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New Method of Identifying Real-Time Predictive Lateral Load Transfer Ratio for Rollover Prevention Systems

Vasilios Tsourapas [1], Damrongrit Piyabongkarn [1], Alexander C. Williams [2] and Rajesh Rajamani [3]

***Abstract*— Vehicle rollover accounts for a significant percentage of fatal accidents in the USA and worldwide. In this paper, two rollover indexes are proposed and analyzed. The first rollover index estimates the actual Lateral Transfer Ratio (LTR) while the second, referred to as 'Predictive Lateral Transfer Ratio' (PLTR), incorporates the predictive influence of the driver's steering input, thus allowing the actuator to respond faster to the rollover scenario. Both algorithms are compared in open loop and closed loop simulation environment using a baseline control system. The PLTR is shown to be superior in preventing rollover under a number of different cornering maneuvers. The vehicle used for simulation is a large SUV with a top-loaded condition. Preliminary experimental results comparing the two algorithms are also presented.**

I. INTRODUCTION

Vehicle rollover has been identified as the vehicle crash with highest fatality. According to the National Highway Traffic Safety Administration (NHTSA) [1], vehicle rollovers account for approximately 3% of passenger vehicle crashes annually. However, 33% of fatalities involved with all passenger vehicle crashes are related to rollover accidents. Continued popularity of high-CG vehicles such as SUVs and trucks lends reason to further development of anti-rollover systems as these types of vehicles are most easily associated with incidents involving vehicle rollover. Along with the financial toll, the human cost associated with vehicle rollover is the main motivation behind the development of active-rollover warning systems such as the one discussed in this paper.

Rollover prevention and detection have been studied by many researchers [2,3,4,5,6,7]. Several different means of detecting vehicle rollover have been introduced such as the lateral load transfer ratio (LTR) [8,9,10] and the time-to-rollover (TTR) metric [11,12]. The latter method needs precise vehicle modeling parameters in order to be able to predict the vehicle roll angle ahead of time. The LTR defines vehicle rollover as the moment either the left or right side of the vehicle experiences lift-off from the ground. This index only gives a snap shot of vehicle dynamics by detecting instantaneous load transfer due to lateral acceleration, regardless of steering patterns. Analytical calculations as well as experimental data contribute to the definition of a threshold for determining rollover threat based on estimated values of the LTR. If the threshold is set too low, the LTR will give a warning, or activate the rollover prevention system even during safe normal driving. If the threshold is set too high, preventive action may activate too late to avoid vehicle rollover. Identifying a good LTR threshold is difficult because of dynamic changes and unexpected disturbances, which cannot be captured using only a lateral acceleration based LTR.

This paper proposes a new method to estimate a predictive LTR (PLTR). This predictive index is based on factors that occur over a time horizon. It will indicate future vehicle rollover propensity based on the current LTR and steering angle pattern. The decision to use the steering wheel angle as a means of prediction in the PLTR can further be supported by the findings of Liu *et. al.* [13] who concluded that the factors associated with steering had the greatest capability of providing the earliest rollover warning.

The outline of this paper is as follows: First, a derivation of the LTR, based on a half-car vehicle dynamic model and the use of sensors to detect lateral acceleration, yaw rate, and sprung mass roll angle will be shown. Simulations conducted in CarSim on various car types to better define the precision and accuracy of the new LTR index follows. Validation with experimental data is also provided. Further estimation of the new LTR based solely on the body-fixed lateral acceleration of the vehicle is also discussed. Later, the new predictive rollover index, PLTR, is derived based on the simplified LTR presented earlier. The derivation of the PLTR is followed by simulation results that conclusively show the effectiveness of this new index to predict impending rollover. Finally, the effectiveness of anti-rollover control using the PLTR and LTR implemented on a vehicle equipped with actively controlled electro-hydraulic limited slip differentials is presented.

II. VEHICLE MODEL

The variables used for this vehicle model are defined in the Nomenclature. Figure 1 displays the lateral dynamics of the vehicle. The intended vehicle direction as well as instantaneous yaw rate and velocity are also indicated on this diagram. Figure 2 displays the roll dynamics of the vehicle.

