{aligned}\dot{x_e}(k)&=A_{edc}x_e(k)+B_{edc}\omega(k),\\ z(k)&=C_ex_e(k),\end{aligned}\tag{33}$$

where $A_{edc} = \begin{bmatrix} A & B_2(C_k+\Delta C_k) \\ B_kC_2 & A_k \end{bmatrix}, B_{edc} = \begin{bmatrix} B_1 \\ B_kD_{21} \end{bmatrix}$, and C_e is the same as the one in (4).

Theorem 3: Consider system (1), $\gamma > 0$, and $\delta_a > 0$ are constants. If there exist matrices $\hat{A}$, $\hat{B}, \hat{C}$, $X > 0, Y > 0$, and a constant $\varepsilon_c > 0$ such that the following LMI holds:

$$\left[\begin{array}{cccccccc} -X & -I & 0 & AX+B_2\hat{C} & A & B_1 & B_2M_c & 0 \\ * & -Y & 0 & \hat{A} & YA+\hat{B}C_2 & YB_1+\hat{B}D_{21} & YB_2M_c & 0 \\ * & * & -I & C_1X+D_{12}\hat{C} & C_1 & 0 & D_{12}M_c & 0 \\ * & * & * & -X & -I & 0 & 0 & \varepsilon_c\delta_aXE_c^T \\ * & * & * & * & -Y & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_cI & 0 \\ * & * & * & * & * & * & * & -\varepsilon_cI \end{array}\right] < 0. \quad (34)$$

Then controller (32) with

$$A_k = (X^{-1}-Y)^{-1}(\hat{A}-YAX-\hat{B}C_2X-YB_2\hat{C})X^{-1},$$
$$B_k = (X^{-1}-Y)^{-1}\hat{B}, \quad C_k = \hat{C}X^{-1} \quad (35)$$

solves the non-fragile H_∞ control problem for system (1).

Proof: Due to the limit of the space, it is omitted here. ■

Remark 4: Theorem 3 shows that the non-fragile controller design problem with $\Delta A_k = 0$, $\Delta B_k = 0$ and ΔC_k in the norm-bounded form defined by (30) can be converted into a convex one depending a single parameter $\varepsilon_c > 0$.

D. Algorithm

Combining the results in Subsection A and Subsection C, a two-step procedure is summarized as follows:

Algorithm 1: *Step 1:* Minimize γ subject to $X > 0, Y > 0$ and LMI (34). Denote the optimal solutions as $X = X_{opt}$ and $\hat{C} = \hat{C}_{opt}$. Then by (35), $C_{kopt} = \hat{C}_{opt}X_{opt}^{-1}$.

Step 2: Let $C_k = C_{kopt}$, minimize γ subject to $F_A, F_B, N, S, \bar{P}_{12}, \bar{P}_{11} > 0, \bar{P}_{22} > 0$, and LMIs (24), (25). Denote the optimal solutions as $N = N_{opt}$, $F_A = F_{Aopt}$ and $F_B = F_{Bopt}$. Then according to (21), we obtain $A_k = (N^T)^{-1}F_{Aopt}$, $B_k = (N^T)^{-1}F_{Bopt}$. The resulting A_k, B_k and C_k will form the non-fragile dynamic output feedback H_∞ controller gains.

E. Evaluation of H_∞ performance index

In Theorem 2, we restrict the slack variable matrix G with the structure (14) for obtaining the convex design condition, which may result in more conservative evaluation of the H_∞ performance index bound. So, in this subsection, for a designed controller, the matrix G without the restriction is exploited for obtaining less conservative evaluation of the H_∞ performance index bound.

When the controller parameter matrices A_k, B_k and C_k are known, the problem of minimizing γ subject to (3) for a given $\delta_a > 0$ and $\| G_{z\omega}(z) \| < \gamma$ can be converted into the one: minimize γ^2 subject to the following LMIs:

$$\begin{bmatrix} P-G-G^T & 0 & G^TA_e & G^TB_e \\ * & -I & C_e & 0 \\ * & * & -P & 0 \\ * & * & * & -\gamma^2I \end{bmatrix} < 0, \quad (36)$$
$$\delta_{aij},\ \delta_{bik},\ \delta_{clj} \in \{-\delta_a, \delta_a\},$$
$$i,j = 1,\cdots,n;\ k = 1,\cdots,p;\ l = 1,\cdots,q,$$

where A_e, B_e and C_e are defined as in (4).

