2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Adaptive Asymptotic Tracking Control of a Class of Discrete-Time Nonlinear Systems with Parametric and Nonparametric Uncertainties

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Abstract— **In this paper, adaptive control is studied for a class of nonlinear discrete-time systems in parameter-strict-feedback form with both parametric and non-parametric uncertainties. The non-parametric uncertainty function is assumed to satisfy the Lipschitz condition. To achieve asymptotical tracking performance, estimation of both uncertainties is constructed. Future states are predicted to overcome the noncausal problem. Based on future states prediction and uncertainties estimation, a novel adaptive control is proposed. An augmented tracking error of equal growth order of the output tracking error is used in the parameter estimation law. The proposed adaptive control achieves asymptotical tracking performance and guarantees the boundedness of all closed-loop signals. The effectiveness of the proposed control law is demonstrated in the simulation.**

I. INTRODUCTION

Adaptive control of continuous-time systems has been extensively studied for many years. Compared with continuous-time systems, adaptive control design for discrete-time systems is much more difficult. One reason is that there are less mathematical tools available for discrete-time systems and the other one may lie in the limitation of feedback mechanism in discrete-time. As shown in [1], when the growth rate of the uncertain nonlinearity is larger than a certain number, even a simple first order discrete-time system cannot be globally stabilized. Due to these difficulties, only a few discrete-time counterparts of continuous-time systems have been explored. One example is the strict-feedback nonlinear system which has been extensively studied in continuous-time with backstepping design. Its discrete-time counterpart has also attracted much research interests recently.

In [2], [3], [4], discrete-time backstepping has been studied for a class of strict-feedback systems in which control gains are all ones. However, for more general strict-feedback systems with unknown control gains, the coordinate transformation based backstepping mentioned above is not directly applicable. Therefore, future states prediction based adaptive control using discrete Nussbaum gain to deal with unknown control directions has been developed in [5] where prediction errors was made of smaller growth order of the tracking error. In [6], the prediction based adaptive control has also been exploited for controlling output-feedback systems. The prediction based adaptive control was inspired by our earlier research results in [7], [8], [9], where strict-feedback systems with unknown nonlinear system functions have been studied using neural network (NN) control and prediction function have been proposed to avoid noncausal problem in control design.

In adaptive control of discrete-time systems, robustness has been the subject of much research. By employment of projection algorithm in the parameter update law which guarantees the boundedness of parameter estimates, robust adaptive control of strict-feedback systems perturbed by small growth nonlinear uncertainties was presented in [3], [4]. Due to the universal approximation ability of neural network (NN), many control designs have been carried on by using NN to compensate for the nonlinear uncertainties. By modeling a class of nonlinear systems as a linear part with an additive nonlinear part, multi-model adaptive control has been proposed in [10] with NN employed to compensate for the unknown nonlinear part which is considered to be bounded, while in [11], by assuming the nonlinear part is of small growth rate, generalized minimum variance (GMV) control was presented using NN to deal with the nonlinearity.

It is well known that sliding model control results in invariance properties to matched uncertainties and offers robustness to the closed-loop controlled system. Slide mode control of discrete-time linear system with nonlinear uncertainties have been well studied in [12], [13] and later, in [14], [15], adaptive control have been combined with slide mode to deal with parametric uncertainties in the linear model. It is noted that in these discrete-time sliding mode control results, to guarantee global stability, the nonlinear uncertainties are also required to bounded or of small growth rate. On the contrary, it is easy to construct a sliding mode control in continuous-time to eliminate the effect no matter how large growth rate it has.

The above mentioned results in robust adaptive control may only partially eliminate the effect of the nonparametric uncertainty. For system perturbed by nonlinear nonparametric uncertainties, most of the existing robust adaptive control results are not able to achieve asymptotic tracking. However, from the view point of academic exploration, it is of great research interest in discrete-time adaptive control to fully compensate the nonparametric nonlinear uncertainty such that asymptotic tracking performance can be obtained. One recent successful attempt to eliminate a class of nonparametric nonlinear uncertainty was made in [16] for a simple first order system. Later, an extension has been developed for general minimum phase linear system [17]. Based on the previous results, in this paper we are going to further study the fully compensation of nonparametric nonlinear uncertainty in adaptive control of a class of strict-feedback

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978-1-4244-4524-0/09/$25.00 ©2009 AACC

system.

The main contributions of the paper lie in:

(i) A novel lemma for nonparametric uncertainty estimation is proposed.

(ii) Both parametric and non-parametric uncertainties are estimated.

(iii) A novel adaptive control is constructed based on future states prediction and estimation of both uncertainties.

Throughout this paper, the following notations are used.

- $\|\cdot\|$ denotes the Euclidean norm of vectors and induced norm of matrices.
- Z_0^+ represents the set of all nonnegative integers.
- $\mathbf{0}_{[\mathbf{p}]}$ stands for p-dimension zero vector.
- $(\hat{\ })$ and $(\tilde{\ })$ denote the estimate of unknown parameter and estimation error, respectively.