[1] **Vasilios Tsourapas and Damrongrit (Neng) Piyabongkarn are with Eaton Corporation, Innovation Center, Email: VasiliosTsourapas@eaton.com, NengPiyabongkarn@eaton.com**

[2] **Alexander Williams is with the Dept. of Mechanical Engineering, Stanford University, Email: acwillia@stanford.edu**

[3] **Rajesh Rajamani is with the University of Minnesota, Minneapolis, MN 55455, Email: rajamani@me.umn.edu**

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The effect of the vehicle's un-sprung mass on roll dynamics is neglected. Distances associated with the roll center location as well as vehicle roll angle and base bank angle are also specified in the image.

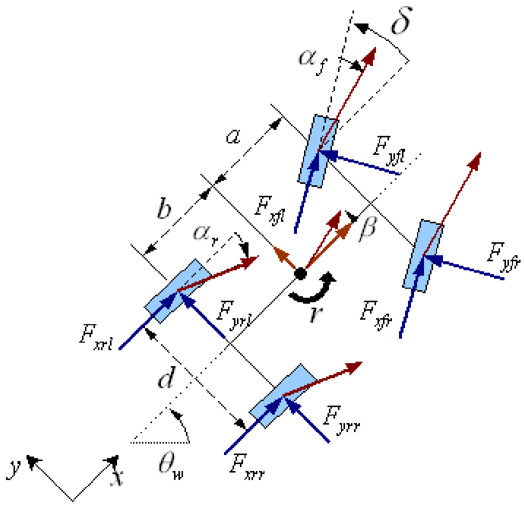

Fig. 1 Vehicle lateral dynamics

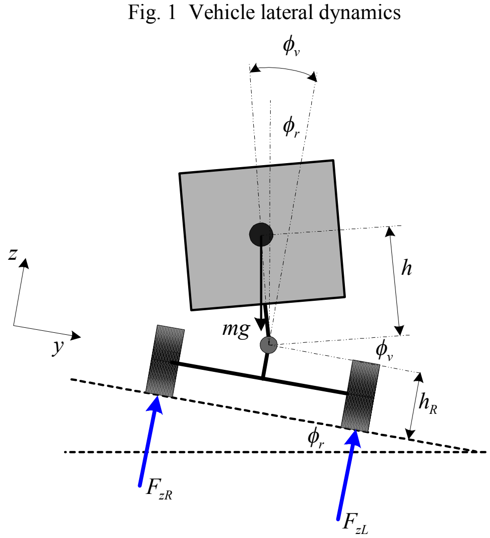

Fig. 2 Vehicle roll dynamics

The vehicle lateral dynamics can be written as

$$\sum F_y = mA_y = F_{yrl} + F_{yrr} + \left(F_{yfl} + F_{yfr}\right)\cdot \cos\delta + \left(F_{xfl} + F_{xfr}\right)\cdot \sin\delta \quad (1)$$

where

$$A_y = \dot{v} + ru - g\cdot\sin\phi_r + hr^2\cdot\sin\phi_v + h\dot{\phi}_v^2\cdot\sin\phi_v - h\ddot{\phi}_v\cdot\cos\phi_v\,.$$

The developed expression for ΣF_y seen in equation (1) is based with respect to the un-sprung mass referenced in Fig.1. ΣF_y is estimated to act with equal and opposite force on the sprung mass as demonstrated below in the derivation of the roll dynamics.

$$\begin{aligned}\left(I_{xx} + mh^2\right)\cdot\left(\ddot{\phi}_v - \ddot{\phi}_r\right) &= \left(F_{zL} - F_{zR}\right)\cdot\frac{d}{2} + \sum F_y\cdot h\cdot\cos\phi_v \\ &+ mgh\cdot\sin\phi_v\cdot\cos\phi_r - mgh\cdot\cos\phi_v\cdot\sin\phi_r \\ &+ \left[(I_{yy} - I_{zz}) - mh^2\right]\cdot r^2\cdot\sin(\phi_v - \phi_r)\cos(\phi_v - \phi_r) \qquad (2)\end{aligned}$$

