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Phase Transition in Complex Networks

Dan Wang, Na Cai, Yuanwei Jing, Siying Zhang

***Abstract*—We investigate numerically the influence of complex network topological structure on the traffic delivery, by incorporating local traffic information into the shortest path routing policy. As free traffic delivery on the communication networks is important to their efficient functioning, we find the network capacity that can be measured by the critical value of phase transition from free flow to congestion. Here, we present three kinds of traffic models based on the information processing capacity of individual nodes to study three kinds of networks, respectively.**

I. INTRODUCTION

MODERN society increasingly depends on large commu-nication networks such as the Internet and WWW [1]-[2]. The need for information spreading pervades our lives and its efficient handling and delivery is becoming one of the most important practical problems. To ensure uncongested traffic flows on a complex network is naturally of great interest. Our particular interest is to understand under what conditions traffic congestion can occur on a complex network and to explore possible ways of control to alleviate the congestion.

The past few years have witnessed a hectic activity devoted to the characterization and understanding of networked structures as diverse as ecological and biological systems or the Internet and the WWW. These networks generally exhibit complex topological properties such as the small-world phenomenon [3] and scale-free behavior [4]. It is thus of paramount interest to study the effect of network topology on traffic flow [5]-[9], which is the key feature that distinguishes our work from the existing ones. While our model is for communication networks, we expect it to be relevant to other practical networks in general, such as the postal service network or the airline transportation network. Our studies may be useful for designing communication protocols for complex networks.

In this paper, we construct three kinds of traffic models to study the influence of network topological structure on the traffic delivery by the traffic awareness routing strategy. In the first model, the capacity of packet delivery of each node is proportional to its degree; in the second model, it is proportional to the number of shortest paths passing through the nodes, while in the third model, it is proportional to the ratio of the nodes betweenness to the total number of the nodes in the network. The quantity of interest is the critical value R_c of information generations as measured by the number of packets created within the network in unit timed at which a phase transition occurs from free to congested traffic flow. Simulation results show that, in the case of identical average degree, small-world network with small connecting probability p is significantly more susceptible to traffic cong-estion than random networks and scale-free networks in model I, while the capacities of all kinds of networks are enhanced greatly in model II, especially for WS small-world network. For protocol based on model III, the critical value R_c is roughly the same for all kinds of networks.

The structure of the paper is as follows: The network model is described in Sec. II. In Sec. III, the traffic awareness routing strategy is introduced with the detailed dynamic process of packet transportation. Simulations and discussions are presented in Sec. IV. The conclusion is given in Sec. V.

II. NETWORK MODEL

It would be natural to regard the real network as a random graph at first sight. However, real networks show statistical properties that are far from being completely random. In this paper, we grow three different network topologies for comparison.

The random graph is constructed according to the well known model proposed by Erdös and Rényi [10]. A network with N labeled vertices is connected by M edges, which are connected each couple of nodes with a probability $0 < p < 1$. Also, we grow a small-world model introduced by Watts and Strogatz [3]. The model is based on a rewiring procedure of the edges implemented with probability p. The starting point is a N nodes ring, in which each node is symmetrically connected to its $2m$ nearest neighbors for a total of $K = 2m$ edges. Then, for every node, each link connected to a clockwise neighbor is rewired to a randomly chosen node with probability p. Next, we grow a scale-free model with tunable clustering introduced by Holme and Kim [11]. The model modifies the Barabási-Albert (BA) algorithm [4] by adding an additional step: triangle formation with probability p. And the model also shows the power-law degree distri

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Dan Wang is with the School of information Science and Engineering, Northeastern University Shenyang, China. (E-mail: wangdan0307@126.com).

Yuanwei Jing is with the School of Information Science and Engineering, Northeastern University, Shenyang, China (E-mail: ywjjing@mail.neu.edu.cn).

Siying Zhang is with the School of Information Science and Engineering, Northeastern University, Shenyang, China (E-mail: syzhang@mail.neu.edu.cn).

-bution $P(k) \sim k^{-\gamma}$, with an exponent $\gamma = 3$.

