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Robust Stabilization of Nonholonomic Moving Robots with Uncalibrated Visual Parameters

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Abstract— **This paper discusses visual servoing stabilization of nonholonomic mobile robots with uncertain camera parameters. To cope with the nonholonomic property of the system, we propose a time varying feedback controller for robustly stabilizing the position and orientation of the mobile robot to desired ones using visual feedback when the depth of the image features and the camera parameters are not known. This controller is developed based on a new formulation of the problem in the image space. The stability of the time varying controller is proved rigorously. We have carried out simulations whose results confirmed good performance of the proposed method.**

I. Introduction

A mobile robot is one of the well-known systems with nonholonomic constraints[13][14]. By the theorem of R. Brockett(1983)[15], a nonholonomic system cannot be stabilized at a single equilibrium point by a smooth feedback controller. To solve this problem, lots of methods have been considered, such as chained form methods[16][17], tracking control[18] and discontinuous feedback control [19] etc. In the control of nonholonomic mobile robots, it is usually assumed that the robot states are available and exactly reconstructed using proprioceptive and exteroceptive sensor measurements. But in practical mobile robot applications, there are several ideal conditions that can not be satisfied, such as uncertainties in the kinematic model, mechanical limitations, noise and so on. The estimation of the robot state from sensor measurements can be affected by these perturbations.

Visual feedback is an important approach to improve the control performance of manipulators since it mimics the human sense of vision and allows for operating on the basis of noncontact measurement and unstructure of the environment. Since the late 1980s, tremendous effort has been made to visual servoing and vision-based manipulations[22].

The nonholonomic control problem results to be more involved because of the visual feedback. Designing the feedback at the sensor level increases system performances especially when uncertainties and disturbances affect the robot model and the camera calibration, see [23] and therein references.

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Based on the success of image extraction/interpretation technology and advances in control theory, research has focused on the use of monocular camera-based vision systems for navigating a mobile robot [4][10][12]. A significant issue with monocular camera-based vision systems is the lack of depth information. From a review of literature, various approaches have been developed to address the lack of depth information inherent in monocular vision systems. For example, using consecutive image frames and an object database, Kim et al. [5] recently proposed a mobile robot tracking controller based on a monocular visual feedback strategy. To achieve their result, they linearized the system equations using a Taylor series approximation, and then applied extended Kalman filtering (EKF) techniques to compensate for the lack of depth information [5]. Dixon et al. [2] used feedback from an uncalibrated, fixed (ceiling-mounted) camera to develop an adaptive tracking controller for a mobile robot that compensated for the parametric uncertainty in the camera and the mobile robot dynamics. Dixon et al. exploit Lyapunov-based adaptive techniques to compensate for the unknown depth information [2]. However, to employ these techniques, they require the depth from the camera to the mobile robot plane of motion to remain constant (i.e., the camera plane and the mobile robot plane must be parallel). This assumption reduces the nonlinear pinhole camera model to a decoupled linear transformation; however, it also restricts the applicability of the controller. Recently, Chen et al. [1] developed a mobile robot visual servo tracking controller when the camera is onboard. An advantage of the result in [1] is that the mobile robot is not constrained to a planar application and an adaptive estimate is provided to compensate for unknown time-varying depth information. However, the development in [1] and [2] cannot be applied to solve the mobile robot regulation problem due to restrictions on the mobile robot reference velocity (i.e., the reference linear velocity cannot converge to zero). Wang et al. [11] also exploit a Lyapunov-based adaptive technique to compensate for a constant unknown depth parameter for a monocular mobile robot tracking problem. While the approach in [11] may be well suited for tracking applications, the stability analysis requires the same restrictions on the reference trajectory of the mobile robot as in [2], and hence, cannot be applied to solve the regulation problem.

