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# Cooperative Target-capturing Strategy for Multi-vehicle Systems with Dynamic Network Topology

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***Abstract*— This paper deals with cooperative target-capturing problem for multi-vehicle systems with dynamic network topology. Firstly, we introduce a dynamic network topology that depends on relative distance between the vehicles. Secondly, we propose the target-capturing strategy based on consensus seeking with dynamic network topology. In proposed strategy, at least one vehicle can acquire the information of the target-object and network topology among vehicles is time-varying but always connected. To analyze the convergence of target-capturing behavior with dynamic network topology, algebraic graph theory and matrix theory are utilized. Finally, numerical simulation results and experimental results are provided that demonstrate the effectiveness of the proposed method.**

## I. INTRODUCTION

In recent years, there have been increasing research interests in the distributed cooperative control of networked multi-vehicle systems. Several research groups developed the coordination control strategies that achieve a capturing formation around a target-object (specific area) by multiple mobile vehicles using neighbor information [1]-[6]. Owing to the broad range of applications ($e.g.$ investigations in hazardous environments, mobile sensor networks and security systems), the task of capturing target-object is investigated in the distributed cooperative control of networked multi-vehicle systems.

The capturing the target-object is divided into two problems, grasping behavior and enclosing behavior. The grasping behavior is the object-closure condition in decentralized form in [3]. On the other hand, the enclosing behavior is that multiple vehicles are controlled in a distributed manner to converge to an assigned formation while tracking the moving target object. Kobayashi *et al.* [4] proposed the decentralized grasping control law using the concept of force-closure and enclosing control law based on a gradient decent method for multiple vehicles with the local information in a plane. In their method, each vehicle requires the local information of target-object and two neighbor vehicles. Marshall *et al.* [5] proposed a cyclic pursuit based formation control strategies for multiple mobile vehicles moving in a plane. They showed that the multiple vehicles finally can assemble in a circular formation that is similar to that of [4]. In [6], Kim *et al.* proposed a distributed cooperative control method based on a cyclic pursuit strategy in a target-capturing task in 3 dimensional space by multi-vehicle systems. In the above method, each vehicle's behavior is decided using the local information of the target-object and one neighbor vehicle. In their method, all vehicles require the information of the target-object. In addition, the network topology among the vehicles is limited to the cycle graphs. $i.e.$ enclosing the target-object cannot be achieved with the network topology except cycle graphs. Consensus algorithm based formation control strategies for multi-vehicle systems are proposed in [7]-[8]. Ren [7] proposed the formation control strategies for multi-vehicle systems where the states of each vehicle approach a common time-varying reference state. Similarly, Namerikawa *et al.* [8] proposed a formation control strategies based on consensus algorithm for multi-vehicle systems. In this paper, the controller gains are designed by consensus algorithm, Lyapunov stability theorem and algebraic graph theory. From the techniques of these researches, we proposed the consensus seeking based target-capturing strategy for multi-nonholonomic vehicle systems in 2 dimensional plane [9]. In our frameworks, at least one vehicle can observe the target-object and the target-object is included in the network topology which has a directed spanning tree. Therefore the network topology that can be treated is wider than these previous researches. However, we assumed that framework in fixed network topology in these papers. In many scenarios, networked cooperative systems can possess a dynamic network topology that is time-varying due to node and link failures/creations, packet-loss, state-dependence and formation reconfiguration[10]-[14]. In many of these researches, consensus problems and the problems that states approximate to one point are treated. However, time-varying or dynamic reference signal (*i.e.* the target-object in this paper) is not treated in these researches.

In this paper, we propose the target-capturing strategy for networked multiple vehicles with dynamic network topology which are controlled to converge to the formation while they are tracking the target-object moving in 3 dimensional space. We first define a dynamic network topology that depends on the relative distance between the vehicles. Secondly, we propose the capturing control law based on the consensus seeking algorithm to each vehicle. Finally, the effectiveness of the proposal method is verified by the numerical simulations and experiments.

This paper is organized as follows. Section II introduces the multi-vehicle systems and target-object in three dimensions, network topology that depends on relative distance between the vehicles and control objectives. Section III describes the proposed consensus seeking based target-capturing strategies for multiple vehicles. Section IV de-

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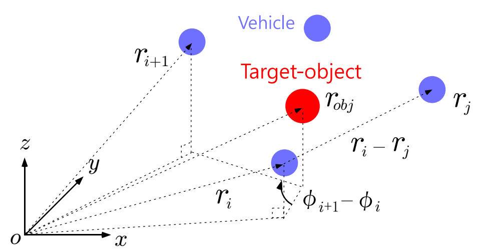



Fig. 1. Coordinate frames and notations

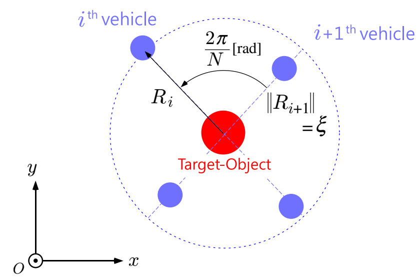



Fig. 2. An example of Target-capturing behavior (in $xy$ plane)

scribes the results by numerical simulations. In Section V, we validate our proposed results by experiments. Finally, we summarize the obtained results in Section VI.