II. Problem Formulation and Preliminaries

A. System Representation

Consider a class of parameter-strict-feedback nonlinear discrete-time systems with both parametric and non-parametric uncertainties in the following form:

$$\begin{cases} \xi_i(k+1) = \Theta_i^T \Phi_i(\bar{\xi}_i(k)) + \xi_{i+1}(k) \\ \qquad\qquad i = 1, 2, \ldots, n-1 \\ \xi_n(k+1) = \Theta_n^T \Phi_n(\bar{\xi}_n(k)) + u(k) + \nu(\bar{\xi}_n(k)) \\ y(k) = \xi_1(k) \end{cases} \tag{1}$$

where $\bar{\xi}_j(k) = [\xi_1(k), \xi_2(k), \ldots, \xi_j(k)]^T$ are system states, $\Theta_j \in R^{p_j}$, $j = 1, 2, \ldots, n$, are parametric uncertainties (p_j's are positive integers), $\Phi_j(\bar{\xi}_j(k)) : R^j \to R^{p_j}$ are known vector-valued functions, and $\nu(\bar{\xi}_n(k))$ is nonparametric uncertainty function. The control objective is to make the output $y(k)$ track a bounded reference trajectory $y_d(k)$ and to guarantee the boundedness of all the closed-loop signals.

Remark 1: It is noted that the nonparametric nonlinear uncertainty appear in the control range, i.e., the uncertainty is matched. Though matched uncertainties have been extensively studied in the robust control literature [12], [13], [14], [15], few results completely eliminate the effect of nonlinear nonparametric uncertainty and achieve asymptotic output tracking.

Assumption 1: The nonparametric uncertain function $\nu(\cdot)$, is Lipschitz function, i.e., $\|\nu(\varepsilon_1) - \nu(\varepsilon_2)\| \le L\|\varepsilon_1 - \varepsilon_2\|$, $\forall \varepsilon_1, \varepsilon_2 \in R^n$, where $L < \gamma^*$ with γ^* be defined later in (31). The system functions $\Phi_i(\cdot)$, $i = 1, 2, \ldots, n$, are also Lipschitz functions with Lipschitz coefficients L_i.

Remark 2: It is usual in discrete-time control to assume that the nonparametric nonlinear uncertainty is of small growth rate [18], [13], [3], [14], [11], [15] or globally bounded [10], [19]. The Lipschitz uncertainty function $\nu(\bar{\xi}_n(k))$ has been addressed in the nonlinear systems literature, e.g. [1], [20], which can be used to describe a class of nonlinear dynamics of control systems. As shown in [1], it is impossible to obtain global stability results for discrete-time control system in the presence of nonparametric nonlinear uncertainty with large growth rate. Differing from the robust control where uncertainty is not estimated, the uncertainty Lipschitz coefficient L will be estimated in the paper.

B. Preliminaries

Definition 1: [10] Denote $PC_{[0,\infty)}$ the set of all real piecewise continuous functions with bounded discontinuities defined on $[0, \infty)$. Let $x_1(k) : R \to R \in PC_{[0,\infty)}$, and $x_2(k) : R \to R \in PC_{[0,\infty)}$.

- We denote $x_1(k) = O[x_2(k)]$, if there exist positive constants m_1, m_2 and k_0 such that $\|x_1(k)\| \le m_1 \max_{\tau \le k} \|x_2(\tau)\| + m_2, \quad \forall k > k_0$.
- We denote $x_1(k) = o[x_2(k)]$, if there exists a discrete-time function $\alpha(k)$ satisfying $\lim_{k\to\infty} \alpha(k) \to 0$ and a constant k_0 such that $\|x_1(k)\| \le \alpha(k) \max_{\tau \le k} \|x_2(\tau)\|, \quad \forall k > k_0$.
- We denote $x_1(k) \sim x_2(k)$ if they satisfy $x_1(k) = O[x_2(k)]$ and $x_2(k) = O[x_1(k)]$.

Lemma 1: [21] For some given real scalar sequences $s(k)$, $b_1(k)$, $b_2(k)$ and vector sequence $\sigma(k)$, if the following conditions hold:

(i) $\lim_{k\to\infty} \frac{s^2(k)}{b_1(k)+b_2(k)\sigma^T(k)\sigma(k)} = 0$,

(ii) $0 < b_1(k) < K$ and $0 \le b_2(k) < K$, $\forall k \ge 1$, with a finite K,

(iii) $\sigma(k) = O[s(k)]$.

Then, we have

(a) $\lim_{k\to\infty} s(k) = 0$, and (b) $\sigma(k)$ is bounded.

