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Control Scheme for Human-Robot Co-manipulation of Uncertain, Time-varying Loads

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Abstract— **The collaboration between a human operator and a robotic manipulator for performing a joint task is the main topic of this article. One of the most common tasks involves transporting and positioning a load. The proposed control strategy allows the co-manipulation of loads with unknown, time-varying mass. The control algorithm is of a cyclic nature, where each cycle is subdivided into two stages. The first phase is the estimation of the human force, since the only connection between the robot and the human is via the load. In the second phase, a scaled version of the estimated human force is applied to the load. The proposed approach is illustrated by means of simulation results for a two-link robot manipulator.**

I. INTRODUCTION

In some human activities, such as those at building sites, in logistics, medical care and agriculture, human workers have to move a load between two points. In many cases, this load is too heavy for a human to transport; therefore the use of a robotic manipulator to aid the human operator seems natural. On the other hand, the human intelligence is essential in performing the positioning task (path planning and control) and the direct interaction with the environment; therefore the human operator is a desirable factor in the loop.

The use of a crane-like robot which is teleoperated from a cockpit is a possible solution, but this kind of setup has disadvantages concerning the costs and the space such a robot will occupy. A more elegant solution is to let the human and the robot manipulate the load together in order to achieve the common positioning goal. For this reason the robot must, firstly, determine the force that the human applies to the load and, secondly, amplify it. Note that mounting force sensors on the end effector of the robot is not feasible since the human operator will typically interact with the load directly (and not with the robot end-effector). Various strategies have been considered for such force-sensor-less control schemes which estimate the human force. [1] proposes an adaptive disturbance observer scheme, while [2] uses a set of tests for model identification to tune the disturbance observer. [3] and [4] propose a H^{∞} estimation algorithm. The problem of cooperative motion control by a human and a robot is tackled in [5] using the "interactive virtual impedance". In [6], a discussion on the state of the art in force-sensor-less power assist control is presented with an emphasis on the estimation of the human force using linear models for the robot with the load. All the control strategies discussed above assume more or less a perfect knowledge of the dynamics of the robot with the load, i.e. the mass of the load is considered known, or with very small uncertainty, and constant.

The current study, which extends a result presented in [7], introduces a control strategy for a robotic device that amplifies the human force. As mentioned before, the robot does not have any direct coupling with the human operator except via the load, i.e. the robot is supposed to help the human operator by scaling the force the human applies to the load. Here we face two problems: Firstly, the robot should amplify the human force, which is unknown. Namely, the force of the human operator can not be measured because the robot is in contact only with the load, while the human also acts only on the load. This means that the human force has to be estimated. Secondly, in many cases the human deals with loads of different mass, which are also generally unknown and could also be time-varying. We emphasize here that the estimation of the human force using an inverse dynamics model together with measurements of the encoders in the robot joints (see [2], [6]) is not feasible in our problem setting since the mass of the load is unknown. This study focuses on designing a control strategy which can solve both problems simultaneously without using an adaptive control algorithm which present high computational complexity and many parameters to be chosen and tuned. Moreover, our algorithm is robust for large uncertainties in mass of the load.

This paper is structured as follows: The problem statement is discussed in Section III. The controller design dealing with the issues of unknown operator force and unknown time-varying loads is proposed in Section IV. In Section V, the effectiveness of this method is illustrated by application to a two-link robot manipulator. In the final section of this paper, the conclusions and some perspectives on future research are discussed.

II. PRELIMINARIES

In this section, we recall some definitions and results concerning the property of input-to-state stability as introduced by Sontag in [8], see also [9]. The input-to-state stability property of nonlinear systems is exploited in the proof of the main result of this article. Consider the general nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)), \; x(0) = x_0, \tag{1}$$

with solutions $\varphi(t, x_0, u)$, where $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is continuously differentiable. The set of all the measurable locally bounded functions $u : \mathbb{R}^+ \to \mathbb{R}^m$, endowed with the supremum norm $\sup\{|u(t)|, t \geq 0\} \leq \infty$ is denoted as L^m_∞. A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is called a class $\mathscr{K}$-function, i.e. $\gamma \in \mathscr{K}$, if it is continuous, strictly increasing and $\gamma(0) = 0$. A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is called a class $\mathscr{K}_\infty$-function if $\gamma \in \mathscr{K}$ and $\gamma(s) \to \infty$ as $s \to \infty$. A function $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is a class $\mathscr{KL}$-function if for each fixed $t \geq 0$, $\beta(\cdot, t) \in \mathscr{K}$ and for each fixed $s \geq 0$, $\beta(s, t)$ is decreasing to zero as $t \to \infty$. The concept of input-to-state stability introduced in [8] states that system (1) is input-to-state stable (ISS) if there exist a function $\beta \in \mathscr{KL}$ and a function $\gamma \in \mathscr{K}_\infty$ such that, for each input $u \in L^m_\infty$, all the initial values x_0 and for any $t \geq 0$ the inequality $|\varphi(t, x_0, u)| \leq \beta(|x_0|, t) + \gamma(\sup_{0 \leq \tau \leq t} |u(\tau)|)$ holds.

