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Continuous Model Reference Adaptive Control with Sliding Mode for a Class of Nonlinear Plants with Unknown State Delay

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Abstract— **In this paper, we develop a model reference adaptive control (MRAC) scheme with sliding mode for a class of nonlinear dynamic systems with state delay which is robust with respect to an unknown plant delay, to a nonlinear perturbation, and to an external disturbance with unknown bounds. A novel Lyapunov-Krasovskii type functional is introduced to design the adaptive controller with smooth control action, and the stability proof.**

I. INTRODUCTION

The robust sliding mode control technique applied to uncertain systems with time-delays is a research area that is receiving considerable attention during the last few years, see e.g. [1], [2], [3], [4], [5] and the references therein.

The use of conventional pure robust sliding mode control for plants with delays entails two well-known drawbacks connected to the sliding mode approach itself, namely: (i) all uncertainty bounds need to be available to the designer in advance, and this knowledge is a necessary condition for the closed-loop stability proof; (ii) the control is in general discontinuous, since the control law contains the sign function, and hence, the direct application of such a sliding mode controller may give rise to undesirable chattering.

The relaxation of the first shortcoming of the pure sliding control method motivates the combination of sliding mode robust control with adaptive approaches, whereby the knowledge of the upper bounds of the perturbations will not be required. The adaptive approaches may offer an effective tool to solve this problem. Only few combined adaptive—sliding mode results for delay systems have been published, see e.g. [6], [7], [8], [9], [11].

In [6] an adaptive state feedback robust stabilization problem is considered for a class of state delay systems with input nonlinearities. But also that paper restrictively assumes that the adaptive controller *knows* the bounds of the non-linear perturbation terms, and the bounds of the sector in which the input non-linearity resides. [7] deals with the state feedback stabilization of linear systems with *known* state delays, subject to bounded external disturbance with *unknown* bounds. The design guarantees convergence to a small ball. In [8] the variable structure MRAC problem is considered for a class of stable, input delayed plants. A state feedback stabilizing memoryless controller for linear systems with known parameter matrices in the presence of unknown norm-bounded delayed nonlinear perturbation was studied in [9]. A backstepping method was used to construct the controller.

Discontinuous sliding mode coordinated adaptive decentralized tracking for a class of nonlinear systems with state delays was considered in [10], but under the restrictive assumption that the controller knows the time delays.

In our recent paper [11], a sliding mode discontinuous model reference adaptive control (MRAC) scheme was considered for a class of nonlinear dynamic systems which is robust with respect to an unknown state delay, to a nonlinear perturbation, and to an external disturbance with unknown bounds.

From the previous discussion, one can conclude that also in the adaptive case the discontinuous character of the control action is not avoided, and that may not be desirable because of chattering. Practical and efficient approaches to overcome chattering have been reported based mainly on some approximation of the sign function see, e.g. [17]. Then the trajectory will remain in some neighbourhood of the sliding surface, thus decreasing the tracking accuracy. The designer has to trade-off chattering against tracking accuracy.

In order to alleviate chattering, higher order sliding mode (HOSM) control that is able to generate robust continuous/smooth control actions was proposed in [12], [13], [14]. Both classical SMC and HOSM control are robust to unknown bounded disturbances with known bounds. As we mentioned above, there exist high frequency switching classical SMC algorithms with adaptation that address the problem of unknown bounds of the disturbances. However, we are not aware of smooth/continuous classical SMC or HOSM control that is able to provide adaptation to the unknown bounds of the disturbances especially for nonlinear systems with unknown time delays. Therefore, the design of adaptive continuous SMC that is robust to disturbances with unknown bounds for systems with unknown time delays is a challenge.

In the present paper we extend the result of [11] to the case of smooth control action with sliding mode. We introduce a new continuous adaptive robust tracking scheme for a class of nonlinear dynamic systems with state delay. The proposed adaptive controller parametrization admits model reference adaptive designs with zero asymptotical errors, without knowledge of the time delay, and with robustness properties with respect to state dependent delayed nonlinear

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perturbations, also in the presence of an unknown disturbance. Some cases of *a priori* knowledge about the state dependent delayed non-linearity are considered.

