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Repetitive Process based Iterative Learning Control designed by LMIs and Experimentally Verified on a Gantry Robot

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Abstract— **In this paper we use a 2D systems setting to develop new results on iterative learning control for linear single-input single-output (SISO) plants, where it is well known in the subject area that a trade-off exists between speed of convergence and the response along the trials. Here we give new results by designing the control scheme using a strong form of stability for repetitive processes/2D linear systems known as stability along the pass (or trial). The design computations are in terms of Linear Matrix Inequalities (LMIs) and results from experimental verification on a gantry robot are also given.**

I. INTRODUCTION

Iterative learning control (ILC) is a technique for controlling systems operating in a repetitive (or pass-to-pass) mode with the requirement that a reference trajectory $y_{ref}(t)$ defined over a finite interval $0 \leq t \leq \alpha$ is followed to a high precision. Examples of such systems include robotic manipulators that are required to repeat a given task, chemical batch processes or, more generally, the class of tracking systems.

Since the original work [1] in the mid 1980s, the general area of ILC has been the subject of intense research effort. Initial sources for the literature here are the survey papers [2] and [3]. The analysis of ILC schemes is firmly outside standard, or 1D, control theory, although it still has a significant role to play in certain cases of practical interest. In this paper we deal with ILC schemes that can be represented as a repetitive process [4].

In ILC, a major objective is to achieve convergence of the trial-to-trial error and often this has been treated as the only one that needs to be considered. It is, however, possible that enforcing fast convergence could lead to unsatisfactory performance along the trial, and here we address this problem by first showing that ILC schemes can be designed for a class of discrete linear systems by extending techniques developed for linear repetitive processes. This allows us to use the strong concept of stability along the pass (or trial) for these processes, in an ILC setting, as a possible means of dealing with poor/unacceptable transients in the along the trial dynamics. The results developed give control law design algorithms that can be implemented via LMIs, and results from their experimental implementation on a gantry robot executing a pick and place operation are also given.

The remainder of this paper begins with a simulation study which demonstrates that it is possible for trial-to-trial error convergence to occur where the along the trial response is very poor. This is followed by analysis which shows how the design of a class of ILC laws can be formulated in a repetitive process setting and designed via LMIs to ensure stability along the trial, with the possibility of tuning to give desired along the trial performance. Finally, the experimental results are given.

In this paper, the null and identity matrices with the required dimensions are denoted by 0 and I respectively. Also $\Gamma \succ 0$ and $\Gamma \prec 0$ respectively are used to denote symmetric matrices which are positive definite and negative definite respectively. The symbol $\mathrm{r}(\cdot)$ is used to denote the spectral radius of a given matrix. In particular if M is a $p \times p$ matrix with eigenvalues $\lambda_i, 1 \leq i \leq p$, then $r(M) = \max_{i \leq i \leq p} |\lambda_i|$.

II. BACKGROUND

Consider the case when the plant to be controlled can be modeled as a single-input, single-output differential linear time-invariant system with state-space model defined by $\{A_c, B_c, C_c\}$. In an ILC setting this is written as

$$\begin{array}{rcl} \dot{x}_k(t) & = & A_c x_k(t) + B_c u_k(t), 0 \leq t \leq \alpha, \\ y_k(t) & = & C_c x_k(t), \end{array} \quad (1)$$

where on trial k, $x_k(t) \in \mathbb{R}^n$ is the state vector, $y_k(t) \in \mathbb{R}^m$ is the output vector, $u_k(t) \in \mathbb{R}^r$ is the vector of control inputs, and $\alpha < \infty$ is the trial length. If the signal to be tracked is denoted by $y_{ref}(t)$ then $e_k(t) = y_{ref}(t) - y_k(t)$ is the error on trial k, and the most basic requirement is to force the error to converge in k. It is, however, possible that trial-to-trial convergence will occur but produce along the trial performance which is far from satisfactory for many practical applications. Consider, for example, a gantry robot executing the following set of operations: collect an object from a location and place it on a moving conveyor, ii) return to the original location and collect the next one and place it on the conveyor, and iii) repeat i) and ii) for the next one and so on. Then if the object has an open top and is filled with liquid, and/or is fragile in nature, unwanted vibrations during the transfer time could have very detrimental effects. Hence in such cases there is also a need to control the along the trial dynamics and in this paper the method used is a strong form of stability theory for linear repetitive processes.

