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Output Tracking Control for Continuous-Time Networked Control Systems with Communication Constraints

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Abstract— **This paper studies the problem of H_∞ output tracking control for continuous-time networked control systems (NCSs) with communication constraints. By using the continuous Jensen inequality, linear matrix inequality (LMI)-based H_∞ output tracking controller design for nominal NCSs with controller-to-actuator communication constraints is presented, and a new method is proposed to design H_∞ output tracking controllers for NCSs with both controller-to-actuator and sensor-to-controller communication constraints. The simulation results illustrate the effectiveness of the proposed H_∞ output tracking controller design for NCSs with communication constraints.**

I. INTRODUCTION

Networked control systems are spatially distributed systems in which the communication between sensors, actuators, and controllers occurs through a shared band-limited digital communication network.

The use of a multipurpose shared network to connect spatially distributed elements results in flexible architectures and generally reduces installation and maintenance costs. However, the introduction of communication network will inevitably lead to time delay, packet dropout, communication constraints, etc.

Many researchers have studied stability/stabilization for NCSs [1]-[3]. In [4], by using the Lyapunov-Razumikhin function techniques, delay-dependent condition on the stabilization of NCSs was obtained, and stabilizing state feedback controllers were also constructed. For other methods dealing with time delay and packet dropout, see also [5]-[9]. Recently, there have been considerable research efforts on H_∞ control for systems with delay [10]-[14].

As we can see, the main topic of the literature presented above is time delay and packet dropout, few studies the problem of communication constraints in NCSs. In fact, NCSs with communication constraints are also a hot topic in recent years. [15] investigated a state estimation problem involving finite communication capacity constraints. In [16], a method for optimal off-line scheduling of a limited resource used for control purposes was presented. [17] reviewed several recent results on estimation, analysis, and controller synthesis for NCSs, and the problem of band-limited channels was also discussed. For other results on systems with communication constraints, see also [18]-[20].

Synthesizing feedback controllers to achieve asymptotic tracking of prescribed reference outputs while rejecting disturbances is a fundamental problem in control. The main objective of tracking control is to make the output of the plant, via a controller, track the output of a given reference model as close as possible. For the problem of output tracking control, [21] studied the reliable robust tracking controller design problem against actuator faults and control surface impairment for aircraft. [22] solved the tracking and disturbance rejection problem for infinite-dimensional linear systems, with reference and disturbance signals that were finite superpositions of sinusoids. For other results on output tracking control, see also [23]-[25] and the references therein.

The study on output tracking control and NCSs keeps attracting considerable attention due to the demands from practical dynamic processes in industry [26]-[27]. To the best of our knowledge, however, few works pay attention to the problem of output tracking control for networked control systems with communication constraints, which motivates the present study.

By using the continuous Jensen inequality, LMI-based H_∞ output tracking controller design for nominal NCSs with controller-to-actuator communication constraints is presented in this paper, and a new method is proposed to design H_∞ output tracking controllers for NCSs with both controller-to-actuator and sensor-to-controller communication constraints. The proposed controller design methods can guarantee asymptotic tracking of prescribed reference outputs while rejecting disturbances.

This paper is organized as follows. Section 2 presents the closed-loop models of H_∞ output tracking NCSs with communication constraints. Section 3 is dedicated to the H_∞ output tracking controller design for nominal NCSs with communication constraints. The results of numerical simulation are presented in Section 4. Conclusions are stated in Section 5.

Notation. Throughout this paper, M^T represents the transpose of matrix M. I and 0 represent identity matrices and zero matrices with appropriate dimensions, respectively. $*$ denotes the entries of matrices implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate

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dimensions.

II. Preliminaries and problem statement

Consider a linear time-invariant plant described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1u(t) + B_2\omega(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (1)$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^r$, $\omega(t) \in R^q$ are the state vector, control input vector, measured output, and disturbance input, respectively, and $\omega(t)$ is assumed to belong to $L_2[0, \infty)$. A, B_1, B_2, C, D are known constant matrices of appropriate dimensions.

The main objective of this paper is to make the output $y(t)$ of the plant, via a controller, track the output of a given reference model as close as possible. Suppose the reference signal $y_r(t)$ is generated by

$$\begin{aligned} \dot{x}_r(t) &= Gx_r(t) + r(t) \\ y_r(t) &= Hx_r(t) \end{aligned} \quad (2)$$

where $x_r(t)$ and $r(t)$ are reference state and energy bounded reference input, respectively, $x_r(t) \in R^l$, $r(t) \in R^l$, $y_r(t) \in R^p$, G and H are known constant matrices of appropriate dimensions with G Hurwitz stable.

Suppose h is the length of sampling period, the control inputs which are based on plant states at the instants $i_kh,\ i_{k+1}h,\ \cdots\ (k = 1,\ 2,\ \cdots)$ are transmitted to the actuator successfully, the others are dropped, then $\{i_1h,\ i_2h,\ \cdots\}$ is a subset of $\{h,\ 2h,\ \cdots\}$. In this paper, we suppose the packet dropout is stochastic, and the latest available control inputs will be used if there exists packet dropout.

