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A Control Performance Benchmark Subject to Output Variance/Covariance Upper Bound and Pole Placement Constraint

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***Abstract*— User-specified benchmark has been reported in the literature for control loop performance assessment. A structured closed-loop response which is subject to simultaneous output covariance upper bound and pole placement constraint is proposed in this paper and served as an improved user-specified benchmark against which the existing controller performance can be compared. An LMI-based approach is used to formulate this problem and a cone complementarity linearization (CCL) algorithm is applied to find a global solution. An associated model approximation problem is solved in the sense of H_2-norm in contrast to the use of H_∞-norm of the previous work to obtain an optimal solution. The results are evaluated in a dry process rotary cement kiln example.**

I. INTRODUCTION

Most of current performance assessment algorithms are based on minimum variance control (MVC) as the benchmark. It provides an absolute theoretical lower bound of variance against which real controllers can be compared.This lower bound of variance can be estimated directly from closed-loop routine operating data and *a priori* known process time delay. The popularity is due to both the computational and conceptual simplicity and the less amount of information needed [1] [12]. But the MVC is seldom implemented in practice because of its lack of robustness to model uncertainty and use of excessive input actions [11]. More realistic user-specified benchmarks have been investigated. These benchmarks compare the variance of the current closed-loop dynamics to the variance of the user-specified closed-loop dynamics directly. In any closed-loop response there is a feedback control invariant term that can not be changed by controllers. The remaining part that can be determined by the user is the feedback dependent part of the closed-loop dynamics. Therefore, this latter part can be replaced by a user-specified dynamics which satisfies some desired performance requirements. The obtained closed-loop response is referred to as the structured closed-loop response [8]. It can then be used as a more realistic benchmark to assess the performance of the current system.

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The structured closed-loop dynamics can be specified by some performance requirements, such as time constant, decay rates, impulse response coefficients bounds, pole and zero locations and so on. For example, a modified index was proposed which considered the performance requirements due to non-minimum phase zeros and unstable poles [16]. Another modified index compared the current closed-loop output variance to a modified achievable variance which corresponds to placing one pole of the closed-loop system arbitrarily by the user [1]. In [17], an acceptable performance is described which is expressed by constrains on the closed-loop transfer function impulse response coefficients. Huang and Shah ([3]) presented the user-specified benchmark from a more general point of view including non-minimum phase systems. Another user-specified benchmark for systems with linear time varying disturbance is proposed in [7], in which the user-specified dynamics is determined to minimize the sum of weighted output variances of all different disturbances. Xu et al. [8] proposed a user-specified benchmark in which the structured closed-loop dynamics is subject to the output variance/covariance upper bound.

Pole location plays an important role in determining performance of a dynamic system. The transient response of a linear system is related to the location of its poles ([2]). For example, in the z plane of a discrete-time system, the closer the pole is to the origin, the faster the decay speed of the transient response of the system will be. Inspired by this fact, an improved user specified benchmark is proposed in this paper from the viewpoint of output variance/covariance and pole location. The structured closed-loop response is determined to satisfy both the output variance/covariance upper bound and the pole placement constraint by replacing the controller dependent part of the origin closed-loop dynamics with a user-specified response whose poles lie within a predefined region. However there may exist many feasible solutions to this problem. It is necessary to choose a practical one. Xu et al. ([8]) proposed a solution by minimizing the difference between the feedback controller dependent part of the structured benchmark closed-loop dynamics and that of the original closed-loop dynamics in the sense of H_∞-norm. The smaller the difference is, the easier the controller tuning will be to satisfy the user-specified requirements. However, the small value of the H_∞-norm only indicates that the two dynamics are close in frequency domain. In this paper, H_2-

norm is considered in contrast to H_∞-norm since the H_2-norm of the transfer function of a system bounds the output energy of the system in the time domain. It is an intuitive way to determine how close the two dynamics are. In addition, linear matrix inequality (LMI) technique is used to formulate this user-specified benchmark and an LMI region method ([15] [6]) is applied to solve the pole placement constraint.

The outline of this paper is as follows. Problem statement is discussed in Section II. Special formulation of the structured closed-loop response is proposed in Section III. Solutions to the structured closed-loop response based on an LMI approach are presented in Section IV. A model approximation problem is discussed in Section V. Simulation results are shown in Section VI, followed by conclusion in Section VII.