Definition 1: [8] A smooth function $V : \mathbb{R}^n \to \mathbb{R}$ is called an ISS Lyapunov function for system (1) if there exist the functions $\alpha_1, \alpha_2 \in \mathscr{K}_\infty$, $\alpha_3, \chi \in \mathscr{K}$ such that

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \tag{2}$$

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and

$$|x| \geq \chi(|u|) \Rightarrow \dot{V}(t) \leq -\alpha_3(|x|) \quad (3)$$

hold for any $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$.

The quantitative aspects regarding the existence of an ISS Lyapunov function have been developed in [8] and [10]. These results are synthesized by the following theorem:

Theorem 1: If an ISS Lyapunov function exists for system (1), then the system (1) is input-to-state stable with $\beta(\cdot,t) = \alpha_1^{-1} \circ \mu(\alpha_2(\cdot),t)$ (where $\circ$ is the function composition operator) and $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \chi$, where μ is the solution of the differential equation:

$$\frac{d}{dt}\mu(r,t) = -\alpha_3 \circ \alpha_2^{-1}(\mu(r,t)) \quad (4)$$

with the initial condition $\mu(r,0) = r$.

III. PROBLEM FORMULATION

The problem tackled here is robot-assisted load carrying by human operators. The main goal of the robot is to scale the force that the human operator applies to the mass. In that way, the human will 'feel' a load with lower mass but will still be in charge of the position control of the load. When designing a robot control scheme for this purpose we face the following problems:

- The mass of the load is unknown and possibly time-varying. Applications with time-varying mass can be encountered in the case of dispersing liquids from a container (e.g. painting or concrete pouring on building sites);
- The force that the human operator applies is unknown, since there are no force sensors on the load; the human operator is in direct contact with the load to be transported. The only measurements available are the position coordinates of the robot links.

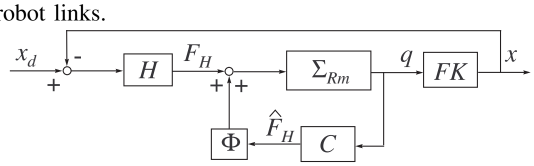

Fig. 1. Problem Setup.

In Figure 1, we present the problem setup in more detail. The human operator has a desired trajectory x_d (in Cartesian coordinates of the load) in mind and establishes a position control strategy H so that, using the (visual) feedback loop, he can achieve the positioning goal. Using this strategy, the human operator will apply the force F_H to the load with the time-varying mass $m(t)$. The problem is that in many applications the mass is too heavy for the human to transport or the speed achieved is too low. The assisting robotic device with the load $m(t)$ is represented by the dynamic block Σ_{Rm}. The controller C, which will be developed in the next section, estimates the human force, F_H, by $\hat{F}_H$, using the measurements of the motor encoders from the joints of the robot. This estimated force is amplified by a factor Φ and the resulting force is applied to the load, thereby amplifying the human operator power. The block FK represents a forward kinematics block from the joint coordinates to Cartesian coordinates.

IV. MAIN RESULT

The unknown variables in the problem discussed in the previous section are the mass of the load and the human operator's force. The only measurements available are the joint coordinates of the robotic device. Using this partial information, we have to estimate the human force and the robot should apply an additive force which scales the human force. As the available measurements do not allow a direct control strategy due to unknown parameters and signals (a force control is dependent on the mass of the load), we propose to tackle the problem in two temporal steps:

1) Estimate the human operator force;
2) Apply the scaled force.

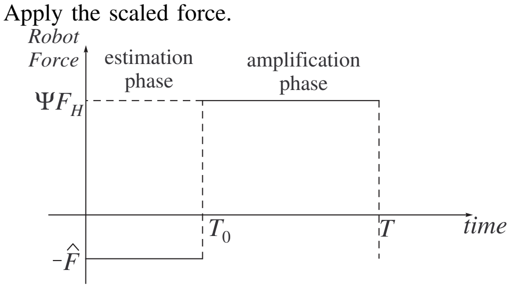

Fig. 2. Temporal division of the control strategy.

The question which arises is how to obtain this temporal division in the algorithm (see Figure 2). In this respect, it is important to note the difference between the frequencies with which a human operator and the robot can perform their tasks. Studies [11] and [12] have shown that a human can perform a task with a frequency of up to 6Hz, which is much slower than the typical sampling frequency used in a robotic control scheme. This means that if the frequency with which the two steps of our procedure are implemented is significantly higher than 6Hz, then the robotic device can correctly track the force of the human operator and apply the scaled force to achieve its goal.

The force generated by the robot is a signal similar to a Pulse Width Modulation (PWM) signal. Such an input signal generates a series of accelerations and decelerations with a frequency of $\frac{1}{T}$, with T the length of a cycle. This frequency should be set above the maximal frequency that a human can perceive to avoid that the operator feels a possibly disturbing vibration induced by the algorithm. In [11], it has been shown that a human subject can feel a vibrating object with frequencies up to 300Hz. Unfortunately, no research has been done for the perception of signals other than sinusoidal ones. Moreover, human perception greatly depends on the amplitude of the vibration since for higher amplitudes the perception limit is 300Hz, while for lower amplitudes the sensitivity limit decreases to 40Hz. This information should also be taken into consideration when choosing the cycle period T.