## II. Problem Statement

### A. Multi-vehicle Systems and Target-Object

In this paper, we consider the following $n \geq 1$ mobile vehicles in Cartesian Coordinates [1](see Fig. 1).

$$\dot{r}_i = u_i, \qquad i = 1, 2, \cdots, n. \tag{1}$$

where $r_i = [x_i \ y_i \ z_i]^T \in \mathbb{R}^3$ is the position of $i^{\text{th}}$ vehicle and $u_i \in \mathbb{R}^3$ is the control input of $i^{\text{th}}$ vehicle. Next, the target-object is assumed by the following equation.

$$\dot{r}_{obj} = f(t, r_{obj}, r_1, \cdots, r_n) \tag{2}$$

where $r_{obj} = [x_{obj} \ y_{obj} \ z_{obj}]^T \in \mathbb{R}^3$ is the position of the target-object.

In this paper, the target-object moves depending own states and the states of the neighbor vehicles. Moreover, the target-object has the following *Assumption 1*.

**Assumption 1** *$f(t, r_1, \ldots, r_n, r_{obj})$ is piecewisely continuous in $t$ and locally Lipschitz in $r_{obj}$, $r_1$, $\cdots$, $r_n$.*

The target-objet will be expressed by the following *Examples 1, 2 and 3*.

**Example 1**

$$\dot{r}_{obj} = g\left(r_{obj}, t\right) \tag{3}$$

In Example 1, the target-object moves that depends on own states. The target-object does not use the states of neighbor vehicles. However, the motion of this target-object is unnatural. In actual environment, we have to assume the target-object that obtains the information of the neighbor vehicles to escape from the multi-vehicle systems. Then, the following model was assumed in [2].

**Example 2** *[2]*

$$\dot{r}_{obj} = k^1_{obj} \sum_{j=1}^{n} a_{obj,j}\left(r_{obj} - r_j\right) \tag{4}$$

$$a_{obj,j} = \begin{cases} 1, & \text{if target can observe } j^{\text{th}}\text{vehicle} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

*where $k^1_{obj} \in \mathbb{R}$ is the positive constant gain.*

[1]In 2D plane, nonholonomic vehicles such as two-wheeled vehicles used that can be expressed as the linear first-order system by feedback linearization using the virtual structures [9].

If $r_{obj} = r_j, {}^\forall j$ is satisfied, then $\dot{r}_{obj} = 0$. In Example 2, the target-object moves to escape from the neighbor vehicles. Therefore, it is guessed that the capturing the target-object becomes difficult in Example 2. In this model, the more the away from the multi-vehicles, the more the target-object accelerates. Therefore, we here introduce the following model.

**Example 3**

$$\dot{r}_{obj} = \nabla U_{obj} \tag{6}$$

$$U_{obj} = k^2_{obj} \sum_{j=1}^{n} a_{obj,j} e^{-\|r_{obj} - r_j\|} \tag{7}$$

*where $k^2_{obj} \in \mathbb{R}$ is constant gain and $\|\cdot\|$ is Euclidean norm. The potential function $U_{obj} \to 0$ as $\|r_{obj} - r_j\| \to \infty$.*

In Example 3, the target-object moves to escape from the neighbor vehicles. In this model, if the distance between the target-object and the neighbor vehicle, then the more quickly the target-object runs away. Therefore, Example 3 is the model near more actual phenomenon compared with Example 2.

### B. Network Topology

Information exchange between the vehicles or between the vehicle and the target-object can be represented as a graph. We give here some basic some terminology and definitions from graph theory. Let $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ denoted a graph with the set of vertices $\mathscr{V} = \{1, 2, \cdots, n\}$ and the set of edges $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$. The graph is divided into undirected graphs and directed graphs (digraphs). The set of neighbors of vertex $i$ is denoted by

$$\mathcal{N}_i(\|r_{ij}\|) = \{j : \|r_{ij}\| \leq \rho\} \subseteq \{1, 2, \cdots, n\}. \tag{8}$$

where $r_{ij} = r_i - r_j$ and $\rho \in \mathbb{R}_+$ is an interaction range.

