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Adaptive Fault-tolerant H_∞ Compensation Controller Design with Actuator Failures

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Abstract— **In this paper, the problem of designing adaptive fault-tolerant H_∞ compensation controllers for linear time-invariant continuous-time systems is presented. Linear matrix inequalities (LMIs) are developed with multiple Lyapunov functions to find a stabilizing controller gain such that the disturbance attenuation performance is optimized. Direct adaptive-state feedback control schemes are proposed to estimate the unknown controller parameters on-line for actuator fault and perturbation compensations. Then a class of adaptive robust state feedback controllers is constructed relying on the LMI result and the updated values of these estimations. Base on the Lyapunov stability theory, it shows that the resulting closed-loop system can guarantee to be ε-stable and suboptimal H_∞ performances in the presence of faults on actuators and external perturbations. A numerical example of rocket fairing structural-acoustic model and its simulation results are given.**

I. Introduction

In most practical control systems, component failures including sensors, actuators and even the plant itself, may occur in uncertain time and the size of faults is also unknown. The faults may lead to performance deterioration or even instability of the system. Therefore, the study of fault-tolerant control (FTC) system design has received considerable attention over the past two decades (see e.g., [1] – [20]), which keeps the systems safe to achieve proper performances whenever components are healthy or faulted. The existing faut-tolerant design approaches can be broadly classified into two groups, namely passive approach [1] – [7] and active approach [8] – [20]. Since the active FTC system offers the flexibility to select different controllers, the most suitable controller can be chosen for the situation and the better performance can be obtained than the passive FTC system. There are primary two typical approaches for fault compensations in active fault-tolerant, such as adaptive approach [8] – [17] and fault detection and isolation (FDI) [18] – [20].

In recent years, many researchers have focused on the development of adaptive fault-tolerant control methods. In [9], the perfect performance tracking results are obtained when considering the fault model of loss actuator effectiveness. In [13] – [16], the results in adaptive fault-tolerant control are based on model reference adaptive control, where the outputs of closed-loop systems can track the prescribed referent outputs. However, as we know that perturbations play an important role in the system, some of above works, such as [9] – [13], have not consider the perturbations within the system, and the proposed methods may not suitable for the FTC system if there exist perturbations. Moreover, [15] – [16] consider the perturbations under some special conditions, such as $\lim_{t\to\infty} z(t) = 0$ ($z(t)$ is perturbation) [15] and constant perturbation [16]. Therefore, the capability of perturbation rejection for the above FTC systems is very weak. On the other hand, direct adaptive method proposed in [10] can compensate the time-varying parameterizable actuator failures, but for the unparameterizable failures, an approximations of the actuator failures must be employed and the closed-loop system can be guarantee stability rather than asymptotically stable [12]. Furthermore, [13] considers the unparameterizable failures in the system, but the requirement of knowledge of upper bounds of failures is needed and asymptotic tracking cannot also be ensured. In this paper, the new proposed robust adaptive schemes can solve the problem of FTC with a general actuator failure model, which make sure the system can be uniformly ultimately bounded under the influence of actuator unparameterizable time-varying failures and perturbations.

Although there existed many results for fault-tolerant H_∞ control in reliable control area [1], [2], [4] – [6], and also fault compensations in active FTC area [9], [12], [13] – [16], few efforts are made to consider the problem of addressing perturbation attenuation performances of systems with LMI method and compensating actuator fault effects with adaptive method simultaneously. Recently, the adaptive H_∞ performance of FTC system have been addressed in [8], but the fault model of stuck is not considered and the method is fail when the fault effect factor is a time-varying scalar. Besides, there is conservative design for use of a common Lyapunov matrix for different system modes (fault modes). [5] considers the performance of system with stuck faults via an LMI method, but the design is also conservative with using a common Lyapunov matrix for different fault modes. In order to reduces the conservatism of the design, multiple Lyapunov functions will be introduced to develop a stabilizing controller gain such that the H_∞ performance is optimized.