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As an example to illustrate this last point consider the case of a linear continuous-time system whose dynamics are modeled by the transfer-function

$$G(s) = \frac{(s+1)(s+5)}{(s+3)(s^2+4s+29)}, \tag{2}$$

which is to be controlled in the ILC setting using the P-type law

$$u_{k+1}(t) = u_k(t) + Le_{k+1}(t), \tag{3}$$

with, in particular, $L = 3$ which is easily shown to result in trial-to-trial error convergence. Fig. 1 shows the response of the controlled system over 50 trials when the reference signal ($y_{ref}(t)$) is a unit step function of 2 seconds duration applied at $t = 0$. Fig. 2 shows the performance of the controlled system for the 30th trial. These responses confirm that trial-to-trial error convergence occurs but along the trial performance can be very poor.

The unique characteristic of a repetitive, or multipass [4], process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique control problem that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < \infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(t), 0 \le t \le \alpha$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(t),\ 0 \le t \le \alpha,\ k \ge 0$.

Attempts to control these processes using standard (or 1D) systems theory and algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass-to-pass (k direction) and along a given pass (t direction) and also the initial conditions are reset before the start of each new pass. To remove these deficiencies, a rigorous stability theory has been developed [4] based on an abstract model of the dynamics in a Banach space setting that includes a very large class of processes with linear dynamics and a constant pass length as special cases, including those described by (4) below. In terms of their dynamics, it is the pass-to-pass coupling (noting again their unique feature) which is critical. This is of the form $y_{k+1} = L_\alpha y_k$, where $y_k \in E_\alpha$ (E_α a Banach space with norm $||\cdot||$) and L_α is a bounded linear operator mapping E_α into itself.

Consider now discrete linear repetitive processes described by the following state-space model over $p = 0, 1, \ldots, \alpha - 1, k \ge 1$,

$$\begin{aligned} x_k(p+1) = &\ Ax_k(p) + Bu_k(p) + B_0 y_{k-1}(p), \\ y_k(p) = &\ Cx_k(p) + Du_k(p) + D_0 y_{k-1}(p), \end{aligned} \tag{4}$$

where on pass k, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector, $u_k(p) \in \mathbb{R}^r$ is the control input vector, α is the finite pass length. To complete the process description, it is necessary to specify the initial, or boundary, conditions, i.e. the state initial vector on each pass and the initial pass profile. Here these are taken to be zero. In the next section, we show how a repetitive process setting can be used to analyze ILC schemes and, in particular, how the stability theory of these processes can be employed to develop algorithms for control law design to prevent performance such as that of Fig. 2 from arising.

III. ILC AS A REPETITIVE PROCESS

From this point onwards we work in the discrete domain and so assume that the process dynamics have been sampled by the zero-order hold method at a uniform rate T_s seconds to produce a discrete state-space model with matrices $\{A, B, C\}$. Also introduce

$$\begin{aligned} \eta_{k+1}(p+1) &= x_{k+1}(p) - x_k(p), \\ \Delta u_{k+1}(p) &= u_{k+1}(p) - u_k(p), \end{aligned} \tag{5}$$

and let $e_k(p) = y_{ref}(p) - y_k(p)$ denote the current trial error. Then it is possible to proceed as in [5] and use an ILC law which requires the current trial state vector $x_k(p)$ of the plant using

$$\Delta u_{k+1}(p) = K_1 \eta_{k+1}(p+1) + K_2 e_k(p+1), \tag{6}$$

and hence the controlled system dynamics can be written as

$$\begin{aligned} \eta_{k+1}(p+1) &= \hat{A}\eta_{k+1}(p) + \hat{B}_0 e_k(p), \\ e_{k+1}(p) &= \hat{C}\eta_{k+1}(p) + \hat{D}_0 e_k(p), \end{aligned} \tag{7}$$

where

$$\begin{aligned} \hat{A} &= A + BK_1, \\ \hat{B}_0 &= BK_2, \\ \hat{C} &= -C(A + BK_1), \\ \hat{D}_0 &= (I - CBK_2), \end{aligned} \tag{8}$$