Lemma 2: Under Assumption 1, the states and input of system (1) satisfy

$$\begin{aligned} \bar{\xi}_i(k+j) &= O[\bar{\xi}_n(k)],\ j = n-i \\ \bar{\xi}_i(k) &= O[y(k+i-1)],\ i = 1, 2, \ldots, n-1 \\ u(k) &= O[y(k+n)] \end{aligned} \tag{2}$$

Proof: See Appendix A. ■

Lemma 3: Given two bounded sequences $X(k), Y(k) \in R^m$ satisfying $\lim_{k\to\infty} \|X(k) - Y(k)\| = 0$, where m can be any positive integer, and a fixed positive integer τ. Define

$$l_k = \arg\min_{l \le k-\tau} \|X(k) - Y(l)\| \tag{3}$$

Then, we have

$$\lim_{k\to\infty} \|X(k) - Y(l_k)\| = 0$$
$$\lim_{k\to\infty} \|Y(k) - Y(l_k)\| = 0$$

Proof: See Appendix B. ■

C. Future States Prediction

In this subsection, prediction of future states $\bar{\xi}_i(k+j)$, $i = 1, 2, \ldots, n-1$, $j = 1, 2, \ldots, n-i$, which are independent of control input $u(k)$, are proceeded based on estimation of parameters. For convenience, we denote $\Phi_i(\bar{\xi}_i(k))$ as $\Phi_i(k)$ and $\nu(\bar{\xi}_n(k))$ as $\nu(k)$ without ambiguity.

Let $\hat{\Theta}_i(k)$ denote the estimate of Θ_i and $\tilde{\Theta}_i(k) = \hat{\Theta}_i(k) - \Theta_i$ as estimation error.

Define one-step ahead prediction $\hat{\xi}_i(k+1|k)$, $i = 1, 2, \ldots, n-1$, as the estimation of $\xi_i(k+1)$ at the k-th step

$$\hat{\xi}_i(k+1|k) = \quad \hat{\Theta}_i^T(k-n+2)\Phi_i^T(k) + \xi_{i+1}(k) \tag{4}$$

Define $\bar{\hat{\xi}}_i(k+j|k) = [\hat{\xi}_1(k+j|k), \ldots, \hat{\xi}_i(k+j|k)]^T$, where j-step prediction $\hat{\xi}_i(k+j|k)$ $(2 \le j \le n-1)$, $i = 1, 2, \ldots, n-j$, as the estimation of $\xi_i(k+j)$ at the k-th step, is defined as

$$\begin{aligned}\hat{\xi}_i(k+j|k) &= \hat{\Theta}_i^T(k-n+j+1)\hat{\Phi}_i^T(k+j-1|k) \\ &\quad + \hat{\xi}_{i+1}(k+j-1|k)\end{aligned} \tag{5}$$

where $\hat{\Phi}_i^T(k+j-1|k)$ are defined on the $(j-1)$-step predictions:

$$\hat{\Phi}_i^T(k+j-1|k) = \Phi_i^T(\bar{\hat{\xi}}_i(k+j-1|k)) \tag{6}$$

The estimated parameters are obtained by the following update law:

$$\begin{aligned}\hat{\Theta}_i(k+1) &= \hat{\Theta}_i(k-n+2) - \frac{\tilde{\xi}_i(k+1|k)\Phi_i(k)}{1+\|\Phi_i(k)\|^2} \\ \tilde{\xi}_i(k+1|k) &= \hat{\xi}_i(k+1|k) - \xi_i(k+1) \\ i &= 1, 2, \ldots, n-1\end{aligned} \tag{7}$$

Remark 3: The parameter update law (7) is presented at the $(k+1)$-th step when $\bar{\xi}_n(k+1)$ are all available. The control input $u(k)$ is designed at the k-th step and only depends on $\hat{\Theta}_i(j)$, $j \le k$.

Considering the future states prediction in (4), and (5) and parameter update law (7), we have the following lemma:

Lemma 4: [5] The parameter estimates $\hat{\Theta}_i(k)$ in (7) are bounded and $\|\bar{\tilde{\xi}}_i(k+j|k)\| = o[O[y(k+i+j-2)]]$, $i = 1, 2, \ldots, n-1$, $j = 1, 2, \ldots, n-i$, where $\bar{\tilde{\xi}}_i(k+j|k) = \bar{\hat{\xi}}_i(k+j|k) - \bar{\xi}_i(k+j)$.