II. PLANT MODEL AND PROBLEM FORMULATION

We consider a class of nonlinear uncertain systems with state delays, suitably initialized, of the form

$$\dot{x}(t) = Ax(t) + bu(t) + bf(x(t), x(t-\tau), t) + bd(t) \quad (1)$$

where $x \in R^n$ is the state vector, $u(t) \in R$ is the control input, and $d(t) \in R$ is a bounded disturbance. The constant matrices $A \in R^{n\times n}$ and $b \in R^n$ have *unknown* elements. $f(x(t), x(t-\tau), t)$ is regarded as an *uncertain* state dependent nonlinear perturbation. $\tau \in R^+$ is an *unknown* time delay.

Our objective is to design a state feedback controller for (1) such that the closed-loop system is stable, and the states $x(t)$ asymptotically exact track the states of the non-delayed stable reference model

$$\dot{x}_r(t) = A_r x_r(t) + b_r r(t) \quad (2)$$

where $x_r(t) \in \mathbb{R}^n$ is the state vector, and $r \in \mathbb{R}$ is the reference input which is assumed to be a uniformly bounded and piecewise continuous function of time. The matrices A_r, b_r are known constant matrices of appropriate dimensions.

The following is assumed regarding the plant and the reference model:

(A1) There exist an *unknown* constant vector $\theta_e^* \in R^n$ and a nonzero constant scalar θ_o such that the following equations are satisfied,

$$A = A_r - b\theta_e^{*T}, \; b_r = b\theta_o. \quad (3)$$

(A2) The sign of θ_o is known and, without loss of generality, positive.

(A3) The external disturbance $d(t)$ is bounded by an *unknown* constant $\|d(t)\| < d^*$.

(A4) For the nonlinear perturbation $f(x(t), x(t-\tau), t)$ we assume that the nonlinear function $f(x(t), x(t-\tau), t)$ is such that there exist nonnegative, but *unknown*, numbers ξ_{1l}^* and ξ_{2l}^* such that

$$|f(x(t), x(t-\tau), t)| \leq \xi_1^* \|x(t)\| + \xi_2^* \|x(t-\tau)\| \quad (4)$$

III. ADAPTIVE CONTROLLER DESIGN

The following procedure of design a *continuous* adaptive control was inspired by an idea from conventional sliding mode theory for discontinuous control, see e.g. [18], [17].

A. Sliding surface

First, we definite a sliding surface in the conventional way:

$$S(t) = Ge(t) = 0, \quad G = b_r^T P \quad (5)$$

where $e(t) = x(t) - x_r(t)$ is the tracking error.

The vector G defines a stable sliding motion, where the matrix $0 < P^T = P \in \mathbb{R}^{n\times n}$ is obtained by solving the Lyapunov equation

$$PA_r + A_r^T P + Q = 0 \quad (6)$$

for any chosen constant matrix $Q \in \mathbb{R}^{n\times n}$ such that $Q = Q^T > 0$. Different choices of Q will not affect boundedness and the asymptotic behavior of the closed loop signals, but they will affect the transient responses in the sliding mode, see e.g. [18], [17].

B. Control law parametrization

The objective is now to propose an adaptive control law such that: (i) all signals in the closed-loop system are bounded; (ii) the tracking error $e(t)$ converges to zero asymptotically with time for any bounded reference input $r(t)$.

We look for a control law parametrization of the form

$$u(t) = \theta_e^T(t)e(t) + \theta_I(t) \quad (7)$$

with the updating laws

$$\begin{aligned} \dot{\theta}_e(t) &= -\Gamma S(t)e(t) \\ \dot{\theta}_I(t) &= -\gamma S(t), \quad \theta_I(0) = 0 \end{aligned} \quad (8)$$

where $\theta_e(t) \in \mathbb{R}^n$ and $\theta_I(t) \in \mathbb{R}$ are the vector and scalar adaptation gains, $\Gamma = \Gamma^T > 0$ and $\gamma > 0$ are some constant design matrix and scalar respectively.

Remark 1: The controller has a simple and conceptually clear structure, uses feedback action only, and thus does not require a direct measurement of the command signals as is usually the case in MRAC.

C. Basic tracking error

Next, it is necessary, as always in model reference adaptive control (MRAC) theory [15], [16], to express the closed-loop system in terms of the tracking error $e(t) = x(t) - x_r(t)$, and some parameter errors.