The second issue of this design is to determine the real amplification coefficient, Ψ (see Figure 2). Since the desired amplification coefficient is Φ, which relates to the entire period of the cycle T, we must determine a new scaling coefficient Ψ because the amplification period lasts only for $T - T_0$. Assuming that the effect of the robot action should be the same in both cases, i.e. the average robot force is the same during one cycle period, one can determine the scaling factor Ψ.

Next, the algorithm for the estimation of the human force is discussed in Section IV-A, whereas the algorithm effectuating the amplification of the human force is presented in Section IV-B.

A. Human Force Estimation

Consider the nonlinear system dynamics of a robot with an additional load of unknown, and possibly time-varying, mass $m(t)$ at its end-effector, which are described by:

$$M(q,t)\ddot{q} + D(q,\dot{q},t) = \tau + J^T(q)F_H, \quad (5)$$

where $q \in \mathbb{R}^n$ is the vector of generalized joint displacements, $\dot{q} \in \mathbb{R}^n$ is the generalized joint velocity vector, $\ddot{q} \in \mathbb{R}^n$ is the generalized joint acceleration vector, $\tau \in \mathbb{R}^n$ is the robot torque vector, $F_H \in \mathbb{R}^d$ is the human operator's force vector (d is the space dimension, $d = 2$ for 2D or $d = 3$ for 3D), $M \in \mathbb{R}^{n \times n}$ is the symmetric, positive

definite inertia matrix, $D \in \mathbb{R}^n$ is the vector containing the sum of centripetal, Coriolis, friction and gravitational forces/torques and $J \in \mathbb{R}^{n\times d}$ is the Jacobian matrix relating the end-effector velocity $\dot{x} \in \mathbb{R}^d$ to the generalized joint velocity $\dot{q}$ by $\dot{x} = J(q)\dot{q}$.
For the sake of simplicity, we adopt the following assumption:

Assumption 1: The Jacobian matrix J is nonsingular at all times of operation.

Remark 1: The above assumption implies that we do not consider redundant robots, i.e. $d = n$, and no kinematic singularities are encountered.

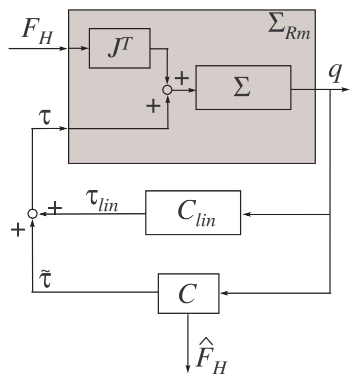

Fig. 3. Human Force Estimation Scheme.

The objective of this phase is to determine an estimate of the human force, $\hat{F}_H \in \mathbb{R}^n$. Hereto, we design an estimation controller strategy as schematically depicted in Figure 3, where the controller C_{lin} compensates for the robot dynamics without the load and the controller C estimates the human force $\hat{F}_H$, with $\tau = \tau_{lin} + \tilde{\tau}$, where τ_{lin} and $\tilde{\tau}$ are the outputs of the controllers C_{lin} and C, respectively. Assuming the system can be linearly parameterized with respect to the time-varying mass $m(t)$ of the load (the inertial, the gravitational, the centripetal, the Coriolis and the friction forces are typically linear with respect to the mass $m(t)$), then (5) can typically be written as:

$$M_R(q)\ddot{q} + D_R(q,\dot{q}) + m(t)P_M(\ddot{q},\dot{q},q) = \tau + J^T(q)F_H, \tag{6}$$

where M_R and D_R contain the information concerning the robot dynamics without the end-effector load, $m(t)$ is the unknown mass of the load and P_M represents the remaining terms which depend on the mass of the load.
The controller C_{lin} is designed based on the idea of partial feedback linearization:

$$\tau_{lin} = M_R(q)\ddot{q} + D_R(q,\dot{q}). \tag{7}$$

Introducing relation (7) in (5) leads to:

$$m(t)P_M(\ddot{q},\dot{q},q) = \tilde{\tau} + J^T(q)F_H, \tag{8}$$

where P_M and J are known and we have to design $\tilde{\tau}$, the output of controller C, such that, independent of the magnitude of the unknown mass of the load, estimation of the human force F_H can be achieved. Here, we assume that $m(t) \in [M_{min}, M_{max}]$, $\forall t \in \mathbb{R}^+$. By defining $\hat{F} := J^{-T}\tilde{\tau}$, relation (8) can be written as:

$$m(t)J^{-T}(q)P_M(\ddot{q},\dot{q},q) = \hat{F} + F_H. \tag{9}$$

If we define $\eta^{(p)} := J^{-T}(q)P_M(\ddot{q},\dot{q},q)$, with $p \geqslant 1$ a constant integer and $\eta^{(p)}$ denoting the p^{th} time-derivative of η, then equation (9) is equivalent to the linear differential equation:

$$m(t)\eta^{(p)} = \hat{F} + F_H. \tag{10}$$

If we consider the controller strategy:

$$\hat{F} = -\sum_{i=0}^{p} K_i \eta^{(i)}, \tag{11}$$

with $K_i = diag\left(K_{i,1},\ldots,K_{i,j},\ldots,K_{i,n}\right) \in \mathbb{R}^{n\times n}$, $i = 0,\ldots,p$, then the output $\hat{F}_H = K_0\eta$ represents the estimated human force. The choice of parameters K_i should be made such that $K_i \geq 0$, for $i = 0,1,\ldots,p-1$ and $m(t)I_n + K_p > 0$ for all $m(t) \in [M_{min}, M_{max}]$, where I_n is the $n\times n$ identity matrix. The control strategy is depicted in Figure 4 using a chain of p integrators. Due to the diagonal structure of the matrices K_i, $i = 0,\ldots,p$, relation (10) can be written as a juxtaposition of equations:

$$m(t)\eta_j^{(p)} = -\sum_{i=0}^{p} K_{i,j}\eta_j^{(i)} + F_{H,j}, \tag{12}$$

where η_j and $F_{H,j}$, $j = 1,\ldots,n$, are the j^{th} components of the vector η and the human force vector F_H, respectively. This means that system (10) can be seen as a decoupled system where the input $F_{H,j}$ only affects η_j and its time-derivatives.

In Figure 4, a block diagram of the estimation controller is

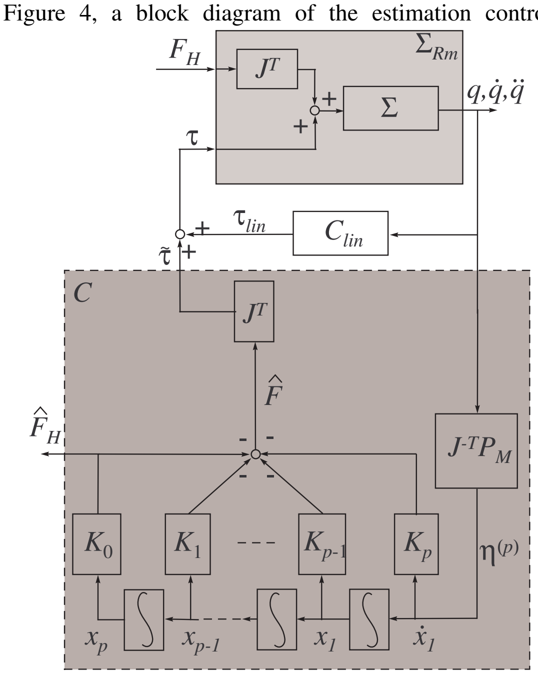

Fig. 4. Human Force Estimation Controller.

presented. For each of the decoupled differential equation in (12), we can write the state-space representation of the single input-single output system as:

$$\left\{\begin{array}{l}
\dot{x}^j(t) = \begin{pmatrix} -\frac{K_{p-1,j}}{m(t)+K_{p,j}} & \cdots & \cdots & \cdots & -\frac{K_{0,j}}{m(t)+K_{p,j}} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix} x^j(t) \\
\qquad + \begin{pmatrix} \frac{1}{m(t)+K_{p,j}} \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix} u_j(t) \\
y_j(t) = K_{0,j}x_p^j(t),
\end{array}\right. \tag{13}$$

where $u_j = F_{H,j}$ is the j^{th} component of the human force vector, F_H, the input of the system, $x^j = (\eta_j^{(p-1)}, \ldots, \eta_j)^T$ is the state vector and $y_j = \hat{F}_{H,j}$ is the j^{th} component of the estimated force, the output of the system. Note that the desired behavior for system (13) is $y_j(t) \to u_j(t)$ as $t \to \infty$.

Let us now define the estimation error $e_j := u_j - y_j$ and the new state vector $\varepsilon^j := (\varepsilon_1^j, \ldots, \varepsilon_p^j)^T := (e_j, \dot{e}_j, \ldots, e_j^{(p-1)})^T$ containing the estimation error and its derivatives. Rewriting system (13) in terms of this new state variable ε^j, leads to:

$$\dot{\varepsilon}^j = A^j(t)\varepsilon^j + B^j(t)v_j(t),\ j = 1, \ldots, n, \tag{14}$$

where

$$A^j(t) := \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & \ldots & 0 & 1 \\ -\frac{K_{0,j}}{m(t)+K_{p,j}} & \ldots & \ldots & \ldots & -\frac{K_{p-1,j}}{m(t)+K_{p,j}} \end{pmatrix},\ B^j(t) := \begin{pmatrix} 0 & \ldots & \ldots & 0 \\ \vdots & \ldots & \ldots & \vdots \\ \vdots & \ldots & \ldots & \vdots \\ 0 & \ldots & \ldots & 0 \\ \frac{K_{1,j}}{m(t)+K_{p,j}} & \ldots & \frac{K_{p-1,j}}{m(t)+K_{p,j}} & 1 \end{pmatrix}$$

and $v_j(t) := (\dot{u}_j, \ldots, u_j^{(p)})^T$.

The following technical result shows that the error dynamics (14) are input-to-state stable (ISS) with respect to $v_j(t)$, which also implies the global uniform asymptotic stability (GUAS) of $\varepsilon^j = 0$ for constant $u_j(t)$, i.e. $v_j(t) = 0$, for $m(t) \in [M_{min}, M_{max}]$, $\forall t$.