The vertical dynamics of sprung mass can be expressed as:

$$m\ddot{z} = m\cdot\left(\dot{\phi}_v^2 h\cdot\cos\phi_v + \ddot{\phi}_v h\sin\phi_v\right) = \left(F_{zL} + F_{zR}\right) - mg\cdot\cos\phi_r \quad (3)$$

Finally, equations (1)-(3) are modified such that the bank angle is neglected as follows,

$$\sum F_y = m\cdot\left(\dot{v} + ru + hr^2\cdot\sin\phi_v + h\dot{\phi}_v^2\cdot\sin\phi_v - h\ddot{\phi}_v\cdot\cos\phi_v\right) \quad (4)$$

$$\begin{aligned}\left(I_{xx} + mh^2\right)\cdot\ddot{\phi}_v &= \left(F_{zL} - F_{zR}\right)\cdot\frac{d}{2} + \sum F_y\cdot h\cdot\cos\phi_v \\ &+ mgh\cdot\sin\phi_v + \left[\left(I_{yy} - I_{zz}\right) - mh^2\right]\cdot r^2\cdot\sin\phi_v\cdot\cos\phi_v \qquad (5)\end{aligned}$$

$$m\ddot{z} = m\cdot\left(\dot{\phi}_v^2 h\cdot\cos\phi_v + \ddot{\phi}_v h\sin\phi_v\right) = \left(F_{zL} + F_{zR}\right) - mg \quad (6)$$

III. LATERAL LOAD TRANSFER RATIO

A common expression used to indicate the vehicle rollover propensity is the lateral transfer ratio (LTR) defined as:

$$LTR := \frac{F_{zR} - F_{zL}}{F_{zL} + F_{zR}} \quad (7)$$

This index utilizes vertical tire forces F_{zL} and F_{zR}. The LTR defines vehicle rollover as the moment either the left or right side of the vehicle experiences lift-off from the ground. The LTR varies from –1 to 1, where –1 and 1 refer to either the left or right vehicle tires losing contact with the ground, and 0 refers to equal vertical forces on both sides of the vehicle (zero roll). This index, like most currently studied indexes, only gives a snap shot of vehicle dynamics by detecting vehicle dynamics, such as the dominant lateral acceleration trait, regardless of steering patterns. Through the use of this method, the time between detection of potential rollover characteristics and the moment rollover occurs may sometimes be too small for a rollover prevention system to stop the vehicle from rolling over.

Utilizing (5) and (6), assuming $\ddot{\phi}_v$ and $\dot{\phi}_v$ are zero and substituting into equation (7) the following expression is obtained:

$$LTR = \frac{2}{d}\cdot\frac{h\cdot(\cos\phi_v\cdot(\dot{v} + ru) + hr^2\cdot\sin\phi_v + g\cdot\sin\phi)}{g} \quad (8)$$

Using the following assumptions: $\cos^2\phi_v \approx 1$, $hr^2 \approx 0$ and $\dot{v} + ru = A_{y\_meas}\cdot\cos\phi_v$, the final expression of the Lateral Transfer Ratio is obtained as

$$LTR_e = \frac{2h}{dg}\left[A_{y\_meas} + g \cdot \sin\phi_v\right] \quad (9)$$

where A_{y_meas} is the measured lateral acceleration of the vehicle. In order to validate the above expression for estimating the Lateral Transfer Ratio, the CarSim© simulation tool is utilized. The vehicle simulated is a large SUV with the following parameters:

Table 1: Simulation Vehicle Particulates

Quantity	**Symbol**	**Value**	**Unit**
Mass	m	4400	*Kg*
C.G. Height	h	0.94	m
Trackwidth	d	1.819	m