An undirected graph is called connected if there is an edge between any distinct pair of vertices. A directed graph is called strongly connected if there is a directed path from every vertex to every other vertex. A directed tree is a directed graph, where every vertex has exactly one parent except for one vertex, called root, which has no parent, and the root has a directed path to every other vertex. Note that in a directed tree, each edge has a natural orientation away from the root, and no cycle exists. In the case of undirected graphs, a tree is a graph in which every pair of vertices is connected by exactly one path. A directed spanning tree of a directed tree formed by graph edges that connect all of the vertices of the graph. Note that the condition that a digraph has a directed spanning tree is equivalent to the case that there exists as least a vertex having a directed path to all of the other vertices. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. A graph with no edges (but at least one vertex) is called empty.

The adjacency matrix $\mathcal{A}(\mathscr{G}) = [a_{ij}] \in \mathbb{R}^{n\times n}$ is defined as $a_{ii} = 0$ and $a_{ij} = 1$ if $(j,i) \in \mathscr{E}$ where $i \neq j$. The adjacency matrix of a undirected graph is defined accordingly except that $a_{ij} = a_{ji}, ^\forall i \neq j$, since $(j,i) \in \mathscr{E}$ implies $(i,j) \in \mathscr{E}$. The degree of vertex $i$ is the number of its neighbors $|\mathcal{N}_i|$ and is denoted by deg($i$). The degree matrix of graph $\mathscr{G}$ is diagonal matrix defined as $\mathcal{D}(\mathscr{G}) = [d_{ij}] \in \mathbb{R}^{n\times n}$ where

$$d_{ij} = \begin{cases} \deg(i) = \sum_{j=1, j\neq i}^{n} a_{ij} & , i = j \\ 0 & , i \neq j \end{cases} \tag{9}$$

graph Laplacian of the graph $\mathscr{G}$ is defined by

$$\mathcal{L}(\mathscr{G}) = \mathcal{D}(\mathscr{G}) - \mathcal{A}(\mathscr{G}) = [l_{ij}] \in \mathbb{R}^{n\times n} \tag{10}$$

It is well known that the graph Laplacian have several fundamental properties as follows.

**Property 1** *All the row sums of $\mathcal{L}$ are zero and thus $\mathbf{1} = [\begin{array}{cccc} 1 & 1 & \cdots & 1 \end{array}]^T \in \mathbb{R}^n$ is eigenvector of $\mathcal{L}$ associated with the eigenvalue $\lambda(\mathcal{L}) = 0$.*

**Property 2** *For a connected graph, the graph Laplacian $\mathcal{L}$ is symmetric positive semi-definite and the eigenvalue $\lambda(\mathcal{L}) = 0$ is unique.*

Now, we assume that the following network topology among vehicles is satisfied.

**Assumption 2** *Network topology among the vehicles is time-varying but always connected.*

### C. Control Objectives

We first define the position in which $i^{\text{th}}$ vehicle encloses the target-object as *capturing position* $R_i \in \mathbb{R}^3$. Note that for the sake of clarity and page limitation, this paper only considers the equal convergence positions for all vehicles; *i.e.*,

$$\|R_1\| = \|R_2\| = \cdots = \|R_n\| = \xi = const. \tag{11}$$

where $\xi \in \mathbb{R}$ is the *capturing radius*. Let $\phi_i = \tan^{-1}(y_i/x_i)$ denotes the counterclockwise angle of $i^{\text{th}}$ vehicle and the center is the target-object. We also define the following target-capturing behavior.

**Definition 1** *(Target-capturing Behavior)*
*The $n$ vehicles are spaced out around the target-object at intervals of the assigned angles and maintain these angles and each vehicle approaches to the target-object and finally keeps the distance $\xi$.*

In other words, the control objectives for the target-capturing behavior can be formulated as follows (see Figure 2);

**[Control Objectives]**

C1) $\lim_{t\to\infty} \|r_i(t) - r_{obj}(t)\| = \xi$,
C2) $\lim_{t\to\infty} \|\dot{r}_i(t) - \dot{r}_{obj}(t)\| = 0$,
C3) $\lim_{t\to\infty} \|\phi_{i+1}(t) - \phi_i(t)\| = \frac{2\pi}{n}$ [rad], $\quad i = 1, 2, \cdots, n$

In control objective C3), if $i = n$, then $n + 1 = 1$.

In the next section, the target-capturing strategy which achieves the control objectives C1)-C3) is developed.

