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Adaptive Control of a Robotic System Undergoing a Non-Contact to Contact Transition with a Viscoelastic Environment

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***Abstract*— Control of a robot interacting with a soft compliant environment is a practically important problem, with potential applications in areas involving human robot interaction (HRI) like rehabilitation, search and rescue, assistive robotics and haptics. The objective, in this paper, is to control a robot as it transitions from a non-contact to a contact state with an unactuated viscoelastic mass-spring system such that the mass-spring is regulated to a desired final position. Because of its simplicity and better physical consistency in explaining the behavior of viscoelastic materials, a Hunt-Crossley nonlinear model is used to represent the viscoelastic contact dynamics. An adaptive Lyapunov based controller is proposed, and shown to guarantee uniformly ultimately bounded (UUB) regulation of the system despite parametric uncertainty throughout the robot and mass-spring systems. The proposed controller only depends on the position and velocity terms, and hence, obviates the need for measuring the impact force and acceleration. Further, the resulting controller is continuous, and the same controller can be used for both non-contact and contact states of the robot with its environment.**

I. INTRODUCTION

Applications such as robot interaction with human tissue in clinical procedures and rehabilitative tasks [1]–[4], cell manipulation [5]–[7], finger manipulation [8]–[11], and etc. [12]–[15] provide practical motivation to study robotic interaction with viscoelastic materials. Motivation is also provided by the complexity of modeling the stress and strain relationships at the contact interface and also by the need to compensate for these effects in a closed-loop controller.

In the last two decades, research efforts have focused on modeling and analyzing the contact with stiff and compliant surfaces. The first contact model, proposed by Hertz [16] over a century ago, used an elastic spring model to relate the impact force to the local indentation. However, the Hertzian contact model does not account for the energy loss effects at contact, and hence, is not suitable for robotic contact with soft environments. Thereafter, a number of contact models were developed which included a damping term to account for energy dissipation at impact such as the linear Kelvin-Voigt spring-damper model [17] and the impact-pair model in [18]. Hunt and Crossley [19] showed the inadequacy of the linear damping models to represent the physical nature of the energy transfer process. They proposed a compliant contact model with nonlinear damping, which eliminated the discontinuous impact forces at initial contact and separation. The literature survey in [20] concluded that a contact model with nonlinear damping (e.g., Hunt-Crossley) was the most suitable for representing the real behavior of the system during impact. The Hunt-Crossley model has found acceptance in the scientific community and is now being used by a number of researchers [20]- [23]. Marhefka and Orin [21] used the Hunt-Crossley model for the simulation of robotic systems and considered a foot contact problem in a walking machine to demonstrate the utility of the model for tasks involving non-contact to contact state transitions. Recently, the Hunt-Crossley model was used in [23] an online estimation algorithm to estimate the mechanical impedance parameters during contact. The researchers in [23] used a soft silicone gel that exhibited substantial energy dissipation to experimentally prove their result.

In addition to modeling the contact interaction between the robot and its environment, another challenge is to effectively compensate for the non-contact to contact transition with a closed-loop robot controller. Control in the presence of a non-contact to contact transition is challenging because the impact forces can be destabilizing, especially for high-speed transitions, and due to the inevitable uncertainty in the interaction. Based on these motivating factors, a variety of techniques have been developed to control the robot motion in the presence of a contact transition [24]- [38].

In this paper, Lyapunov-based adaptive backstepping methods are used to control a robot from a non-contact initial state to a final contact state that requires the robot to collide with a viscoelastic object and regulate it to a desired position. A contribution of this work with respect to previous literature is that a single continuous controller is used for the robot in non-contact, during the transition from non-contact to contact, and then after contact when the two systems are coupled. Another advantage is that only position and velocity measurements are required and no acceleration or force measurements are used in the design of the controller. Moreover, the regulation result is achieved despite the fact that the parametrically uncertain robot collides with a parametrically uncertain Hunt-Crossley viscoelastic material. The Hunt-Crossley model used in this paper differs from the linear elastic models used in our previous work in [35]–[37] where the impact was considered with a stiff environment with no damping at contact. The use of stiffness and damping terms in the model is motivated by the fact that the contact is assumed to exhibit energy dissipation at impact. The differences in the contact models result in