Theorem 2: Consider systems (14). If there exist matrices $P_j = P_j^T > 0$ and scalars $\rho_j > 0$, $j = 1, \ldots, n$, such that the following linear matrix inequalities (LMIs) are satisfied:

$$P_j A_i^j + \left(A_i^j\right)^T P_j \leq -\rho_j P_j,\ i \in \{1, 2\} \text{ and } j \in \{1, \ldots, n\}, \tag{15}$$

with

$$A_1^j = \begin{pmatrix} 0 & & I \\ -K_{0,j}\underline{\alpha^j} & \ldots & -K_{p-1,j}\underline{\alpha^j} \end{pmatrix}, \tag{16}$$

$$A_2^j = \begin{pmatrix} 0 & & I \\ -K_{0,j}\overline{\alpha^j} & \ldots & -K_{p-1,j}\overline{\alpha^j} \end{pmatrix} \tag{17}$$

and $\underline{\alpha^j} = \frac{1}{M_{max}+K_{p,j}}$, $\overline{\alpha^j} = \frac{1}{M_{min}+K_{p,j}}$, then the systems (14) are ISS with respect to the input $v_j(t)$ for each $j \in \{1, \ldots, n\}$. In particular, functions β^j and γ^j (see Theorem 1) are respectively given by:

$$\beta^j(r,t) = \sqrt{\frac{\lambda_{max}^j}{\lambda_{min}^j}}\, r e^{-\frac{\rho_j}{4}t} \tag{18}$$

and

$$\gamma^j(r) = \frac{4}{\rho_j} \frac{\lambda_{max}^j}{\lambda_{min}^j} r, \tag{19}$$

where $\lambda_{min}^j = min(eig(P_j))$ and $\lambda_{max}^j = max(eig(P_j))$.

Proof: Let $\alpha^j(t) = \frac{1}{m(t)+K_{p,j}} \in [\underline{\alpha^j}, \overline{\alpha^j}]$, $j = 1, \ldots, n$. Then $\alpha^j(t) = \lambda^j(t)\underline{\alpha^j} + (1 - \lambda^j(t))\overline{\alpha^j}$, $j = 1, \ldots, n$, with $0 \leq \lambda^j(t) \leq 1, \forall t \geq 0$. Hence the time-varying matrix $A^j(t)$ can be written as a convex combination of two matrices A_1^j and A_2^j:

$$A^j(t) = \lambda^j(t)A_1^j + (1 - \lambda^j(t))A_2^j, \forall t \in \mathbb{R}^+, j = 1, \ldots, n, \tag{20}$$

with A_1^j and A_2^j as in (16) and (17), respectively, and $\lambda^j(t) \in [0, 1]$. Since $\exists P_j = P_j^T > 0$ and $\rho_j > 0$ such that (15) is satisfied, it holds that $P_j A^j(t) + \left(A^j\right)^T(t)P_j = P_j(\lambda^j(t)A_1^j + (1 - \lambda^j(t))A_2^j) + (\lambda^j(t)\left(A_1^j\right)^T + (1 - \lambda^j(t))\left(A_2^j\right)^T)P_j = \lambda^j(t)(P_j A_1^j + \left(A_1^j\right)^T P_j) + (1 - \lambda^j(t))(P_j A_2^j + \left(A_2^j\right)^T P_j) \leq -\lambda^j(t)\rho_j P_j - (1 - \lambda^j(t))\rho_j P_j = -\rho_j P_j$.

Let us define $|x|_P := \sqrt{x^T P x}$ and consider the candidate ISS Lyapunov functions $V_j = \frac{1}{2}|\varepsilon^j|_{P_j}^2$. The time-derivative of V_j along the solutions of (14) satisfies:

$$\dot{V}_j = \frac{1}{2}\left(\varepsilon^j\right)^T (P_j A^j(t) + \left(A^j(t)\right)^T P_j)\varepsilon^j + \left(\varepsilon^j\right)^T P_j B^j(t) v_j(t). \tag{21}$$

Let $\overline{v}_j(t) := B^j(t)v_j(t)$. Then using the LMIs (15), (21) can be written as:

$$\dot{V}_j \leq -\frac{1}{2}\rho_j|\varepsilon^j|_{P_j}^2 + |\varepsilon^j|_{P_j} \sup_{t \in \mathbb{R}^+} |\overline{v}_j(t)|_{P_j} \tag{22}$$

$$\Rightarrow \dot{V}_j \leq -\frac{1}{4}\rho_j|\varepsilon^j|_{P_j}^2 + |\varepsilon^j|_{P_j}(-\frac{1}{4}\rho_j|\varepsilon^j|_{P_j} + \sup_{t \in \mathbb{R}^+} |\overline{v}_j(t)|_{P_j}), \tag{23}$$

which means that:

$$|\varepsilon^j|_{P_j} \geq \frac{4}{\rho_j} \sup_{t \in \mathbb{R}^+} |\overline{v}_j(t)|_{P_j} \Rightarrow \dot{V}_j \leq -\frac{1}{4}\rho_j|\varepsilon^j|_P^2. \tag{24}$$