In this paper, the fault-tolerant H_∞ compensation control problem for linear time-invariant continuous-time systems against actuator faults is studied. The adaptive H_∞ compensation design approach will be used for a general failure of

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actuator fault model, which covers the cases of normal operation, loss of effectiveness, outage, and stuck. Each control effectiveness is assumed to be unknown. A notion of H_∞ performance index is obtained by LMI approach, and compensation controller gain will be updated via adaptive laws. The adaptive approach and LMI approach to robust control are combined successfully to give adaptive fault-tolerant H_∞ compensation controller design methods for state feedback case. Then, the controllers are constructed relying on the LMI result and the updated values of these estimations. Based on the Lyapunov stability theory, the adaptive closed-loop system can be guaranteed to be uniformly ultimately bounded and to obtain suboptimal H_∞ performance in the presence of failures on actuators and external perturbations.

The rest of the paper is organized as follows. The FTC problem formulation is described in Section 2. In Section 3 the direct adaptive state feedback controller is developed. Section 4 gives a numerical example of rocket fairing structural-acoustic model and its simulation results. Finally, conclusion is given in Section 5.

II. PRELIMINARIES AND PROBLEM STATEMENT

We first introduce our notation and gather some elementary facts. R stands for the set of real numbers and for a real matrix E, $\|E\|$ represents the induced norm. With $\mathrm{Tr}[E]$ we denote the trace of E, i.e. the sum of the diagonal entries. Given matrices $M_k, k = 1,\ldots,n$, the notation $\mathrm{diag}_{k=1}^{n}[M_k]$ denotes the block-diagonal matrix with M_k along the diagonal and denoted $\mathrm{diag}_k[M_k]$ for brevity. For the sake of easing the notation of partitioned symmetric matrices, the symbol $(*)$ denotes generically each of its symmetric blocks.

In this paper, we consider a linear time-invariant continuous-time model captured the following state-space equation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) + B_3 d(t) \\ z(t) &= Cx(t) + Du(t) \end{aligned} \qquad (1)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $z(t) \in R^s$ is the regulated output, $w(t) \in L_2^q[0,\infty]$ is the exogenous disturbance, and $d(t) \in R^p$ is a bounded continuous vector function which represents the perturbations for the system, and we assume there exist two known positive constants $\underline{d}$, $\bar{d}$ such that $\underline{d} \le \|d(t)\| \le \bar{d}$. A, B_1, B_2, B_3, C and D are known real constant matrices with appropriate dimensions.

In this paper, we consider actuator faults including loss of effectiveness, outage and stuck. Let $u_{ij}^F(t)$ represent the signal from the ith actuator that has failed in the jth faulty mode. Then we denote a general actuator fault model as:

$$u_{ij}^F(t) = \rho_i^j(t) u_i(t) + \sigma_i^j u_{si}(t), \quad i = 1\ldots m, j = 1\ldots L \qquad (2)$$

where $\rho_i^j(t)$ is the actuator efficiency factor, the index j denotes the jth faulty mode and L is the total faulty modes, and $\underline{\rho}_i^j$ and $\bar{\rho}_i^j$ represent the known lower and upper bounds of $\rho_i^j(t)$, respectively. $u_{si}(t)$ is an unparametrizable bounded time-varying stuck-actuator fault in the ith actuator [13], and

TABLE I

FAULT MODEL

fault model	$\underline{\rho}_i^j$	$\bar{\rho}_i^j$	σ_i^j
normal	1	1	0
outage	0	0	0
loss of effectiveness	>0	<1	0
stuck	0	0	1

we assume there exist two known positive constants $\underline{u}_s$, $\bar{u}_s$ such that $\underline{u}_s \le \|u_s(t)\| \le \bar{u}_s$. Note the practical case, we have $0 \le \underline{\rho}_i^j \le \rho_i^j(t) \le \bar{\rho}_i^j$, and σ_i^j is unknown constant defined as:

$$\sigma_i^j = \begin{cases} 0, & \rho_i^j > 0 \\ 0 \text{ or } 1, & \rho_i^j = 0. \end{cases}$$

Then, Table 1 can be given to illustrate the fault model.

Denote

$$u_j^F(t) = [u_{1j}^F(t), u_{2j}^F(t), \cdots, u_{mj}^F(t)]^T = \rho^j(t)u(t) + \sigma^j u_s(t)$$

where $\rho^j(t) = \mathrm{diag}[\rho_1^j(t), \rho_2^j(t), \cdots, \rho_m^j(t)]$, $\rho_i^j(t) \in [\underline{\rho}_i^j, \bar{\rho}_i^j]$, $\sigma^j = \mathrm{diag}[\sigma_1^j, \sigma_2^j, \cdots, \sigma_m^j]$, $j = 1,2,\ldots,L$.