If there does not exist any communication constraints, suppose τ_k denotes the time from the instant i_kh when sensor samples plant state to the instant when actuator transfers data to the plant, then the state feedback controller takes the following form

$$u(t) = K_1x(i_kh) + K_2x_r(i_kh) \quad (3)$$

where $t \in [i_kh + \tau_k,\ i_{k+1}h + \tau_{k+1})$, K_1 and K_2 are the state feedback controller gains which will be designed in this paper.

To notice that $i_kh = t - (t - i_kh)$, define $t - i_kh = \tau(t)$, then $i_kh = t - \tau(t)$. Suppose the lower-bound and upper-bound of τ_k are τ_m and τ_M, respectively, and the upper-bound of $(i_{k+1} - i_k)h + \tau_{k+1}$ is η_M, then $\tau_m \le \tau_k \le \tau(t) < (i_{k+1} - i_k)h + \tau_{k+1} \le \eta_M$.

In this paper, we consider the problem of H_∞ output tracking control for nominal NCSs with sensor-to-controller and controller-to-actuator communication constraints.

A. NCSs with controller-to-actuator communication constraints

Suppose the shared communication medium can simultaneously provide W_θ $(1 \le W_\theta < m)$ controller-to-actuator communication channels, and there do not exist any communication constraints in the sensor-to-controller communication channels. Let the binary-valued variables θ_i $(i = 1,\ 2,\ \cdots,\ m)$ denote the medium access status of the ith element of $u(t)$, i.e. $\theta_i:\ \mathscr{R} \to \{0,\ 1\}$, where 1 implies "accessing" and 0 means "not accessing". If $\theta_i = 0$, we let the actuator ignore the ith element of $u(t)$ by assuming a zero-value.

Based on the above communication protocol, we can see that the control input $u(t)$ for NCSs with controller-to-actuator medium access constraints can be described as follows

$$u(t) = \widetilde{W}_\theta[K_1x(i_kh) + K_2x_r(i_kh)] \quad (4)$$

where $t \in [i_kh + \tau_k,\ i_{k+1}h + \tau_{k+1})$, $\widetilde{W}_\theta = diag(\theta_1,\ \theta_2,\ \cdots,\ \theta_m)$, and $\theta_i = 1$ or $\theta_i = 0$ $(i = 1,\ 2,\ \cdots,\ m)$.

Define $\xi(t) = \begin{bmatrix} x(t) \\ x_r(t) \end{bmatrix}$, $K = [K_1\ K_2]$, the control input $u(t)$ presented in (4) can also be described as follows

$$u(t) = \widetilde{W}_\theta K\xi(t - \tau(t)) \quad (5)$$

Considering the effect of time delay and packet dropout, from (1)-(2) and (5), we can get the following augmented closed-loop system

$$\begin{aligned} \dot{\xi}(t) &= \Psi_1\xi(t) + \Psi_2\widetilde{W}_\theta K\xi(t - \tau(t)) + \Psi_3\nu(t) \\ e(t) &= \widetilde{\Psi}_1\xi(t) + \widetilde{\Psi}_2\widetilde{W}_\theta K\xi(t - \tau(t)) \end{aligned} \quad (6)$$

where $e(t) = y(t) - y_r(t)$, $\nu(t) = \begin{bmatrix} \omega(t) \\ r(t) \end{bmatrix}$, $\Psi_1 = \begin{bmatrix} A & 0 \\ 0 & G \end{bmatrix}$, $\Psi_2 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$, $\Psi_3 = \begin{bmatrix} B_2 & 0 \\ 0 & I \end{bmatrix}$, $\widetilde{\Psi}_1 = [C\ \ -H]$, $\widetilde{\Psi}_2 = D$.

B. NCSs with both controller-to-actuator and sensor-to-controller communication constraints

Suppose the NCSs can simultaneously provide W_θ $(1 \le W_\theta < m)$ controller-to-actuator communication channels and W_δ $(1 \le W_\delta < n + l)$ sensor-to-controller communication channels. Let the binary-valued variables δ_i $(i = 1,\ 2,\ \cdots,\ n + l)$ denote the medium access status of the ith element of $\xi(t)$, i.e. $\delta_i:\ \mathscr{R} \to \{0,\ 1\}$, where 1 implies "accessing" and 0 means "not accessing". If $\delta_i = 0$, we let the controller ignore the ith element of $\xi(t)$ by assuming a zero-value.

Based on the above communication protocol, we can see that the control input $u(t)$ for NCSs with both controller-to-actuator and sensor-to-controller communication constraints can be described as follows

$$u(t) = \widetilde{W}_\theta K\widetilde{W}_\delta\xi(t - \tau(t)) \quad (7)$$

where $t \in [i_kh + \tau_k,\ i_{k+1}h + \tau_{k+1})$, $\widetilde{W}_\theta$ is the same as the one in (5), $\widetilde{W}_\delta = diag(\delta_1,\ \delta_2,\ \cdots,\ \delta_{n+l})$, and $\delta_i = 1$ or $\delta_i = 0$ $(i = 1,\ 2,\ \cdots,\ n + l)$.