Note that the height of the center of gravity was increased from the standard 0.83m in order to simulate a top-loaded vehicle state that results in a rollover scenario during a NHTSA fishhook maneuver. Furthermore, the vehicle is simulated with open differentials and no rollover mitigation properties. Figures 3 and 4 show the simulation results comparing the actual LTR, equation (7), and the estimated LTR, equation (8), during a double lane change and a NHTSA fishhook maneuver respectively. Very good matching is observed between the actual and estimated LTR in both NHTSA fishhook and double lane change maneuvers. Specifically for the NHTSA fishhook, wheel lift off occurs at t=2.8s and the LTR is well estimated up to that point; after wheel lift off occurs the actual LTR stays at the value of one since the tire vertical forces on one side become zero.

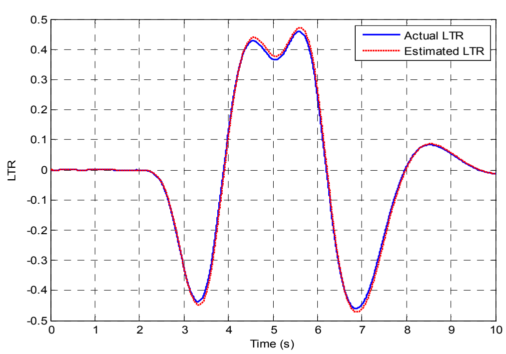

Fig. 3 Actual vs. Estimated LTR during a double lane change maneuver at 80kph

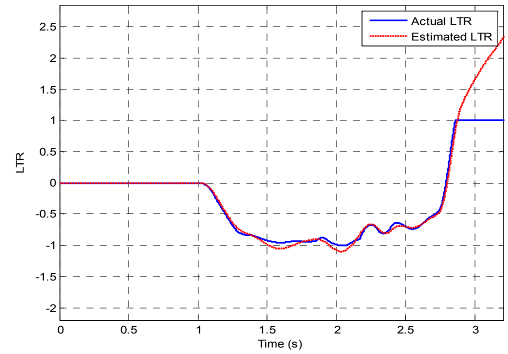

Fig. 4 Actual vs. Estimated LTR during a fishhook maneuver at 70kph

IV. Predictive Lateral Load Transfer Ratio (PLTR)

A typical LTR is estimated from information obtained at a given time. It is like taking a snap shot of a dynamic system. Information gathered at a fixed time is used to determine the immediate, as opposed to future, rollover threat. Analytical calculations as well as experimental data contribute to the definition of a threshold for determining rollover threat based on estimated values of the LTR. If the threshold is set too low, the LTR will give a warning, or activate the rollover prevention system even for normal safe driving. If the threshold is set too high, preventive action may activate too late to avoid the vehicle from rolling over. Identifying a good LTR threshold is difficult because of dynamic changes and unexpected disturbances, which cannot be captured using only a static LTR.

Hence, a new predictive rollover index, predictive LTR (PLTR), is proposed in this paper. This predictive index is based on factors that occur over a time horizon. It will indicate future vehicle rollover based on data collected in the current frame for a wide range of vehicle maneuvers. The PLTR is defined as follows

$$PLTR_{t_0}(\Delta t) = LTR(t_0) + L\dot{T}R(t_0) \cdot \Delta t \quad (13)$$

where Δt is the preview time and t_0 is the current time.

Considering the LTR from equation (9), it can further be simplified for this PLTR derivation as

$$LTR = \frac{2h}{d}\left[\frac{A_y}{g} + \sin\phi\right] \quad (14)$$

Hence, we have

$$PLTR_{t_0}(\Delta t) = LTR(t_0) + \frac{2h}{d} \cdot \frac{d}{dt}\left[\frac{A_y}{g} + \sin\phi\right] \cdot \Delta t \quad (15)$$

or

$$PLTR_{t_0}(\Delta t) = LTR(t_0) + \frac{2h}{d \cdot g} \cdot \left[\dot{A}_y + g\dot{\phi}\right]\Delta t \quad (16)$$