The contribution of this paper is that the characteristic of the system is exploited to craft a robust controller that enables the mobile robot image pose and the orientation regulation despite the lack of depth information and the lack of precise visual parameters provided that the camera plane

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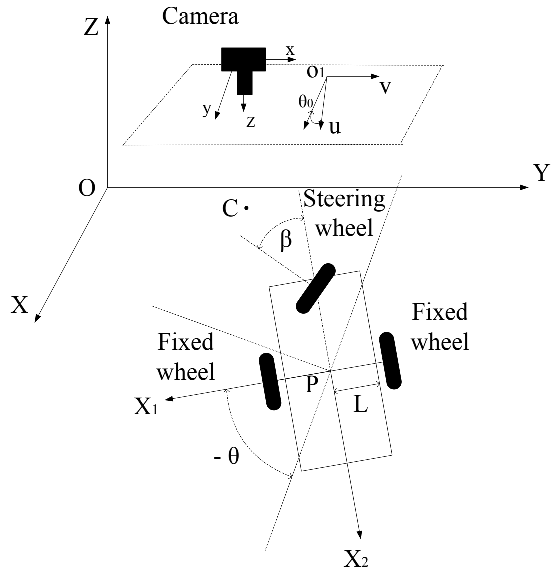

Fig. 1. Wheeled mobile robots with monocular camera

and the mobile robot plane must be parallel. The difficulty in design of the stabilizing controller caused by the non trigonal structure appeared in the nonholonomic kinematic control system chained form is overcome. Due to assumptions on the reference trajectory resulting from the nonholonomic constraint, the aforementioned visual servo tracking control results cannot be applied to solve the regulation problem considered in the current result. See [3][8][9] for a more technically detailed description of the issues and differences associated with developing tracking and regulation controllers for nonholonomic systems. The stabilization of nonholonomic kinematic control systems have been discussed recently in [6] [7]. But the triangular structure in the systems discussed were needed.

The result in this paper is achieved with a monocular vision system with uncalibrated visual parameters, and the control design approach incorporates the full nonholonomic kinematic equations of motion. In addition, the triangular structure in the models proposed in this paper is not satisfied. Simulation results are provided to illustrate the performance of the developed controller.

The paper is organized as follows. Section 2 introduces the camera-object visual model in terms of the planar optical flow equations. In Section 3, the controllers are synthesized. In Section 4, the simulation results carried out to validate the theoretical framework. Finally, in Section 5 the major contribution of the paper is summarized.

II. Problem Statement

A. System Configuration

In the Figure 1, the mobile robot is shown.

Assume that a pinhole camera is fixed to the ceiling and the camera plane and the mobile robot plane are parallel. There are three coordinate frames, namely the inertial frame X-Y-Z, the camera frame x-y-z and the image frame u-O_1-v. Assume that the x-y plane of the camera frame is the identical one with the plane of the image coordinate plane. C is the crossing point between the optical axis of the camera and X-Y plane. Its coordinate relative to X-Y plane is (p_x, p_y), the coordinate of the original point of the camera frame with respect to the image frame is defined by (O_{c1}, O_{c2}), (x, y) is the coordinate of the mass center of the robot with respective to X-Y plane. Suppose that (x_m, y_m) is the coordinate of (x, y) relative to the image frame. Pinhole camera model yields

Pinhole camera model yields

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \mathbf{R} \left[\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} p_x \\ p_y \end{bmatrix} \right] + \begin{bmatrix} O_{c1} \\ O_{c2} \end{bmatrix} \quad (1)$$

where γ_1, γ_2 are constant which are dependent on the depth information, focus length, scalar factors along u axis and v axis respectively.

$$\mathbf{R} = \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix} \quad (2)$$

where θ_0 denotes the angle between u axis and X axis with a positive anticlockwise orientation, where it is assumed that X axis, x axis and u axis have the same orientation.

B. Problem Description

Assume that the geometric center point and the mass center point of the robot are the same. The nonholonomic constraint is defined by

$$\dot{x}\cos\theta + \dot{y}\sin\theta = 0 \quad (3)$$

By this formula, nonholonomic kinematic equation is written by

$$\begin{cases} \dot{x} &= -Lv\sin\theta\sin\beta \\ \dot{y} &= Lv\cos\theta\sin\beta \\ \dot{\theta} &= v\cos\beta \\ \dot{\beta} &= \omega \end{cases} \quad (4)$$

where v and ω denote the velocity of the heading direction of the robot and the angle velocity of the rotation of the robot, respectively.