The notations throughout this paper are standard. I is the identity matrix with appropriate dimensions. $*$ denotes the symmetric part or the block of no concern. $X \succ 0$ (resp. $X \succeq 0$) means that the matrix X is symmetric and positive definite (resp. positive semidefinite). $\left(\begin{array}{c|c} A & G \\ \hline C & F \end{array}\right)$ or $\{A, G, C, F\}$ is the denotation of the state space realization. $\otimes$ denotes the Kronecker product.

II. Problem statement

A. Preliminary

For a SISO discrete linear time invariant system, the closed-loop response can be divided into two parts [9][4]:

$$y_k = F(q^{-1})a_k + q^{-d}R_{cl}(q^{-1})a_k \tag{1}$$

where y_k is the measured output, and a_k is the external disturbance which is a zero mean white noise. It has been shown that if the time delay is d, then the first part of (1) is the feedback controller invariant term, whereas the second part is the feedback controller dependent term, which will vanish under minimum variance control [9]. If the latter part $R_{cl}(q^{-1})$ is replaced by a user-specified response $L_R(q^{-1})$, the corresponding closed-loop response which is known as the structured closed-loop response is then obtained as following:

$$y_k^* = F(q^{-1})a_k + q^{-d}L_R(q^{-1})a_k \tag{2}$$

Similarly, for a MIMO system, the output filtered by the unitary interactor matrix D can be divided into the feedback controller invariant term and the feedback controller dependent term [3].

$$q^{-d}Dy_k = F(q^{-1})a_k + q^{-d}R_{cl}(q^{-1})a_k \tag{3}$$

where d is the order of the interactor matrix D. The structured closed-loop response can then be obtained as:

$$q^{-d}Dy_k^* = F(q^{-1})a_k + q^{-d}L_R(q^{-1})a_k \tag{4}$$

It can be seen that the SISO case is a special one of the MIMO case when $D = q^d$ [20]. Therefore, only MIMO case is considered in the following for simplicity.

The objective of the user-specified benchmark is to find a suitable $L_R(q^{-1})$ such that all the user-specified requirements are satisfied and in the same time the control performance can be realizable by a linear time invariant controller. This structured closed-loop response, if exists, has been used as user-specified benchmark to assess controller performance [8] [10]. In industrial process, however, the complete process model and disturbance model are often unknown. But the closed-loop routine operating data can be obtained easily and then the closed-loop time series model $\hat{G}_{cl}(q^{-1})$ can be identified from the routine operating data by time series analysis. The corresponding $F(q^{-1})$ and $R_{cl}(q^{-1})$ can be obtained from $\hat{G}_{cl}(q^{-1})$ and are denoted here as $\hat{F}(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$, respectively. This procedure is summarized in the following two equations:

$$\hat{y}_k = \hat{G}_{cl}(q^{-1})a_k, \quad q^{-d}D\hat{G}_{cl} = \hat{F}(q^{-1}) + q^{-d}\hat{R}_{cl}(q^{-1}) \tag{5}$$

By replacing $\hat{R}_{cl}(q^{-1})$ with $L_R(q^{-1})$, the structured closed-loop response can be expressed as:

$$q^{-d}D\hat{y}_k^* = \hat{F}(q^{-1})a_k + q^{-d}L_R(q^{-1})a_k \tag{6}$$

which is equivalent to

$$\hat{y}_k^* = q^dD^{-1}\hat{F}(q^{-1})a_k + D^{-1}L_R(q^{-1})a_k \tag{7}$$

B. Problem statement

Definition 1: Given the discrete LTI system, the output variance/covaraince upper bound constraint are defined as ([5]):

$$trace(\lim_{k\to\infty} E(y_k y_k^T)) < \sigma_y^2, \ \lim_{k\to\infty} E(y_k y_k^T) \prec \Phi_y \tag{8}$$

where σ_y^2 and Φ_y are predefined output variance and covariance upper bound respectively.

Definition 2 (LMI Regions [15]): A subset Γ of the complex plane is called an LMI region if there exist a symmetric matrix L and a matrix M such that

$$\Gamma = \{z \in C : f_\Gamma(z) \prec 0\} \tag{9}$$

with

$$f_\Gamma = L + zM + \bar{z}M^T \tag{10}$$

where f_Γ is called the characteristic function of Γ.

With the above definitions, the improved user-specified benchmark problem can be stated as the following.