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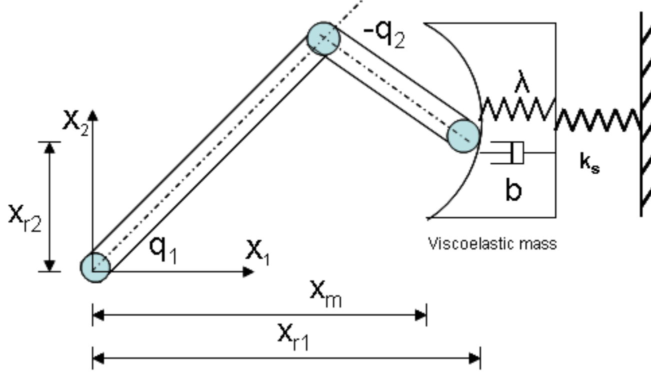

Fig. 1. Robot contact with a viscoelastic mass.

differences in the control development/stability analysis with regard to [35]–[37]. Specifically, hyperbolic trigonometric terms can be eliminated from the control design and stability analysis. In addition, the complicated two stage analysis required to prove the results in [35]–[37] has been eliminated. The control structure in this paper includes a desired robot velocity as a virtual control input to the unactuated viscoelastic mass-spring system, coupled with a control force input to ensure the actual robot position tracks the desired position. Adaptive update laws are designed to compensate for parametric uncertainty in the robot and the Hunt-Crossley model (i.e. stiffness and damping coefficients).

II. Dynamic Model

The development in this paper is motivated by the academic problem illustrated in Fig. 1. While the subsequent control design and stability analysis is developed for a two degree-of-freedom (DOF) system in a planar Cartesian-space, the underlying mathematics can be extended to higher order systems without additional constraints. The dynamic model for a rigid two-link revolute robot in contact with a compliant viscoelastic environment is given by

$$M\left(x_r\right)\ddot{x}_r + C\left(x_r,\dot{x}_r\right)\dot{x}_r + h(x_r) + \begin{bmatrix} F_m \\ 0 \end{bmatrix} = F \quad (1)$$

$$m\ddot{x}_m + k_s(x_m - x_0) = F_m. \quad (2)$$

In (1), $x_r(t)$, $\dot{x}_r(t)$, $\ddot{x}_r(t) \in \mathbb{R}^2$ represent the planar Cartesian position, velocity, and acceleration of the robot links respectively, $M(x_r) \in \mathbb{R}^{2\times 2}$ represents the uncertain inertia matrix, $C(x_r, \dot{x}_r) \in \mathbb{R}^{2\times 2}$ represents the uncertain Centripetal-Coriolis effects, $h(x_r) \in \mathbb{R}^2$ represents uncertain conservative forces (e.g., gravity), $F_m(x_r, \dot{x}_r, x_m, \dot{x}_m) \in \mathbb{R}$ denotes the interaction force between the robot and the mass during impact, and $F(t) \in \mathbb{R}^2$ represents the force control inputs. In (2), $x_m(t), \ddot{x}_m(t) \in \mathbb{R}$ represent the displacement and acceleration of the unknown viscoelastic mass $m \in \mathbb{R}$, $x_0 \in \mathbb{R}$ represents the initial undisturbed position of the mass, and $k_s \in \mathbb{R}$ represents the unknown stiffness of the spring connected to the mass. When the horizontal position of the robot, denoted by $x_{r1}(t) \in \mathbb{R}$, is greater than or equal to the position of the viscoelastic material, x_m (i.e., when $x_{r1}(t) \geq x_m(t)$, see Fig. 1) contact occurs, and the interaction force $F_m(x_{r1}, \dot{x}_{r1}, x_m, \dot{x}_m)$ is modeled as

$$F_m = \Lambda F_v, \quad (3)$$

where $\Lambda(x_{r1}, x_m) \in \mathbb{R}$, a function which switches at impact, is defined as

$$\Lambda \triangleq \begin{cases} 0 & x_{r1} < x_m \\ 1 & x_{r1} \geq x_m, \end{cases} \quad (4)$$

and $F_v(x_{r1}, \dot{x}_{r1}, x_m, \dot{x}_m) \in \mathbb{R}$ denotes the Hunt-Crossley force defined as [19]

$$F_v \triangleq \lambda\delta^n + b\dot{\delta}\delta^n. \quad (5)$$

In (5), $\lambda \in \mathbb{R}$ is the contact stiffness of the viscoelastic mass, $b \in \mathbb{R}$ is the impact damping coefficient, $\delta(x_r, x_m) \in \mathbb{R}$ denotes the local deformation of the material and is defined as