Taking the state transformation

$$\begin{cases} x_0 &= \theta \\ x_1 &= x\cos\theta + y\sin\theta \\ x_2 &= -x\sin\theta + y\cos\theta \\ x_3 &= L\tan\beta - x\cos\theta - y\sin\theta \end{cases}$$

and the input transformation

$$\begin{cases} u_0 &= v\cos\beta \\ u_1 &= v\cos\beta(x\sin\theta - y\cos\theta) + L\omega\sec^2\beta, \end{cases}$$

then we can obtain the common chained form as follows

$$\begin{cases} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= x_2u_0 \\ \dot{x}_2 &= x_3u_0 \\ \dot{x}_3 &= u_1 \end{cases}$$

However, if (x,y) is measured by using a camera shown in fig 1, its image position can be obtained by (1). By using (1) and (4), we have

$$\begin{aligned}\left[\begin{array}{c}\dot{x}_m\\ \dot{y}_m\end{array}\right] &= \left[\begin{array}{cc}\gamma_1 & 0\\ 0 & \gamma_2\end{array}\right] R \left[\begin{array}{c}\dot{x}\\ \dot{y}\end{array}\right]\\ &= \left[\begin{array}{c}-Lv\gamma_1\sin\beta\sin(\theta-\theta_0)\\ Lv\gamma_2\sin\beta\cos(\theta-\theta_0)\end{array}\right]\end{aligned}$$

Therefore,

$$\left[\begin{array}{c}\dot{x}_m\\ \dot{y}_m\\ \dot{\theta}\\ \dot{\beta}\end{array}\right] = \left[\begin{array}{c}-Lv\gamma_1\sin\beta\sin(\theta-\theta_0)\\ Lv\gamma_2\sin\beta\cos(\theta-\theta_0)\\ v\cos\beta\\ \omega\end{array}\right]$$

Take a state transformation

$$\left\{\begin{array}{lcl}x_0 &=& \theta\\ x_1 &=& x_m\cos\theta + y_m\sin\theta\\ x_2 &=& -x_m\sin\theta + y_m\cos\theta\\ x_3 &=& L\tan\beta - x_m\cos\theta - y_m\sin\theta\end{array}\right.$$

and input transformation

$$\left\{\begin{array}{lcl}u_0 &=& v\cos\beta\\ u_1 &=& L\omega\sec^2\beta - x_2 v\cos\beta\end{array}\right.$$

We have the following system

$$\left\{\begin{array}{lcl}\dot{x}_0 &=& u_0\\ \dot{x}_1 &=& x_2u_0 + (x_1+x_3)u_0\chi_1\\ \dot{x}_2 &=& x_3u_0 + (x_1+x_3)u_0\chi_2\\ \dot{x}_3 &=& u_1\end{array}\right. \tag{5}$$

where $\chi_1 = \gamma_2\sin\theta_0 + (\gamma_2-\gamma_1)\cos x_0\sin(x_0-\theta_0)$, $\chi_2 = \gamma_1\cos\theta_0 + (\gamma_2-\gamma_1)\cos x_0\cos(x_0-\theta_0) - 1$.

Assumption 1. $\theta_0 = 0$, and $\gamma_1 = \gamma_2 = \alpha$ are unknown.

Remark 1. $\theta_0 = 0$ means that the direction of u axis is identical to that one of X axis. $\gamma_1 = \gamma_2 = \alpha$ means that the scalar factor along u axis is the same with that one along v axis. Some CCD cameras are made like this.

Under these assumptions, (5) can be written as

$$\left\{\begin{array}{lcl}\dot{x}_0 &=& u_0\\ \dot{x}_1 &=& x_2u_0\\ \dot{x}_2 &=& x_3u_0 + (x_1+x_3)u_0(\alpha-1)\\ \dot{x}_3 &=& u_1\end{array}\right. \tag{6}$$

This form is different from that one proposed in [25], where $(x_1+x_3)u_0(\alpha-1)$ is required to satisfy that

$$|(x_1+x_3)u_0(\alpha-1)| \le |(x_1,x_2)|\,\phi(x_0,x_1,x_2)$$

where $\phi(x_0,x_1,x_2)$ is a certain non-negative function. It is impossible to guarantee this before the design of the controller.