Equation (16) shows the calculation of the *PLTR* at time t_0 predicted for a future time horizon Δt. A_{y_meas} is typically noisy and it is difficult to obtain a smooth value after derivation. A filtering technique is used to solve this problem shown below:

$$PLTR_{t_0}(\Delta t) = \frac{2h}{d}\left[\frac{A_{y\_meas}}{g} + \sin\phi_{meas}\right] + \frac{2h}{d \cdot g}\left(\frac{s}{\tau s+1}A_{y\_meas} + \frac{\tau s}{\tau s+1}\dot{A}_{y\_meas} + g\dot{\phi}_{meas}\right)\cdot\Delta t \quad (17)$$

where τ is a time constant.

The lateral acceleration can be further estimated from the lateral dynamics equation (1).

By utilizing a linear approximation and the small angle assumption, the lateral dynamics equation can be written as [14,15]:

$$mA_y = -C_0\beta - C_1\frac{r}{u} + 2C_f\delta \quad (18)$$

where $C_0 = 2C_f + 2C_r$ and $C_1 = 2aC_f - 2bC_r$. C_f and C_r are the cornering stiffnesses for the front and the rear tires respectively.

The derivative of equation (18) can be written as

$$\dot{A}_y = \frac{-C_0(A_y - ru) - C_1\dot{r}}{mu} + \frac{2C_f}{m}\frac{1}{\tau_{sw}s+1}\cdot\frac{1}{SR}\dot{\delta}_d \quad (19)$$

where $\frac{\delta_w}{\delta_d} = \frac{1}{SR}\cdot\frac{1}{\tau_{sw}s+1}$; δ_d is the driver's steering wheel angle; τ_{sw} is the steering first-order time constant and *SR* is the steering ratio.

By using this model-based filter, the noise from the derivation of the steering wheel angle can be filtered out using a low-pass filter. Moreover, the driver's steering input information plays an important role in predicting the rollover index due to the inherent delay between the steering input and its influence on vehicle roll

The new PLTR is displayed below:

$$\begin{aligned} PLTR_{t_0}(\Delta t) &= \frac{2h}{d}\left[\frac{A_{y\_meas}(t_0)}{g} + \sin\phi_{meas}\right] \\ &+ \frac{2h}{d \cdot g}\left\{\frac{s}{\tau s+1}A_{y\_meas}(t_0) + \frac{\tau s}{\tau s+1}\cdot\frac{-C_0(A_y - ru) - C_1\dot{r}}{mu} + \ldots\right. \\ &\left.\ldots + \frac{\tau s}{\tau s+1}\cdot\frac{2C_f}{m}\frac{s}{\tau_{sw}s+1}\cdot\frac{1}{SR}\delta_d(t_0) + g\dot{\phi}_{meas}\right\}\cdot\Delta t \end{aligned} \quad (20)$$

The filter $\frac{\tau s^2}{(\tau s+1)(\tau_{sw}s+1)}$, is used on the driver's steering angle. The prediction time Δt needs to be selected to be long enough to cover the rollover prevention system response time.

Based on the relatively small magnitude of the $\sin\phi_{meas}$ term, it can be replaced with a constant, k, for simplicity. Finally, the new PLTR is shown below,

$$\begin{aligned} PLTR_{t_0}(\Delta t) &= \frac{2h}{d}(1+kg)A_{y\_meas}(t_0) \\ &+ \frac{2h}{d \cdot g}\left\{\frac{s}{\tau s+1}A_{y\_meas}(t_0) + \frac{\tau s}{\tau s+1}\cdot\frac{-C_0(A_y - ru) - C_1\dot{r}}{mu} + \ldots\right. \\ &\left.\ldots + \frac{\tau s}{\tau s+1}\cdot\frac{2C_f}{m}\cdot\frac{s}{\tau_{sw}s+1}\cdot\frac{1}{SR}\delta_d(t_0) + g\dot{\phi}_{meas}\right\}\cdot\Delta t \end{aligned} \quad (21)$$

This new rollover index has the following advantages over the typical LTR.