$$\delta \triangleq x_{r1} - x_m. \quad (6)$$

Also, in (5), $\dot{\delta}(t)$ is the relative velocity of the contacting bodies, and n is the Hertzian compliance coefficient which depends on the surface geometry of contact. The model in (3) is a continuous contact force-based model wherein the contact forces increase from zero upon impact and return to zero upon separation. Also, the energy dissipation during impact is a function of the damping constant which can be related to the impact velocity and the coefficient of restitution [19], thus making the model more consistent with the physics of contact. The contact is considered to be direct-central and quasi-static (i.e., all the stresses are transmitted at the time of contact and sliding and friction effects during contact are negligible) where plastic deformation effects are assumed to be negligible.

The dynamic model in (1)-(5) exhibits the following properties that will be utilized in the subsequent analysis.

Property 1: The inertia matrix $M(x_r)$ is symmetric, positive definite, and can be lower and upper bounded as

$$a_1||\xi||^2 \leq \xi^T M \xi \leq a_2||\xi||^2, \qquad \forall \xi \in \mathbb{R}^2 \quad (7)$$

where $a_1, a_2 \in \mathbb{R}$ are positive constants and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: The following skew-symmetric relationship is satisfied

$$\xi^T(\frac{1}{2}\dot{M}(x_r) - C(x_r, \dot{x}_r))\xi = 0 \qquad \forall \xi \in \mathbb{R}^2. \quad (8)$$

Property 3: The robot dynamics in (1) can be linearly parameterized as

$$Y(x_r,\ x_m,\ \dot{x}_r,\ \dot{x}_m,\ \ddot{x}_r)\theta = M\left(x_r\right)\ddot{x}_r + C\left(x_r,\dot{x}_r\right)\dot{x}_r + h(x_r) + \begin{bmatrix} F_m \\ 0 \end{bmatrix},$$

where $\theta \in \mathbb{R}^p$ contains the constant unknown system parameters, and $Y(x_r,\ x_m,\ \dot{x}_r,\ \dot{x}_m,\ \ddot{x}_r) \in \mathbb{R}^{2\times p}$ denotes the known regression matrix.

Property 4: The expression for the interaction force $F_m(x_r,\ \dot{x}_{r,}\ x_m,\ \dot{x}_m)$ in (3) can be written, using (4) and (6), as

$$F_m = \begin{cases} 0 & \delta < 0 \\ \lambda\delta^n + b\dot{\delta}\delta^n & \delta \geq 0. \end{cases} \tag{9}$$

Based on the fact that

$$\lim_{\delta \to 0^-} F_m = 0 \qquad \lim_{\delta \to 0^+} F_m = 0, \tag{10}$$

the interaction force F_m is continuous.

Assumption 1: The robot and mass spring positions, $x_r(t)$ and $x_m(t)$, are measurable, and the corresponding velocities, $\dot{x}_r(t)$ and $\dot{x}_m(t)$, are determined numerically using the standard backwards difference algorithm. Further, it is assumed that the robot trajectory $x_r(t)$ is bounded due to the geometry of the robot.

Assumption 2: The Hertzian compliance coefficient, n, in (5), is assumed to be known.

Assumption 3: The local deformation of the viscoelastic material during contact, $\delta(x_r, x_m)$, defined in (6), is assumed to be upper bounded, and hence δ^n can be upper bounded as

$$\delta^n < \mu, \tag{11}$$

where $\mu \in \mathbb{R}$ is a positive bounding constant.

Assumption 4: We assume that the mass and the spring constant of the mass-spring system can be lower and upper bounded as

$$\underline{m} < m < \overline{m} \qquad \underline{k_s} < k_s < \overline{k_s} \tag{12}$$

where $\underline{m}$, $\overline{m}$, $\underline{k_s}$, $\overline{k_s} \in \mathbb{R}$ denote known positive bounding constants. The damping constant b, in (5), is assumed to be upper bounded as

$$b < \overline{b} \tag{13}$$

where $\overline{b} \in \mathbb{R}$ denotes a known positive bounding constant.