## III. Proposed Control Strategy

In this section, we discuss the case that a portion of vehicles have access to the target-object (*i.e.* the networked leader-follower systems). It is assumed that it is generally difficult for each vehicle to get the information of the target-object in actual environment. We first introduce the following assumption.

**Assumption 3** *At least one vehicle can get the information of target-object.*

We propose the following target-capturing control laws.

$$u_i = \kappa_i \Bigg[ a_{iobj}\left(\|r_{iobj}\|\right)\left\{-k\left(\hat{r}_i - r_{obj}\right) + \dot{r}_{obj}\right\} + \sum_{j=1}^{n} a_{ij}\left(\|r_{ij}\|\right)\left\{-k\left(\hat{r}_i - \hat{r}_j\right) + \dot{\hat{r}}_j\right\}\Bigg] \tag{12}$$

$$a_{iobj}(\|r_{iobj}\|) = \begin{cases} 1, & (\|r_i - r_{obj}\| \leq \rho_{obj}) \\ 0, & (\|r_i - r_{obj}\| > \rho_{obj}) \end{cases} \tag{13}$$

$$\kappa_i = \frac{1}{a_{iobj}(\|r_{iobj}\|) + \sum_{j=1}^{n} a_{ij}(\|r_{ij}\|)} \tag{14}$$

$$R_i = \xi \begin{bmatrix} \cos\alpha_i \sin\beta_i \\ \sin\alpha_i \sin\beta_i \\ \cos\beta_i \end{bmatrix}, \quad \alpha_i = \frac{2\pi(i-1)}{n} \text{ [rad]} \tag{15}$$

where $k \in \mathbb{R}$ is constant gain, $R_{ij} = R_i - R_j$, $\hat{r}_i = r_i - R_i$, $\rho_{obj} \in \mathbb{R}$, $\alpha_i \in [0, 2\pi)$ and $\beta_i \in (-\pi/2, \pi/2)$ are the desired capturing angles. And, $a_{iobj}$ is the variable that represents whether vehicles can recognize the target-object. Actually, $\rho_{obj}$ is the sensor range. In two dimensional plane, we design the capturing position $R_i$ as $\beta_i = 0$ [rad][2].

From Eq. (15), the control objective C1) can be rewritten as follows

$$\begin{aligned} \lim_{t\to\infty} \|r_i - r_{obj}\| &= \|R_i\| = \xi, \\ \lim_{t\to\infty} (\hat{r}_i - r_{obj}) &= \lim_{t\to\infty} (r_i - R_i - r_{obj}) = 0. \end{aligned} \tag{16}$$

and

$$\begin{aligned} \lim_{t\to\infty} \|r_i - r_j\| &= R_{ij} (= R_i - R_j), \\ \lim_{t\to\infty} (\hat{r}_i - \hat{r}_j) &= 0 \end{aligned} \tag{17}$$

Furthermore, we consider the desired capturing angle $\alpha_i$. The angles between $i + 1$ and $i$ can be represented as follows

$$\|\alpha_{i+1} - \alpha_i\| = \left|\frac{2\pi i}{n} - \frac{2\pi(i-1)}{n}\right| = \frac{2\pi}{n} \text{ [rad]}$$

[2]If we design $\beta_i = 0$, we can get the same capturing position $R_i = \xi[\cos\alpha_i \;\; \sin\alpha_i]^T \in \mathbb{R}^2$ in [9]

Hence, the control objectives C1)-C3) can be rewritten the following new control objectives C'1)-C'2) :

C1') $\lim_{t\to\infty}\|\hat{r}_i(t)-\hat{r}_j(t)\|=0,\ \lim_{t\to\infty}\|\hat{r}_i(t)-r_{obj}(t)\|=0,$

C2') $\lim_{t\to\infty}\|\dot{r}_i(t)-\dot{r}_j(t)\|=0,\ \lim_{t\to\infty}\|\dot{r}_i(t)-\dot{r}_{obj}(t)\|=0,$ for $i,j\ (i\neq j)=1,\cdots,n.$

In this paper, we consider the case when the graph topology is not constant. The switching signal $\sigma:[0,\infty)\to\mathcal{P}$ is the right continuous switching signal and $\mathcal{P}=\{1,2,\dots,r\};r\in\mathbb{N}$ is the finite index set associated with the elements of $\mathscr{G}_N=\{\mathscr{G}^1,\dots,\mathscr{G}^r\}$. We assume that the switching signal is piecewise continuous and denote by $t_w, w=1,2,\dots$ the consecutive discontinuities of the switching signal $\sigma(t)$.

Here, we have the following *Theorem 1*.