In view of (1), (2) and Assumptions (A1) and (A2) we obtain, after some manipulations,

$$\begin{aligned} \dot{e}(t) = &A_r e(t) - b\theta_e^{*T} e(t) - b\theta_e^{*T} x_r(t) - b_r r(t) \\ &+ bf(x(t), x(t-\tau), t) + bd(t) + bu(t) \end{aligned} \quad (9)$$

Then by using (7) and in view that $b_r = b\theta_0$, see Assumption (A1), we have the following basic tracking error equation

$$\begin{aligned} \dot{e}(t) = &A_r e(t) + b_r\theta_o^{-1}\tilde{\theta}_e^T(t)e(t) + b_r\theta_o^{-1}\theta_I(t) - b_r r(t) \\ &- b_r\theta_o^{-1}\theta_e^{*T} x_r(t) + b_r\theta_o^{-1} f(\star) + b_r\theta_o^{-1} d(t) \end{aligned} \quad (10)$$

The parameter error $\tilde{\theta}_e(t) \in \mathbb{R}^n$ is

$$\tilde{\theta}_e(t) = \theta_e(t) - \theta_e^* \quad (11)$$

with the unknown vector θ_e^* from (A1).

D. The Lyapunov-Krasovskii functional

For stability analysis the following Lyapunov-Krasovskii functional is proposed

$$\begin{aligned} V = &V_1 + V_2 + V_3; \\ V_1 = &S^2(t) + e^T(t)Pe(t) + \int_{t-\tau}^{t} \nu \|e(t)\|^2 \\ V_2 = &c_o\theta_o^{-1}\tilde{z}^T(t)\Gamma^{-1}\tilde{z}(t) \\ V_3 = &c_o\theta_o^{-1}\gamma^{-1}\left(\theta_I(t) - \theta_I^*\operatorname{sgn}(S(t))\right)^2 \end{aligned} \quad (12)$$

where

$$\tilde{z}(t) = \tilde{\theta}_e(t) + z_0, \quad z_o = \frac{\theta_o}{2c_o}\rho P b_r \tag{13}$$

and the scalar design parameter $\gamma > 0$. The matrix P is from (6) and $c_o = b_r^T P b_r + 1$. The constants $\nu > 0$, $\rho > 0$ and θ_I^* will be defined later. The sign $\|\star\|$ denotes the Euclidian norm. The sign function $\mathrm{sgn}(S(t))$ of a signal $S(t)$ is defined as

$$\mathrm{sgn}(S(t)) = \begin{cases} 1, & S(t) > 0; \\ 0, & S(t) = 0; \\ -1, & S(t) < 0. \end{cases}$$

Remark 2: The main feature and difference from traditional Lyapunov-Krasovskii functionals used in sliding mode adaptive design is the simultaneously occuring terms based on the error norm and the norm of $S(t)$. It will be shown that such a combination makes it possible to get smooth control action.

E. The Lyapunov-Krasovskii functional derivatives

Invoking (3), (5) and P from Lyapunov's equation (6), the time derivative of V_1 along (10) can be written as

$$\begin{aligned}\dot{V}_1(t)|_{(10)} = &-e^T(t)Qe(t) + 2e^T(t)Pb_r b_r^T PA_r e(t) \\ &+ 2c_o\theta_o^{-1}S(t)\tilde{\theta}_e^T(t)e(t) + 2c_o\theta_o^{-1}S(t)\theta_I(t) \\ &+ \nu\|e(t)\|^2 - \nu\|e(t-\tau)\|^2 - 2c_o S(t)r(t) \\ &- 2c_o\theta_o^{-1}S(t)\theta_e^{*T}x_r(t) + 2c_o\theta_o^{-1}S(t)d(t) \\ &+ 2c_o\theta_o^{-1}S(t)f(x(t),x(t-\tau),t)\end{aligned} \tag{14}$$