III. Control Development

In the subsequent control development, the desired robot velocity is designed as a virtual control input to the unactuated viscoelastic mass. The desired velocity is designed to ensure that the robot impacts and then regulates the mass to a desired position. Since it is not possible to directly control the mass trajectory, backstepping methods are then used to develop a control force input to ensure that the robot tracks the desired trajectory despite the non-contact to contact transition and parametric uncertainties in the robot and the viscoelastic mass-spring system. The viscoelastic model requires that the backstepping error be developed in terms of the desired robot velocity. A challenge to backstep on the desired robot velocity is that the desired velocity is premultiplied by $\Lambda(x_{r1}, x_m)$, which is zero when the robot and material are not in contact. Hence, a strategic combination of nonlinear damping and adaptive backstepping is used in the subsequent development.

A. Control Objective

The control objective is to regulate the position of a viscoelastic mass attached to a spring via a two-link planar robot that transitions from non-contact to contact with the mass-spring assembly through an impact collision. To quantify the control objective, the following errors are defined

$$e_r \triangleq x_{rd} - x_r \qquad e_m \triangleq x_{md} - x_m, \tag{14}$$

where $e_r(t) \triangleq [e_{r1}(t), e_{r2}(t)]^T \in \mathbb{R}^2$ and $e_m(t) \in \mathbb{R}$ denote the errors for the end-point of the second link of the robot and mass-spring system (MSR) (see Fig. 1), respectively. In (14), $x_{md} \in \mathbb{R}$ denotes the constant known desired position of the mass, and $x_{rd}(t) \triangleq [x_{rd1}(t), x_{rd2}]^T \in \mathbb{R}^2$ denotes the desired position of the end-point of the second link of the robot. To facilitate the subsequent control design and stability analysis, filtered tracking errors for the robot and the mass-spring, denoted by $r_r(t) \in \mathbb{R}^2$ and $r_m(t) \in \mathbb{R}$ respectively, are defined as

$$\begin{aligned} r_r &\triangleq \dot{e}_r + \alpha e_r \\ r_m &\triangleq \dot{e}_m + \gamma_1 e_m + \gamma_2 e_f, \end{aligned} \tag{15}$$

where $\alpha \in \mathbb{R}^{2\times 2}$ is a positive, diagonal, constant gain matrix, $\gamma_1,\ \gamma_2 \in \mathbb{R}$ are positive constant gains and $e_f\ (t) \in \mathbb{R}$ is an auxiliary filter variable designed as.

$$\dot{e}_f = -\gamma_3 e_f + \gamma_2 e_m - k_1 r_m, \tag{16}$$

where $k_1,\ \gamma_3 \in \mathbb{R}$ are positive constant control gains.

B. Closed-Loop Error System

The open-loop error system for the mass can be obtained by multiplying (15) by m and then taking its time derivative as

$$\begin{aligned} m\dot{r}_m &= \Lambda\delta^n b\theta_m + k_s d - \Lambda b\dot{\delta}\delta^n + \chi - m\gamma_1^2 e_m \\ &\quad -m\gamma_2 k_1 r_m, \end{aligned} \tag{17}$$

where (2), (3), (5), (14), (15), and (16) were used, $d = x_{md} - x_0 \in \mathbb{R}$ is a positive constant denoting the desired displacement of the mass-spring system, and $\theta_m \in \mathbb{R}$ is defined as

$$\theta_m \triangleq -\frac{\lambda}{b}. \tag{18}$$

It can be seen from (18), that θ_m contains constant unknown system parameters, λ and b. In (17), $\chi\,(e_m,\ r_m,\ e_f,\ t) \in \mathbb{R}$ is an auxiliary term defined as

$$\chi \triangleq m\gamma_1 r_m + (m\gamma_2^2 - k_s)e_m - (m\gamma_1\gamma_2 + m\gamma_2\gamma_3)e_f. \tag{19}$$

The auxiliary expression $\chi\,(e_m,\ r_m,\ e_f,\ t)$ defined in (19) can be upper bounded as

$$\chi \leq \zeta_1 \left\|z\right\| \tag{20}$$

where $\zeta_1 \in \mathbb{R}$ is a known positive constant and $z(t) \in \mathbb{R}^3$ is defined as

$$z \triangleq \left[\begin{array}{ccc} r_m & e_m & e_f \end{array}\right]. \tag{21}$$

To facilitate the subsequent backstepping-based design, the desired velocity of the robot is designed as a virtual control input to the unactuated mass-spring system as

$$\begin{aligned} \dot{x}_{rd1} &= \hat{\theta}_m + \dot{x}_m \qquad (22)\\ \dot{x}_{rd2} &= 0; \qquad x_{rd2}(0) = \varepsilon. \end{aligned}$$