This state-space model is of the form (4) and hence the repetitive process stability theory can be applied to this ILC control scheme.

The stability theory for linear repetitive processes with constant pass length consists of two distinct concepts. Asymptotic stability, i.e. bounded-input bounded-output (BIBO) stability over the fixed finite pass length $\alpha > 0$, requires the existence of finite real scalars $M_\alpha > 0$ and $\lambda_\alpha \in (0, 1)$ such that $||L_\alpha^k|| \le M_\alpha \lambda_\alpha^k,\ k \ge 0$, where $||\cdot||$ also denotes the induced operator norm. For processes described by (4) it has been shown elsewhere, see, for example, Chapter 3 of [4], that this property holds if, and only if, $\mathrm{r}(D_0) < 1$. When applied to the ILC (where the term pass is replaced by trial) state-space model (7) this requires that $\mathrm{r}(\hat{D}_0) = \mathrm{r}(I - CBK_2) < 1$.

This last condition is precisely that obtained by applying 2D discrete linear systems stability theory to (7), as first proposed in [6], to ensure trial-to-trial error convergence only. Using the repetitive process setting, however, provides a means of examining what happens after a 'very large' number of trials have elapsed if this form of stability holds. The method of doing this is by the so-called limit profile for

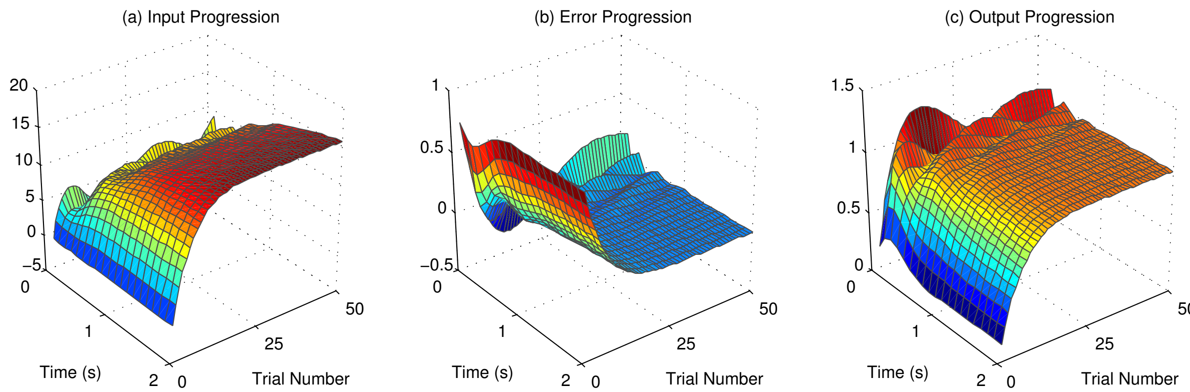

Fig. 1. Responses produced by (2) under the ILC law (3) with $L = 3$.