With known unitary interactor matrix D and and given the estimated closed-loop model (5), find a suitable $L_R(q^{-1})$ such that all the poles of $L_R(q^{-1})$ lie within a predefined LMI region Γ (which is called pole placement constraint) and the structured closed-loop response satisfies the output covariance upper bound constraint (8).

Once $L_R(q^{-1})$ is solved, the structured closed-loop response can be used as a benchmark to assess the current controller performance against which the output covariance is compared. For a MIMO system, the overall control performance index and the individual performance index for the $i-th$ output are defined as

$$\hat{\eta} = \frac{trace(cov(\hat{y}_k^*))}{trace(cov(\hat{y}_k))}, \; \hat{\eta}_i = \frac{trace(cov(\hat{y}_k^{*(i)}))}{trace(cov(\hat{y}_k^{(i)}))} \tag{11}$$

III. Formulation of the structured closed-loop response

Following the approach in [8], define the state space realization of D^{-1},$qD^{-1}\hat{F}(q^{-1})$,and $L_R(q^{-1})$ as $\left(\begin{array}{c|c} A_D & G_D \\ \hline C_D & F_D \end{array}\right)$, $\left(\begin{array}{c|c} A_F & G_F \\ \hline C_F & F_F \end{array}\right)$, and $\left(\begin{array}{c|c} A_R & G_R \\ \hline C_R & F_R \end{array}\right)$ respectively. Then the state space realization of the structured closed-loop response (7) can be written as $\left(\begin{array}{c|c} A_S & G_S \\ \hline C_S & F_S \end{array}\right)$, where

$$\begin{array}{ll} A_S = \begin{pmatrix} A_F & 0 & 0 \\ 0 & A_D & G_D C_R \\ 0 & 0 & A_R \end{pmatrix}, & G_S = \begin{pmatrix} G_F \\ G_D F_R \\ G_R \end{pmatrix} \\ C_S = \begin{pmatrix} C_F & C_D & F_D C_R \end{pmatrix}, & F_S = F_F + F_D F_R \end{array} \tag{12}$$

A. Formulation of output covariance upper bound constraint

Lemma 1: ([20]) The structured closed-loop response (7) satisfying the output covariance upper bound constraint (8) with y_k replaced by $\hat{y}_k^*$ is feasible if and only if there exists a solution $\{\Sigma, A_R, G_R, C_R, F_R\}$ such that

$$\begin{pmatrix} -\Sigma & A_S & G_S \\ A_S^T & -\Sigma^{-1} & 0 \\ G_S^T & 0 & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{13}$$

$$\begin{pmatrix} -\Phi_y & C_S & F_S \\ C_S^T & -\Sigma^{-1} & 0 \\ F_S^T & 0 & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{14}$$

where Σ is a positive definite symmetric matrix satisfying $\Sigma \succ \Sigma_{cl}$, Σ_{cl} is the closed-loop steady state covariance matrix. Ω is the variance matrix of the white noise.

With Lemma 1, if there is a state space realization $\{A_R, G_R, C_R, F_R\}$ of $L_R(q^{-1})$ which satisfies (13) and (14), then the structured closed-loop response satisfies the output covariance upper bound constraint.

B. Formulation of pole placement constraint

Assume Γ is a predefined LMI region. A dynamic system with the state matrix A is called Γ-stable if all its poles lie within Γ, i.e., all eigenvalues of the matrix A lie within Γ. The pole placement constraint restricts all the poles of $L_R(q^{-1})$ within the LMI region Γ. Hence, if the state matrix of $L_R(q^{-1})$ is Γ-stable, then the $L_R(q^{-1})$ subject to the pole placement constraint can be solved. The following lemma provides a path to solve this problem.

Lemma 2: ([15]) The matrix A is Γ-stable if and only if there exists a symmetric positive definite matrix U such that

$$M_\Gamma(A,U) \triangleq L \otimes U + M \otimes (AU) + M^T \otimes (AU)^T \prec 0 \tag{15}$$

where matrix L and M are defined in (10).