Similar to (6), we can get the following augmented closed-loop system

$$\begin{aligned} \dot{\xi}(t) &= \Psi_1\xi(t) + \Psi_2\widetilde{W}_\theta K\widetilde{W}_\delta\xi(t - \tau(t)) + \Psi_3\nu(t) \\ e(t) &= \widetilde{\Psi}_1\xi(t) + \widetilde{\Psi}_2\widetilde{W}_\theta K\widetilde{W}_\delta\xi(t - \tau(t)) \end{aligned} \quad (8)$$

where $\xi(t)$, $e(t)$, $v(t)$, Ψ_1, Ψ_2, Ψ_3, $\widetilde{\Psi}_1$, $\widetilde{\Psi}_2$ are the same as the ones in (6).

This paper is devoted to H_∞ output tracking controller design for NCSs with communication constraints.

Remark 1. As shown in the output tracking NCSs (6) and (8), time delay, packet dropout and communication constraints are taken into consideration simultaneously, which is different from the existing results in the literature.

The following continuous Jensen inequality will be used in the sequel.

Lemma 1 [28]. For any symmetric positive definite matrix $M \in R^{s*s}$, scalars $\beta_1 < \beta_2$, a vector function $\mathscr{C} : [\beta_1,\ \beta_2] \to R^s$ such that the integrals in the following are well defined, then

$$\begin{aligned}&-(\beta_2-\beta_1)\int_{\beta_1}^{\beta_2}\mathscr{C}^T(s)M\mathscr{C}(s)ds\\&\le -\Big(\int_{\beta_1}^{\beta_2}\mathscr{C}(s)ds\Big)^T M\Big(\int_{\beta_1}^{\beta_2}\mathscr{C}(s)ds\Big)\end{aligned} \tag{9}$$

III. H_∞ Output Tracking Controller Design for NCSs with Communication Constraints

This section is concerned with the problem of H_∞ output tracking controller design for the augmented closed-loop systems (6) and (8).

A. NCSs with controller-to-actuator communication constraints

Theorem 1. For given scalars η_M and τ_m, if there exist symmetric positive definite matrices W, $\widetilde{Q}_1$, $\widetilde{Q}_2$, $\widetilde{R}_1$, $\widetilde{R}_2$, matrix V, scalar $\gamma > 0$, such that the following LMIs hold for every feasible value of $\widetilde{W}_\theta$

$$\begin{bmatrix}\widetilde{\Omega}_{11} & 0 & \widetilde{R}_1 & \widetilde{\Omega}_{14} & \Psi_3 & W\Psi_1^T & W\Psi_1^T & W\widetilde{\Psi}_1^T\\ * & -\widetilde{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \widetilde{\Omega}_{33} & \widetilde{R}_2 & 0 & 0 & 0 & 0\\ * & * & * & -2\widetilde{R}_2 & 0 & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{48}\\ * & * & * & * & -\gamma I & \Psi_3^T & \Psi_3^T & 0\\ * & * & * & * & * & \widetilde{\Omega}_{66} & 0 & 0\\ * & * & * & * & * & * & \widetilde{\Omega}_{77} & 0\\ * & * & * & * & * & * & * & -\gamma I\end{bmatrix} < 0 \tag{10}$$

where

$$\begin{aligned}\widetilde{\Omega}_{11} &= \Psi_1 W + W\Psi_1^T + \widetilde{Q}_1 + \widetilde{Q}_2 - \widetilde{R}_1 - \widetilde{R}_2\\ \widetilde{\Omega}_{14} &= \widetilde{R}_2 + \Psi_2\widetilde{W}_\theta V\\ \widetilde{\Omega}_{33} &= -\widetilde{Q}_2 - \widetilde{R}_1 - \widetilde{R}_2\\ \widetilde{\Omega}_{46} &= V^T\widetilde{W}_\theta\Psi_2^T\\ \widetilde{\Omega}_{48} &= V^T\widetilde{W}_\theta\widetilde{\Psi}_2^T\\ \widetilde{\Omega}_{66} &= \eta_M^{-2}(\widetilde{R}_1 - 2W)\\ \widetilde{\Omega}_{77} &= (\eta_M-\tau_m)^{-2}(\widetilde{R}_2 - 2W)\end{aligned}$$

then with the control law

$$u(t) = \widetilde{W}_\theta K\xi(t-\tau(t)), \quad K = VW^{-1} \tag{11}$$

the augmented closed-loop system described by (6) is asymptotically stable with H_∞ output tracking performance γ.