**Theorem 1** *: Consider the system of $n$ vehicles (1) and the target-object (2). We apply the capturing control laws (12)-(15) to the system. If the system satisfies $k>0$ and [Assumptions 1-3], then the system asymptotically achieves the control objective C'1).*

*Proof:* Substituting Eq. (12)-(15) into Eq. (1).

$$\dot{r}_i = \kappa_i\Bigg[a_{iobj}\left(\|r_{iobj}\|\right)\left\{-k\left(\hat{r}_i-r_{obj}\right)+\dot{r}_{obj}\right\} + \sum_{j=1}^{n}a_{ij}\left(\|r_{ij}\|\right)\left\{-k\left(\hat{r}_i-\hat{r}_j\right)+\dot{\hat{r}}_j\right\}\Bigg] \tag{18}$$

We assume the target-object as $n+1^{\text{th}}$ vehicle. Let $r_{obj}\doteq\hat{r}_{n+1}=r_{n+1}-R_{n+1}$, $R_{n+1}\doteq 0$ and $a_{iobj}\doteq a_{in+1}$. Here, we can get the following closed loop system.

$$\begin{bmatrix}\sum_{j=1}^{n+1}a_{1j} & 0 & \cdots & 0\\ 0 & \sum_{j=1}^{n+1}a_{2j} & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & 0 & 1\end{bmatrix}\otimes I_3\begin{bmatrix}\dot{\hat{r}}_1\\ \dot{\hat{r}}_2\\ \vdots\\ \dot{\hat{r}}_{n+1}\end{bmatrix}$$
$$=-k\begin{bmatrix}\sum_{j=1}^{n+1}a_{1j} & -a_{12} & \cdots & -a_{1(n+1)}\\ -a_{21} & \sum_{j=1}^{n+1}a_{2j} & \ddots & -a_{2(n+1)}\\ \vdots & \ddots & \ddots & \vdots\\ 0 & 0 & 0 & 0\end{bmatrix}\otimes I_3\begin{bmatrix}\hat{r}_1\\ \hat{r}_2\\ \vdots\\ \hat{r}_{n+1}\end{bmatrix}$$
$$+\begin{bmatrix}0 & a_{12} & \cdots & a_{1(n+1)}\\ a_{21} & 0 & \ddots & a_{2(n+1)}\\ \vdots & \ddots & \ddots & \vdots\\ 0 & 0 & 0 & 1\end{bmatrix}\otimes I_3\begin{bmatrix}\dot{\hat{r}}_1\\ \dot{\hat{r}}_2\\ \vdots\\ \dot{\hat{r}}_{n+1}\end{bmatrix} \tag{19}$$

Eq. (19) can be rewritten as

$$(\mathcal{L}_\sigma\otimes I_3)\dot{\hat{r}}=-k(\mathcal{L}_\sigma\otimes I_3)\hat{r} \tag{20}$$

where $\mathcal{L}_\sigma$, $\mathcal{D}_\sigma$ and $\mathcal{A}_\sigma$ are graph Laplacian, degree matrix and adjacency matrix that depend on the switching signal $\sigma(t)$, $\otimes$ is Kronecker product and $\hat{r}\in\mathbb{R}^{3(n+1)}$ is $\hat{r}=\begin{bmatrix}\hat{r}_1^T & \hat{r}_2^T & \cdots & \hat{r}_{n+1}^T(=r_{obj}^T)\end{bmatrix}^T$.

[3] Next, we introduce error variables $\hat{r}_{ei}\doteq\hat{r}_i-\hat{r}_{n+1}$. Thus the following error system is obtained.

$$\begin{bmatrix}\sum_{j=1}^{n+1}a_{1j} & -a_{12} & \cdots & -a_{1n}\\ -a_{21} & \sum_{j=1}^{n+1}a_{2j} & -a_{23} & -a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ -a_{n1} & \cdots & -a_{n(n-1)} & \sum_{j=1}^{n+1}a_{nj}\end{bmatrix}\otimes I_3\begin{bmatrix}\dot{\hat{r}}_{e1}\\ \dot{\hat{r}}_{e2}\\ \vdots\\ \dot{\hat{r}}_{en}\end{bmatrix}$$
$$=-k\begin{bmatrix}\sum_{j=1}^{n+1}a_{1j} & -a_{12} & \cdots & -a_{1n}\\ -a_{21} & \sum_{j=1}^{n+1}a_{2j} & -a_{23} & -a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ -a_{n1} & \cdots & -a_{n(n-1)} & \sum_{j=1}^{n+1}a_{nj}\end{bmatrix}\otimes I_3\begin{bmatrix}\hat{r}_{e1}\\ \hat{r}_{e2}\\ \vdots\\ \hat{r}_{en}\end{bmatrix} \tag{21}$$

Eq. (21) can be rewritten as

$$(\mathcal{M}_\sigma\otimes I_3)\dot{\hat{r}}_e=-k(\mathcal{M}_\sigma\otimes I_3)\hat{r}_e \tag{22}$$

where $\hat{r}_e=\begin{bmatrix}(\hat{r}_1-\hat{r}_{n+1})^T & \cdots & (\hat{r}_n-\hat{r}_{n+1})^T\end{bmatrix}^T\in\mathbb{R}^{3n}$ and $\mathcal{M}_\sigma\in\mathbb{R}^{n\times n}$ is new matrix to represent the network topology.