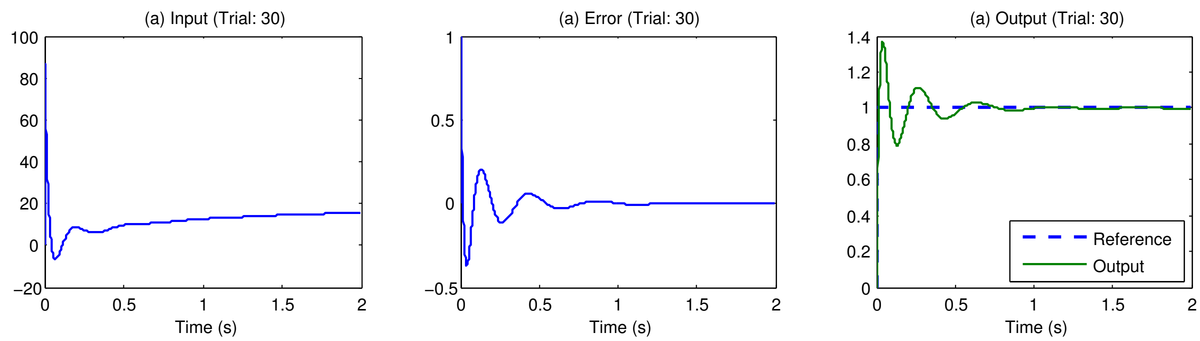

Fig. 2. Responses produced by (2) under the ILC law (3) with $L = 3$ and $k = 30$.

asymptotically stable linear repetitive processes, which we now introduce in terms of (4).

Suppose that $\mathrm{r}(D_0) < 1$ for a discrete linear repetitive process described by (4). Suppose also that the input sequence applied $\{u_{k+1}\}_k$ converges strongly as $k \to \infty$ (i.e. in the sense of the norm on the underlying function space) to u_∞. Then the strong limit $y_\infty := \lim_{k\to\infty} y_k$ is termed the limit profile corresponding to this input sequence and its dynamics (with $D = 0$ for ease of presentation) are described by

$$\begin{aligned} x_\infty(p+1) &= (A + B_0(I - D_0)^{-1}C)x_\infty(p) \\ &+ Bu_\infty(p), \\ y_\infty(p) &= (I - D_0)^{-1}Cx_\infty(p). \end{aligned} \tag{9}$$

Note, however, that the finite pass length means that the dynamics of this limit profile can be unacceptable. In particular, over a finite duration even an unstable 1D linear system can only produce a bounded output. Hence $r(A+B_0(I-D_0)^{-1}C) \geq 1$ is possible, e.g. in the case that $A = -0.5$, $B = 0$, $B_0 = 0.5 + \beta$, $C = 1$, $D = 0$, $D_0 = 0$ and $\beta > 0$ is a real scalar such that $|\beta| \geq 1$.

Even if $r(A+B_0(I-D_0)^{-1}C) < 1$ the process may still have a transient response that is unacceptable for a given application, e.g. a gantry robot placing open top containers containing liquid on a moving conveyor belt. In cases where such features are not acceptable, the stronger concept of stability along the pass must be used. In effect, for the model (4), this requires that the BIBO stability property holds uniformly with respect to the pass length α.

For the discrete linear repetitive processes considered here, there are a wide range of stability along the pass tests but here we use an LMI based condition since, see also below, it leads immediately to algorithms for control law design, a feature which is not present in alternatives. We require the following preliminary results.

Lemma 1: [4] A discrete linear repetitive process described by (4) (with the pairs $\{A, B_0\}$ and $\{C, A\}$ controllable and observable respectively) is stable along the pass if, and only if,

i) $\mathrm{r}\,(D_0) < 1$,
ii) $\mathrm{r}\,(A) < 1$,
iii) all eigenvalues of $G(z) = C(zI - A)^{-1}B_0 + D_0$ have modulus strictly less than unity $\forall\,|z| = 1$.

Theorem 1: [7] Consider a single-input single-output (SISO) (controllable and observable) discrete linear system with transfer-function $G(z) = C(zI - A)^{-1}B + D$. Then the following are equivalent

i) $|G(z)| < 1$, $\forall \omega \in [0, 2\pi]$ and $z = e^{j\omega}$,
ii) there exist $Q \succ 0$ and a symmetric matrix P such that

$$\begin{bmatrix} H_1 & APC^T - QC^T & B \\ CPA^T - CQ & CPC^T - I & D \\ B^T & D^T & -I \end{bmatrix} \prec 0, \tag{10}$$

where

$$H_1 = APA^T - P - QA^T - AQ + 2Q.$$

The following result is the basis of the ILC design developed in this paper.