Then, the set of operators with above structure is denoted by

$$\Delta_{\rho^j} = \{\rho^j(t) : \rho^j(t) = \mathrm{diag}[\rho_1^j(t), \rho_2^j(t), \ldots, \rho_m^j(t)],\ \rho_i^j(t) \in [\underline{\rho}_i^j, \bar{\rho}_i^j]\}, \qquad (3)$$

and we also denote the following set

$$N_{\rho^j} = \{\rho^j(t) : \rho^j(t) = \mathrm{diag}[\rho_i^j(t), \rho_2^j(t), \ldots, \rho_m^j(t)],\ \rho_i^j(t) = \underline{\rho}_i^j \text{ or } \rho_i^j(t) = \bar{\rho}_i^j\} \qquad (4)$$

where $i = 1,2,\ldots,m$, $j = 1,2,\ldots,L$. Thus, the set N_{ρ^j} contains a maximum of 2^m elements.

For the sake of convenience description, for all possible faulty modes L, the following uniform actuator fault model is exploited:

$$u^F(t) = \rho(t)u(t) + \sigma u_s(t) \qquad (5)$$

where $\rho(t) = \mathrm{diag}_i[\rho_i(t)] \in \{\rho^1(t), \ldots, \rho^L(t)\}$.

Hence, the dynamics with actuator faults (1) is described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_2\rho(t)u(t) + B_2\sigma u_s(t) + B_1 w(t) + B_3 d(t) \\ z(t) &= Cx(t) + D\rho(t)u(t). \end{aligned} \qquad (6)$$

Remark 1: Here, $z(t)$ is defined as the regulated output. But the inputs being stuck are uncontrollable. So we do not consider the stuck inputs in $z(t)$.

The following notions are needed in formulating the considered problem.

Definition 1: Consider the system (6). The system is said to be *ε-stable* if for any $x(0) \in R^n$, the corresponding state satisfies

$$\lim_{t\to\infty} \|x(t)\| = \varepsilon,$$

where ε is a small positive scalar, and we define $\|x_{\min}\| := \varepsilon$.

Definition 2:[8] Consider the following closed-loop system under state-feedback design

$$\begin{aligned} \dot{x}(t) &= A_a x(t) + B_a w(t) \\ z(t) &= C_a x(t). \end{aligned} \tag{7}$$

Let $\gamma > 0$ be a given constant, then the system (7) is said to be stable with γ-disturbance attenuation, if for any $\tau > 0$, the output $z(t)$ of the system (7) under $x(0) = 0$ satisfies

$$\int_0^\infty z^T(t)z(t)dt \le \gamma^2 \int_0^\infty w^T(t)w(t)dt + \tau. \tag{8}$$

Remark 2: By the above definition, let $\tau = \eta^2$, for $\int_0^\infty w^T(t)w(t)dt > \eta$, then we have

$$\int_0^\infty z^T(t)z(t)dt \le (\gamma^2 + \eta) \int_0^\infty w^T(t)w(t)dt.$$

For $\int_0^\infty w^T(t)w(t)dt \le \eta$, it follows

$$\int_0^\infty z^T(t)z(t)dt \le \gamma^2\eta + \eta^2,$$

which shows that the adaptive H_∞ performance index is close to the standard H_∞ performance index when η is sufficiently small.

The following well-known bounded real lemma can be stated for the close-loop system (7).

Lemma 1: [26] Consider the closed-loop system (7), let $G(s) = C_a(sI - A_a)B_a$, if there exist a real matrix $P = P^T > 0$ and a positive scalar γ such that

$$\begin{bmatrix} A_a P + P A_a^T & * & * \\ B_a^T & -\gamma^2 I & * \\ C_a P & 0 & -I \end{bmatrix} < 0, \tag{9}$$

then the system is stable and $G(s)$ satisfies $\|G(s)\|_\infty \le \gamma$.