The problem to be investigated in this paper is how to design controller u_0 and u_1 such that system (6) can be stabilized with unknown α.

Assumption 2. For positive unknown vision parameter α, there exist known positive α_1 and α_2 such that

$$\alpha_1 \le \alpha \le \alpha_2 \tag{7}$$

This assumption is not rigorous. Commonly, the position upper and lower bounds of the scalar factor can be estimated in advance.

III. Controller Design

First choose control input u_0 as follows:

$$u_0 = -\lambda_0 x_0 \tag{8}$$

where λ_0 is a positive parameter to be designed.

From (6) and (8), we have

$$x_0(t) = e^{-\lambda_0 t}x_0(0) \tag{9}$$

where $x_0(0)$ is the initial value of $x_0(t)$ when $t=0$.

If $x_0(0) \neq 0$, $x_0(t) \neq 0$ for all t. We can take the transformation as follows:

$$y_1 = \frac{x_1}{x_0^2}, y_2 = \frac{x_2}{x_0}, y_3 = x_3 \tag{10}$$

Then

$$\begin{array}{lcl}\dot{y}_1 &=& \dfrac{x_2u_0x_0^2 - 2x_0x_1u_0}{x_0^4} = \dfrac{(-\lambda_0x_0)[x_2x_0^2 - 2x_0x_1]}{x_0^4}\\ &=& -\lambda_0y_2 + 2\lambda_0y_1\\ \dot{y}_2 &=& \dfrac{x_0[x_3u_0 + (x_1+x_3)u_0(\alpha-1)] - x_2u_0}{x_0^2}\\ &=& \dfrac{\lambda_0x_0x_2}{x_0^2} + \dfrac{x_3u_0 + (x_1+x_3)u_0(\alpha-1)}{x_0}\\ &=& \lambda_0y_2 - \lambda_0x_3 - \lambda_0(x_1+x_3)(\alpha-1)\\ &=& \lambda_0y_2 - \lambda_0y_3 - \lambda_0(x_0^2y_1+y_3)(\alpha-1)\\ &=& \lambda_0y_2 - \lambda_0\alpha y_3 - \lambda_0x_0^2y_1\\ \dot{y}_3 &=& u_1\end{array}$$

So,

$$\left\{\begin{array}{l}\dot{y}_1 = 2\lambda_0y_1 - \lambda_0y_2\\ \dot{y}_2 = \lambda_0y_2 - \lambda_0\alpha y_3 - \lambda_0x_0^2y_1(\alpha-1)\\ \dot{y}_3 = u_1\end{array}\right. \tag{11}$$

Take control input u_1 as follows:

$$u_1 = k_1y_1 + k_2y_2 + k_3y_3 \tag{12}$$

where $k_i (i=1,2,3)$ are parameters to be designed.

Substituting (12) into (11), we have

$$\dot{\mathbf{Y}} = (\ \mathbf{A} + \mathbf{B}(t))\,\mathbf{Y}$$

where $\mathbf{Y} = [y_1, y_2, y_3]^{\mathbf{T}}$, and

$$\begin{array}{lcl}\mathbf{A} &=& \left(\begin{array}{lll}2\lambda_0 & -\lambda_0 & 0\\ 0 & \lambda_0 & -\lambda_0\alpha\\ k_1 & k_2 & k_3\end{array}\right),\\ \mathbf{B} &=& \left(\begin{array}{ccc}0 & 0 & 0\\ 0 & 0 & -\lambda_0x_0^2(\alpha-1)\\ 0 & 0 & 0\end{array}\right).\end{array}$$

By using lemma in [24], system (11) is asymptotically stable if and only if A is Hurwitz and $B(t)$ converges to zero exponentially. It is obviously seen that $B(t)$ converges to zero exponentially by using (9).