Using the inequality from (A4) and boundedness of the reference signals ($|r(t)| \le r^*, \|x_r\| \le x_r^*$) we can write the following estimates for some terms of (14)

$$\begin{aligned}-2c_o\theta_o^{-1}S(t)\theta_e^{*T}x_r(t) \le& 2c_o|S(t)|\,\theta_o^{-1}|S(t)|\,\|\theta_e^*\|\,\|x_r(t)\| \\ \le& 2c_o|S(t)|\,\theta_o^{-1}\mu_1\end{aligned} \tag{15}$$

$$\begin{aligned}-2c_o S(t)r(t) \le& 2c_o|S(t)|\,|r(t)| \\ \le& 2c_o\theta_o^{-1}|S(t)|\,\mu_2\end{aligned} \tag{16}$$

$$\begin{aligned}2c_o\theta_o^{-1}S(t)d(t) \le& 2c_o\theta_o^{-1}|S(t)|\,|d(t)| \\ \le& 2c_o\theta_o^{-1}|S(t)|\,\mu_3\end{aligned} \tag{17}$$

$$\begin{aligned}2c_o\theta_o^{-1}S(t)f(\star) \le& 2\hat{\xi}_1|S(t)|\,\|e(t)\| \\ &+ 2\hat{\xi}_2|S(t)|\,\|e(t-\tau)\| \\ &+ 2c_o\theta_o^{-1}|S(t)|\,\mu_4\end{aligned} \tag{18}$$

$$2e^T(t)Pb_r b_r^T PA_r e(t) \le \hat{\xi}_3\|e(t)\|^2 \tag{19}$$

where $\mu_1 = \|\theta_e^*\|x_r^*$, $\mu_2 = \theta_o r^*$, $\mu_3 = d^*$, $\mu_4 = \left(\xi_1^* + \xi_2^*\right)x_r^*$, $\hat{\xi}_1 = \theta_o^{-1}c_o\xi_1^*$, $\hat{\xi}_2 = \theta_o^{-1}c_o\xi_2^*$ and $\hat{\xi}_3 = 2\left\|Pb_r b_r^T PA_r\right\|$.

By using (15) - (19) we have from (14)

$$\begin{aligned}\dot{V}_1(t)|_{(10)} \le &-e^T(t)Qe(t) + \hat{\xi}_3\|e(t)\|^2 + \nu\|e(t)\|^2 \\ &- \nu\|e(t-\tau)\|^2 + 2c_o\theta_o^{-1}S(t)\tilde{\theta}_e^T(t)e(t) \\ &+ 2c_o\theta_o^{-1}S(t)\theta_I(t) + 2|S(t)|\,\hat{\xi}_1\|e(t)\| \\ &+ 2|S(t)|\,\hat{\xi}_2\|e(t-\tau)\| - 2c_o\theta_o^{-1}|S(t)|\,\theta_I^* \\ &- \rho|S(t)|^2 + \rho|S(t)|^2\end{aligned} \tag{20}$$

where $-\theta_I^* = \mu_1 + \mu_2 + \mu_3 + \mu_4$.

For convenience, let Q from (6), ν from (12) and ρ from (20) be $Q = q_1 I + q_2 I$, $\nu = \nu_1 + \nu_2$, $\rho = \rho_1 + \rho_2 + \rho_3$, where q_1, q_1, ν_1, , ρ_1, ρ_2 and ρ_3 are positive constants and I is the identity matrix. Then, combining $-\rho_2|S(t)|^2$ and $2|S(t)|\,\hat{\xi}_1\|e(t)\|$, $-\rho_3|S(t)|^2$ and the $2|S(t)|\,\hat{\xi}_2\|e(t-\tau)\|$ terms of (20), completing the squares, and dropping negative terms, we obtain from (20)

$$\begin{aligned}\dot{V}_1(t)|_{(10)} \le &-q_1\|e(t)\|^2 - \nu_1\|e(t-\tau)\|^2 - \rho_1|S(t)|^2 \\ &- q_2\|e(t)\|^2 + \hat{\xi}_3\|e(t)\|^2 + \nu\|e(t)\|^2 + \frac{2\hat{\xi}_1^2}{\rho_2}\|e(t)\|^2 \\ &- \nu\|e(t-\tau)\|^2 + \frac{2\hat{\xi}_2^2}{\rho_3}\|e(t-\tau)\|^2 + \rho|S(t)|^2 \\ &+ 2c_o\theta_o^{-1}S(t)\tilde{\theta}_e^T(t)e(t) + 2c_o\theta_o^{-1}S(t)\theta_I(t) \\ &- 2c_o\theta_o^{-1}|S(t)|\,\theta_I^*\end{aligned} \tag{21}$$