Then, the main objective of this paper is to construct an adaptive state feed-back controller $u(t)$ such that the system ε-stable, where the states converge in a tube of ray ε asymptotically or in finite time, with suboptimal H_∞ performances even in the cases of failures on actuators and perturbations.

III. Adaptive fault-tolerant H_∞ compensation control system design

In this section, we develop an LMI-based method for design of a suboptimal H_∞ controller gain, and also present adaptive laws to updating the controller parameters for compensating actuator faults and perturbations. Then, a method for designing adaptive fault-tolerant H_∞ compensation controllers via state feedback is presented in Theorem 1.

We assume all the states of system are available at every instant. Thus, consider a linear time-invariant FTC model described by (6) and controller model

$$u(t) = K_1 x(t) + K_2(t)x(t). \tag{10}$$

Then the closed-loop FTC system model can be written by

$$\begin{aligned} \dot{x}(t) &= (A + B_2\rho K_1)x(t) \\ &\quad + B_2\rho K_2(t)x(t) + B_2\sigma u_s(t) + B_1 w(t) + B_3 d(t) \\ z(t) &= (C + D\rho K_1 + D\rho K_2(t))x(t). \end{aligned} \tag{11}$$

To ensure the achievement of fault-tolerant control objective, a basic requirement is that the system $(A, B_2\rho(t))$ is uniformly completely controllable for any actuator failure mode $\rho(t) \in \{\rho^1(t)\ldots\rho^L(t)\}$ under consideration. Besides, the following assumptions in adaptive H_∞ FTC design are also assumed to be valid.

Assumption 1: $\text{rank}[B_2\rho(t)] = \text{rank}[B_2]$ for any actuator failure mode $\rho(t) \in \{\rho^1(t)\ldots\rho^L(t)\}$.

Remark 3: Assumption 1 introduces a condition of actuator redundancy in the system, and it seems necessary to completely compensate the stuck-actuator faults. The reason can be explained as follows. We first assume the perturbation $d(t)$ does not exist in system. Then, according to (11), in order to compensating the stuck-actuator faults $u_s(t)$, we should design a control law $K_2(t)$ to make the equation $B_2\rho(t)K_2(t)x(t) = -B_2\sigma u_s(t)$ holds true. Following the knowledge of linear algebra theory, $K_2(t)$ has a solution under the condition of $\text{rank}(B_2\rho(t)) = \text{rank}(B_2\rho(t), -B_2\sigma u_s(t))$ holds true. On the other hand, if without Assumption 1, it is easy to see that $\text{rank}(B_2\rho(t)) \le \text{rank}(B_2)$. Therefore, due to the special relationship of $\rho(t)$ and σ (see Table 1), we can obtain $\text{rank}(B_2\rho(t)) \le \text{rank}(B_2\rho(t), -B_2\sigma u_s(t))$. Obviously, there exist some very special constant u_s which can guarantee the equation holds, and it is almost impossible to make the equation hold for time-varying stuck. Thus, the Assumption 1 seems necessary to compensate the stuck fault $u_s(t)$. There are also many mechanical systems belonging to this class of systems and some works $[10]-[11]$ had also been proposed based on the redundancy condition. Although it is still under the condition, a novel FTC will be proposed.

Assumption 2: For FTC system (11), there exists a matrix function F of appropriate dimensions such that

$$B_3 = B_2 F. \tag{12}$$

Remark 4: With the same reason of compensating the stuck-actuator faults, Assumption 2 seems also necessary to compensating the perturbation. Actually, letting $u_s = 0$, we should let $B_2\rho(t)K_2(t)x(t) = B_3 d(t)$ for compensating the perturbation, and there must exists an appropriate dimensions matrix F such that $B_3 = B_2 F$ for guaranteeing the equation holds true. Many systems also satisfy this marching condition for robust control problem, such as [24].

Now, we denote a set for any time-varying matrix $K(t)$:

$$\begin{aligned} \Delta_K = \{K(t) : &\,\text{Tr}[K^T(t)K(t)] \\ &\in \{\min_t\{\text{Tr}[K^T(t)K(t)]\}, \max_t\{\text{Tr}[K^T(t)K(t)]\}\}\}, \end{aligned} \tag{13}$$

for $t \ge 0$.