Similar to the design condition given in Theorem 1, the above method is also with the numerical computation problem. To solve the problem, the following lemma provides a solution using the structured vertex separator approach.

Denote

$$G_{a1} = [g_{a11}\ g_{a12}\ \cdots\ g_{a1l_a}], G_{a2} = [g_{a21}^T\ g_{a22}^T\ \cdots\ g_{a2l_a}^T]^T. \quad (37)$$

where

$$g_{a1k} = \begin{bmatrix} (\mathbf{0}_{1\times n} & e_i^T) G & \mathbf{0}_{1\times q} & \mathbf{0}_{1\times 2n} & \mathbf{0}_{1\times r} \end{bmatrix}^T,$$
$$g_{a2k} = \begin{bmatrix} \mathbf{0}_{1\times 2n} & \mathbf{0}_{1\times q} & \mathbf{0}_{1\times n} & e_j^T & \mathbf{0}_{1\times n} \end{bmatrix},$$
for $k = (i-1)n+j,\ i,j = 1,\cdots,n.$
$$g_{a1k} = \begin{bmatrix} (\mathbf{0}_{1\times n} & e_i^T) G & \mathbf{0}_{1\times q} & \mathbf{0}_{1\times 2n} & \mathbf{0}_{1\times r} \end{bmatrix}^T,$$
$$g_{a2k} = \begin{bmatrix} \mathbf{0}_{1\times 2n} & \mathbf{0}_{1\times q} & h_j^TC_2 & \mathbf{0}_{1\times n} & h_j^TD_{21} \end{bmatrix},$$
for $k = n^2+(i-1)p+j,\ i = 1,\cdots,n, j = 1,\cdots,p.$
$$g_{a1k} = \begin{bmatrix} ((B_2g_i)^T & \mathbf{0}_{1\times n}) G & (D_{12}g_i)^T & \mathbf{0}_{1\times 2n} & \mathbf{0}_{1\times r} \end{bmatrix}^T,$$
$$g_{a2k} = \begin{bmatrix} \mathbf{0}_{1\times 2n} & \mathbf{0}_{1\times q} & \mathbf{0}_{1\times n} & e_j^T & \mathbf{0}_{1\times n} \end{bmatrix},$$
for $k = n^2+np+(i-1)n+j,\ i = 1,\cdots,q, j = 1,\cdots,n.$

Then we have

Lemma 7: Consider system (1). Let $\gamma > 0, \delta_a > 0$ be constants, and controller parameter matrices A_k, B_k, C_k be given. Then $\| G_{z\omega} \| < \gamma$ holds for all δ_{aij}, δ_{bit} and δ_{clj} satisfying (3), if there exist a matrix G, a positive definite matrix $P > 0$ and a symmetric matrix Θ with the structure described by (23) such that (25) and the following LMI hold:

$$\begin{bmatrix} Q_s & G_{a1} \\ G_{a1}^T & 0 \end{bmatrix} + \begin{bmatrix} G_{a2} & 0 \\ 0 & I \end{bmatrix}^T \Theta \begin{bmatrix} G_{a2} & 0 \\ 0 & I \end{bmatrix} < 0 \quad (38)$$

where

$$Q_s = \begin{bmatrix} P-G-G^T & 0 & G^TA_{e0} & G^TB_{e0} \\ * & -I & C_{e0} & 0 \\ * & * & -P & 0 \\ * & * & * & -\gamma^2I \end{bmatrix}$$

with A_{e0}, B_{e0} and C_{e0} are defined by (7).

Proof: It is similar to the proof of Theorem 2, and omitted here. ■

Remark 5: For evaluating the H_∞ performance bound of the transfer function from ω to z, the condition given in Lemma 7 usually is less conservative than that given in Theorem 2 because no structure constraint on the slack variable matrix G in Lemma 7 is imposed.

IV. Example

In the following, an example is given to illustrate the effectiveness of the proposed method.