Proof: Let us consider the following Lyapunov function

$$V(t) = V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t) \tag{12}$$

where

$$\begin{aligned}V_1(t) &= \xi^T(t)P\xi(t)\\ V_2(t) &= \int_{t-\tau_m}^{t}\xi^T(\alpha)Q_1\xi(\alpha)d\alpha\\ V_3(t) &= \int_{t-\eta_M}^{t}\xi^T(\alpha)Q_2\xi(\alpha)d\alpha\\ V_4(t) &= \eta_M\int_{-\eta_M}^{0}\int_{t+\beta}^{t}\dot{\xi}^T(\alpha)R_1\dot{\xi}(\alpha)d\alpha d\beta\\ V_5(t) &= (\eta_M-\tau_m)\int_{-\eta_M}^{-\tau_m}\int_{t+\beta}^{t}\dot{\xi}^T(\alpha)R_2\dot{\xi}(\alpha)d\alpha d\beta\end{aligned}$$

P, Q_1, Q_2, R_1 and R_2 are symmetric positive definite matrices with appropriate dimensions.

From Lemma 1, we can see that

$$\begin{aligned}&-\eta_M\int_{t-\eta_M}^{t}\dot{\xi}^T(\alpha)R_1\dot{\xi}(\alpha)d\alpha\\&\le -[\xi(t)-\xi(t-\eta_M)]^T R_1[\xi(t)-\xi(t-\eta_M)]\end{aligned} \tag{13}$$

On the other hand,

$$\begin{aligned}&-(\eta_M-\tau_m)\int_{t-\eta_M}^{t-\tau_m}\dot{\xi}^T(\alpha)R_2\dot{\xi}(\alpha)d\alpha\\ &= -(\eta_M-\tau_m)\int_{t-\tau(t)}^{t-\tau_m}\dot{\xi}^T(\alpha)R_2\dot{\xi}(\alpha)d\alpha\\ &\quad -(\eta_M-\tau_m)\int_{t-\eta_M}^{t-\tau(t)}\dot{\xi}^T(\alpha)R_2\dot{\xi}(\alpha)d\alpha\\ &\le -[\xi(t)-\xi(t-\tau(t))]^T R_2[\xi(t)-\xi(t-\tau(t))]\\ &\quad -[\xi(t-\tau(t))-\xi(t-\eta_M)]^T R_2[\xi(t-\tau(t))-\xi(t-\eta_M)]\end{aligned} \tag{14}$$

To notice that

$$\dot{V}_1(t) = 2\xi^T(t)P\dot{\xi}(t) \tag{15}$$

$$\dot{V}_2(t) = \xi^T(t)Q_1\xi(t) - \xi^T(t-\tau_m)Q_1\xi(t-\tau_m) \tag{16}$$

$$\dot{V}_3(t) = \xi^T(t)Q_2\xi(t) - \xi^T(t-\eta_M)Q_2\xi(t-\eta_M) \tag{17}$$

$$\dot{V}_4(t) = \eta_M^2\dot{\xi}^T(t)R_1\dot{\xi}(t) - \eta_M\int_{t-\eta_M}^{t}\dot{\xi}^T(\alpha)R_1\dot{\xi}(\alpha)d\alpha \tag{18}$$

$$\begin{aligned}\dot{V}_5(t) =& (\eta_M-\tau_m)^2\dot{\xi}^T(t)R_2\dot{\xi}(t)\\ &-(\eta_M-\tau_m)\int_{t-\eta_M}^{t-\tau_m}\dot{\xi}^T(\alpha)R_2\dot{\xi}(\alpha)d\alpha\end{aligned} \tag{19}$$

Considering the definition of $e(t)$, for any nonzero $\tilde{\xi}(t)$, we have

$$\gamma^{-1}e^T(t)e(t) - \gamma v^T(t)v(t) = \tilde{\xi}^T(t)\Xi\tilde{\xi}(t) \tag{20}$$

where

$$\tilde{\xi}(t) = [\xi^T(t)\ \xi^T(t-\tau_m)\ \xi^T(t-\eta_M)\ \xi^T(t-\tau(t))\ v^T(t)]^T$$

$$\Xi = \begin{bmatrix}\gamma^{-1}\widetilde{\Psi}_1^T\widetilde{\Psi}_1 & 0 & 0 & \gamma^{-1}\widetilde{\Psi}_1^T\widetilde{\Psi}_2\widetilde{W}_\theta K & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ \gamma^{-1}K^T\widetilde{W}_\theta\widetilde{\Psi}_2^T\widetilde{\Psi}_1 & 0 & 0 & \gamma^{-1}\mathscr{X} & 0\\ 0 & 0 & 0 & 0 & -\gamma I\end{bmatrix},$$

and $\mathscr{X} = (\widetilde{\Psi}_2\widetilde{W}_\theta K)^T(\widetilde{\Psi}_2\widetilde{W}_\theta K)$.