The characteristic polynomial of $\mathbf{A}$ is

$$|\lambda\mathbf{I}-\mathbf{A}| = \begin{vmatrix} \lambda-2\lambda_0 & \lambda_0 & 0 \\ 0 & \lambda-\lambda_0 & \alpha\lambda_0 \\ -k_1 & -k_2 & \lambda-k_3 \end{vmatrix}$$

$$\begin{aligned} &= (\lambda-2\lambda_0)\left(\lambda^2-(\lambda_0+k_3)\lambda+\lambda_0k_3+\alpha\lambda_0k_2\right)-k_1\alpha\lambda_0^2 \\ &= \lambda^3-(3\lambda_0+k_3)\lambda^2+(3k_3+\alpha k_2+2\lambda_0)\lambda_0\lambda- \\ &\quad \lambda_0^2(2k_3+2\alpha k_2+\alpha k_1) \end{aligned}$$

Then matrix $\mathbf{A}$ is a Hurwitz matrix if and only if

$$\begin{cases} \lambda_0>0 \\ k_3+3\lambda_0<0 \\ 3k_3+\alpha k_2+2\lambda_0>0 \\ 2k_3+2\alpha k_2+\alpha k_1<0 \\ (k_3+3\lambda_0)(3k_3+\alpha k_2+2\lambda_0) \\ <\lambda_0(2k_3+2\alpha k_2+\alpha k_1) \end{cases} \quad (13)$$

Under assumption 2, there exists a solution k_1, k_2, k_3 satisfying the group of inequality (13). The proof of this argument can be seen in the appendix.

To sum up, we have the following main result.

Theorem 1. Under assumption 2, for system (6), choose λ_0, k_1, k_2, k_3 such that (13), then the controller is chosen by

$$\begin{cases} u_0=-\lambda_0x_0 \\ u_1=k_1y_1+k_2y_2+k_3y_3 \end{cases} \quad (14)$$

Then system (6) can be stabilized.

Proof. By the argument above, x_0, y_1, y_2 and y_3 converge to zero as t goes to infinity. (10) can be used to deduce that x_1, x_2 and x_3 converge to zero too as t goes to infinity.

In the discussion above, there is an assumption that $x_0(0)\neq 0$. In fact, this is not rigorously required. In other word, when this condition is not satisfied. A open loop control can be used to make this condition be satisfied in a limit time. The conclusion is stated as follows:

Remark 2. If $x_0(0)=0$, take $u_0=k$(non zero constant). Then $x_0(t)=kt$. It is obvious that $x_0(t)$ will be not zero after a limit time T. Then switch to (14) and system (6) can be stabilized finally.

Remark 3. From theorem 1, it seems that assumption 2 is not used. In fact, the solution λ_0, k_1, k_2, k_3 of the group of inequalities (13) exists if the assumption 2 is valid, which is shown in the appendix.

IV. Simulation

For system (6), take an arbitrarily given initial value $[\pi/3[\text{rad}], 0.5[\text{cm}], -0.6[\text{cm}], -0.3[\text{rad}]]$, which imply that the initial configuration is $[x_0, x_1, \ x_2, x_3]^{\mathrm{T}} = [\pi/3, 0.5, -0.6, -0.3]^{\mathrm{T}}$. The desired configuration is $[0, 0, 0, 0]$. The controller is chosen as the formula in the theorem with parameters $\gamma=0.03, \gamma_1=0.028, \gamma_2=0.032$, $\alpha_0=0.026, k_3=-56$, $k_2=91$, $k_1=41$, $\lambda_0=-1.58$, The trajectories of states and movements of the robot are shown in below fig.2-fig.6, respectively. These figures show the effectiveness of the proposed controller. In addition, if the parameters are chosen to make that the eigenvalues of