Since the state space realization of $L_R(q^{-1})$ is given, if the state matrix A_R is Γ-stable, then the pole placement constraint is satisfied. A disk region Γ of radius r centered at origin is considered in this paper. The disk region Γ can be expressed as

$$\Gamma = \{z \in C : f_\Gamma(z) \;=\; \begin{pmatrix} -r & z \\ \bar{z} & -r \end{pmatrix} \prec 0\} \tag{16}$$

Lemma 3: A solution of $L_R(q^{-1})$ whose poles lie within the disk region Γ is feasible if and only if there is a symmetric positive definite matrix U such that

$$\begin{pmatrix} -r^2 U & A_R \\ A_R^T & -U^{-1} \end{pmatrix} \prec 0 \tag{17}$$

Proof. Following Lemma 2 and the definition of disk region Γ (16), this lemma can be proved. □

Lemma 3 can be combined with Lemma 2 to get the following result.

Theorem 1: A feasible solution of $L_R(q^{-1})$ which is subject to output covariance upper bound constraint and pole placement constraint can be obtained if and only if there exist symmetric positive definite matrices Σ and U such that (13), (14), and (17) are feasible.

Proof. The proof can be obtained by following Lemma 1 and Lemma 3 and is omitted. □

Once $L_R(q^{-1})$ is solved, the structured benchmark closed-loop response can be obtained and used as a benchmark.

IV. LMI solutions to the improved user-specified benchmark problem

It can be seen that in (13), (14), and (17), there are some elements such as A_S, G_S which are linear combinations of the unknown parameters of $L_R(q^{-1})$. Define $K_R = \begin{pmatrix} A_R & G_R \\ C_R & F_R \end{pmatrix}$, so that all the unknown parameters of $L_R(q^{-1})$ can be included into one decision matrix. Extract K_R from these linear-combination-elements, thereby each of these elements becomes an affine function of K_R.

For example, define

$$\bar{G}_S = \begin{pmatrix} G_F \\ 0 \\ 0 \end{pmatrix}, H_1 = \begin{pmatrix} 0 & 0 \\ 0 & G_D \\ I & 0 \end{pmatrix}, H_2 = \begin{pmatrix} 0 \\ I \end{pmatrix}$$

therefore G_S can be rewritten as $\bar{G}_S + H_1 K_R H_2$ which is an affine function of K_R.

However, in (13), (14), and (17) there are some other elements such as Σ^{-1} and U^{-1} which make these inequalities nonlinear. The following procedure however makes the solution of this nonlinear problem simpler.

Note that for any matrices X and Y, if the LMI

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \succeq 0 \tag{18}$$

is feasible, then $trace(XY) \geq n$, and $trace(XY) = n$ if and only if $XY = I$ [8].

Define $\Sigma \triangleq X$ and $\Sigma^{-1} = Y$ such that

$$XY = I \tag{19}$$

and also $U^{-1} = V$ such that

$$UV = I \tag{20}$$

Using the method mentioned above to make all the linear-combination-elements in (13), (14), and (17) to be affine functions of K_R, the inequalities (13), (14), and (17) can be converted into the following LMIs.

$$\begin{pmatrix} -X & A_S & G_S \\ * & -Y & 0 \\ * & * & -\Omega^{-1} \end{pmatrix}, \begin{pmatrix} -\Phi_y & C_S & F_S \\ * & -Y & 0 \\ * & * & -\Omega^{-1} \end{pmatrix}$$
$$\begin{pmatrix} -r^2 U & A_R \\ * & -V \end{pmatrix} \tag{21}$$

Hence, a feasible solution to the problem (19)-(21) can be obtained by solving the following concave minimization problem.

Problem 1:

Minimize$_{\{X,Y,U,V,K_R\}}\{trace(XY) + trace(UV)\}$

$$\text{subject to } (18), (21), \text{ and } \begin{pmatrix} U & I \\ I & V \end{pmatrix} \succeq 0 \tag{22}$$

It can be seen that if the optimal solution of problem 1 satisfies $trace(XY)+tracec(UV) = n_{XY}+n_{UV}$, then (22) is feasible; otherwise is infeasible. Hence, the problem to find a $L_R(q^{-1})$ to satisfy all constraints is converted to finding a globe solution of the minimization problem 1. However, the objective function is nonconvex. A cone complementarity linearization (CCL) algorithm can be used to find a global solution of problem 1 most of time [14] [13].