Let us select values of ρ_2, ρ_3, q_2 and ν_2 from the inequalities

$$\rho_2(q_2 - \hat{\xi}_3 - \nu) > 2\hat{\xi}_1^2, \quad \rho_3\nu_2 > 2\hat{\xi}_2^2 \tag{22}$$

Then we obtain from (21)

$$\begin{aligned}\dot{V}_1(t)|_{(10)} \le &-q_1\|e(t)\|^2 - \nu_1\|e(t-\tau)\|^2 - \rho_1|S(t)|^2 \\ &- 2c_o\theta_o^{-1}|S(t)|\,\theta_I^* + \rho|S(t)|^2 \\ &+ 2c_o\theta_o^{-1}S(t)\tilde{\theta}_e^T(t)e(t) + 2c_o\theta_o^{-1}S(t)\theta_I(t)\end{aligned} \tag{23}$$

In view of (8) and (13), the time derivative of V_2 satisfies

$$\begin{aligned}\dot{V}_2(t)|_{(10)} =& 2c_o|\theta_o|^{-1}\tilde{\theta}_e(t)^T\Gamma^{-1}\dot{\tilde{\theta}}_e(t) + 2c_o|\theta_o|^{-1}z_o^T\Gamma^{-1}\dot{\tilde{\theta}}_e(t) \\ =& -2c_o\theta_o^{-1}\tilde{\theta}_e^T(t)S(t)e(t) - \rho|S(t)|^2\end{aligned} \tag{24}$$

For the time derivative of V_3 we have

$$\begin{aligned}\dot{V}_3(t)|_{S(t)\neq 0} =& -2c_o\theta_o^{-1}S(t)\theta_I(t) + 2c_o\theta_o^{-1}|S(t)|\,\theta_I^* \\ \dot{V}_3(t)|_{S(t)=0} =& 0\end{aligned} \tag{25}$$

Then, invoking (23), (24) and (25) we obtain for the time derivative of V from (12)

$$\begin{aligned}\dot{V}(t)|_{S(t)\neq 0} \le& -q_1\|e(t)\|^2 - \nu_1\|e(t-\tau)\|^2 - \rho_1|S(t)|^2 \\ \dot{V}(t)|_{S(t)=0} \le& -q_1\|e(t)\|^2 - \nu_1\|e(t-\tau)\|^2\end{aligned} \tag{26}$$

This implies that V and, therefore $e(t), S(t), \tilde{\theta}_e(t), \theta_e(t), \tilde{\theta}_I(t), \theta_I(t) \in L_\infty$ and $e(t) \in L_2$ by following the usual arguments in e.g. [15], [16]. The remainder of the stability analysis follows directly using the steps in [15], [16]. Because $e(t)$, $\theta_e(t)$ and $\theta_I(t)$ are bounded it following that $u(t)$ from (7) that $u(t) \in L_\infty$. Furthermore, $x_r(t), e(t) \in L_\infty$ imply that $x(t) \in L_\infty$. Because $x(t) \in L_\infty$ and from Assumption (A4) the function $f(\star)$ is bounded. So all signals are bounded before the controlled system enters the surface $S(t) = 0$.

Now we show that $\lim_{t\to\infty} S(t) = 0$. Because all the signals in the right-hand side of (10) are bounded, from (5) we have that $\dot{S}(t) = b_r P\dot{e}(t) \in L_\infty$ and therefore $S(t)$ is uniformly continuous. After integration the equation (26) for $S(t) \neq 0$ can be rewritten as

$$\int_{t_0}^{t}|S(t)|^2 dt \le V(t_0) - V(t) \;\; \forall t > t_0 \tag{27}$$

and

$$\int_{t_0}^{\infty} |S(t)|^2 dt \le V(t_0) - V_\infty < \infty, \tag{28}$$

i.e., $S(t) \in L_2$. Using $S(t), \dot{S}(t) \in L_\infty$, $S(t) \in L_2$ and e.g. applying Lemma 2.14 [16, p.80] we have $S(t) \to 0$ as $t \to \infty$. A similar argument gives that $e(t) \to 0$ as $t \to \infty$.