In (22), $\hat{\theta}_m(t) \in \mathbb{R}$ is a parameter estimate vector for θ_m, $\varepsilon \in \mathbb{R}$ is an appropriate positive constant selected to ensure that the robot impacts the mass-spring system. The parameter estimate $\dot{\hat{\theta}}_m(t)$ is generated according to the adaptive update law

$$\dot{\hat{\theta}}_m = proj(\Lambda\delta^n\Gamma_m r_m), \qquad (23)$$

where $\Gamma_m \in \mathbb{R}$ is a positive, constant, adaptation gain, and $proj(\cdot)$ denotes a sufficiently smooth projection algorithm [39] utilized to guarantee that $\hat{\theta}_m(t)$ can be bounded as

$$\underline{\theta}_m \leq \hat{\theta}_m \leq \bar{\theta}_m, \qquad (24)$$

where $\underline{\theta}_m$, $\bar{\theta}_m \in \mathbb{R}$ denote known, positive, constant lower and upper bounds for $\hat{\theta}_m(t)$, respectively. After adding and subtracting $\Lambda b\delta^n\dot{x}_{rd1}(t)$ defined in (22) from the open-loop expression in (17), the closed-loop error system for $r_m(t)$ can be obtained as

$$\begin{aligned} m\dot{r}_m &= \Lambda\delta^n b\tilde{\theta}_m + k_s d + \Lambda b\delta^n\dot{e}_{r1} + \chi - m\gamma_1^2 e_m \qquad (25)\\ &\quad -m\gamma_2 k_1 r_m, \end{aligned}$$

where $\tilde{\theta}_m(t) \in \mathbb{R}$ denotes a parameter estimation error defined as

$$\tilde{\theta}_m \triangleq \theta_m - \hat{\theta}_m. \qquad (26)$$

In (25), $k_1 \in \mathbb{R}$ is a positive constant control gain defined as

$$\begin{aligned} k_1 &\triangleq \frac{k_{n1}}{4\gamma_2} + \frac{1}{\gamma_2} + \frac{1}{4\underline{m}\gamma_2}(\zeta_1^2 k_{n2} + k_{n3}) \qquad (27)\\ &\quad + \frac{1}{4\underline{m}\gamma_2}(\bar{b}^2\mu^2 k_{n4} + \bar{b}^2\mu^2 k_{n5}), \end{aligned}$$

where $k_{n1},\ k_{n2},\ k_{n3},\ k_{n4},\ k_{n5} \in \mathbb{R}$ are positive constant nonlinear damping gains and μ, $\underline{m}$, $\bar{b}$, γ_2 and ζ_1 have already been defined in (11)-(13), (15) and (20) respectively. The open-loop robot error system can be obtained by taking the time derivative of $r_r(t)$, premultiplying by the robot inertia matrix M, and utilizing (1), (14), and (15) as

$$M\dot{r}_r = Y_r\theta_r - Cr_r - F, \qquad (28)$$

where $Y_r(x_r, \dot{x}_r, x_m, \dot{x}_m, t)\theta_r \in \mathbb{R}^2$ is defined as

$$\begin{aligned} Y_r\theta_r &\triangleq M\ddot{x}_{rd} + \alpha M\dot{e}_r + h + C\dot{x}_{rd} + \alpha Cx_{rd} \qquad (29)\\ &\quad + \begin{bmatrix} \Lambda(\lambda\delta^n + b\dot{\delta}\delta^n) \\ 0 \end{bmatrix} - \alpha Cx_r, \end{aligned}$$

where $Y_r(x_r, \dot{x}_r, x_m, \dot{x}_m, t) \in \mathbb{R}^{2\times P}$ denotes a known regression matrix of measurable quantities, and $\theta_r \in \mathbb{R}^P$ denotes an unknown constant parameter vector. The appendix provides an expression for $\ddot{x}_{rd1}(t)$ to illustrate that the second derivative of the desired trajectory is continuous and does not require acceleration measurements. Based on (28) and the subsequent stability analysis, the robot force control input is designed as

$$F = Y_r\hat{\theta}_r + e_r + k_2 r_r, \qquad (30)$$

where $k_2 \in \mathbb{R}$ is a positive constant control gain, and $\hat{\theta}_r(t) \in \mathbb{R}^P$ is an estimate for θ_r generated by the following adaptive update law:

$$\dot{\hat{\theta}}_r = proj(\Gamma_r Y_r^T r_r) \qquad (31)$$

In (31), $\Gamma_r \in \mathbb{R}^{P\times P}$ is a positive definite, constant, diagonal, adaptation gain matrix, and $proj(\cdot)$ denotes a projection algorithm utilized to guarantee that the $i-th$ element of $\hat{\theta}_r(t)$ can be bounded as

$$\underline{\theta}_{ri} \leq \hat{\theta}_{ri} \leq \bar{\theta}_{ri}, \qquad (32)$$

where $\underline{\theta}_{ri}$, $\bar{\theta}_{ri} \in \mathbb{R}$ denote known, constant lower and upper bounds for each element of $\theta_r(t)$, respectively. The closed-loop error system for $r_r(t)$ can be obtained after substituting (30) into (28) as

$$M\dot{r}_r = Y_r\tilde{\theta}_r - k_2 r_r - Cr_r - e_r, \qquad (33)$$

where the parameter estimation error $\tilde{\theta}_r(t) \in \mathbb{R}^P$ is defined as

$$\tilde{\theta}_r \triangleq \theta_r - \hat{\theta}_r. \qquad (34)$$

IV. Stability Analysis

Theorem: The controller given by (22), (23), (30), and (31) ensures uniformly ultimately bounded regulation of the MSR system in the sense that

$$|e_m(t)|,\ \|e_r(t)\| \to \varepsilon_0\exp(-\varepsilon_1 t) + \varepsilon_2 \qquad (35)$$

provided k_{n3} is chosen sufficiently large and the control gains $k_{n1},\ k_{n2},\ k_{n4},\ k_{n5},\ \gamma_1$ and α are selected according to the sufficient gain conditions

$$\begin{aligned} k_{n1} &> \frac{\gamma_1^2 k_1^2}{\gamma_3}, \quad k_{n4} > \frac{1}{k_2}, \quad k_{n5} > \alpha, \qquad (36)\\ k_{n2} &> \frac{1}{\underline{m}\min\left\{\gamma_1^3,\ \left(\gamma_3\gamma_1^2 - \frac{\gamma_1^4 k_1^2}{k_{n1}}\right),\ 1\right\}}, \end{aligned}$$

where $\varepsilon_0,\ \varepsilon_1,\ \varepsilon_2 \in \mathbb{R}$ denote positive constants; $k_{n1},\ k_{n2}$, k_{n4} and k_{n5} are defined in (27); α and γ_1 are defined in (15) and k_2 is defined in (30).

Proof: Let $V(t) \in \mathbb{R}$ denote a non-negative, radially unbounded function (i.e., a Lyapunov function candidate) defined as

$$\begin{aligned} V &= \frac{1}{2}r_r^T Mr_r + \frac{1}{2}e_r^T e_r + \frac{1}{2}\tilde{\theta}_r^T\Gamma_r^{-1}\tilde{\theta}_r \qquad (37)\\ &\quad + \frac{1}{2}b\Gamma_m^{-1}\tilde{\theta}_m^2 + \frac{1}{2}mr_m^2\\ &\quad + \frac{1}{2}\gamma_1^2 me_f^2 + \frac{1}{2}\gamma_1^2 me_m^2. \end{aligned}$$

and it follows directly from the bounds given in (7), (24) and (32), that $V(r_r, e_r, e_m, e_f, r_m, \tilde{\theta}_r, \tilde{\theta}_m, t)$ can be upper and lower bounded as

$$\lambda_1 \|y\|^2 \leq V(t) \leq \lambda_2 \|y\|^2 + \eta, \tag{38}$$

where λ_1, λ_2, $\eta \in \mathbb{R}$ are known positive bounding constants, and $y(t) \in \mathbb{R}^7$ is defined as

$$y = \begin{bmatrix} r_r^T & e_r^T & z^T \end{bmatrix}^T. \tag{39}$$

After using (8), (15), (16), (23), (25), (31), and (33), the time derivative of (37) can be determined as

$$\begin{aligned} \dot{V} \ = \ & r_r^T(Y_r\tilde{\theta}_r - k_2 r_r - Cr_r - e_r) + \frac{1}{2} r_r^T \dot{M} r_r \\ & + e_r^T(r_r - \alpha e_r) - \tilde{\theta}_r^T Y_r^T r_r - \Lambda\delta^n b\tilde{\theta}_m r_m \\ & + r_m(\Lambda\delta^n b\tilde{\theta}_m + k_s d + \Lambda\delta^n b\dot{e}_{r1}) \\ & + r_m(\chi - m\gamma_1^2 e_m - m\gamma_2 k_1 r_m) \\ & + \gamma_1^2 m e_f(-\gamma_3 e_f + \gamma_2 e_m - k_1 r_m) \\ & + \gamma_1^2 m e_m(r_m - \gamma_1 e_m - \gamma_2 e_f). \end{aligned} \tag{40}$$