Following the terms $A_a P + P A_a^T$ and $C_a P$ in (9), it is well recognized that there are closely interrelation between the Lyapunov matrix P and the controller gain matrix, such as the existence of product terms. The following Lemma, which alleviate the interrelation between the Lyapunov matrix and controller gain matrix using multiple Lyapunov functions, can help us to reduce the conservatism of the design.

Lemma 2: ([22], [23]) Consider the closed-loop system (7), if there exist symmetric positive definite matrices P and matrices $F \in R^{n\times n}$ and $G \in R^{n\times n}$ such that

$$\begin{bmatrix} FA_a + A_a^T F^T & P - F + A_a^T G & FB_a & C_a^T \\ * & -(G+G^T) & G^T B_a & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (14)$$

then (9) holds.

Hence, consider the closed-loop FTC system described by (11), the following Lemma is stated for suboptimal H_∞ performance index γ with multiple Lyapunov functions.

Lemma 3: Consider the closed-loop FTC system (11). For given positive scalars ξ, λ and γ, If there exist symmetric positive definite matrices X^j for any $\bar{K}_2 \in \Delta_K$, $\rho \in \Delta_{\rho^j}$, $j = 1,2,\ldots,L$ and any appropriately dimensioned matrices S and L such that

$$\begin{bmatrix} \Xi_{11}^j & \Xi_{12}^j & B_1 & \Xi_{14}^j & \Xi_{15}^j \\ * & -\lambda(S+S^T) & \lambda B_1 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\xi^{-1} I \end{bmatrix} < 0 \quad (15)$$

where

$$\begin{array}{l} \Xi_{11}^j = AS + S^T A^T + B_2\rho^j L + L^T \rho^j B_2^T \\ \Xi_{12}^j = X^j - S + \lambda(L^T\rho^j B_2^T + S^T A^T) \\ \Xi_{14}^j = L^T \rho^j D^T + S^T C^T + S^T \bar{K}_2^T \rho^j D^T \\ \Xi_{15}^j = S^T \bar{K}_2^T \rho^j D^T \end{array} \quad (16)$$

are feasible, then there exist a controller gain $K_1 = LS^{-1}$ such that it is an FTC H_∞ controller.

Proof: Following (11) and (14), we let $A_a = A + B_2\rho^j K_1$, $B_a = B_1$ and $C_a = C + D\rho^j K_1 + D\rho^j K_2(t)$, and set $G = \lambda F^T$. Since (14) implies that $G + G^T > 0$ then G is nonsingular matrix. Adding $[\bar{K}_2^T \rho^j D^T, 0,0,0,-\xi^{-1}I]^T$ and it's transpose as the fifth column and the fifth low of the LMI (14), respectively. Then, Pre- and postmultiplying (14) by $\mathrm{diag}[F^{-1}, F^{-T}, I, I, I]$ and $\mathrm{diag}[F^{-T}, F^{-1}, I, I, I]$, respectively, and letting $S = F^{-T}$, $X^j = F^{-1}P^j F^{-T}$, we have (15). ■

Remark 5: The advantage of LMIs (15) lies in the fact that by introducing additional free weighting matrices to express the relationship between the terms of the system equation [23]. But the term Ξ_{15}^j in (15) brings conservatism of optimizing the H_∞ performance and we just can obtain suboptimal H_∞ performance. However, we can choose small ξ to reduce the conservatism.

Now, consider the controller model (10), $K_2(t)$ is given by the following function:

$$K_2(t) = \frac{-B_2^T P_{\max} b \parallel x^T P_{\min} B_2 \parallel \hat{k}_3(t)}{\parallel x^T P_{\min} B_2 \parallel^2 a + \zeta} \quad (17)$$

where $P_{\max} = \max_j(P^j)$, $P_{\min} = \min_j(P^j)$, and $P^j > 0$, $j = 1,2,\ldots,L$, ζ is an arbitrary small positive constant, and a, b are suitable positive constants which satisfied:

$$\parallel x^T P_{\min} B_2 \parallel^2 a + \zeta \leq \parallel x^T P_{\min} B_2 \sqrt{\underline{\rho}^j} \parallel^2 b \quad (18)$$