Theorem 2: [7] A SISO discrete linear repetitive process of the form (7) (with the pairs $\{A, B_0\}$ and $\{C, A\}$ controllable and observable respectively) is stable along the pass if, and only if, there exist $\bar{r} \succ 0$, $\bar{S} \succ 0$, $Q \succ 0$ and a symmetric matrix P such that the following LMIs are feasible

i) $\hat{D}_0^T \bar{r} \hat{D}_0 - \bar{r} \prec 0$,

ii) $\hat{A}^T \bar{S} \hat{A} - \bar{S} \prec 0$,

iii)

$$\begin{bmatrix} \hat{A}P\hat{A}^T - P - Q\hat{A}^T - \hat{A}Q + 2Q & \hat{A}P\hat{C}^T - Q\hat{C}^T & \hat{B}_0 \\ \hat{C}P\hat{A}^T - \hat{C}Q & \hat{C}P\hat{C}^T - I & \hat{D}_0 \\ \hat{B}_0^T & \hat{D}_0^T & -I \end{bmatrix} \prec 0. \tag{11}$$

Proof: The first two conditions follow immediately from Lyapunov stability theory for 1D discrete linear systems. The third LMI is the result of a direct application of Lemma 1. ■

IV. LMI BASED ILC DESIGN

In the ILC setting Theorem 2 cannot be directly applied to the controlled process as the resulting conditions are not in LMI form. The following result allows direct application of this theorem.

Theorem 3: The SISO version of (7) is stable along the trial if there exist matrices $S \succ 0$, N_S and K_2 such that the following LMIs are feasible

$$\begin{bmatrix} -CBK_2 & 0 \\ 0 & CBK_2 - 1 - \lambda \end{bmatrix} \prec 0, \tag{12}$$

$$\begin{bmatrix} -S & SA^T + N_S^T B^T \\ AS + BN_S & -S \end{bmatrix} \prec 0, \tag{13}$$

$$\begin{bmatrix} -S - AS\gamma - (AS\gamma)^T - BN_S\gamma - (BN_S\gamma)^T + 2S\gamma & (S\gamma)^T A^T C^T + (N_S\gamma)^T B^T C^T & BK_2 & AS + BN_S \\ CAS\gamma + CBN_S\gamma & -I & I - CBK_2 & -CAS - CBN_S \\ K_2^T B^T & I - K_2^T B^T C^T & -I & 0 \\ SA^T + N_S^T B^T & -SA^T C^T - N_S^T B^T C^T & 0 & -S \end{bmatrix} \prec 0, \tag{14}$$

where $\gamma > 0$ and $0 < \lambda \leq 1$. If these LMIs are feasible then

$$K_1 = N_S S^{-1}. \tag{15}$$

Computational examples suggest that γ should be very small, e.g. 0.001, and λ should be close to 0.9.

We now show that Theorem 2 is equivalent to Theorem 3. To simplify the proof, we consider each LMI of the previous result separately.

1) First LMI: First note that both $\bar{r}$ and $\hat{D}_0$ are real numbers and hence

$$\bar{r}(\hat{D}_0^2 - 1) < 0,$$

with $\bar{r} > 0$. Hence using (8) it is obvious that

$$(1 - CBK_2)^2 - 1 < 0,$$

or

$$CBK_2(CBK_2 - 2) < 0.$$

Hence we require $0 < CBk_2 < 2$. Note also that the value of CBK_2 greatly influences the trial-to-trial error convergence. In particular, if CBK_2 is very close to 2, convergence is very slow. It is therefore beneficial to introduce a stronger constraint on the permissible values of K_2. Hence we select the stability margin $0 < \lambda \leq 1$ such that $CBK_2 < 1 + \lambda$, which is equivalent to (12) since here CBK_2 is a scalar.