After straightforward computations, we ultimately arrive at the following implications:

$$|\varepsilon^j| \geq \frac{4}{\rho_j}\sqrt{\frac{\lambda_{max}^j}{\lambda_{min}^j}} \sup_{t \in \mathbb{R}}(\overline{v}_j(t)) \Rightarrow \dot{V}_j \leq -\frac{\rho_j}{4}\lambda_{max}^j|\varepsilon^j|^2. \tag{25}$$

Define the functions:

$$\begin{aligned} \alpha_1^j(r) &= \tfrac{\lambda_{min}^j}{2} r^2, & \alpha_2^j(r) &= \tfrac{\lambda_{max}^j}{2} r^2, \\ \alpha_3^j(r) &= \tfrac{\rho_j}{4}\lambda_{max}^j r^2, & \chi^j(r) &= \tfrac{4}{\rho_j}\sqrt{\tfrac{\lambda_{max}^j}{\lambda_{min}^j}}\, r. \end{aligned} \tag{26}$$

Then, the solution of the differential equation (4) is: $\mu(r,t) = re^{-\frac{\gamma}{2}t}$. Using these definitions, Definition 1 and Theorem 1, we can conclude that system (14) is ISS with the functions β^j and γ^j defined as in (18) and (19) respectively. ■

Corollary 1: Consider system (13) with a constant input $u_j(t) = U_j$. Under the conditions of Theorem 2, the equilibrium point $x^j = (0, \ldots, 0, \frac{U_j}{K_{0,j}})^T$ is global uniformly asymptotically stable (GUAS). Using Corollary 1, we prove that the system output $y(t) \to u(t)$ when $t \to \infty$, for a constant input signal $u(t)$ and time-varying parameter $m(t) \in [M_{min}, M_{max}]$, $\forall t$.

Remark 2: The estimation algorithm provides exact tracking of a constant human force (see Corollary 1). As the estimation is much faster than the variation of the human force, we can assume that the human force is approximately constant during one cycle, which means that this algorithm can provide a very good approximation of the input signal.

Theorem 2 shows that for a non-constant human force the estimation error remains bounded and that this bound can be related to the time-derivatives of the human force via a linear ISS gain relation (see (19)). Moreover, the function $\beta^j(r,t)$ as in (18) reflects a bound on the transient convergence of the estimation algorithm. This knowledge is instrumental in choosing T_0 (i.e. the length of the estimation interval) as the estimation algorithm should reach sufficient accuracy within this time-slot of the cyclic algorithm, see Figure 2.

B. Scaling the human force

Based on the maximal human operation frequency, we can determine the period T of one cycle of the algorithm, which includes the estimation stage and amplification stage, see Figure 2. Using the analysis in Theorem 2, one can obtain an upper bound for the settling time for the estimator (from the expression of the function β^j in (18)). If we consider that the output has settled if the error has dropped below 5% of the initial value, then the settling time for function β^j as in (18) is given by $T_s^j \geq -\frac{4}{\rho_j} ln(0.05) = -\frac{4}{\rho_j}(-2.9957) \simeq 3\frac{4}{\rho_j}$. The duration of the estimation phase T_0 is chosen to be longer than the maximum settling time for each input-output channels, i.e. $\max_{j=1,\dots,n}\left\{3\frac{4}{\rho_j}\right\} < T_0 < T$. This means that the scaled force is applied for $T - T_0$ during one cycle. The strategy we proposed in Section III has set a required scaling factor Φ, but during one cycle the scaled force is applied for only a fraction of time $(T - T_0)$. Therefore, we have to determine the new scaling factor Ψ, which leads to an overall scaling factor Φ. The human applies the average force $\frac{1}{T}\int_{kT}^{(k+1)T} F_H dt$, over the cycle k. For the sake of simplicity, we consider the following assumption:

Assumption 2: F_H is constant during each cycle, i.e. $F_H(t) = F_H(kT)$, $\forall t \in [kT,(k+1)T)$.

Since $\frac{1}{T}$ is chosen to be significantly larger than the maximum frequency of human operator, this is a reasonable assumption. Under this assumption, the human applies the force $F_H(kT)$ and the robot should apply the force $\Phi F_H(kT)$, presumably with $\Phi > 0$. The robot applies the force F_R, with:

$$F_R(t) = \begin{cases} \hat{F}(t), kT \leq t < kT + T_0 \\ \Psi \hat{F}_H(kT+T_0), kT+T_0 \leq t < (k+1)T \end{cases}. \quad (27)$$

Under the Assumption 2, systems (13) reach the equilibrium point $(\eta_j^{(p-1)},\dots,\eta_j)^T = (0,\dots,0,\frac{F_{H,j}}{K_{0,j}})^T$. This means that $\hat{F} = -\sum_{i=0}^{p} K_i \eta^{(i)} = K_0\eta = -\hat{F}_H$. Consequently, the average force supplied by the robot is approximately given by:

$$F_R = \frac{1}{T}\left(\int_{kT}^{kT+T_0}(-1)\hat{F}_H(t)dt + \int_{kT+T_0}^{(k+1)T} \Psi \hat{F}_H(kT+T_0)dt\right). \quad (28)$$