Here, Eq. (20) and Eq. (22) are equivalent equations. If *Assumptions 2, 3* is satisfied, then $(\mathcal{M}_\sigma\otimes I_3)\hat{r}_e=0$ has an evident solution $\hat{r}_e=0$ and $\mathcal{M}_\sigma$ is nonsingular matrix.

From the following relationship, we can verify whether $\mathcal{M}_\sigma$ is nonsingular matrix.

$$\mathcal{M}_\sigma \text{ is positive definite} \Longrightarrow \mathcal{M}_\sigma \text{ is nonsingular}$$

Here, we introduce a nonzero vector $x=[x_1\ x_2\ \cdots x_n]\in\mathbb{R}^n$ to prove it. If the network of system is connected graph $(a_{ij}=a_{ji},\ \forall i,j(i,j\neq n+1))$, then $x^T\mathcal{M}_\sigma x$ can be expressed as follows.

$$x^T\mathcal{M}_\sigma x=\sum_{l=1}^{n}a_{ln+1}x_l^2+\sum_{j=1}^{n}a_{ij}\left(x_i-x_j\right)^2 \tag{23}$$

And furthermore, if at least one vehicle can get the information of target-object $(a_{in+1}=1)$ and $x\neq\gamma\mathbf{1}$, then we can get the following condition.

$$\sum_{l=1}^{n}a_{ln+1}x_l^2>0 \tag{24}$$

[3] If the network topology is fixed, then the following equations are obtained from Eq. (20).

$$(\mathcal{L}\otimes I_3)\hat{r}\to 0 \quad as\ t\to\infty$$
$$\hat{r}_i-r_{obj}\to 0 \quad as\ t\to\infty,\quad i\in\mathscr{V}$$

where $\gamma \in \mathbb{R}$ is any scalar constant. From *property 1* of graph Laplacian, the second term of Eq. (23) is represented as

$$\sum_{j=1}^{n} a_{ij} (x_i - x_j)^2 = x^T \mathcal{L}_\sigma x \geq 0 \tag{25}$$

From *property 2* of graph Laplacian, when the eigenvector of graph Laplacian and $\mathbf{1} = [1\ 1 \cdots\ 1]^T$ are only linearly dependent, the eigenvalue of graph Laplacian is zero.

If the second term of the right side of Eq. (23) is zero, then the first term is strictly positive. Conversely, the first term of the right side of Eq. (23) is zero, then the second term is always positive. Here, we can get the following condition.

$$x^T \mathcal{M}_\sigma x = \sum_{l=1}^{n} a_{ln+1} x_l^2 + \sum_{j=1}^{n} a_{ij} (x_i - x_j)^2 > 0 \tag{26}$$

Accordingly, if assumption 2 is satisfied, $\mathcal{M}_\sigma$ is strictly positive definite and nonsingular.

Multiply the both sides of Eq. (22) by $(\mathcal{M}_\sigma \otimes I_3)^{-1}$. Thus, we can get the following equations.

$$\dot{\hat{r}}_e = -k (\mathcal{M}_\sigma \otimes I_3)^{-1} (\mathcal{M}_\sigma \otimes I_3) \hat{r}_e \tag{27}$$

$$\dot{\hat{r}}_e = -k \hat{r}_e \tag{28}$$

This closed loop system is the system that does not depend on the network topology. Therefore if the network topology is the time-varying, then the closed loop system representation does not change. Convergence speed of vehicles is decided only by controller gain $k$ and choice of the gain is easy. From the control gain is positive ($k > 0$), we can get:

$$\hat{r}_e \rightarrow 0, \quad \dot{\hat{r}}_e \rightarrow 0 \qquad as \ \ t \rightarrow \infty, \tag{29}$$

$$\hat{r} \rightarrow \mathbf{1} \otimes \hat{r}_{n+1}, \quad \dot{\hat{r}} \rightarrow \mathbf{1} \otimes \dot{\hat{r}}_{n+1} \quad as \ t \rightarrow \infty. \tag{30}$$