2) Second LMI: Substituting $\hat{A}^T \bar{S} \hat{A} - \bar{S} \prec 0$ in this LMI gives

$$(A + BK_1)^T \bar{S} (A + BK_1) - \bar{S} \prec 0,$$

or, on applying the Schur's complement formula followed by an obvious congruence transform,

$$\begin{bmatrix} -\bar{S}^{-1} & \bar{S}^{-1}(A + BK_1)^T \\ (A + BK_1)\bar{S}^{-1} & -\bar{S}^{-1} \end{bmatrix} \prec 0,$$

and (13) follows on setting $S = \bar{S}^{-1}$ and $N_S = K_1 S$.

3) Third LMI: First set $P = \bar{S}^{-1}$ in (11) and then, by the Schur's complement formula, this condition is equivalent to

$$\begin{bmatrix} -S - Q\hat{A}^T - \hat{A}Q + 2Q & -Q\hat{C}^T & \hat{B}_0 & \hat{A} \\ -\hat{C}Q & -I & \hat{D}_0 & \hat{C} \\ \hat{B}_0^T & \hat{D}_0^T & -I & 0 \\ \hat{A}^T & \hat{C}^T & 0 & (-S)^{-1} \end{bmatrix} \prec 0, \tag{16}$$

or, on applying an obvious congruence transform,

$$\begin{bmatrix} -S - Q\hat{A}^T - \hat{A}Q + 2Q & -Q\hat{C}^T & \hat{B}_0 & \hat{A}S \\ -\hat{C}Q & -I & \hat{D}_0 & \hat{C}S \\ \hat{B}_0^T & \hat{D}_0^T & -I & 0 \\ S\hat{A}^T & S\hat{C}^T & 0 & -S \end{bmatrix} \prec 0. \tag{17}$$

Introducing (8) and simplifying we obtain

$$\begin{bmatrix} -S - QA^T - QK_1^T B^T - AQ - BK_1 Q + 2Q & QA^T C^T + QK_1^T B^T C^T & BK_2 & AS + BK_1 S \\ CAQ + CBK_1 Q & -I & I - CBK_2 & -CAS - CBK_1 S \\ k_2^T B^T & I - K_2^T B^T C^T & -I & 0 \\ SA^T + SK_1^T B^T & -SA^T C^T - SK_1^T B^T C^T & 0 & -S \end{bmatrix} \prec 0. \tag{18}$$

Then lastly use $N_S = K_1 S$ and assume $Q = S\gamma$, $\gamma > 0$, to obtain (14).

Finally, to apply the control law of (6) note that after simple algebraic manipulations we obtain

$$\begin{aligned} u_k(p) &= u_{k-1}(p) + K_1(x_k(p) - x_{k-1}(p)) \\ &+ K_2(y_{ref}(p+1) - y_{k-1}(p+1)). \end{aligned} \tag{19}$$

V. An Example — Simulation and Experimental Results

The new ILC control law design algorithms developed in this paper have been experimentally validated using a multi-axis gantry robot, see Fig. 3, previously used for testing and comparing the performance of other ILC algorithms, see, for example, [8] and this section gives some of the results obtained together with supporting discussion. Each axis (their orientation is marked in Fig. 3) of the gantry robot is controlled individually and the models of all were obtained by means of frequency response tests that determined the continuous-time transfer-functions.

Fig. 3. The gantry robot

The resulting transfer-function for the X-axis is

$$G(s) = \frac{13077183.4436(s+113.4)}{s(s^2+61.57s+1.125\cdot 10^4)(s^2+227.9s+5.647\cdot 10^4)} \cdot \frac{(s^2+30.28s+2.13\cdot 10^4}{(s^2+466.1s+6.142\cdot 10^5)}. \tag{20}$$

The required reference trajectory was designed to simulate a "pick and place" process and this reference signal has been used in all previous algorithm tests allowing comparison of obtained results. The X-axis component of this trajectory is shown in Fig. 4. Completing the design with $\gamma = 0.1$ and