where Ψ is the scaling factor we have to determine and we have ignored the torque corresponding to the controller C_{lin} since the human force is supposed to move only the load and not the robot links. Let us suppose that $\hat{F}_H(kT+T_0) = F_H(kT+T_0)$, i.e. the estimation is working, then the right-hand side of relation (28) is equivalent to:

$$\frac{1}{T}\left(\Psi(T-T_0)F_H(kT) - \int_{kT}^{kT+T_0}\hat{F}_H(t)dt\right), \quad (29)$$

where the second term is approximately equal to $\frac{T_0}{T}F_H(kT)$. Note that the lower the settling time for the estimation procedure, the better the approximation. Using this approximation and the requirement that $\frac{1}{T}\int_{kT}^{(k+1)T} F_R dt = \Phi F_H(kT)$, we obtain the following equation from which we can determine the scaling factor Ψ:

$$\Phi F_H(kT) = \frac{1}{T}\left(\Psi(T-T_0)F_H(kT) - T_0 F_H(kT)\right), \quad (30)$$

or

$$\Psi = \frac{\Phi T + T_0}{T - T_0}. \quad (31)$$

The estimation/force scaling algorithm is now fully defined and the design goals have been reached.

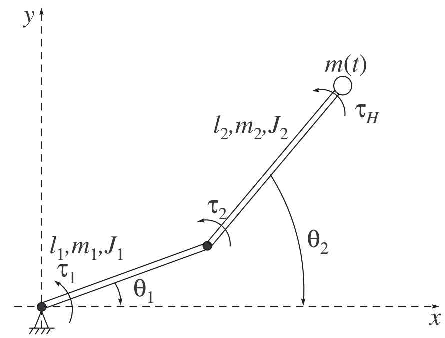

Fig. 5. A two-link robot.

V. ILLUSTRATIVE EXAMPLE

In this section, we will apply the estimation and control design proposed in the previous section to a two-link robot in the horizontal plane, see Figure 5. We assume that the links are rigid and the joints are frictionless. The dynamics of the robot can be described by equation (6), where

$$M_R = \begin{pmatrix} J_1 + \frac{m_1 l_1^2}{4} + m_2 l_1^2 & \frac{m_2 l_1 l_2}{2} cos(\theta_2 - \theta_1) \\ \frac{m_2 l_1 l_2}{2} cos(\theta_2 - \theta_1) & J_2 + \frac{m_2 l_2^2}{4} \end{pmatrix} \quad (32)$$

$$D_R = \begin{pmatrix} -\frac{m_2 l_1 l_2}{2}\dot{\theta}_2^2 sin(\theta_2 - \theta_1) \\ \frac{m_2 l_1 l_2}{2}\dot{\theta}_1^2 sin(\theta_2 - \theta_1) \end{pmatrix} \quad (33)$$

$$P_M = \begin{pmatrix} l_1^2\ddot{\theta}_1 + l_1 l_2 \ddot{\theta}_2 cos(\theta_2-\theta_1) - l_1 l_2 \dot{\theta}_2^2 sin(\theta_2-\theta_1) \\ l_1 l_2 \ddot{\theta}_1 cos(\theta_2-\theta_1) + l_2^2\ddot{\theta}_2 + l_1 l_2 \dot{\theta}_1^2 sin(\theta_2-\theta_1) \end{pmatrix} \quad (34)$$

$$J = \begin{pmatrix} -l_1 sin\theta_1 & -l_2 sin\theta_2 \\ l_1 cos\theta_1 & l_2 cos\theta_2 \end{pmatrix}. \quad (35)$$

Herein l_i, m_i and J_i are the length, mass and moment of inertia about the center of mass of link i, $i = 1,2$, respectively. Moreover, $m(t)$ represents the mass of the load.

For simulation purposes, we consider the following parametric settings: $l_1 = l_2 = 0.6$ m, $m_1 = m_2 = 2$ kg, $J_1 = J_2 = \frac{m_1 l_1^2}{12} = 0.06$ kgm^2 for the robot links. We assume that the mass of the load varies between $M_{min} = 10$ kg and $M_{max} = 50$kg by the law $m(t) = 40e^{-\frac{t}{2}} + 10$. Knowing that a human operator can not generate signals with a frequency greater that 6Hz, the cycle period for our design is $T = 0.01s$ ($\frac{1}{T} = 100 \gg 6$).

For the estimation phase, we have chosen only one integrator ($p = 1$) per input-output channel with $K_1 = -5I_2$ and $K_0 = 10^5 I_2$. We have solved the LMIs (15) for $\rho_j = 4000$ yielding $P_j = 1$, $j = 1,2$. Consequently, the estimation error dynamics (14) is ISS with respect to $v_j(t)$, $j = 1,2$. Now the ISS result in Theorem 2 provides an ultimate bound on the estimation error of $\frac{4}{\rho_j}\sup(\bar{v}_j(t)) = 0.001\sup(\dot{u}_j(t))$, $j = 1,2$. The ISS property also provides some important insights for the design of the global controller because it allows to determine the period of the estimation cycle T_0. The function β^j, $j = 1,2$, as in (18), is providing a bound on the convergence rate for the system: $\beta(r,t) = re^{-1000t}$ and the settling time is $T_s = \max_{j=1,2}\left\{3\frac{4}{\rho_j}\right\} = 0.003$ s. As a consequence we have chosen $T_0 = 0.005$ s $> T_s$.