for any $\underline{\rho}^j = \mathrm{diag}_i[\underline{\rho}_i^j] \in \Delta_{\rho^j}$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,L$ when the system states can converge to a small set as time goes to infinity; $\hat{k}_3(t) \in R$ is updated by the following adaptive law:

$$\frac{d\hat{k}_3(t)}{dt} = \mathrm{Proj}_{[\underline{k}_3,\bar{k}_3]}\{\hat{k}_3\} \begin{cases} \bar{k}_3, & \hat{k}_3 = c\bar{k}_3 \\ r \parallel x^T \parallel, & \text{otherwise} \end{cases} \quad (19)$$

where constant $c > 1$ is a weighting of $\hat{k}_3$, and

$$\underline{k}_3 = \min_j(\|\sigma^j\|\underline{u}_s + \|F\|\underline{d}), \quad j = 1,2,\ldots,L \quad (20)$$

$$\bar{k}_3 = \max_j(\|\sigma^j\|\bar{u}_s + \|F\|\bar{d}), \quad j = 1,2,\ldots,L \quad (21)$$

$r > 0$ is the adaptive law gain to be designed according to practical application, $\hat{k}_3(t_0)$ is finite, and from (19), we can see $\hat{k}_3(t) \geq 0$ if $\hat{k}_3(t_0) \geq 0$; $\mathrm{Proj}\{\cdot\}$ denotes the projection operator [25], whose role is to project the estimates $k_3(t)$ to the interval $[\underline{k}_3, \bar{k}_3]$.

On the other hand, letting

$$\tilde{k}_3(t) = \hat{k}_3(t) - k_3. \quad (22)$$

Since k_3 is a unknown constant, we can write the following error system

$$\frac{d\tilde{k}_3(t)}{dt} = r \parallel x^T P_{\min} B_2 \parallel. \quad (23)$$

In the following, by $(x,\tilde{k}_3)(t)$ we denote a solution of the closed-loop system and the error system. Then, the following theorem can be obtained which shows the globally boundedness of the solutions of the adaptive closed-loop system described by (11) and (23).

Theorem 1. Consider the adaptive closed-loop system described by (11) and (23) under Assumptions 1-2. If there exist matrices $X^j > 0$, $j = 1,2,\ldots,L$ and any appropriately dimensioned matrices S and L such that LMI (15) hold. Then the state feedback controller $u(t)$ described in (10) with controller parameters $K_1 = LS^{-1}$ and $K_2(t)$ given by (17) and adaptive law $\hat{k}_3$ determined according to (19), can guarantee that the closed-loop fault-tolerant system is uniformly ultimately bounded and have suboptimal H_∞ performance for any $\rho(t) \in \Delta_{\rho^j}$ satisfies, in normal case, i.e., $\rho(t) = 1$,

$$\int_0^\infty z^T(t)z(t)dt \leq \gamma_n^2 \int_0^\infty w^T(t)w(t)dt + \frac{1}{r}\tilde{k}_3^2(0), \text{ for } x(0) = 0 \quad (24)$$

and in actuator faults cases, i.e., $\rho \in \{\rho^1, \ldots, \rho^L\}$, satisfies

$$\int_0^\infty z^T(t)z(t)dt \leq \gamma_f^2 \int_0^\infty w^T(t)w(t)dt + \frac{1}{r}\tilde{k}_3^2(0), \text{ for } x(0) = 0. \quad (25)$$

Proof: Due to the space limitations, we omit the proof. ■

Corollary 1: Assume that LMIs (15) hold for $\gamma_f > \gamma_n > 0$, adaptive update laws and control gain functions $K_2(t)$ are given by (19), (17), respectively. Then the closed-loop system (11) is stable and with H_∞ performance indexes no large than γ_n and γ_f for normal and actuator failure cases, respectively.

Proof: Let $F(0) = r^{-1}\tilde{k}_3^2(0)$. Then, following (19), it shows that $k_3 \in [\underline{k}_3, \bar{k}_3]$, and we can also see that $\hat{k}_3(t)$ is also bounded from (19). Thus, $\tilde{k}_3(0)$ is bounded, such that $\tilde{k}_3(0) \in [0, \bar{k}_3 - \underline{k}_3]$. Therefore, we can choose r sufficiently large so that $F(0)$ is sufficiently small. Thus, the conclusion follows from (24), (25) and Definition 1. ■

From Theorem 1 and Corollary 1, we have the following algorithm to optimize the adaptive H_∞ performance in normal and fault cases.