Combining (13)-(20) together, we have

$$\gamma^{-1}e^T(t)e(t)-\gamma v^T(t)v(t)+\dot{V}(t)\le \tilde{\xi}^T(t)\Lambda\tilde{\xi}(t)$$

In the following, we will prove that $\gamma^{-1}e^T(t)e(t)-\gamma v^T(t)v(t)+\dot{V}(t)<0$, that is $\Lambda<0$. Using the Schur complement, we can see that $\Lambda<0$ is equivalent to

$$\begin{bmatrix}\Omega_{11} & 0 & R_1 & \Omega_{14} & P\Psi_3 & \Psi_1^T & \Psi_1^T & \widetilde{\Psi}_1^T\\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \Omega_{33} & R_2 & 0 & 0 & 0 & 0\\ * & * & * & -2R_2 & 0 & \Omega_{46} & \Omega_{46} & \Omega_{48}\\ * & * & * & * & -\gamma I & \Psi_3^T & \Psi_3^T & 0\\ * & * & * & * & * & \Omega_{66} & 0 & 0\\ * & * & * & * & * & * & \Omega_{77} & 0\\ * & * & * & * & * & * & * & -\gamma I\end{bmatrix}<0 \tag{21}$$

where

$$\begin{aligned}\Omega_{11}&=P\Psi_1+\Psi_1^TP+Q_1+Q_2-R_1-R_2\\ \Omega_{14}&=R_2+P\Psi_2\widetilde{W}_\theta K\\ \Omega_{33}&=-Q_2-R_1-R_2\\ \Omega_{46}&=K^T\widetilde{W}_\theta\Psi_2^T\\ \Omega_{48}&=K^T\widetilde{W}_\theta\widetilde{\Psi}_2^T\\ \Omega_{66}&=-\eta_M^{-2}R_1^{-1}\\ \Omega_{77}&=-(\eta_M-\tau_m)^{-2}R_2^{-1}\end{aligned}$$

Pre- and post-multiply (21) by $diag(P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, I, I)$ and $diag(P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, I, I)$, define $P^{-1}=W$, $P^{-1}R_1P^{-1}=\widetilde{R}_1$, $P^{-1}R_2P^{-1}=\widetilde{R}_2$, $P^{-1}Q_1P^{-1}=\widetilde{Q}_1$, $P^{-1}Q_2P^{-1}=\widetilde{Q}_2$, $KP^{-1}=V$, then (21) is equivalent to

$$\begin{bmatrix}\widetilde{\Omega}_{11} & 0 & \widetilde{R}_1 & \widetilde{\Omega}_{14} & \Psi_3 & W\Psi_1^T & W\Psi_1^T & W\widetilde{\Psi}_1^T\\ * & -\widetilde{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \widetilde{\Omega}_{33} & \widetilde{R}_2 & 0 & 0 & 0 & 0\\ * & * & * & -2\widetilde{R}_2 & 0 & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{48}\\ * & * & * & * & -\gamma I & \Psi_3^T & \Psi_3^T & 0\\ * & * & * & * & * & \Omega_{66} & 0 & 0\\ * & * & * & * & * & * & \Omega_{77} & 0\\ * & * & * & * & * & * & * & -\gamma I\end{bmatrix}<0 \tag{22}$$

where $\widetilde{\Omega}_{11}$, $\widetilde{\Omega}_{14}$, $\widetilde{\Omega}_{33}$, $\widetilde{\Omega}_{46}$, $\widetilde{\Omega}_{48}$ are the same as the ones in (10), Ω_{66} and Ω_{77} are the same as the ones in (21).

On the other hand, for symmetric positive definite matrices P and R_i $(i=1,\ 2)$, we have $(P-R_i)R_i^{-1}(P-R_i)\ge 0$, which is equivalent to $-R_i^{-1}\le P^{-1}R_iP^{-1}-2P^{-1}$. From the definition of $\widetilde{R}_i$ and W, we can see that if (10) is satisfied, (22) is also feasible. Then, for any nonzero $\tilde{\xi}(t)$, if (10) is satisfied, we have $\gamma^{-1}e^T(t)e(t)-\gamma v^T(t)v(t)+\dot{V}(t)<0$. From $\gamma^{-1}e^T(t)e(t)-\gamma v^T(t)v(t)+\dot{V}(t)<0$ and the definition of H_∞, it is easy to prove that the augmented closed-loop system described by (6) with $K=VW^{-1}$ is asymptotically stable with H_∞ output tracking performance γ, this completes the proof. ■

In the following, we will design H_∞ output tracking controllers for NCSs with both sensor-to-controller and controller-to-actuator communication constraints.

B. NCSs with both controller-to-actuator and sensor-to-controller communication constraints

For convenience of understanding, define $J_i=[\underbrace{0\ \cdots\ 0\ 1}_{i}\ \cdots\ 0]$, $\tilde{\delta}_i=[\underbrace{0\ \cdots\ 0\ \delta_iI}_{i}\ \cdots\ 0]$, where δ_i is the same as the one defined in Section 2.