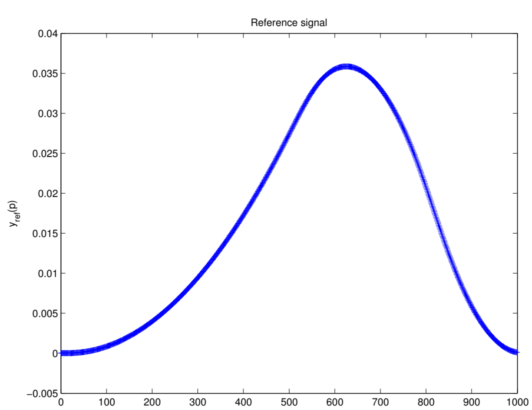

Fig. 4. The X-axis reference trajectory

$\lambda = 0.9$ gives the control law matrices

$$K_1 = [-0.05799 \quad 0.1421 \quad 1.622 \quad 8.688 \quad -8.974 \quad -25.33 \quad -72.95],$$
$$K_2 = 139.3.$$

Fig. 5 compares the mean squared error (mse) plots between simulation and experiment. (There are, of course, differences between predicted and measured values but note the low values of these.) Fig. 6 shows simulated results of the controlled process over the first 20 trials and Fig. 7 the corresponding experimentally measured results. These results demonstrate, in particular, that the new ILC design algorithm developed here is capable of preventing undesirable along the trial dynamics without requiring excessive control action.

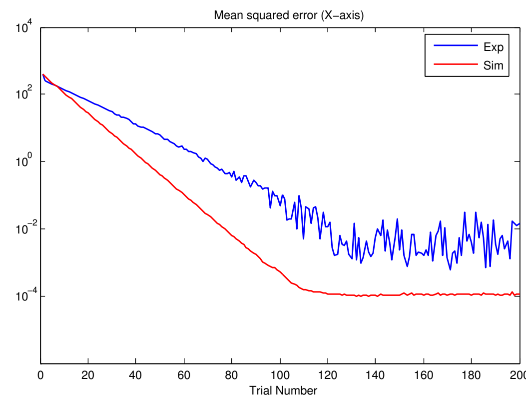

Fig. 5. Mean squared error

VI. Conclusions

This paper has considered the design of ILC schemes using a discrete linear repetitive processes setting. This allows a stability theory to be employed that demands uniformly bounded along the pass (or trial) dynamics (whereas previous approaches only demand bounded dynamics over the finite pass length). Here we have shown that this approach leads to stability conditions expressed in terms of LMIs with immediate formulas for computing the control law matrices. This is a potentially powerful approach in this general area and also a significant step forward in the application of repetitive process systems theory.

The results here establish the basic feasibility of this approach in terms of both theory and experimentation. There is a significant degree of flexibility in the resulting design and current work is undertaking a detailed investigation of how this can be fully exploited. Particular aspects relating to tuning the control law parameters to obtain the best performance and, in particular, examining the role of varying $\gamma > 0$ and $0 < \lambda \leq 1$ of (Theorem 3) in this respect. Note here that the necessary and sufficient stability conditions at present only apply to SISO examples and not for control law design. Further work is also required to extend and improve this key aspect and hence reduce (possible) conservativeness resulting from sufficient, as opposed to necessary and sufficient, control design algorithms. Finally, note that the LMI based results here do not require an SISO plant.