Algorithm 1: Let γ_n and γ_f denote the adaptive H_∞ performance bounds for the normal case and fault cases of the closed-loop system (11), respectively. Then γ_n and γ_f are minimized if the following optimization problem is solvable

$$\min \alpha\eta_n + \beta\eta_f \quad \text{s.t.}(15) \tag{26}$$

where $\eta_n = \gamma_n^2$, $\eta_f = \gamma_f^2$, and α and β are weighting coefficients. Since systems are operating under the normal condition most of time, we can choose $\alpha > \beta$ in (26).

From Lemma 3, we need the knowledge of bound of K_2 for obtain the H_∞ performances γ_n and γ_f. Thus, assume at initial condition $x(0) = 0$, the low bound of $K_2(t)$ is equal to 0 from (17). On the other hand, the upper bound of $K_2(t)$ depends on $\|x_{\min}\|$, $\bar{k}_3$, and P^j, $j = 1, 2, \ldots, L$. The following algorithm is introduced to choose a P^j for obtaining the upper bound of $K_2(t)$, so that we can optimize the H_∞ performances.

Algorithm 2:

Step 1: Assume at iteration $k = 0$, give any initial solution P_0 and $K_{1,0}$ is available, obtain $k_{3,0}$ from (19). Then get upper bound of $K_{2,0}$ from (17).

Step 2: Solve problem (15) to get P_k^j and $K_{1,k}$, then obtain $k_{3,k}^j$ from (19), let $\bar{k}_{3,k} = \max_j(k_{3,k}^j)$, $j = 1, 2, \ldots, L$, and then get $\bar{K}_{2k}$.

Step 3: Set δ_k:=trace$[\bar{K}_{2,k}^T \bar{K}_{2,k}]$. If $\bar{k}_{3,k} > \bar{k}_{3,k-1}$ or $\delta_k > \delta_{k-1}$, then go back to Step 2. Otherwise, stop and obtain P^j, K_1, γ_n and γ_f.

Remark 6: Using the fact with spectral norm inequality, the proposed method has solved the actuator faults such as unparametrizable time-varying bounded stuck faults successfully. Obviously, it is a more effective method than existing direct adaptive methods for actuator failure compensation problem introduced in [10] and [11], where the schemes must be improved for the unparametrizable failures. Moveover, under the proposed method, the FTC system also has the capability of perturbation rejection, while other FTC systems [8] − [13] have not the capability.

IV. Numerical Example

We consider a rocket fairing structural-acoustic model with perturbation input and regulated output added [10]:

$$A = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415 \\ -0.1980 & -0.115 & -0.0318 & 0.3 \\ -3.0500 & 1.1880 & -0.4650 & 0.9 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix},$$

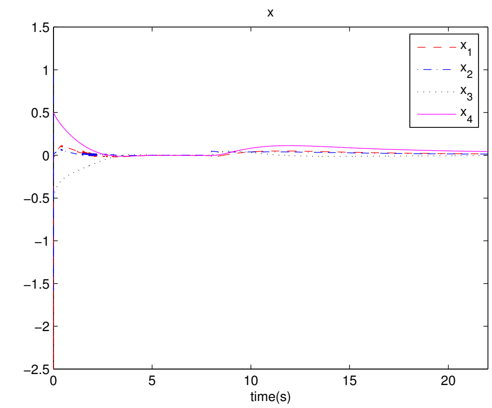

Fig. 1. Responses curves of the system states x(t).