Theorem 2. For given scalars η_M and τ_m, if there exist symmetric positive definite matrices P_i $(i=1,\ 2,\ \cdots,\ n+l)$, $\widetilde{Q}_1$, $\widetilde{Q}_2$, $\widetilde{R}_1$, $\widetilde{R}_2$, matrices V_i $(i=1,\ 2,\ \cdots,\ n+l)$, scalar $\gamma>0$, such that the following LMIs hold for every feasible value of $\widetilde{W}_\theta$ and $\tilde{\delta}_i$

$$\begin{bmatrix}\widetilde{\Omega}_{11} & 0 & \widetilde{R}_1 & \widetilde{\Omega}_{14} & \Psi_3 & \widetilde{\Omega}_{16} & \widetilde{\Omega}_{16} & \widetilde{\Omega}_{18}\\ * & -\widetilde{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \widetilde{\Omega}_{33} & \widetilde{R}_2 & 0 & 0 & 0 & 0\\ * & * & * & -2\widetilde{R}_2 & 0 & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{48}\\ * & * & * & * & -\gamma I & \Psi_3^T & \Psi_3^T & 0\\ * & * & * & * & * & \widetilde{\Omega}_{66} & 0 & 0\\ * & * & * & * & * & * & \widetilde{\Omega}_{77} & 0\\ * & * & * & * & * & * & * & -\gamma I\end{bmatrix}<0 \tag{23}$$

where

$$\begin{aligned}\widetilde{\Omega}_{11}&=\Psi_1\sum_{i=1}^{n+l}J_i^TP_iJ_i+\sum_{i=1}^{n+l}J_i^TP_iJ_i\Psi_1^T+\widetilde{Q}_1+\widetilde{Q}_2-\widetilde{R}_1-\widetilde{R}_2\\ \widetilde{\Omega}_{14}&=\widetilde{R}_2+\Psi_2\widetilde{W}_\theta\sum_{i=1}^{n+l}V_i\tilde{\delta}_i\\ \widetilde{\Omega}_{16}&=\sum_{i=1}^{n+l}J_i^TP_iJ_i\Psi_1^T\\ \widetilde{\Omega}_{18}&=\sum_{i=1}^{n+l}J_i^TP_iJ_i\widetilde{\Psi}_1^T\\ \widetilde{\Omega}_{33}&=-\widetilde{Q}_2-\widetilde{R}_1-\widetilde{R}_2\\ \widetilde{\Omega}_{46}&=(\Psi_2\widetilde{W}_\theta\sum_{i=1}^{n+l}V_i\tilde{\delta}_i)^T\\ \widetilde{\Omega}_{48}&=(\widetilde{\Psi}_2\widetilde{W}_\theta\sum_{i=1}^{n+l}V_i\tilde{\delta}_i)^T\\ \widetilde{\Omega}_{66}&=\eta_M^{-2}(\widetilde{R}_1-2\sum_{i=1}^{n+l}J_i^TP_iJ_i)\\ \widetilde{\Omega}_{77}&=(\eta_M-\tau_m)^{-2}(\widetilde{R}_2-2\sum_{i=1}^{n+l}J_i^TP_iJ_i)\end{aligned}$$

then with the control law

$$u(t)=\widetilde{W}_\theta K\widetilde{W}_\delta\xi(t-\tau(t)),\ K=[V_1P_1^{-1}\ V_2P_2^{-1}\ \cdots\ V_{n+l}P_{n+l}^{-1}] \tag{24}$$

the augmented closed-loop system described by (8) is asymptotically stable with H_∞ output tracking performance γ.

Proof: Similar to the proof of Theorem 1, we can see that if (25) is satisfied, the augmented closed-loop system described by (8) is asymptotically stable with H_∞ output tracking performance γ.

$$\begin{bmatrix}\widetilde{\Omega}_{11} & 0 & \widetilde{R}_1 & \widetilde{\Omega}_{14} & \Psi_3 & W\Psi_1^T & W\Psi_1^T & W\widetilde{\Psi}_1^T\\ * & -\widetilde{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \widetilde{\Omega}_{33} & \widetilde{R}_2 & 0 & 0 & 0 & 0\\ * & * & * & -2\widetilde{R}_2 & 0 & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{46} & \widetilde{\Omega}_{48}\\ * & * & * & * & -\gamma I & \Psi_3^T & \Psi_3^T & 0\\ * & * & * & * & * & \widetilde{\Omega}_{66} & 0 & 0\\ * & * & * & * & * & * & \widetilde{\Omega}_{77} & 0\\ * & * & * & * & * & * & * & -\gamma I\end{bmatrix}<0 \tag{25}$$

where

$$\begin{aligned}
\widetilde{\Omega}_{11} &= \Psi_1 W + W\Psi_1^T + \widetilde{Q}_1 + \widetilde{Q}_2 - \widetilde{R}_1 - \widetilde{R}_2 \\
\widetilde{\Omega}_{14} &= \widetilde{R}_2 + \Psi_2 \widetilde{W}_\theta K \widetilde{W}_\delta P^{-1} \\
\widetilde{\Omega}_{33} &= -\widetilde{Q}_2 - \widetilde{R}_1 - \widetilde{R}_2 \\
\widetilde{\Omega}_{46} &= P^{-1}\widetilde{W}_\delta K^T \widetilde{W}_\theta \Psi_2^T \\
\widetilde{\Omega}_{48} &= P^{-1}\widetilde{W}_\delta K^T \widetilde{W}_\theta \widetilde{\Psi}_2^T \\
\widetilde{\Omega}_{66} &= \eta_M^{-2}(\widetilde{R}_1 - 2W) \\
\widetilde{\Omega}_{77} &= (\eta_M - \tau_m)^{-2}(\widetilde{R}_2 - 2W)
\end{aligned}$$