Hence, the states of the system converge to the target-object.

$$\hat{r}_i \rightarrow r_{obj}, \quad \dot{\hat{r}}_i \rightarrow \dot{r}_{obj} \quad as \ \ t \rightarrow \infty, \quad i \in \mathscr{V}, \tag{31}$$

$$\hat{r}_i \rightarrow \hat{r}_j, \quad \dot{\hat{r}}_i \rightarrow \dot{\hat{r}}_j \quad as \ \ t \rightarrow \infty, \quad i, j \in \mathscr{V}. \tag{32}$$

In other words, the control objectives C'1) and C'2) are achieved. ■

Now, we can get the following remark.

**Remark 1** *If all vehicles cannot get information of the target-object, then* $\mathcal{M}_\sigma$ *has strictly zero eigenvalue* $\lambda(\mathcal{M}_\sigma) = 0$ *and* $\mathcal{M}_\sigma$ *is not nonsingular matrix.*

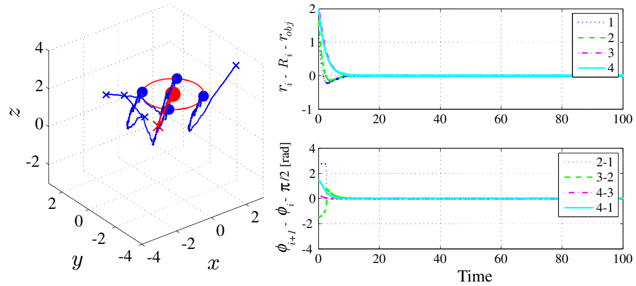
## IV. Numerical Simulations

In this section, the performances of the target-capturing strategy is evaluated by numerical simulations. To illustrate the capturing performances of the proposed method, the simulations are carried out in which $n = 4$ vehicles described by Eq. (1) and one target-object. Now, we show the following two cases.

Case 1) the target moves at random.

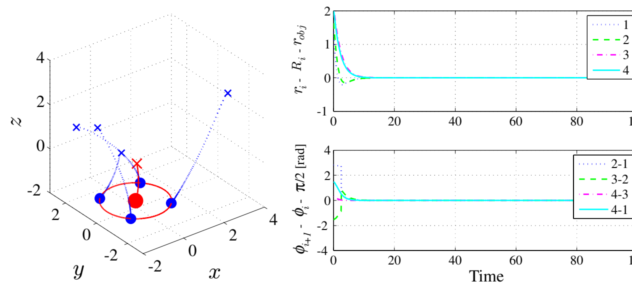
Case 2) the target moves according to the model in Ex. 3.

The numerical simulations were done assuming $k = 0.1$, $\rho = \rho_{obj} = 4$, $\xi = 1$, $k_{obj}^2 = 0.1$, $\beta_i = 0$. The simulation results are shown in Figures 3-4. Figure 3(a)




(a) (b)

Fig. 3. Simulation results of Case 1




(a) (b)

Fig. 4. Simulation results of Case 2

illustrates the trajectories of four vehicles and the target-object in Case 1). In figure 3(a), '×' is the initial position of each vehicle, '•' is the final position of each vehicle. Figure 3(b) illustrates time plots of the states of the vehicles and the target-object in Case 1). Figure 4 illustrates trajectories of the vehicles and the target-object and time plots of the states of the vehicles and the target-object in Case 2). These results show that all vehicles converge to a circular formation around the target-object. From the simulation results, the control objectives C1)-C3) are achieved.

## V. Experimental Evaluation

In experiments (see Figure 5), $n = 3$ two-wheeled vehicles (nonholonomic vehicles) for vehicles and the same one vehicle for the target-object are used. The vehicles used in the experiments are controlled by a digital signal processor (DSP) from dSPACE Inc., which utilizes a PowerPC running at 3.2 [GHz]. Control programs are written in MATLAB/Simulink, and implemented on the DSP using the Real-Time Workshop and dSPACE software which includes ControlDesk, Real-Time Interface. A CCD camera is mounted above the vehicles. The video signals are acquired by a frame grabber board PicPort and image processing software HALCON. The sampling time of this system is $T = 0.2$ [s]. The states of the vehicles are calculated by using the image processing.