III. Adaptive Control Design

Let us rewrite system (1) as

$$\begin{cases} \xi_1(k+n) = \Theta_1^T\Phi_1(\bar{\xi}_1(k+n-1)) + \xi_2(k+n-1) \\ \xi_2(k+n-1) = \Theta_2^T\Phi_2(\bar{\xi}_2(k+n-2)) + \xi_3(k+n-2) \\ \quad\quad\vdots \\ \xi_n(k+1) = \Theta_n^T\Phi_n(\bar{\xi}_n(k)) + u(k) + \nu(\bar{\xi}_n(k)) \\ y(k) = \xi_1(k) \end{cases}$$

and then we combine the n equations above together by iterative substitution and we obtain

$$y(k+n) = \Theta^T\Phi(k+n-1) + u(k) + \nu(\bar{\xi}_n(k)) \tag{8}$$

where

$$\begin{aligned}\Theta &= [\Theta_1^T, \ldots, \Theta_n^T]^T \\ \Phi(k+n-1) &= [\Phi_1^T(\bar{\xi}_1(k+n-1)), \Phi_2^T(\bar{\xi}_2(k+n-2)), \\ &\quad \ldots, \Phi_n^T(\bar{\xi}_n(k))]^T\end{aligned} \tag{9}$$

Using the predicted future states, the future states dependent function $\Phi(k+n-1)$ defined in (8) can be estimated as

$$\begin{aligned}\hat{\Phi}(k+n-1|k) = [&\Phi_1^T(\hat{\xi}_1(k+n-1|k)), \\ &\Phi_2^T(\bar{\hat{\xi}}_2(k+n-2|k)), \ldots, \Phi_n^T(\bar{\xi}_n(k))]^T\end{aligned} \tag{10}$$

According to Lemma 3, let us define

$$\begin{aligned}X(k) &= [\hat{\xi}_1(k+n-1|k), \bar{\hat{\xi}}_2(k+n-2|k), \ldots, \\ &\quad \bar{\hat{\xi}}_{n-1}(k+1|k), \bar{\xi}_n(k)] \\ Y(k+n-1) &= [\xi_1(k+n-1), \bar{\xi}_2(k+n-2), \ldots, \\ &\quad \bar{\xi}_{n-1}(k+1), \bar{\xi}_n(k)]\end{aligned} \tag{11}$$

and

$$l_k = \arg\min_{l \le k-1} \|X(k) - Y(l)\| \tag{12}$$

Then, from (8), we define an auxiliary output $y_a(k)$ as

$$y_a(k+n-1) = \Theta^T\Phi(k+n-1) + \nu(k) \tag{13}$$

which leads to

$$y(k+n) = y_a(k+n-1) + u(k) \tag{14}$$

Then, it is easy to derive

$$\begin{aligned}&y_a(k+n-1) \\ &= y_a(k+n-1) - y_a(l_k) + y_a(l_k) \\ &= \Theta^T[\Phi(k+n-1) - \Phi(l_k)] + \nu(k) - \nu(l_k-n+1) \\ &\quad + y_a(l_k)\end{aligned} \tag{15}$$

Let us introduce the estimate of unknown parameter Θ, $\hat{\Theta}(k)$, which will be calculated in (22). Then, we define the following estimate of $y_a(k+n-1)$

$$\hat{y}_a(k+n-1) = \hat{\Theta}^T(k)[\hat{\Phi}(k+n-1|k) - \Phi(l_k)] + y_a(l_k) \tag{16}$$

where l_k is defined in (12). It should be noted that the estimation of $y_a(k+n-1)$ includes the estimation of both parametric uncertainty and nonparametric uncertainty based on the predicted future states.

Then, by certainty equivalent principle, the adaptive control is designed as

$$u(k) = -\hat{y}_a(k+n-1) + y_d(k+n) \tag{17}$$

Defining the output tracking error $e(k) = y(k) - y_d(k)$ and combining adaptive control in (17), estimate of auxiliary output in (16) and system described in (14) together, we obtain the error dynamics as

$$\begin{aligned}e(k+n) &= -\tilde{\Theta}^T(k)[\Phi(k+n-1) - \Phi(l_k)] + \nu(k) \\ &\quad - \nu(l_k-n+1) - \beta(k+n-1)\end{aligned} \tag{18}$$

where

$$\begin{aligned}\beta(k+n-1) &= \hat{\Theta}^T(k)[\hat{\Phi}(k+n-1|k) - \Phi(k+n-1)] \\ \tilde{\Theta}(k) &= \hat{\Theta}(k) - \Theta\end{aligned} \tag{19}$$

According to Assumption 1, we have

$$|\nu(k) - \nu(l_k-n+1)| \le 2L\max_{k' \le k}\{\|\bar{\xi}_n(k')\|\} + 2c_\nu \tag{20}$$

where $c_\nu = |\nu(0)|$. Now define

$$\hat{c}(k) = 2\lambda\max_{k' \le k}\{\|\bar{\xi}_n(k')\|\} + 2\hat{c}_\nu(k) \tag{21}$$

where λ can be any constant satisfying $L \le \lambda < \lambda^*$, with λ^* defined later in (31).