Suppose $P^{-1} = diag(P_1,\ P_2,\ \cdots,\ P_{n+l})$, $K = [K_1\ K_2\ \cdots\ K_{n+l}]$, define $K_iP_i = V_i$ $(i = 1,\ 2,\ \cdots,\ n+l)$. Considering $\widetilde{W}_\delta = diag(\delta_1,\ \delta_2,\ \cdots,\ \delta_{n+l})$, we have

$$\begin{aligned}
&\Psi_2 \widetilde{W}_\theta K \widetilde{W}_\delta P^{-1} \\
&= \Psi_2 \widetilde{W}_\theta \{V_1[\delta_1 I\ \ 0\ \cdots\ 0] + V_2[0\ \delta_2 I\ \ 0\ \cdots\ 0] \\
&\quad + \cdots + V_{n+l}[0\ \cdots\ 0\ \ \delta_{n+l} I]\} \\
&= \Psi_2 \widetilde{W}_\theta \sum_{i=1}^{n+l} V_i \tilde{\delta}_i
\end{aligned}$$

where $\tilde{\delta}_i = [\underbrace{0\ \cdots\ 0\ \delta_i I}_{i}\ \ \cdots\ \ 0]$. Similarly, we have $P^{-1}\widetilde{W}_\delta K^T \widetilde{W}_\theta \Psi_2^T = (\Psi_2 \widetilde{W}_\theta \sum_{i=1}^{n+l} V_i \tilde{\delta}_i)^T$, $P^{-1}\widetilde{W}_\delta K^T \widetilde{W}_\theta \widetilde{\Psi}_2^T = (\widetilde{\Psi}_2 \widetilde{W}_\theta \sum_{i=1}^{n+l} V_i \tilde{\delta}_i)^T$.

On the other hand, from the proof of Theorem 1, we can see that $W = P^{-1} = diag(P_1,\ P_2,\ \cdots,\ P_{n+l}) = \sum_{i=1}^{n+l} J_i^T P_i J_i$, where $J_i = [\underbrace{0\ \cdots\ 0\ 1}_{i}\ \ \cdots\ \ 0]$, substitute W, $\Psi_2 \widetilde{W}_\theta K \widetilde{W}_\delta P^{-1}$, $P^{-1}\widetilde{W}_\delta K^T \widetilde{W}_\theta \Psi_2^T$, $P^{-1}\widetilde{W}_\delta K^T \widetilde{W}_\theta \widetilde{\Psi}_2^T$ of (25) by $\sum_{i=1}^{n+l} J_i^T P_i J_i$, $\Psi_2 \widetilde{W}_\theta \sum_{i=1}^{n+l} V_i \tilde{\delta}_i$, $(\Psi_2 \widetilde{W}_\theta \sum_{i=1}^{n+l} V_i \tilde{\delta}_i)^T$, $(\widetilde{\Psi}_2 \widetilde{W}_\theta \sum_{i=1}^{n+l} V_i \tilde{\delta}_i)^T$, respectively, we can see that (25) is equivalent to (23), this completes the proof. ∎

In the following, we will show the effectiveness of the proposed controller design methods for NCSs with communication constraints.

IV. Numerical Examples

Example 1. To illustrate the effectiveness of the proposed H_∞ output tracking controller design, we present an open loop unstable system as follows

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -0.1999 & 0.4078 \\ 0.3450 & 0.3560 \end{bmatrix} x(t) + \begin{bmatrix} -0.8020 \\ 0.1287 \end{bmatrix} \omega(t) \\
&\quad + \begin{bmatrix} 0.6451 & 0.5954 & -0.0099 \\ 0.3343 & -0.6012 & -0.0784 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} -0.5282 & 0.7076 \end{bmatrix} x(t) \\
&\quad + \begin{bmatrix} -0.4025 & -0.3986 & 0.0576 \end{bmatrix} u(t)
\end{aligned} \tag{26}$$

The reference model is described as follows

$$\begin{aligned}
\dot{x}_r(t) &= -x_r(t) + r(t) \\
y_r(t) &= 0.8x_r(t)
\end{aligned} \tag{27}$$