1) It acts as part of a warning system to predict, rather than detect, vehicle rollover.
2) It is easier to set the threshold, since the trade-off between false alarms and safety is reduced.
3) More vehicle information (steering angle, yaw rate, roll rate) is used in the index. At the same time, all of the required feedback variables can be easily measured with inexpensive sensors.
4) It can be used in rollover prevention systems that include both torque management or brake-based stability control systems.

A simulation study was performed on an SUV to show the effectiveness of the PLTR with 0.3s predictive time. As can be seen in Fig. 6 and 7, PLTR result matches very well with the actual LTR on a double lane change and a fishhook maneuver. The overshoot observed in the fishhook maneuver is due to the fact that the steering pattern for the fishhook maneuver is more abrupt.

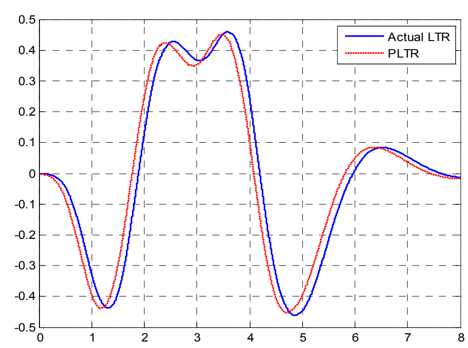

Fig. 6 PLTR (double lane change simulation)

Figure 8 shows the calculation of the LTR and the PLTR from actual vehicle experimental data for a double lane change at 90 kph. Good correlation between the simulation study and the actual implementation was achieved.

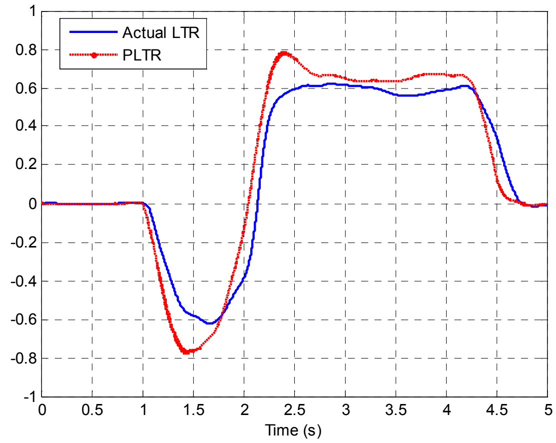

Fig. 7 PLTR (fishhook simulation)

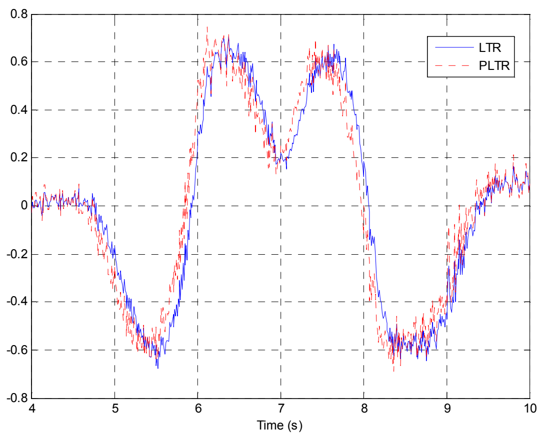

Fig. 8 Double lane change (experiment)

V. Closed-Loop Simulation Comparison

This section presents the results acquired from closed loop simulation utilizing the different rollover indexes proposed in this paper. The simulation model includes the vehicle model in CarSim feeding into a baseline control scheme and in turn into an active differential configuration. The active differentials are electro-hydraulically actuated and have the ability to operate proportionally from fully open to fully locked, thus effectively control the vehicle's yaw and roll motions. A schematic of the differentials is shown in Fig. 9.