III. Dynamical Process and Routing Strategy

We assume that the capacities for processing information are distinctive for different nodes, depending on the numbers of links or the number of shortest paths passing through them or the ratio of the betweenness to the number of all nodes. Our traffic-flow model is described as follows. At each time step, there are R packets generated in the network, with randomly chosen sources and destinations, and a node i delivers at most C_i packets one step toward their destinations, where $C_i = k_i$ in model I, $C_i = B_i$ in model II and $C_i = B_i / N$ in model III, k_i is the degree of node i, and B_i is its betweenness. Once a packet is generated, it is placed at the end of the node queue, which contains the undelivered packets created at current time steps or transmitted from the other nodes. The queue length of each node is assumed to be unlimited [12]-[13]. For simplicity, we treat all nodes as both hosts and routers for generating and delivering packets [6]. To deliver packets, each node performs a local search among its neighbors. If the packet's destination is found within the searched area, the packet is delivered directly to its target and then removed from the network. Otherwise, the next routing node is selected for the packet according to a traffic awareness routing strategy as follows.

The shortest path routing is a widely used routing strategy. This strategy is simple but has its limitation in that it takes no account of the node state it delivers to. Even if the selected node is overloaded and packets will wait a long time to be processed at that node, this routing policy makes no change. In order to make the routing policy be aware of the traffic of the network, we introduce a traffic awareness routing strategy as follows [14]-[16]. Let us assume that node s holds a packet that should be delivered to node t. We first compute the weight d_i of a neighbor node i of s. This weight, which can be viewed as the cost of each packet to pass through node i, is defined as

$$d_i = \alpha l_i + (1-\alpha) f_i \qquad 0 \le \alpha \le 1, \tag{1}$$

where l_i is the shortest path length from node i to target t, α is a tunable parameter that accounts for the degree of traffic awareness incorporated in the delivery algorithm, $f_i = q_i / c_i$ is the estimated waiting time at node i. The queued packet information q_i is changed dynamically in each time step according to the local traffic dynamics. c_i is the capacity of packet delivery of each node. After computing the weight of each neighbor node of node s, we select the next router node with the minimum weight among the neighbors. If there is more than one node with the minimum weight, we select one of them randomly. At each time step, the weight d_i will be calculated dynamically according to the current traffic information in the network and the minimum cost node is selected as the next router. By tuning the value of α, the weight of l_i and f_i will vary and the effectiveness of this routing strategy will also change. It is worth noting that, when $\alpha = 1$, the traffic awareness strategy reduces to the shortest path routing strategy.

IV. Simulations and Discussions

In order to characterize the phase transition, we introduce the order parameter:

$$H(R) = \lim_{t \to \infty} \frac{W(t+\tau) - W(t)}{\tau R}, \tag{2}$$

where $W(t)$ is the total number of packets in the network at time t, and τ is the observation time. The order parameter represents the ratio between the outflow and the inflow of packets during a time window τ. For $R < R_c$, the network is in the steady phase, where the newly created packets is less than the delivered packets. The number of packets $W(t)$ in the network is balanced, leading to a steady free traffic flow. For $R > R_c$, the number of packets $W(t)$ in the network is increased with time and will lead to traffic congestion. Therefore a phase transition occurs at $R = R_c$ and R_c is the maximal generating rate under which the system can maintain its normal and efficient functioning. In other words, the maximal handling and delivering capacity of the system is measured by R_c.

As an appropriate measure of the efficiency of the process, we monitor the aggregation of packets in the network, given by the number of packets $W(t)$ that have not reached their destinations at each time step t. Fig. 1 shows the results obtained for different values of R for the BA model with $N = 100$, $m = m_0 = 3$. In Fig. 1, the continuous line stands for values of $R < R_c$ ($R = 150$) and the dotted line corresponds to $R > R_c$ ($R = 210$, $R = 250$). As it can be seen, when the external driving is applied at low rates (i.e., small R), the protocol allow for a stationary state. In this state, the system is able to balance the in-flow of packets with the flow of packets that reach their destinations. The stationary state, where no macroscopic sign of congestion is observed, corresponds to a free flow phase. The situation changes when the rate at which new packets are introduced increases. As we can see from Fig.1, there is a critical value R_c beyond which a congested phase shows up. Let us now note that for the traffic awareness algorithm ($\alpha = 0.8$) (Fig. 1 dotted black line), when $R > R_c$ ($R = 250$), $W(t)$ grows linearly with time t. On the other hand, when R is close to the critical point R_c ($R = 210$), we observe that $W(t)$ grows at short times and then becomes constant as time goes on (Fig. 1, dash dotted red line). The state also represents steady phase. However, with the increasing of R above the critical value, the curve of $W(t)$ becomes steeper as time goes on. In this state, the system comes into congestion phase.