However, the feasible solution of $L_R(q^{-1})$ which satisfies all the constraints is not unique and sometimes is not practical [20]. The optimal $L_R(q^{-1})$ is the one which is closest to $\hat{R}_{cl}(q^{-1})$ [20], the original feedback controller dependent term. This is because the fact that the closer $L_R(q^{-1})$ is to $\hat{R}_{cl}(q^{-1})$, the less tuning effort to the existing controller is required. If the $\hat{R}_{cl}(q^{-1})$ makes the closed-loop response satisfy all the constraints, then there is no need to construct a structured closed-loop response. In this case the original closed-loop response can be served as a benchmark directly. With availability of the closed-loop routine operating data and the unitary interactor matrix, the response of $\hat{R}_{cl}(q^{-1})$ can be obtained from the estimated closed-loop time series model. Hence, the problem to find an unknown model $L_R(q^{-1})$ which is as close as possible to a known model $\hat{R}_{cl}(q^{-1})$ is same as the model approximation problem.

V. MODEL APPROXIMATION PROBELM

In the literatures of model approximation, H_∞-norm is often used to measure the distance between the unknown model and the known one [18] [14] [21]. However, the interpretation of H_∞-norm is mainly in frequency domain, i.e., the approximation accuracy is measured in frequency domain. In our problem, our interest is in the time domain and it will be more desirable to know how close the two models are in time-domain response. Hence, H_2-norm is considered here. The advantage will be demonstrated in the simulations. The model approximation problem can be stated as to find a optimal $L_R(q^{-1})$ such that

$$\|L_R(q^{-1}) - \hat{R}_{cl}(q^{-1})\|_2 < \gamma \tag{23}$$

where γ is a predefined value to measure the difference between the two models.

The following lemma gives the formulation of the model approximation problem.

Lemma 4: ([19]) $\{A_M, G_M, C_M, F_M\}$ is a state space realization for a transfer function T. A is stable and $\|T\|_2 < \gamma$ if and only if there exist symmetric matrix P and matrix Π such that

$$\begin{aligned} &\begin{pmatrix} -P & A_M & G_M \\ A_M^T & -P^{-1} & 0 \\ G_M^T & 0 & -I \end{pmatrix} \prec 0, \\ &\begin{pmatrix} -\Pi & C_M & F_M \\ C_M^T & -P^{-1} & 0 \\ F_M^T & 0 & -I \end{pmatrix} \prec 0, \\ &trace(\Pi) < \gamma \end{aligned} \tag{24}$$

With the state space model of $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$ defined as $\left(\begin{array}{c|c} A_R & G_R \\ \hline C_R & F_R \end{array}\right)$ and $\left(\begin{array}{c|c} \hat{A}_R & \hat{G}_R \\ \hline \hat{C}_R & \hat{F}_R \end{array}\right)$, we have

$$\begin{aligned} L_R(q^{-1}) - \hat{R}_{cl}(q^{-1}) &= \left(\begin{array}{cc|c} A_R & 0 & G_R \\ 0 & \hat{A}_R & \hat{G}_R \\ \hline C_R & \hat{C}_R & F_R - \hat{F}_R \end{array}\right) \\ &= \left(\begin{array}{c|c} A_M & G_M \\ \hline C_M & F_M \end{array}\right) \end{aligned} \tag{25}$$

Using the method mentioned in Section IV, the A_M, G_M, C_M, and F_M in (24) can be rewritten as affine functions of K_R. Then following Lemma 4, the model approximation problem can be solved if (24) is feasible. It can be seen that there are still nonlinear elements in (24). Define $Q = P^{-1}$ such that $PQ = I$ and then the model approximation problem is readily incorporated with the problem 1. The final problem to be solved is:

Problem 2:

$$\underset{\{X,Y,U,V,P,Q,K_R\}}{\text{Minimize}} \{trace(XY) + trace(UV) + trace(PQ)\}$$

$$\text{subject to } (18), (21), (24), \begin{pmatrix} U & I \\ I & V \end{pmatrix} \succeq 0,$$

$$\text{and } \begin{pmatrix} P & I \\ I & Q \end{pmatrix} \succeq 0$$

Problem 2 can also be solved using CCL algorithm. A feasible solution to problem 2 can be found which implies an optimal $L_R(q^{-1})$ that satisfies the output covariance upper bound constraint and the pole placement constraint simultaneously, and in addition, the model approximation constraint is obtained. Thus the corresponding structured closed-loop response can be served as a practical benchmark to assess the controller performance.