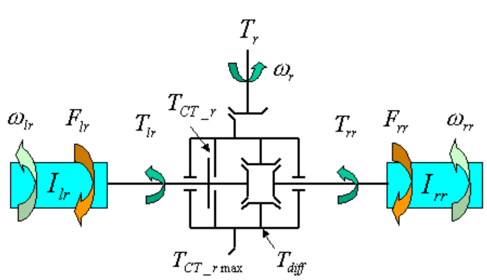

Fig. 9 Electro-hydraulically Actuated Differentials

The adopted baseline control law is given in (22). When the estimated LTR reaches the value of 0.6 the differentials are engaged 50% and then the engagement is increased proportionally to full lock, i.e., 100% engagement, when LTR reaches a value of 0.8.

$$u = \underset{100}{sat}\left[a \cdot \underset{[0\ 0.6]}{deadzone}\{LTR\} + b \right], \tag{22}$$

where $(a,b) = (250,-100)$.

However, the threshold setting in equation (22), e.g. 0.6, may not be suitable for various maneuver types due to the steady state nature of the LTR calculation. It could be conservative or safe to prevent a rollover for a certain normal driving situations. PLTR is proposed to capture the dynamics of the LTR and address the challenge to set the threshold for the control law. Hence, the robustness to the maneuver variation can be improved.

The closed loop performance of the vehicle is compared utilizing the rollover angle, the yaw rate, the X-Y vehicle position and the Actual LTR indexes for a NHTSA fishhook maneuver at 50kph as shown in Fig. 10. The vehicle with no closed loop control as well as the vehicle utilizing the LTRr index for closed loop control result into rollover. The vehicle utilizing PLTR for closed loop control manages to handle the maneuver with no wheel lift (i.e. -1<LTR<1) or rollover.

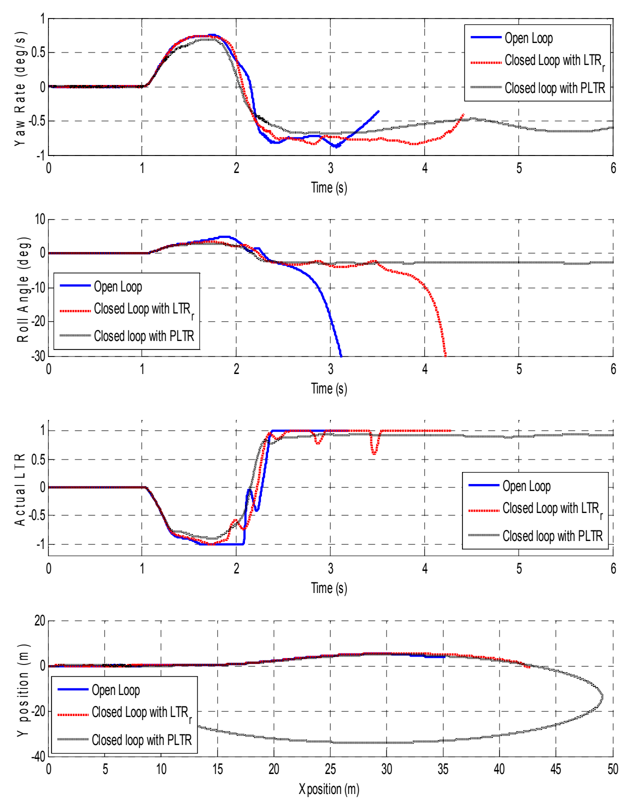

Fig. 10 Open and Closed Loop Performance comparison during NHTSA Fishhook maneuver at 50kph

VI. CONCLUSION

In this paper, the Predictive Lateral Transfer Ratio (PLTR) approach is proposed and compared against the commonly used LTR estimation algorithm that utilizes lateral acceleration and roll angle. It is shown the PLTR when used in conjunction with a closed loop control scheme manages to prevent vehicle rollover where with the typical LTR methodology the vehicle would roll over.