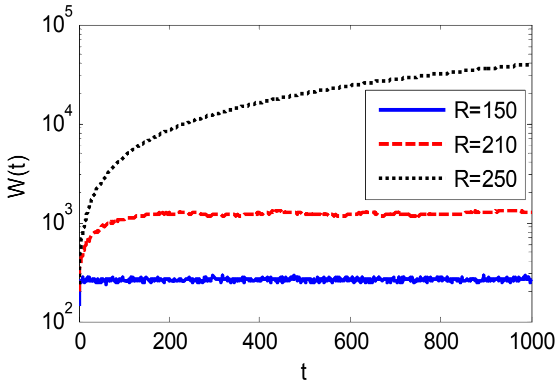

Fig. 1. Total number of active packets as a function of time steps with different packet creation rate.

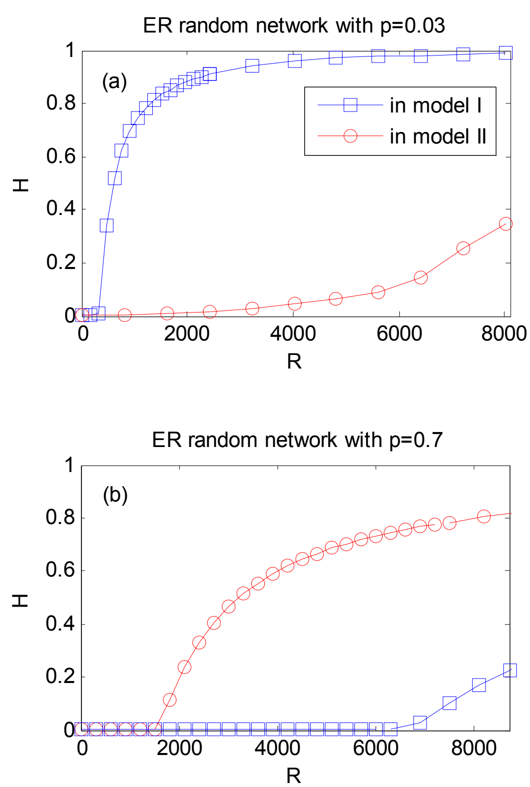

Fig. 2. The order parameter H versus R for the ER network in model I and model II with $\alpha = 0.8$.

The primary goal of our simulation is to understand the behavior of the phase transition, which leads to traffic congestion, with respect to the network topology. As we can see from Fig. 2 to 4, the order parameter H versus R correspond to the traffic awareness routing with $\alpha = 0.8$ for

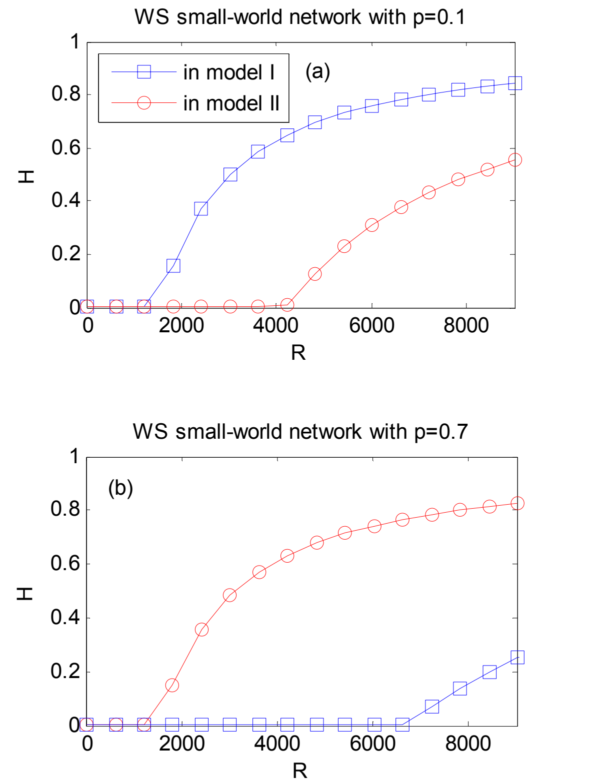

Fig. 3. The order parameter H versus R for the WS network in model I and model II with $\alpha = 0.8$.