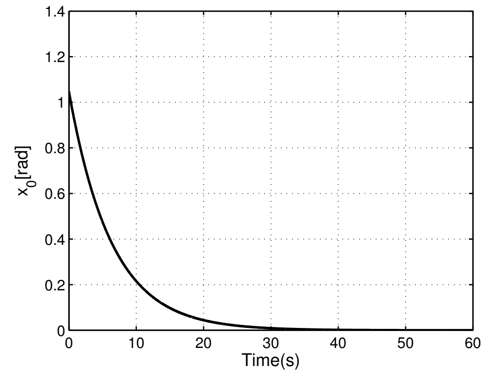

Fig. 2. The trajectory of state x_0 with respect to time

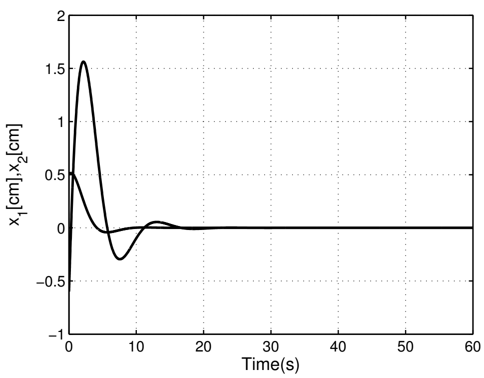

Fig. 3. The trajectory of state x_1 and x_2 with respect to time

$\mathbf{A}$ are far away from image axis on the left side of complex plane, the convergent velocities are faster in the stabilizing procedure.

V. Conclusions

A new kind of feedback stabilizing problem is proposed for the kinematic model of nonholonomic mobile robots based on visual servoing feedback with uncalibrated vision parameters. The stabilizing controllers are investigated for the case $\theta_0=0$, $\gamma_1=\gamma_2$ unknown. As for other cases, such that $\theta_0\neq 0$, $\alpha_1\neq\alpha_2$ are all unknown, the future work will discuss it. In addition, dynamic problems with uncertain parameters are not neglected for high performance of a practical control systems. It will also be dealt with in the future.

VI. ACKNOWLEDGMENTS

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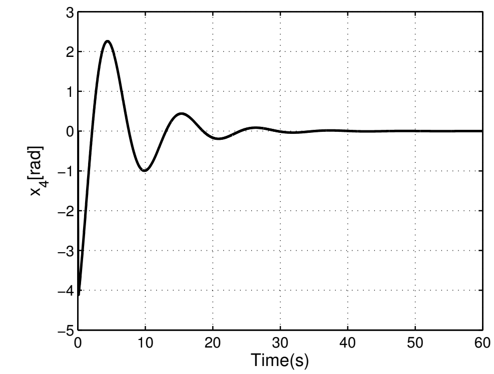

Fig. 4. The trajectory of state x_3 with respect to time

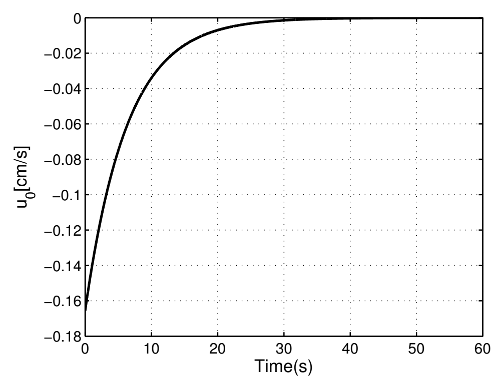

Fig. 5. The trajectory of control input u_0 with respect to time

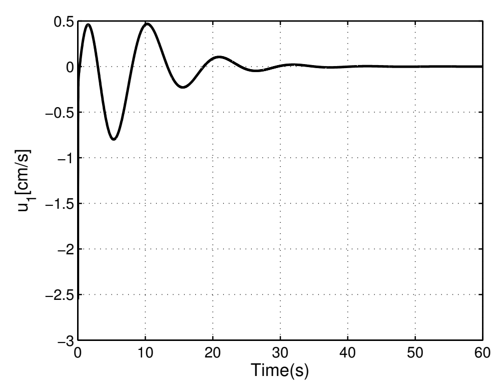

Fig. 6. The trajectory of control input 1_0 with respect to time

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APPENDIX

The proof of the previous argument will be done after a lemma is introduced.