VI. Simulation Results

Consider a dry process rotary cement kiln with a capacity of 1000 t of clinker a day, which is taken from [8]. The model was given as

$$y_k + \begin{pmatrix} -0.914 & -0.08 \\ 0.216 & -0.917 \end{pmatrix} y_{k-1} = \begin{pmatrix} 2.091 & -0.07044 \\ -0.211 & -0.0156 \end{pmatrix} u_{k-1} + a_k + \begin{pmatrix} 0 & 0 \\ 0 & 0.715 \end{pmatrix} a_{k-1} \tag{26}$$

where $E(a_k a_k^T) = \begin{pmatrix} 0.0644 & 0.000257 \\ 0.000257 & 0.0214 \end{pmatrix}$ and the sampling time is 5 min. It is required that large variations in $y_k^{(1)}$ should be avoided in order to ensure steady state operation of the plant. Small variance of $y_k^{(2)}$ will make it possible to operate the process closer to the limit which specifies the maximum free lime content of the clinker. This will result in reduced energy consumption [8]. The unitary interactor matrix is calculated as $D = \begin{pmatrix} 0 & q \\ q & 0 \end{pmatrix}$.

Assume that an output feedback controller has been implemented to control the dry process rotary cement kiln.

$$u_k = \begin{pmatrix} -0.177 & 0.125 \\ 1.84 & 2.09 \end{pmatrix} y_k \tag{27}$$

Four thousand routine operating output data points are collected and the corresponding closed-loop time series model $\hat{G}_{cl}$ can be identified by time series analysis.

Consider to find a second-order $L_R(q^{-1})$ to satisfy the following constraints: 1) output covariance upper bound constraint with $\Phi_y = \begin{pmatrix} 0.0939 & * \\ * & 0.193 \end{pmatrix}$, 2) pole placement constraint with the disk radius range from 0.1 to 0.8, and 3) model approximation constraint with flexible γ.

γ is flexible because the pole placement constraint also restricts the distance between $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$. If the user-specified pole position of $L_R(q^{-1})$ is very far away from that of the original $\hat{R}_{cl}(q^{-1})$, then γ cannot be an arbitrary value but a flexible value of $L_R(q^{-1})$ to get the closest $L_R(q^{-1})$ from $\hat{R}_{cl}(q^{-1})$ when pole position is restricted to lie in a specified disk region.

TABLE I

SIMULATION RESULTS

rad	Poles of $L_R(q^{-1})$	γ	η_1	η_2	η_0
0.1	0.0724 $\pm$ 0.0689 i	1.8	0.8322	0.6590	0.7133
0.2	0.1795 $\pm$ 0.0881 i	1.6	0.8716	0.6928	0.7489
0.3	0.2858 $\pm$ 0.0911 i	1.4	0.8801	0.7163	0.7676
0.4	0.3921 $\pm$ 0.0792 i	1.2	0.8804	0.7561	0.7951
0.5	0.4967 $\pm$ 0.0573 i	0.9	0.8979	0.8093	0.8371
0.6	0.5982 $\pm$ 0.0461 i	0.7	0.9256	0.8208	0.8537
0.7	0.6728 $\pm$ 0.0174 i	0.6	0.9434	0.8439	0.8751
0.8	0.7806, 0.5351	0.51	0.9262	0.8471	0.8719

Solving problem 2 using the CCL algorithm, with the constraints mentioned above, we have the simulation results shown in table I where γ is the smallest difference between $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$, rad is the radius of disk region, and η_1, η_2, η_0 are the individual performance index of $y_k^{(1)}$ and $y_k^{(2)}$ and the whole performance index respectively. In addition the poles of $\hat{R}_{cl}(q^{-1})$ can be calculated as $0.7891 \pm 0.0672i$, 0.4466. The output variances are calculated from $\hat{G}_{cl}(q^{-1})$ as $var(\hat{y}_k^{(1)}) = 0.0969$, $var(\hat{y}_k^{(2)}) = 0.2123$.

It can be seen from table I that all the solutions of $L_R(q^{-1})$ with different rad satisfy the pole placement constraint. With the radius rad being increased, the pole position of corresponding $L_R(q^{-1})$ is closer to that of $\hat{R}_{cl}(q^{-1})$, then the value of γ is smaller which means the benchmark is closer to the original closed-loop response, so the performance index is nearer to 1.