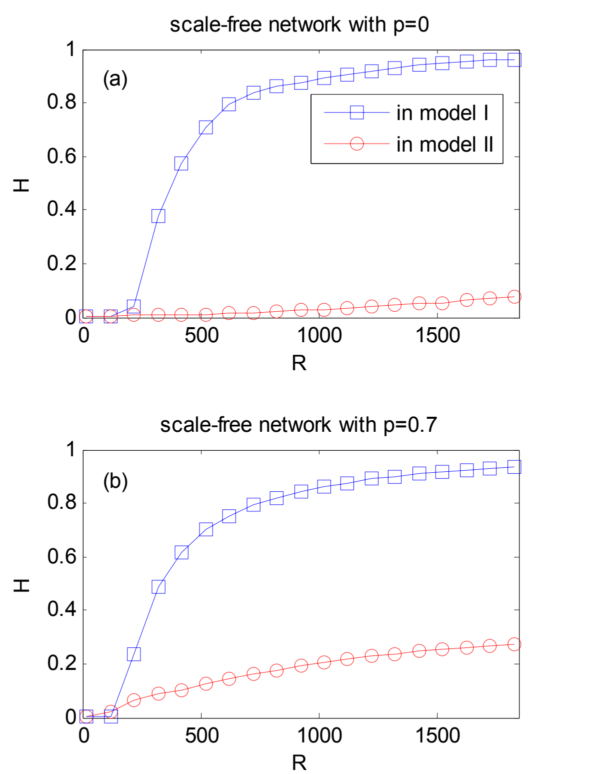

Fig. 4. The order parameter H versus R for the BA network in model I and model II with $\alpha = 0.8$.

three network structures in model I and II. We find, in the case of model I, which is the delivery capacity of each node is proportional to its degree, random and scale-free networks are more tolerant to congestion than WS small world networks. This is because, WS small world networks with $p = 0.1$, the most congested nodes have large betweenness, but very small number of links, that is to say, the ratio $k_{L\max} / B_{L\max}$ is much smaller than those in random and scale-free networks. In model II, the capacity of delivery of each node is proportional to its betweenness. We can see that for small probability p, the values of R_c are greatly enhanced for all kinds of networks considered. These simulation results thus show that the protocol based on our model II are more tolerant to congestion for all kinds of networks with small probability p, especially for WS small world network. One the other hand, for large probability p, compared to the model II, the network capacity by adopting model I is enhanced greatly in ER random network and WS small-world network, as we can see from Fig.2(b) and Fig.3(b). However, the critical value is maximal in model II, no matter what the tunable probability p is in scale-free network in Fig.4.

Now, we present simulation results with model III. Here, the delivery capacity of each node is proportional to the ratio of its betweenness to the total number of nodes, $C_i = B_i / N$ in Eq. (1). Fig. 5 shows the order parameter H versus R for the capacity parameters in ER random network model, WS small world network model, and BA scale-free network model, respectively. We can see that values of R_c based on model III are roughly the same for all kinds of networks considered.

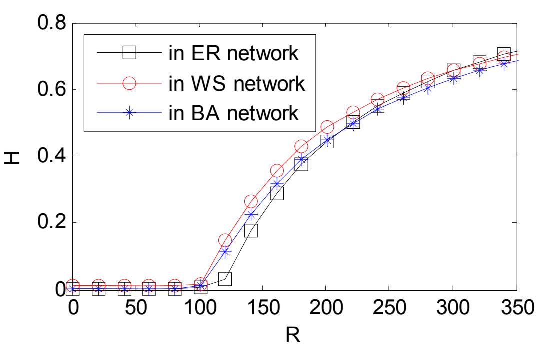

Fig. 5. For model III, the order parameter H versus the packet-generating value R in three network models.

V. Conclusions

We study the effects of network topological structure on the traffic delivery on complex networks. Here, we present three traffic models to study random, small-world and scale-free networks, respectively. The simulation results show that the network capacity depends on the information generation rate, and the underlying network structure. Broadly speaking, the phase transition can occur in the sense that free traffic flow can be guaranteed for low rates of information generation but large rates above a critical value can result in traffic congestions. Considering the feasibility and the cost of changing the real network topology to enhance the network performance, it is comparatively easy to adjust the routing protocols in real communication systems.

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