The estimated parameters in the control law are updated by the following adaptation law

$$\begin{aligned}
\epsilon(k) &= e(k)+\beta(k-1)\\
\hat{\Theta}(k) &= \hat{\Theta}(k-n)+\gamma\frac{a(k)\epsilon(k)[\Phi(k-1)-\Phi(l_{k-n})]}{D(k-n)}\\
\hat{c}_{\nu}(k) &= \hat{c}_{\nu}(k-n)+\frac{a(k)\gamma|\epsilon(k)|}{D(k-n)} \qquad (22)\\
D(k) &= 1+\frac{1}{2}\|\Phi(k+n-1)-\Phi(l_k)\|^2
\end{aligned}$$

where $0<\gamma<1$ and the deadzone is defined as

$$a(k)=\begin{cases}1-\frac{\hat{c}(k-n)}{|\epsilon(k)|} & \text{if } |\epsilon(k)|\geq\hat{c}(k-n)\\ 0 & \text{otherwise}\end{cases} \qquad (23)$$

IV. Stability Analysis

This subsection is devoted to the stability analysis of the closed-loop system. Firstly, the main result of the paper is summarized in the following theorem.

Theorem 1: Consider the adaptive closed-loop system consisting of system (1), states prediction laws defined in (4) and (5) with parameter estimation law (7), control (17) and parameters adaptation law (22). All the signals in the closed-loop system are bounded and the tracking error $e(k)$ is made to converge to zero.

Proof: Substituting the error dynamics (18) into the augmented error $\epsilon(k)$, we have

$$\begin{aligned}
\epsilon(k) &= -\tilde{\Theta}^T(k-n)[\Phi(k-1)-\Phi(l_{k-n})]\\
&\quad+\nu(k-n)-\nu(l_{k-n}-n+1) \qquad (24)
\end{aligned}$$

Choose a positive definite function $V(k)$ as

$$V(k)=\sum_{j=1}^{n}\tilde{\Theta}^T(k-n+j)\tilde{\Theta}(k-n+j)+2\sum_{j=1}^{n}c_{\nu}^2(k)$$

we have the difference equation of $V(k)$ as follows:

$$\begin{aligned}
&\Delta V(k)=V(k)-V(k-1)\\
&=\tilde{\Theta}^T(k)\tilde{\Theta}(k)-\tilde{\Theta}^T(k-n)\tilde{\Theta}(k-n)\\
&\quad+2[\tilde{c}_{\nu}^2(k)-\tilde{c}_{\nu}^2(k-n)]\\
&=\frac{a^2(k)\gamma^2\epsilon^2(k)\|\Phi(k-1)-\Phi(l_{k-n})\|^2}{D^2(k-n)}\\
&\quad+\tilde{\Theta}^T(k-n)[\Phi(k-1)-\Phi(l_{k-n})]\\
&\quad\times\epsilon(k)\frac{2a(k)\gamma}{D(k-n)}\\
&\quad+\frac{2a^2(k)\gamma^2\epsilon^2(k)}{D^2(k-n)}+\frac{4a(k)\gamma|\tilde{c}_{\nu}(k-n)\epsilon(k)|}{D(k-n)}
\end{aligned}$$

Noting $L\leq\lambda$, from inequality (24), we have

$$\begin{aligned}
&\tilde{\Theta}^T(k-n)[\Phi(k-1)-\Phi(l_{k-n})]\epsilon(k)\\
&=-\epsilon^2(k)+\epsilon(k)[\nu(k-n)-\nu(l_{k-n}-n+1)]\\
&\leq|\epsilon(k)|[2L\max_{k'\leq k-n}\|\bar{\xi}_n(k')\|+2c_{\nu}]-\epsilon^2(k)\\
&\leq|\epsilon(k)|[2\lambda\max_{k'\leq k-n}\|\bar{\xi}_n(k')\|+2c_{\nu}]-\epsilon^2(k) \qquad (25)
\end{aligned}$$

From the definition of deadzone in (23), we have

$$2a(k)[\hat{c}(k-n)|\epsilon(k)|-\epsilon^2(k)]=-2a^2(k)\epsilon^2(k) \qquad (26)$$

Noting (25), (26), and

$$1+\frac{1}{2}\|\Phi(k-1)-\Phi(l_{k-n})\|^2\leq D(k-n)$$

Then, we have

$$\begin{aligned}
\Delta V(k)&\leq\frac{2a^2(k)\gamma^2\epsilon^2(k)}{D(k-n)}-\frac{2a(k)\gamma\epsilon^2(k)}{D(k-n)}\\
&\quad+\frac{2a(k)\gamma|\epsilon(k)|[2L\max_{k'\leq k-n}\|\bar{\xi}_n(k')\|+2c_{\nu}]}{D(k-n)}\\
&\quad+\frac{2a(k)\gamma|\epsilon(k)|2\tilde{c}_{\nu}(k-n)}{D(k-n)}\\
&=\frac{2a(k)\gamma}{D(k-n)}[|\epsilon(k)|\hat{c}(k-n)-\epsilon^2(k)]\\
&\quad+\frac{2a^2(k)\gamma^2\epsilon^2(k)}{D(k-n)}\\
&\leq-\frac{2\gamma(1-\gamma)a^2(k)\epsilon^2(k)}{D(k-n)} \qquad (27)
\end{aligned}$$