Consider a linear system of the form (1) with

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 0.5 & -1 & 1 \\ 0.5 & -1 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} -0.5 & 0 \\ -0.5 & 0 \\ -1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} -1 & 1 & -3 \end{bmatrix},$$
$$D_{21} = \begin{bmatrix} 0 & 1 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

By the standard H_∞ controller design method [**?**], we obtain the optimal H_∞ performance index for the system as $\gamma_{opt} = 2.1622$. On the other hand, assume that the designed controller is with form (32). Let $\delta_a = 0.05$, by Theorem 3 with $\varepsilon_c = 155.9999$, we obtain

$$C_{k_{ini}} = \begin{bmatrix} 0.2573 & -0.2351 & 0.3380 \end{bmatrix}.$$

Here ε_c is obtained by searching such that the H_∞ performance index is optimal. δ_a is chosen large appropriately such that the designed $C_{k_{ini}}$ can guarantee Step 2 feasible.

A. Existing method given by condition (31)

In this subsection, we design an H_∞ controller by condition (31) with $C_k = C_{k_{ini}}$.
Assume that the designed controller is with norm-bounded additive uncertainties described by (30), by applying condition (31) with $\delta_a = 0.006$ to design a non-fragile controller, the obtained H_∞ performance index of the obtained non-fragile controller is 2.8645.

B. New method given by Theorem 2

In the following, we design an H_∞ controller by Theorem 2 with $C_k = C_{k_{ini}}$.
Assume that the designed controller is with the additive uncertainties described by (3). For this case with $C_k = C_{k_{ini}}$, it is difficult to apply Theorem 1 to design a non-fragile H_∞ controller because the number of the LMI constraints involved in (20) is 2^{15}. However, Theorem 2 is applicable for solving this problem. By applying Theorem 2 with $\delta_a = 0.006$ and $k_i = i, i = 1, \cdots, 15$, i.e., $s_a = 15$ as well as $k_i = 3i, i = 1, \cdots, 5$, i.e., $s_a = 5$ to design a non-fragile H_∞ controller, and the H_∞ performance indexes of the obtained non-fragile controllers are $\gamma = 2.5204$ ($s_a = 15$) and $\gamma = 2.5099$ ($s_a = 5$), respectively.

C. Evaluation of H_∞ performance index by Lemma 7

In this part, for the above designed controllers, Lemma 7 can give better evaluations of the H_∞ performance index bounds.
Firstly, to facilitate the presentation, denote the controller designed by condition (31) as K_{nm} while denote the controllers designed by Theorem 2 as K_{in15} ($s_a = 15$) and K_{in5} ($s_a = 5$).
By Lemma 7, the H_∞ performance indices of the controller K_{nm} are $\gamma = 2.6507$ ($s_a = 15$) and $\gamma = 2.6501$ ($s_a = 5$) while the H_∞ performance indices of the controllers K_{in15} and K_{in5} are $\gamma = 2.4934$ ($s_a = 15$) and $\gamma = 2.4903$ ($s_a = 5$), respectively.

D. Comparison

Firstly, Table 1 shows the H_∞ performance indices achieved by the designs of the existing method (Condition (31)) and the proposed method (Theorem 2).

TABLE I

PERFORMANCE INDEX BY DESIGN WITH $\delta_a = 0.006$

	Condition (31)	Th.2 ($s_a = 15$)	Th.2 ($s_a = 5$)
γ	2.8645	2.5204	2.5099

Secondly, for the designed non-fragile controllers, Lemma 7 gives better performance indices shown in Table 2.

TABLE II

PERFORMANCE EVALUATION BY LEMMA 7 WITH $\delta_a = 0.006$

	K_{nm}	K_{in15}	K_{in5}
$\gamma(s_a = 15)$	2.6507	2.4934	$--$
$\gamma(s_a = 5)$	2.6501	$--$	2.4903

From this example, we can see that the worst case ($s_a = 15$) of Theorem 2 also is more effective than the non-fragile H_∞ controller existence condition (31).

V. CONCLUSION

In this paper, we have investigated the problem of non-fragile dynamic output feedback H_∞ controller design for linear discrete-time systems. A notion of structured vertex separator is proposed to approach this problem, and exploited to develop sufficient conditions for the non-fragile H_∞ controller design via a two-step procedure. The resulting designs guarantee that the closed-loop systems is asymptotically stable and the H_∞ performance from the disturbance to the regulated output is less than a prescribed level.

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