Another simulation is done to see how close the $L_R(q^{-1})$ is to $\hat{R}_{cl}(q^{-1})$ in the sense of H_2-norm and H_∞-norm ([8]) respectively. The $L_R(q^{-1})$ which satisfies all constraints and approaches $\hat{R}_{cl}(q^{-1})$ in the sense of H_2-norm is denoted by L_R-H_2, whereas the $L_R(q^{-1})$ which satisfies all constraints

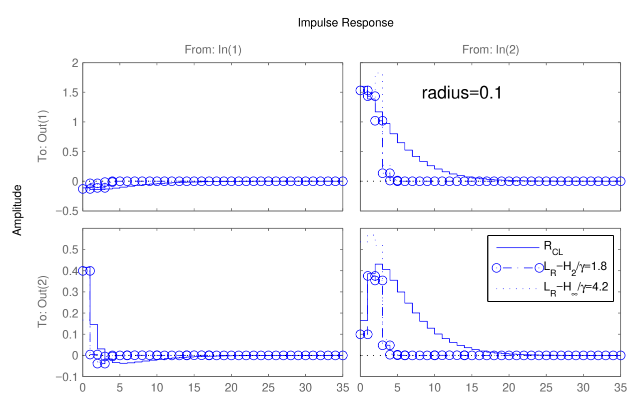

Fig. 1. Impulse responses of two $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$ with radius=0.1

and approaches $\hat{R}_{cl}(q^{-1})$ in the sense of H_∞-norm is denoted by L_R-H_∞. In Fig.1 and 2, the impulse responses of L_R-H_2 and L_R-H_∞ are compared to that of $\hat{R}_{cl}(q^{-1})$ with different disk regions with radius being 0.1 and 0.8 respectively. In the figures, the solid line is the impulse response of $\hat{R}_{cl}(q^{-1})$, the dashed line and the line with circle are impulse responses of L_R-H_2 and L_R-H_∞, respectively.

It can be seen from Fig.1 that when radius of the disk region is equal to 0.1, both of the impulse responses of L_R-H_2 and L_R-H_∞ are not very close to that of $\hat{R}_{cl}(q^{-1})$,which is reasonable because the pole placement constraint is very strict. It can also be seen that the impulse response of L_R-H_2 is closer to that of $\hat{R}_{cl}(q^{-1})$ which implies under this circumstance H_2-norm is more effective than H_∞-norm to show how close the two models are.

From Fig. 2, it can be seen that with the radius being increased, the two responses are all closer to $\hat{R}_{cl}(q^{-1})$ than the responses in Fig. 1, and the response of L_R-H_∞ is closer to that of $\hat{R}_{cl}(q^{-1})$ than L_R-H_2. A larger radius implies the pole placement constraint is less strict; that is to say, when the constraint is not strict, the advantage of L_R-H_2 is not clear.

It has been observed that the L_R-H_2 is always better than L_R-H_∞ at the initial achievable user-specified response whatever the radius is and there are some abrupt jumps at the initial impulse responses of L_R-H_∞. Hence in most cases, the benchmark of L_R-H_2 is better than that of L_R-H_∞.

VII. Conclusion

Control performance assessment problem is considered in this paper from the viewpoint of structured closed-loop response subject to three constraints: 1) output covariance constraint, 2) pole placement constraint, and 3) model approximation constraint. The problem is formulated by LMI and the CCL algorithm is used to solve a concave minimization problem. With closed-loop routine operating data and the unitary interactor matrix, the structured closed-loop response can be obtained with the three constraints being satisfied simultaneously. The resultant feasible structured closed-loop response can serve as a benchmark against which the current control performance can be compared. Case studies compare the benchmark proposed in this paper and benchmark in the sense of H_∞-norm with other constraints being kept same.

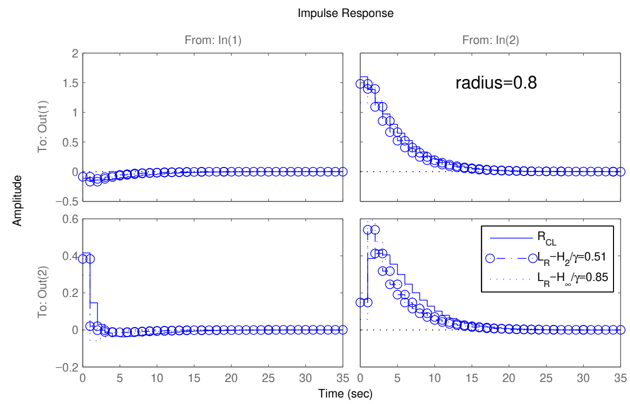

Fig. 2. Impulse responses of two $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$ with radius=0.8

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