Noting that $0<\gamma<1$ and taking summation on both hand sides of (27), we obtain

$$\sum_{k=0}^{\infty}2\gamma(1-\gamma)\frac{a^2(k)\epsilon^2(k)}{D(k-n)}\leq V(0)-V(\infty)$$

which implies

$$\lim_{k\to\infty}\frac{a^2(k)\epsilon^2(k)}{D(k-n)}=0 \qquad (28)$$

and the boundedness of $\hat{\Theta}(k)$ and $\hat{c}_{\nu}(k)$. Now considering the definition of $\beta(k)$ in (19), the definition of $\hat{\Phi}(k+n-1|k)$ in (10), Lemma 4 and Assumption 1, we have $\beta(k-1)=o[O[y(k)]]$. Considering $y(k)\sim e(k)$, we have $\beta(k-1)=o[O[e(k)]]$ and $\epsilon(k)\sim e(k)\sim y(k)$ and further according to Lemma 2, we have

$$\|\bar{\xi}_n(k-n)\|\leq C_1[\max_{k'\leq k}\{|\epsilon(k')|\}+C_2,\ k>k_0$$

where C_1 and C_2 are some constants. From the definition of deadzone in (23), when $|\epsilon(k)|\geq\hat{c}(k-n)$, we have

$$a(k)|\epsilon(k)|=|\epsilon(k)|-\hat{c}(k-n)\geq 0$$

when $|\epsilon(k)|<\hat{c}(k-n)$, we have

$$a(k)|\epsilon(k)|=0>|\epsilon(k)|-\hat{c}(k-n)$$

In summary, we have $|\epsilon(k)|-\hat{c}(k-n)\leq a(k)|\epsilon(k)|$. Together with Lemma 4, we have

$$\begin{aligned}
&\|\bar{\xi}_n(k-n)\|\\
&\leq C_1[\max_{k'\leq k}\{|\epsilon(k')|\}+C_2\\
&=C_1[\max_{k'\leq k}\{|\epsilon(k')|-\hat{c}(k'-n)+\hat{c}(k'-n)\}+C_2\\
&\leq C_1[\max_{k'\leq k}\{a(k')|\epsilon(k')|\}+C_2\\
&\quad+C_1[\max_{k'\leq k}\{\hat{c}(k'-n)\},\ k>k_0 \qquad (29)
\end{aligned}$$

According to the definition of $\hat{c}(k)$ in (21) and the boundedness of $\hat{c}_\nu(k)$, we have

$$\max_{k'\leq k-n}\{\|\bar{\xi}_n(k')\|\} \leq C_1[\max_{k'\leq k}\{a(k')|\epsilon(k')|\} + C_3 \\ +2\lambda C_1 \max_{k'\leq k-n}\{\|\bar{\xi}_n(k')\|\},\ k > k_0 \quad (30)$$

which implies the existence of a small positive constant

$$\lambda^* = \frac{1}{2C_1} \quad (31)$$

such that

$$\max_{k'\leq k-n}\{\|\bar{\xi}_n(k')\|\} \leq \frac{C_1}{1-2\lambda C_1}\max_{k'\leq k}\{a(k')|\epsilon(k')|\} + \frac{C_3}{1-2\lambda C_1},\ k > k_0 \quad (32)$$

holds $\forall\lambda < \lambda^*$, where C_3 is a finite number. It implies $\|\bar{\xi}_n(k-n)\| = O[a(k)\epsilon(k)]$. According to Lemma 4 and Assumption 1, we have $\Phi(k-1) = O[\|\bar{\xi}_n(k-n)\|] = O[a(k)\epsilon(k)]$ and further we have

$$\|\Phi(k-1) - \Phi(l_{k-n})\| = O[a(k)\epsilon(k)] \quad (33)$$

Then, applying Lemma 1 to (28), we have

$$\lim_{k\to\infty} a(k)\epsilon(k) = 0 \quad (34)$$

which guarantees the boundedness of $\bar{\xi}_n(k)$ according to (32), and thus, the bounedness of output $y(k)$ and tracking error $e(k)$. According to Lemma 4, we have $\lim_{k\to\infty}\|\tilde{\bar{\xi}}_i(k+j|k)(k)\| = 0$, $j = n-i$, which implies $\lim_{k\to\infty}\|X(k) - Y(k+n-1)\| = 0$. According to Lemma 3, we see $\lim_{k\to\infty}\|Y(k+n-1) - Y(l_k)\| = 0$ which leads to $\lim_{k\to\infty}\|\Phi(k+n-1) - \Phi(l_k)\| = 0$ and $\lim_{k\to\infty}\|\bar{\xi}_n(k) - \bar{\xi}_n(l_k - n + 1)\| = 0$, and further $\lim_{k\to\infty}|\nu(\bar{\xi}_n(k)) - \nu(\bar{\xi}_n(l_k - n + 1))| = 0$ according to Assumption 1. In (18), we see $\lim_{k\to\infty}\epsilon(k+n-1) = 0$ which leads to $\lim_{k\to\infty} e(k) = 0$. ■