In the experiments, we consider the vehicle whose equations are given as

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \tag{33}$$

where $[x_i\ y_i]^T$ is the position of $i^{\text{th}}$ vehicle, $\theta_i \in (-\pi, \pi]$ is the orientation of $i^{\text{th}}$ vehicle, $v_i$ is the velocity of $i^{\text{th}}$ vehicle and $\omega_i$ is the angular velocity of $i^{\text{th}}$ vehicle. This

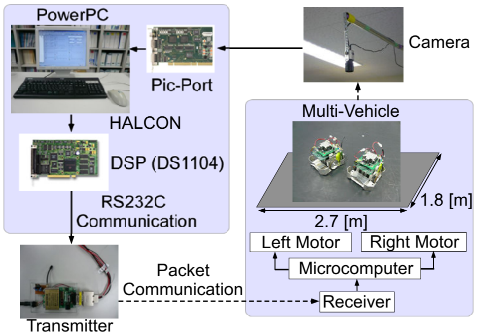



Fig. 5. Experimental setup

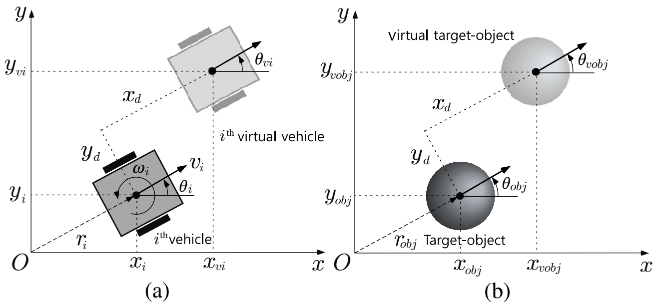



Fig. 6. Definition of virtual structure (a) actual vehicle and virtual vehicle, (b) actual target-object and virtual target-object

vehicle has the nonholonomic constraint. In other words, the vehicle cannot move at horizontal direction. Here, we can use the virtual structure framework from [9]. Now, we define the following virtual structure for feedback linearization as shown in Figure 6.

$$\begin{bmatrix} x_{vi} \\ y_{vi} \\ \theta_{vi} \end{bmatrix} = \begin{bmatrix} x_i + x_d \cos\theta_i - y_d \sin\theta_i \\ y_i + x_d \sin\theta_i + y_d \cos\theta_i \\ \theta_i \end{bmatrix} \tag{34}$$

where $[x_{vi}\ y_{vi}]^T$ is the position of $i^{\text{th}}$ virtual vehicle, $\theta_{vi} \in [-\pi, \pi)$ is the orientation of $i^{\text{th}}$ virtual vehicle, $[x_d\ y_d]^T$ is the distance between actual vehicle and virtual vehicle. Similarly, the virtual target-object is defined from the real target-object in direction $[x_d\ y_d]^T$. Here, we have to choose $x_d \neq 0$. After some manipulations, we can get the linear virtual vehicle and we apply the proposed control laws to each virtual vehicle. From the convergence of the virtual vehicles, the convergence of the actual vehicles can be achieved.

The experiment was done assuming $k = 0.5$, $\rho = 0.5$ [m], $\xi = 0.33$ [m], $[x_d\ y_d]^T = [0.07\ 0]^T$ [m]. The experimental results are shown in Figures 7-8. Figure 7 illustrates the trajectories of three vehicles and the target-object. Figure 8 illustrates time plots of the states of the vehicles and the target-object. The motion of the target-object is set as follow : $[\dot{x}_{obj}\ \dot{y}_{obj}]^T = [0.03\ 0]^T$ [m/s]. At the initial position, the one vehicle cannot observe the target-object. Hence, one vehicle begins to move in roughly 20 [s]. These results show that all vehicles converge to a circular formation around the target-object. From the experimental results, the control objectives are achieved.

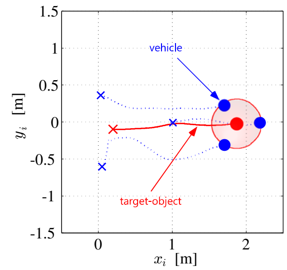



Fig. 7. Experimental result: Trajectory

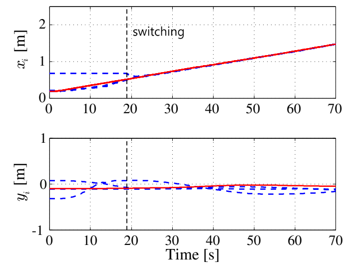



Fig. 8. Experimental result: Time plots of states ($x_i$ and $y_i$)

## VI. Conclusions

In this paper, we have proposed target-capturing strategy for multiple vehicles with dynamic network topology which are controlled to converge to the formation while they are tracking the target-object moving in 3 dimensional space. We first have defined a dynamic network topology that depends on the relative distance between the vehicles. Secondly, we have proposed the capturing control law based on the consensus seeking algorithm to each virtual vehicle. Finally, the effectiveness of the proposal method was verified by the numerical simulations and experiments.

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