Remark 3: We note that the coefficients ρ and ν from (12) are used only for analysis and do not influence the control law. The controller gains adjust automatically to counter the non-desirable effects of the delayed states, the nonlinearity, the disturbance, and parameter uncertainties.

F. Main result

The above arguments constitute the proof of the following result

Theorem 1: Consider system (1) and the reference model (2). Suppose that assumptions (A1)-(A4) hold. Then the adaptive law (7) with update law (8) assures that all signals in the closed-loop plant are bounded and the tracking error $e(t)$ converges to zero asymptotically with time for any reference input $r(t) \in L_\infty$.

Remark 4: Theorem 1 shows that the stability of the closed-loop system and the controller parameters are completely *independent* of the value of the plant time-delay τ. The controller is also robust to an external disturbance.

IV. Some extensions

We now briefly consider some possible extensions of the above design procedure with the new MRAC scheme.

A. The case with multiple and time-varying delays

This paper considers uncertain dynamical systems with one constant time delay. Note that in light of the stability proofs, the method developed here is also applicable to systems with multiple and time-varying delays. In the case of a time-varying delay $\tau(t)$, it is required that $\tau(t) \le \tau^*$ where τ^* is an *unknown* constant. In this case the adaptive control law remains the same: only a slightly modified form of the Lyapunov-Krasovskii functional (12) is required in the above design procedure.

B. Another à priori knowledge about nonlinear term

We consider e.g., the case of a priori knowledge about a nonlinear term of the form

$$|f(x(t), x(t-\tau), t)| \le \theta_\rho^* \rho(x,t), \quad \text{for all } (t, x(t))$$

where $\rho(x,t)$ is a known bounded continuous positive scalar valued function, and θ_ρ^* is a nonnegative unknown scalar.

In this case the design procedure remains the same as above, but we look for a control law parametrization of the form

$$u(t) = \theta_e^T(t)e(t) + \theta_I(t) + \theta_\rho(t)\rho(x,t) \tag{29}$$

with the updating laws for $\theta_e(t)$ and $\theta_I(t)$ from (8) and

$$\dot{\theta}_\rho(t) = \begin{cases} -\gamma_2 S(t)\rho(x,t), & S(t) \ne 0; \\ 0, & S(t) = 0. \end{cases} \quad \theta_\rho(0) = 0 \tag{30}$$

One can see that the control in (7) is modified by adding a special term $\theta_\rho(t)\rho(x,t)$ with the adaptive gain $\theta_\rho(t)$ from (30). For the proof we use a functional of the same type as (12) but with a modification of V_3 by adding the term $c_o\theta_o^{-1}\gamma_2^{-1}\left(\theta_\rho(t) - \theta_\rho^*\mathrm{sgn}(S(t))\right)^2$.

C. Nonlinear uncertain systems without delays

Even in the absence of an unknown state delay, the results of the paper are new. For this case, consider e.g. a class of uncertain nonlinear systems of the form (1) but without the time delay, i.e.

$$\dot{x}(t) = Ax(t) + bu(t) + bf(x(t),t) + bd(t) \tag{31}$$

and Assumption (A4) is e.g. exchanged for

$$|f(x(t),t)| \le \xi_1^* \|x(t)\| \tag{32}$$

with unknown ξ_1^*. The adaptive control law remains *exactly the same* as (7). The design procedure also remains the same, but instead of the Lyapunov-Krasovskii functional (12), we need to use the Lyapunov like function

$$\begin{aligned} V &= V_1 + V_2 + V_3; \quad V_1 = S^2(t) + e^T(t)Pe(t) \\ V_2 &= c_o\theta_o^{-1}\tilde{z}^T(t)\Gamma^{-1}\tilde{z}(t) \\ V_3 &= c_o\theta_o^{-1}\gamma^{-1}\left(\theta_I(t) - \theta_I^*\mathrm{sgn}(S(t))\right)^2 \end{aligned} \tag{33}$$