We assume that the desired value for the scaling parameter Φ is 3. Hence, according to relation (31), $\Psi = 7$.

We have emulated the human behavior by a Proportional-Derivative (PD) controller on each input-output channel with a first-order filter for the frequencies higher than 6Hz and saturation bounds on the human force level. The "human" controller on each Cartesian direction has been emulated by a linear transfer function: $H(s) = \frac{K_d(T_d s+1)}{T_{PL}s+1} = \frac{500(1+s)}{0.1s+1}$, with saturation at $\pm 100N$.

The simulation of the estimation algorithm is presented in Figure 6 for a short time-frame. In this figure, during the amplification phase we consider that the estimated force is zero (in reality no estimation is taking place during this interval). One can observe that the algorithm is tracking accurately the human force for 0.005 seconds, followed by the scaling stage.

The results obtained by the estimation/control algorithm (Human

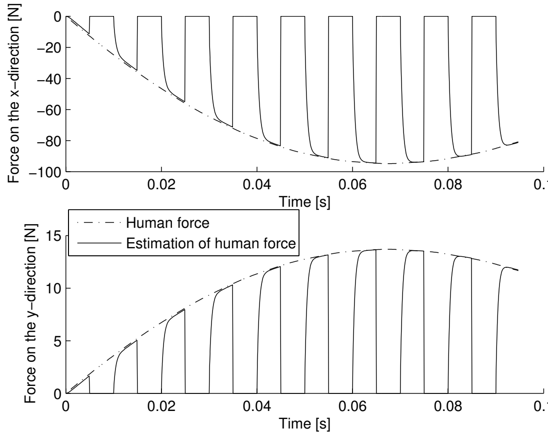

Fig. 6. Human Force Estimation.

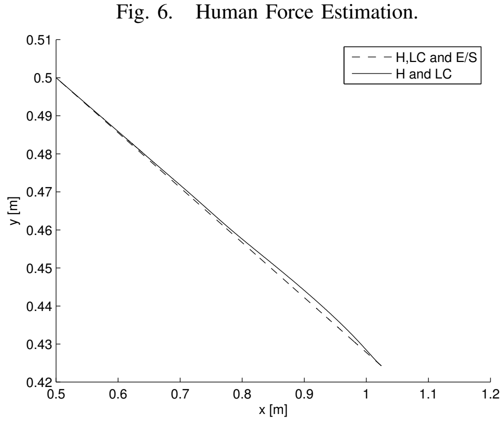

Fig. 7. Load Displacement.

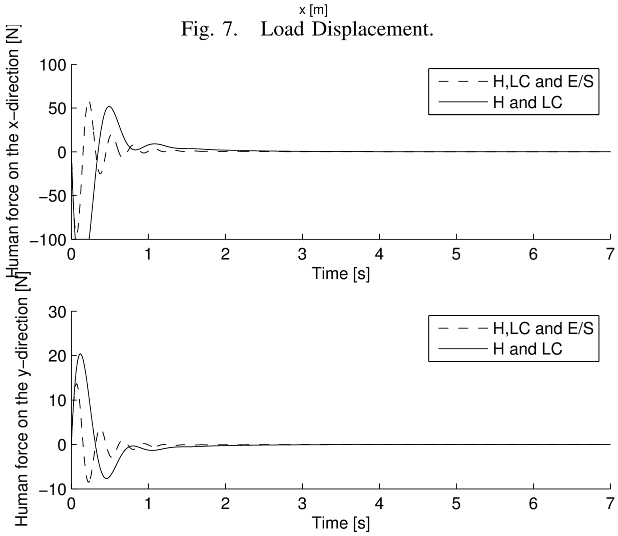

Fig. 8. Human Force.

force (H), compensation for the robot links dynamics (LC) and estimation/scaling algorithm (E/S)) are compared with the use of human force and compensation for the robot links dynamics (H and LC) only.

This simulation focusses on two important issues of this comparison: the end-effector displacement (Figure 7) and the applied human force (Figure 8). In both cases, the desired set point is achieved in the same amount of time because we have used the same PD controller to emulate the human behavior. Concerning the human force applied, we observe that the only case in which the PD controller does not saturate is when the estimation/scaling controller is used. This means that the human does not need to use the maximal force, i.e. there is a reserve of force which can be used to achieve the desired set point faster.

VI. CONCLUSIONS AND PERSPECTIVES

In this paper, we have proposed a control strategy for robots aiding humans to lift/move heavy loads of unknown, time-varying mass. We consider situations in which human intelligence is taking care of both the path planning and position control. Therefore, the robot should merely amplify the human force, thereby, firstly, aiding the human speed up its tasks and, secondly, alleviating the human efforts.

The key issues in tackling this problem are, firstly, the fact that the mass of the load is unknown and time-varying and, secondly, the force applied by the human is unknown. Here, we provide a control /estimation algorithm, which, in a cyclic fashion, firstly, estimates the currently applied human force and, secondly, applies an amplified version of this force. The algorithm can provide a preset amplification of the human force for a range of unknown masses. The proposed strategy is illustrated by application to a two-link robotic example.

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