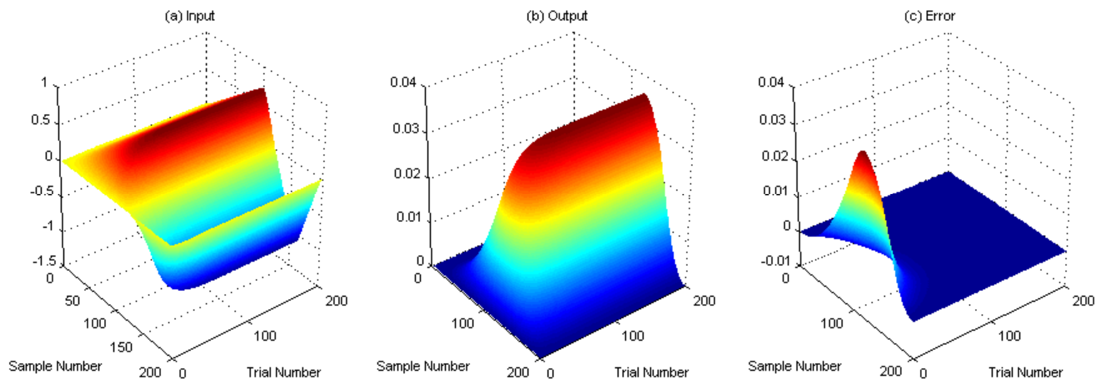

Fig. 6. Simulation results for the first 20 trials in one experiment.

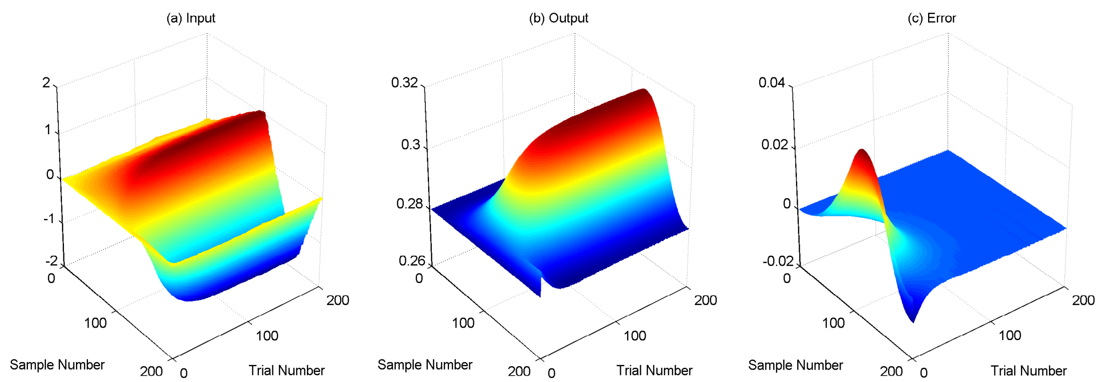

Fig. 7. Experimental results for the first 20 trials in one experiment.

REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operations of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[2] D. A. Bristow, M. Tharayil, and A. G. Alleyne, "A survey of iterative learning control," *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, 2006.

[3] H.-S. Ahn, Y. Chen, and K. L. Moore, "Iterative learning control: brief survey and categorization," *IEEE Transactions on Systems, Man and Cybernetics, Part C*, vol. 37, no. 6, pp. 1109–1121, 2007.

[4] E. Rogers, K. Galkowski, and D. H. Owens, *Control Systems Theory and Applications for Linear Repetitive Processes*, ser. Lecture Notes in Control and Information Sciences. Springer Verlag, 2007, vol. 349.

[5] L. Hladowski, K. Galkowski, Z. Cai, E. Rogers, C. T. Freeman, and P. L. Lewin, "A 2D systems approach to iterative learning control with experimental validation," in *Proc. of the 17th IFAC World Congress*, Soeul, Korea, July 2008, pp. 2832–2837.

[6] J. E. Kurek and M. B. Zaremba, "Iterative learning control synthesis based on 2-D system theory," *IEEE Transactions on Automatic Control*, vol. 38, no. 1, pp. 121–125, 1993.

[7] W. Paszke, P. Rapisarda, E. Rogers, and M. Steinbuch, "Using dissipativity theory to formulate necessary and sufficient conditions for stability of linear repetitive processes," *Research Report, School of Electronics and Computer Science, University of Southampton, UK*, 2008.

[8] J. D. Ratcliffe, P. L. Lewin, E. Rogers, J. J. Hatonen, and D. H. Owens, "Norm-optimal iterative learning control applied to gantry robots for automation applications," *IEEE Transactions on Robotics*, vol. 22, no. 6, pp. 1303–1307, 2006.