V. Simulation example

To illustrate the application of the proposed adaptive scheme, let us consider a plant defined by

$$\begin{aligned} \dot{x}(t) = &\begin{bmatrix} 0 & 1 \\ -0.9 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \big[u(t) + sin(t) \\ &+ cos(x_1(t-\tau))x_2(t) + sin(x_2(t-\tau))x_1(t)\big] \\ x(s) = &[0 \quad 0]^T, s \in [-\tau\, 0), \quad x(0) = [-1 \quad -1]^T \end{aligned} \tag{34}$$

To build the adaptive controller we choose the reference model

$$\begin{aligned} \dot{x}_r(t) &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r(t) \\ x_r(0) &= [0 \quad 0]^T \end{aligned} \tag{35}$$

In this example all the parameters including τ are unknown to the controller. The only information available to the controller is the structural information given in Assumptions A1– A4. The parameter values of the controller are $\Gamma = 200I$ and $\gamma = 200$. The reference input is $r(t) = 2 + 1.5sin(0.5t)rad/sec$.

The simulation results are shown in Figures 1 – 5, where we show the time responses of the control signal $u(t)$, the tracking error $e(t)$, and the tracking errors in the phase plane. In the figures we included, for comparison, the time history of the signals for different values of the time delay τ, namely $\tau = 4$ and $\tau = 6$, with the same controller (7), (8). From the graphs it is clear that the transients are only slightly sensitive to the delay variations. In particular, there are small differences in the responses at the times equalling the plant delay values, $\tau = 4$ and $\tau = 6$.

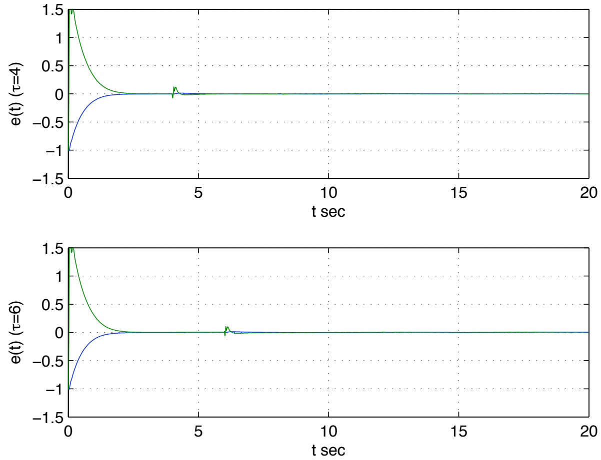

Fig. 1. Simulation of the adaptive control system for the nonlinear plant with unknown state delay. The graphs show the time history of the tracking error $e(t) = [e_1(t), e_2(t)]^T$ for the two values of the state delay, respectively.

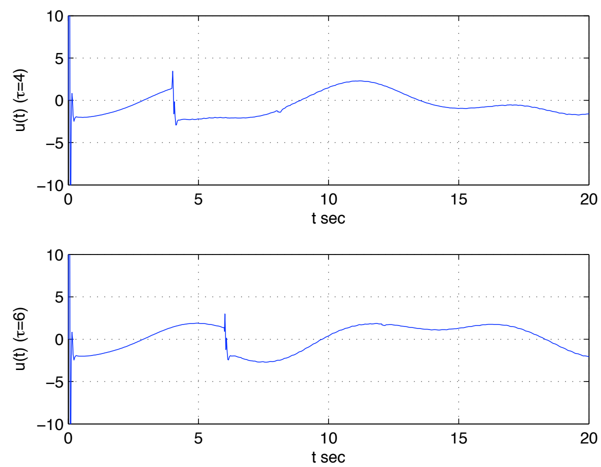

Fig. 2. Simulation of the adaptive control system for the nonlinear plant with unknown state delay. The graphs show the time history of the control $u(t)$ for the two values of the state delay, respectively.