Using (11)-(13), (12), an upper bound can be developed for $\dot{V}$ in (40) as

$$\begin{aligned} \dot{V} \ \leq \ & -k_2 r_r^T r_r - \alpha e_r^T e_r - \gamma_1^3 \underline{m} e_m^2 - \gamma_3 \gamma_1^2 m e_f^2 \\ & - m\gamma_2 k_1 r_m^2 + \zeta_1 |r_m| \|z\| + \gamma_1^2 m k_1 |r_m| |e_f| \\ & + \bar{k}_s d |r_m| + \Lambda\mu\bar{b} \|r_r\| |r_m| + \Lambda\mu\bar{b}\alpha \|e_r\| |r_m|. \end{aligned} \tag{41}$$

Using (27), the expression in (41) can be rewritten as

$$\begin{aligned} \dot{V} \ \leq \ & -k_2 \|r_r\|^2 - \alpha \|e_r\|^2 \\ & -\gamma_1^3 \underline{m} e_m^2 - \gamma_3\gamma_1^2 m e_f^2 - \underline{m} r_m^2 \\ & - \left[\frac{1}{4} m k_{n1} r_m^2 - \gamma_1^2 m k_1 |r_m| |e_f|\right] \\ & - \left[\frac{1}{4}\zeta_1^2 k_{n2} r_m^2 - \zeta_1 |r_m| \|z\|\right] \\ & - \left[\frac{1}{4} k_{n3} r_m^2 - \bar{k}_s d |r_m|\right] \\ & - \left[\frac{1}{4}\mu^2\bar{b}^2 k_{n4} r_m^2 - \Lambda\mu\bar{b} \|r_r\| |r_m|\right] \\ & - \left[\frac{1}{4}\mu^2\bar{b}^2 k_{n5} r_m^2 - \Lambda\mu\bar{b}\alpha \|e_r\| |r_m|\right]. \end{aligned} \tag{42}$$

Completing the squares on the bracketed terms in (42) and using the gain condition on k_{n1} in (36), the expression can be reduced to

$$\begin{aligned} \dot{V} \ \leq \ & -(k_2 - \frac{\Lambda}{k_{n4}}) \|r_r\|^2 - (\alpha - \frac{\Lambda\alpha^2}{k_{n5}}) \|e_r\|^2 \\ & - \left[\underline{m} \min\left\{ \begin{array}{c} \gamma_1^3, \\ \gamma_3\gamma_1^2 - \frac{\gamma_1^4 k_1^2}{k_{n1}}, \\ 1 \end{array} \right\} - \frac{1}{k_{n2}} \right] \|z\|^2 \\ & + \frac{\bar{k}_s^2 d^2}{k_{n3}}. \end{aligned} \tag{43}$$

The expression in (43) can be further upper bounded as

$$\dot{V} \leq -\beta_1 \|y\|^2 + \frac{\bar{k}_s^2 d^2}{k_{n3}}, \tag{44}$$

where $\beta_1 \in \mathbb{R}$, is defined as

$$\beta_1 \triangleq \min\left\{ \begin{array}{c} (k_2 - \frac{\Lambda}{k_{n4}}),\ (\alpha - \frac{\Lambda\alpha^2}{k_{n5}}), \\ \left(\underline{m} \min\left\{ \begin{array}{c} \gamma_1^3, \\ \gamma_3\gamma_1^2 - \frac{\gamma_1^4 k_1^2}{k_{n1}}, \\ 1 \end{array} \right\} - \frac{1}{k_{n2}} \right) \end{array} \right\},$$

Since the inequality in (38) can be utilized to lower bound $\|y\|^2$ as

$$\|y\|^2 \geq \frac{V(t)}{\lambda_2} - \frac{\eta}{\lambda_2}, \tag{45}$$

the inequality in (44) can be expressed as

$$\dot{V}(t) \leq -\frac{\beta_1}{\lambda_2} V(t) + \varepsilon_x, \tag{46}$$

where $\varepsilon_x \in \mathbb{R}$ is a positive constant defined as

$$\varepsilon_x \triangleq \frac{\bar{k}_s^2 d^2}{k_{n3}} + \frac{\beta_1 \eta}{\lambda_2} \tag{47}$$