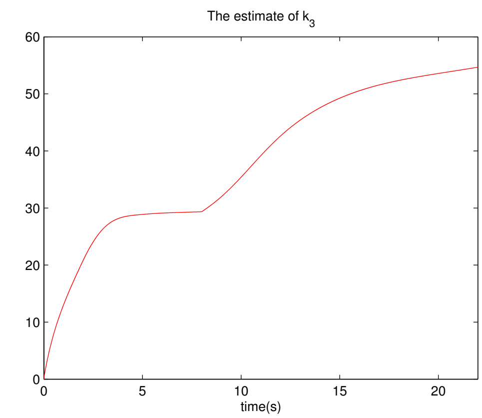

Fig. 2. Responses curves of the estimate of controller parameter k_3.

$$B_2 = \begin{bmatrix} 1 & 1.55 & 0.75 \\ 0.975 & 0.8 & 0.85 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 2 & -2 \\ 1 & -0.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 \\ 0.3 & 0 & 0.5 \\ 0 & 0 & -0.9 \end{bmatrix}.$$

Considering the following four possible faulty modes:

Normal mode 1: All of the actuators are normal, that is, $\rho_1^1 = \rho_2^1 = \rho_3^1 = 1$.

Fault mode 2: The first actuator is outage or stuck, the second and the third actuators may be normal or loss of effectiveness, described by $\rho_1^2 = 0, a_2 \le \rho_2^2 \le 1, a_3 \le \rho_3^2 \le 1, a_2 = 0.5, a_3 = 0.3$, which denotes the maximum loss of effectiveness for the second and the third actuators.

Fault mode 3: The second actuator is outage or stuck, the first and the third actuators may be normal or loss of effectiveness, that is, $\rho_2^3 = 0, b_1 \le \rho_1^3 \le 1, b_3 \le \rho_3^3 \le 1, b_1 = 0.6, b_3 = 0.5$, which denotes the maximum loss of effectiveness for the first actuator and the third actuator.

Fault mode 4: The third actuator is outage or stuck, the first actuators and the second actuators may be normal or loss of effectiveness, that is, $\rho_3^4 = 0, c_1 \le \rho_1^4 \le 1, c_2 \le \rho_2^4 \le 1, c_1 = 0.4, c_2 = 0.3$, which denotes the maximum loss of effectiveness for the first and second actuators.

By using Algorithm 1 and Algorithm 2 with $\alpha = 10$, $\beta = 1$, we obtain the H_∞ performances of closed-loop system are (normal) 1.3405 and 2.7706 (fault) with $\lambda = 0.01$. Then, to verify the effectiveness of the proposed adaptive method, the simulations are given with the following parameters and initial conditions:

$$r = 25, \quad x(0) = [-1, 1, -0.5, 0.5], \quad \hat{k}_3(0) = 0, \quad \xi = 0.2,$$
$$\varepsilon = 0.05, \quad \zeta = 0.0001, \quad a = 1, \quad b = 10, \quad c = 20.$$

The following faulty case is considered in the simulations, that is, before 8 second, the systems operate in normal case, and the perturbations $d(t) = [-0.5,\ 0.5sin(0.2t)]^T$ enter into the system at the beginning $(t \geq 0)$. At 8 second, some faults in actuators have occurs, the third actuator has stuck at $u_{s3}(t) = 1 + 0.5sin(0.1t) + 0.5cos(0.5t)$ and the first actuator loss of effectiveness described by $\rho_3 = 1 - 0.03t$ until loss effectiveness of 40%.

Fig.1 is the responses curves of the system's states with adaptive state feedback controller in above-mentioned faulty case. Fig.2 is the estimated curve of controller parameter $\hat{k}_3$. And the curve will reach $c\bar{k}_3$ as time goes to some sufficiently large number. It is easy to see the estimates can converge and the closed-loop FTC system uniformly bounded even in the presence of faults of actuators and perturbations.

V. Conclusions

This paper has presented a new method for fault-tolerant H_∞ control problem of actuator failure compensation in continuous-time systems. A general actuator failure model is adopted, which covers the cases of normal operation, loss of effectiveness, outage and stuck. The LMIs developed with multiple Lyapunov functions have been proposed to obtain a suboptimal fault-tolerant H_∞ control gain K_1, and the H_∞ performances of resultant closed-loop systems in both normal case and actuator failure cases are optimized. Based on the on-line update adaptation laws to estimate the controller parameters, the direct adaptive control schemes are designed to automatically compensate the effects of faults on actuators and perturbations. The state feedback controllers constructed by LMI-based method and adaptive method can guarantee the system ε-stable in the presence of faults on actuators and perturbations with suboptimal H_∞ performances. A numerical example has shown the effectiveness of the proposed method.

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