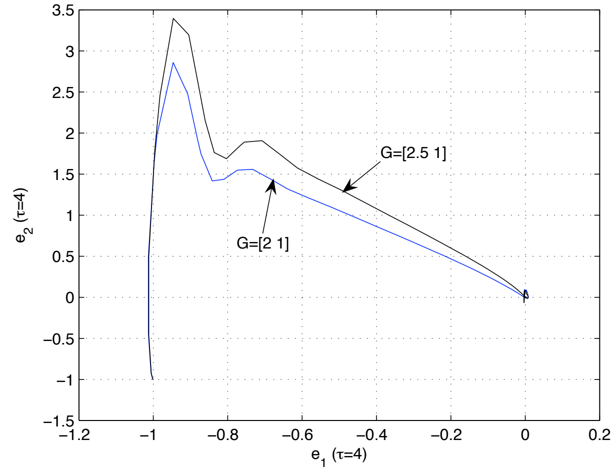

Fig. 3. Simulation of the adaptive control system for the nonlinear plant with unknown state delay. The graphs show the time history of the tracking errors $e_1(t)$ and $e_2(t)$ in the phase plane for the two values of the vector G, respectively.

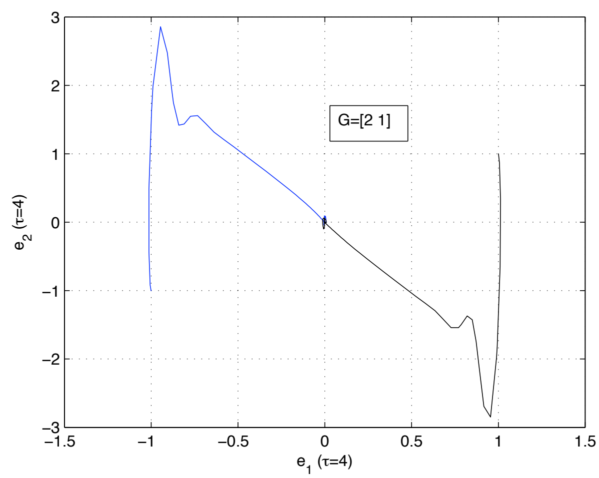

Fig. 4. Simulation of the adaptive control system for the nonlinear plant with unknown state delay. The graphs show the time history of the tracking errors $e_1(t)$ and $e_2(t)$ in the phase plane. Two trajectories starting from two different initial conditions are plotted.

VI. Concluding remarks

A systematic design procedure is proposed for design of new direct adaptive model reference adaptive control schemes for a class of uncertain nonlinear state delay systems with an unknown state delay and an external disturbance. It was shown that using novel adaptive controller parameterizations it is possible to design a state feedback controller with *sliding mode and smooth control action*, which ensures the boundedness of the closed-loop signals, exact asymptotic tracking, and robustness with respect to two cases of nonlinear state dependent perturbations, an external disturbance, and a state delay. The main advantage of the new scheme when applied to a considered class of nonlinear systems: (i) its conceptual clarity and simplicity; (ii) it guarantees smooth control action; (iii) its stability proof, based on a straightforward Lyapunov arguments, is particularly simple.

A suitably selected new Lyapunov-Krasovskii type functional is proposed to design the update mechanism for the controller parameters, and to prove stability. Simulations demonstrate that the MRAC controller with sliding mode and smooth control action has good tracking performance and robustness.

We believe that the new adaptive controller parametrization may be applied to various adaptive tracking problems for plants with and without state delays, such as the case of multiple delays and time-varying delays, and the case of output-feedback adaptive control. These cases are currently under investigation.

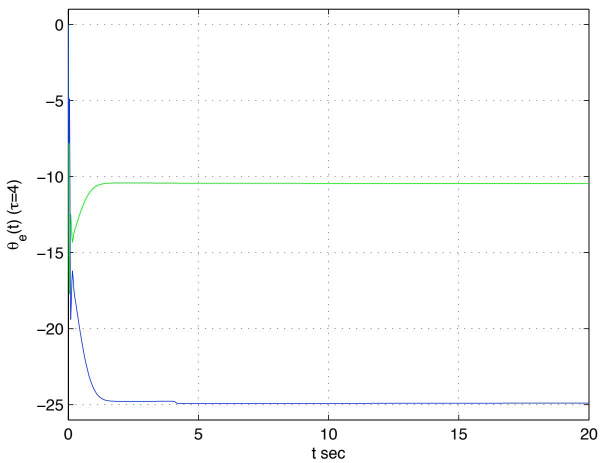

Fig. 5. Simulation of the adaptive control system for the nonlinear plant with unknown state delay. The graphs show the time history of the adjusted parameters vector $\theta_e(t)$.

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