Lemma Under assumption 2, take a and b such that

$$\max\left\{\frac{10}{\alpha_1}, \frac{18}{\alpha_1} - \frac{8}{\alpha_2}\right\} < a < \frac{18}{\alpha_1} \tag{15}$$

$$\frac{18}{\alpha_1} - 3a < b < \frac{8}{\alpha_2} - 2a \tag{16}$$

Then the following group of inequalities

$$\left\{\begin{array}{l} \alpha a > 10 \\ \alpha b < 8 - 2\alpha a \\ 18 < 3a\alpha + \alpha b \end{array}\right. \tag{17}$$

holds.

Proof. By using (15), we have $a > \frac{10}{\alpha_1} > 0$, or $\alpha_1 a > 10$. Hence $\alpha a > \alpha_1 a > 10$, which means that the first formula of (17) is valid. By using (16), we have

$$b + 2a > \frac{18}{\alpha_1} - 3a + 2a = \frac{18}{\alpha_1} - a > 0$$

where (15) is used. Hence $b + 3a > 0$. Therefore

$$\left\{\begin{array}{l} \alpha(2a+b) \leq \alpha_2(2a+b) \\ \alpha(3a+b) \geq \alpha_1(3a+b) \end{array}\right. \tag{18}$$

On the other hand, by using (16), it has

$$2a + b < \frac{8}{\alpha_2}, 3a + b > \frac{18}{\alpha_1}$$

which means

$$\alpha_2(2a+b) < 8, \alpha_1(3a+b) > 18 \tag{19}$$

By using (18), we have

$$\alpha(2a+b) < 8, \alpha(3a+b) > 18$$

which imply that the second and third formulae of (17) are valid. This completes the proof of the lemma $\lhd$.

Next we prove that there exists a solution to the group of inequality (13). For instance, take $\lambda_0 > 0$, $k_3 = -4\lambda_0, k_2 = a\lambda_0, k_1 = b\lambda_0$. Then it can be proved that the group of (13) is valid. In fact,

$k_3 = -4\lambda_0$ means that $k_3 + 3\lambda_0 = -\lambda_0 < 0$. The second inequality of (13) is valid. The first formula of (17) implies that $k_2\alpha > 10\lambda_0$. Therefore,

$$3k_3 + \alpha k_2 + 2\lambda_0 > 3k_3 + 10\lambda_0 + 2\lambda_0 = 0$$

which means that the third second inequality of (13) is valid. In addition, by using the second formula of (17), we have $\alpha k_1 < 8\lambda_0 - 2\alpha k_2$. Hence

$$2k_3+2\alpha k_2+\alpha k_1 < 2k_3+8\lambda_0-2\alpha k_2+2\alpha k_2 = 2k_3+8\lambda_0 = 0$$

which implies that the fourth inequality of (13) is valid.

Finally, by using the third formula of (17), we have $18\lambda_0 < 3\alpha k_2 + \alpha k_1$. Therefore,

$$10\lambda_0 - \alpha k_2 < -8\lambda_0 + 2\alpha k_2 + \alpha k_1 \tag{20}$$

But,

$$\begin{aligned} 10\lambda_0 - \alpha k_2 &= 12\lambda_0 - \alpha k_2 - 2\lambda_0 = -3k_3 - \alpha k_2 - 2\lambda_0 \\ &= -(3k_3 + \alpha k_2 + 2\lambda_0) \end{aligned}$$

By using (20), we have

$$-(3k_3 + \alpha k_2 + 2\lambda_0) < -8\lambda_0 + 2\alpha k_2 + \alpha k_1$$

Hence

$$-\lambda_0(3k_3 + \alpha k_2 + 2\lambda_0) < \lambda_0(-8\lambda_0 + 2\alpha k_2 + \alpha k_1)$$

which implies that

$$(3\lambda_0 + k_3)(3k_3 + \alpha k_2 + 2\lambda_0) < \lambda_0(-8\lambda_0 + 2\alpha k_2 + \alpha k_1)$$

Therefore, the last inequality of (13) holds. This completes the proof $\blacksquare$.