The linear differential inequality in (46) can be solved as

$$V(t) \leq V(0) e^{(-\frac{\beta_1}{\lambda_2})t} + \varepsilon_x \frac{\lambda_2}{\beta_1}\left[1 - e^{(-\frac{\beta_1}{\lambda_2})t}\right]. \tag{48}$$

The inequalities in (38) can now be used along with (47) and (48) to conclude that

$$\|y\|^2 \leq \left[\frac{\lambda_2 \|y(0)\|^2 + \eta}{\lambda_1}\right] e^{(-\frac{\beta_1}{\lambda_2})t} + \left(\frac{\bar{k}_s^2 d^2 \lambda_2}{k_{n3}\lambda_1\beta_1} + \frac{\eta}{\lambda_1}\right). \tag{49}$$

Provided the gain conditions in (36) are satisfied and k_{n3} is chosen sufficiently large, the definitions in (21) and (39), and the expressions in (48) and (49) can be used to prove that $r_r(t)$, $e_r(t)$, $r_m(t)$, $e_m(t)$, $e_f(t) \in \mathcal{L}_\infty$. The parameter error estimates $\tilde{\theta}_r(t)$, $\tilde{\theta}_m(t) \in \mathcal{L}_\infty$, from (24), (26), (32) and (34). Since $e_m(t) \in \mathcal{L}_\infty$, it can be shown from (14), that $x_m(t) \in \mathcal{L}_\infty$. Since $x_r(t) \in \mathcal{L}_\infty$ from assumption 1 and $e_r(t) \in \mathcal{L}_\infty$, it can be shown that $x_{rd1}(t) \in \mathcal{L}_\infty$. Since $r_m(t)$, $e_m(t)$, $e_f(t) \in \mathcal{L}_\infty$, it can be shown from (15) that $\dot{e}_m(t) \in \mathcal{L}_\infty$, and hence $\dot{x}_m(t) \in \mathcal{L}_\infty$ from (14). Due to the fact that $\hat{\theta}_m(t)$, $\dot{x}_m(t) \in \mathcal{L}_\infty$, it can be shown from (22) that $\dot{x}_{rd1}(t) \in \mathcal{L}_\infty$. Since $r_r(t) \in \mathcal{L}_\infty$, linear analysis methods can be used to prove that $\dot{e}_r(t) \in \mathcal{L}_\infty$. Because $\dot{e}_r(t)$, $\dot{x}_{rd1}(t) \in \mathcal{L}_\infty$, (14) can be used to show that $\dot{x}_{r1}(t) \in \mathcal{L}_\infty$. Since $x_{r1}(t), x_m(t), \dot{x}_{r1}(t), \dot{x}_m(t)$, $r_m(t) \in \mathcal{L}_\infty$, it can be shown that $\ddot{x}_{rd1}(t) \in \mathcal{L}_\infty$. It can be further seen from (22) that x_{rd2}, $\dot{x}_{rd2}$, $\ddot{x}_{rd2} \in \mathcal{L}_\infty$.Since $x_r(t), x_m(t), \dot{x}_r(t), \dot{x}_m(t), x_{rd}(t)$, $\dot{x}_{rd}(t)$, $\ddot{x}_{rd}(t)$, $\dot{e}_r(t) \in \mathcal{L}_\infty$, the expression in (29) can be used to prove that that $Y_r(t) \in \mathcal{L}_\infty$. Due to the fact the Y_r, $\hat{\theta}_r$, e_r, $r_r \in \mathcal{L}_\infty$, the control force $F \in \mathcal{L}_\infty$. The result in (35) can be directly obtained from (49).

V. CONCLUSION

In this paper, an adaptive continuous Lyapunov-based controller is proven to control a parametrically uncertain robot moving in free-space through a collision with an uncertain Hunt-Crossley viscoelastic material so that the coupled system converges to a desired setpoint. This result extends our previous work in this area to include a more general contact model, which not only accounts for stiffness but also damping at contact. A restriction of the current work is that the Hertzian compliance coefficient is required to be known. A contribution of the work is that a continuous controller is used to obtain a uniformly ultimately bounded regulation result, and the developed controller does not require force or acceleration measurements. An experimental testbed is currently being developed with a two link direct drive robot that will collide with a human tissue phantom to test the performance of the developed controller.

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