TABLE I

THE H_∞ OUTPUT TRACKING PERFORMANCE BOUNDS

Theorem 1	Theorem 2
0.5411	6.3140

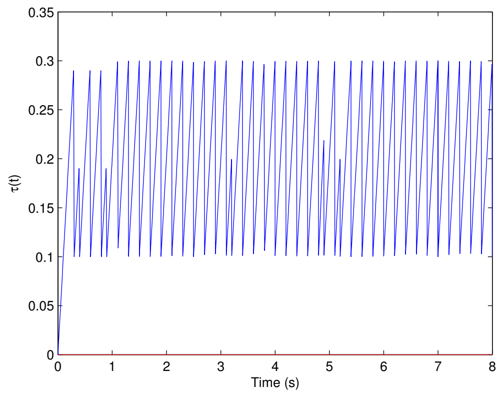

Fig. 1. $\tau(t)$

In this example, we suppose the sampling period $h = 0.1s$, $\eta_M = 0.36s$, $\tau_m = 0.06s$, $\tau_m \le \tau_k \le \tau_M$, the maximum number of consecutive packet dropout is 1, the initial states of the augmented closed-loop systems (6) and (8) are $\xi_0 = [0\ \ 0\ \ 0]^T$.

If there exist controller-to-actuator communication constraints, we suppose $\widetilde{W}_\theta$ may switch between $\widetilde{W}_{\theta 1} = diag(1,\ 1,\ 0)$ and $\widetilde{W}_{\theta 2} = diag(0,\ 1,\ 1)$, if there exist sensor-to-controller communication constraints, we suppose $J_1 = [1,\ 0,\ 0]$, $J_2 = [0,\ 1,\ 0]$, $J_3 = [0,\ 0,\ 1]$, and $\tilde{\delta}_1$, $\tilde{\delta}_2$, $\tilde{\delta}_3$ may switch between $\tilde{\delta}_1 = [1,\ 0,\ 0]$, $\tilde{\delta}_2 = [0,\ 1,\ 0]$, $\tilde{\delta}_3 = [0,\ 0,\ 0]$ and $\tilde{\delta}_1 = [0,\ 0,\ 0]$, $\tilde{\delta}_2 = [0,\ 1,\ 0]$, $\tilde{\delta}_3 = [0,\ 0,\ 1]$.

The H_∞ output tracking performance bounds corresponding to different cases are shown in Table 1. From Table 1, we can see that the less the communication constraints, the better the H_∞ output tracking performance. Table 1 illustrates the effectiveness of the proposed controller design for NCSs

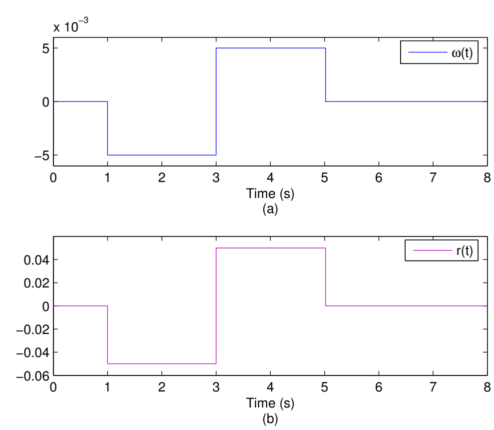

Fig. 2. $\omega(t)$ and $r(t)$

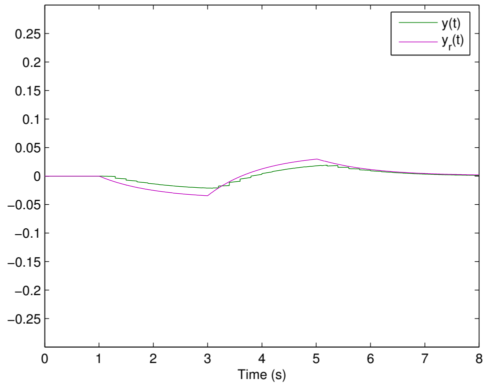

Fig. 3. Outputs $y(t)$ and $y_r(t)$

with communication constraints.

Solving the LMIs in Theorem 1, we can get the controller gain

$$K = \begin{bmatrix} -0.7608 & -0.9845 & -0.3489 \\ -0.3271 & 1.3961 & -0.5978 \\ 7.4717 & 9.6679 & 3.4261 \end{bmatrix}$$

If the system with only controller-to-actuator communication constraints is considered, suppose the sum of time delay and packet dropout $\tau(t)$ is given in Fig. 1, the disturbance input $\omega(t)$ and reference input $r(t)$ are give in Fig. 2 (a) and Fig. 2 (b), respectively, the controller-to-actuator communication constraints matrix $\widetilde{W}_\theta = \widetilde{W}_{\theta 1} = diag(1,\ 1,\ 0)$, then the output $y(t)$ of the system (26) and $y_r(t)$ of the reference model (27) are pictured in Fig. 3, which illustrates the effectiveness of the proposed H_∞ output tracking controller design.

V. Conclusions

In this paper, the problem of H_∞ output tracking control for NCSs with communication constraints has been investigated. By using the continuous Jensen inequality, LMI-based H_∞ output tracking controller design for nominal NCSs with controller-to-actuator communication constraints is presented, the results are also extended to NCSs with both controller-to-actuator and sensor-to-controller communication constraints. Numerical examples have illustrated the effectiveness of the proposed H_∞ output tracking controller design for NCSs with communication constraints.

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