V. Simulation Results

The following second order nonlinear plant is used for simulation.

$$\begin{cases} \xi_1(k+1) = 0.1\xi_1(k)\cos(\xi_1(k)) + 0.3\xi_1(k)\sin(\xi_1(k)) \\ \qquad\qquad + \xi_2(k) \\ \xi_2(k+1) = 0.5\xi_2(k)\frac{\xi_1(k)}{1+\xi_1^2(k)} + 0.4\frac{\xi_2^3(k)}{2+\xi_2^2(k)} + u(k) \\ \qquad\qquad + \nu(\bar{\xi}_2(k)) \\ y(k) = \xi_1(k) \end{cases}$$

where $\nu(\bar{\xi}_2(k)) = 0.01(\cos(0.05k))(\xi_1(k) + \xi_2(k))$. The control objective is to make the output $y(k)$ track the desired reference trajectory $y_d(k) = 1.5\sin(\frac{\pi}{5}kT) + 1.5\cos(\frac{\pi}{10}kT)$, where $T = 0.1$. The initial system states are $\bar{\xi}_2(0) = [0,\ 0]^T$. The control parameter is chosen as $\gamma = 0.09$ and $\lambda = 0.1$. The simulation results are presented in Figs. 1–3. Fig. 1 depicts the reference signal $y_d(k)$ and system output $y(k)$; Fig. 2 illustrates the boundedness of the control input $u(k)$; Fig. 3 demonstrates the boundedness of the estimated parameters $\hat{c}_\mu(k)$ and $\|\hat{\Theta}(k)\|$. From Fig. 1, it can been seen that system output $y(k)$ asymptotically tracks the reference signal $y_d(k)$.

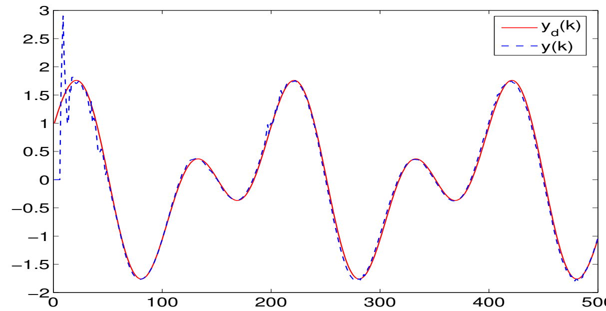

Fig. 1. Reference signal and system output

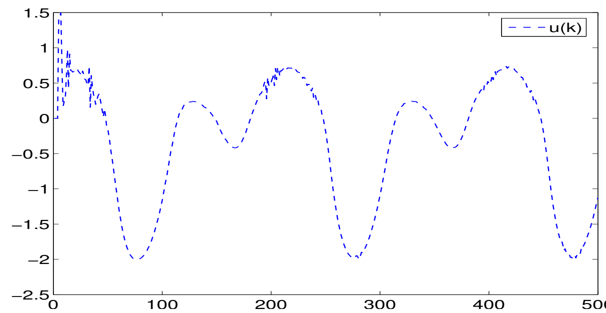

Fig. 2. Control signal

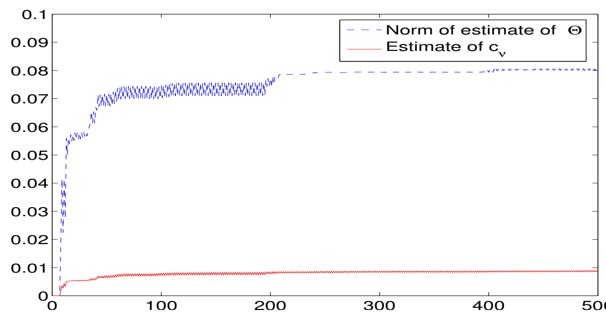

Fig. 3. Boundedness of parameter estimates

VI. Conclusion

In this paper, adaptive control based on future state prediction and estimation of both parametric and nonparametric uncertainties has been studied for a class of nonlinear discrete-time systems in parameter-strict-feedback form. To completely compensate for the uncertainties, an auxiliary output including both parametric and nonparametric uncertainties has been introduced and predicted in the control design. All the signals in the closed-loop system are uniformly bounded and the output tracking error is made to be zero ultimately.

Appendixes

A: Proof of Lemma 2.