NOMENCLATURE

a	longitudinal distance from c.g. to front axle
A_y	vehicle lateral acceleration
A_{y_meas}	measured vehicle lateral acceleration
b	longitudinal distance from c.g. to rear axle
C_f	front tire cornering stiffness
C_r	rear tire cornering stiffness
d	track width
F_{xfl}	front-left tire longitudinal force
F_{xfr}	front-right tire longitudinal force
F_{xrl}	rear-left longitudinal force
F_{xrr}	rear-right longitudinal force
F_{yfl}	front-left lateral force
F_{yfr}	front-right lateral force
F_{yrl}	rear-left lateral force
F_{yrr}	rear-right lateral force
g	gravity acceleration
h	distance from sprung mass CG to roll center
h_R	roll center height
$I_{xx,yy,zz}$	moment of inertia about respected axes
m	*vehicle sprung* mass
r	yaw rate
u	vehicle's longitudinal velocity
v	vehicle's lateral velocity
β	vehicle body slip angle
δ	steering wheel angle
ϕ_r	road bank angle
ϕ_v	road bank angle

REFERENCES

[1] National Highway Traffic Safety Administration (NHTSA) website: www.nhtsa.dot.gov

[2] Li Xu and H. Eric Tseng, "Robust Model-Based Fault Detection for a Roll Stability Control System", IEEE Transactions On Control Systems Technology, vol. 15, no. 3, May 2007

[3] Alexander Hac, "Rollover Stability Index Including Effects of Suspension Design", SAE World Congress 2002, Detroit, Michigan, ISDN 2002-01-0965

[4] Alexander Hac, "Influence of Active Chassis Systems on Vehicle Propensity to Maneuver-Induced Rollovers", SAE World Congress 2002, Detroit, Michigan, ISDN 2002-01-0967

[5] Hanlong Yang and LouisYizhang Liu, "A Robust Active Suspension Controller with Rollover Prevention", SAE World Congress 2003, Detroit, Michigan, ISDN 2003-01-0959

[6] Jamie Gertsch and Oliver Eichelhard, " Simulation of Dynamic Rollover Threshold for Heavy Trucks", SAE Paper 2003-01-3385

[7] Bjorn Johansson and Magnus Gafvert, "Untripped SUV Rollover Detection and Prevention", 43rd IEEE Conference on Decision and Control, December 14-17, 2004

[8] Odenthal D., Bunte T. and Ackermann J., "Nonlinear Steering and Breaking Control for Vehicle Rollover Avoidance," Proceedings of European Control Conference, Karlsruhe, Germany, 1999.

[9] Liu P.J., Rakheja S. And Ahmed A.K.W., "Detection of Dynamic Roll Instability of Heavy Vehicles for Open-Loop Rollover Control, SAE Paper No. 973263, 1997.

[10] J.Y. Lew, D. Piyabongkarn, and J.A. Grogg, "Minimizing Dynamic Rollover Propensity with Electronic Limited Slip Differentials," SAE 2006 Transactions Journal of Passenger Cars: Mechanical Systems, pp. 1183-1190, SAE book number V115-6, 2007.

[11] Bo-Chiuan Chen and Huei Peng, "Differential-Braking-Based Rollover Prevention for Sport Utility Vehicles with Human-in-the-loop Evaluations", Vehicle System Dynamics, Vol 36, No.4-5, November 2001, pp.359-389

[12] Bo-Chiuan Chen and Huei Peng, "A Real-Time Rollover Threat Index for Sports Utility Vehicles", Proceedings of the American Control Conference San Diego, California, 1999, pp.1233-1237

[13] P.J. Lui, S. Rakheja and A. K. W. Ahmed, "Detection of Dynamic Roll Instability of Heavy Vehicles for Open-Loop Rollover Control", SAE International Truck and Bus Meeting, Cleveland, Ohio, pp.105-112.

[14] D. Piyabongkarn, R. Rajamani, J.A. Grogg and J.Y. Lew, "Development and Experimental Evaluation of a Slip Angle Estimation for Vehicle Stability Control," Proceedings of the 2006 American Control Conference, Minneapolis, June 14-16, 2006.

[15] R. Rajamani, Vehicle Dynamics and Control, ISBN 0-387-26396-9, Springer Verlag, New York, 2005.