From system description (1), we can see that

$$\xi_2(k) = \xi_1(k+1) - \Theta_1^T\Phi_1(\xi_1(k))$$

Noting Assumption 1, we have $\xi_2(k) = O[\xi_1(k+1)]$ and further $\|\bar{\xi}_2(k)\| = O[\xi_1(k+1)]$.

In the same way, we can deduce that $\xi_3(k) = O[\|\bar{\xi}_2(k+1)\|]$ and further $\|\bar{\xi}_3(k)\| = O[\xi_1(k+2)]$ and so on. In summary, we have

$$\|\bar{\xi}_i(k)\| = O[\xi_1(k+i-1)], \quad i = 1, 2, \ldots, n$$

and $\xi_n(k+1) = O[\xi_1(k+n)]$. For the control input, we have

$$|u(k)| \leq |\xi_n(k+1)| + |\Theta_n \Phi(\bar{\xi}_n(k))| + |\nu(\bar{\xi}_n(k))|$$

Together with Assumption 1, it implies $u(k) = O[\xi_1(k+n)]$ Let us rewrite system (1) as

$$\begin{aligned} \xi_1(k+n-1) &= \Theta_1^T \Phi_1(\bar{\xi}_1(k+n-2)) + \xi_2(k+n-2) \\ &\vdots \\ \xi_{n-2}(k+2) &= \Theta_{n-2}^T \Phi_{n-2}(\bar{\xi}_{n-2}(k+1)) + \xi_{n-1}(k+1) \\ \xi_{n-1}(k+1) &= \Theta_{n-1}^T \Phi_{n-1}(\bar{\xi}_{n-1}(k)) + \xi_n(k) \end{aligned}$$

From the first equation from bottom, we obtain $\xi_{n-1}(k+1) = O[\|\bar{\xi}_n(k)\|]$. Then, from the second equation from bottom, we obtain $\xi_{n-2}(k+2) = O[\|\bar{\xi}_n(k)\|]$. Continuing backwardly, it follows $\xi_{n-i}(k+j) = O[\|\bar{\xi}_n(k)\|]$ and $\|\bar{\xi}_{n-i}(k+j)\| = O[\|\bar{\xi}_n(k)\|]$, $i = 1, 2, \ldots, n-1$, $j = n-i$. This completes the proof.

B: Proof of Lemma 3.

We will prove it by seeking a contradiction in a similar way as in [1]. Firstly, let us suppose that

$$\overline{\lim}_{k\to\infty} \|X(k) - Y(l_k)\| = \epsilon > 0 \tag{35}$$

where $\overline{\lim}$ denotes the upper limit. Then we can take from $X(k)$ a subsequence $\{X(k_j), j \geq 1\}$ such that

$$\|X(k_j) - Y(l_{k_j})\| > \frac{\epsilon}{2}, \quad k_j - l_{k_j} \geq \tau$$

According to the definition in (3), we have

$$\|X(k_j) - Y(k')\| > \frac{\epsilon}{2}, \quad \forall 0 \leq k' \leq k_j - \tau$$

which implies

$$\begin{aligned} &\|X(k_j) - X(k')\| + \|X(k') - Y(k')\| \\ \geq\ &\|X(k_j) - Y(k')\| > \frac{\epsilon}{2}, \quad \forall 0 \leq k' \leq k_j - \tau \end{aligned}$$

According to $\lim_{k\to\infty} \|X(k) - Y(k)\| = 0$, there exists a finite number N such that $\|X(k) - Y(k)\| < \frac{\epsilon}{4}, \ \forall k > N$, which leads to

$$\|X(k_j) - X(k')\| > \frac{\epsilon}{4}, \quad \forall N < k' \leq k_j - \tau$$

Then, for $N < k_i \leq k_j - \tau$, $i < j$, we have $\|X(k_j) - X(k_i)\| > \frac{\epsilon}{4}$, or equivalently $\|X(k_j) - X(k_i)\| > \frac{\epsilon}{4}$, which means that $\{X(k_j), j \geq 1\}$ is unbounded. This contradicts to the boundedness of $X(k)$. Consequently (35) cannot hold and thus we have

$$\underline{\lim}_{k\to\infty} \|X(k) - Y(l_k)\| = \overline{\lim}_{k\to\infty} \|X(k) - Y(l_k)\| = 0$$

where $\underline{\lim}$ denotes the lower limit. Then, we have

$$\lim_{k\to\infty} \|X(k) - Y(l_k)\| = 0$$

and

$$\begin{aligned} 0 &\leq \lim_{k\to\infty} \|Y(k) - Y(l_k)\| \\ &\leq \lim_{k\to\infty} \|Y(k) - X(k)\| + \lim_{k\to\infty} \|X(k) - Y(l_k)\| = 0 \end{aligned}$$

which leads to $\lim_{k\to\infty} \|Y(k) - Y(l_k)\| = 0$. This completes the proof.

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