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H_∞ Filtering for a Class of Discrete-time Switched linear systems

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***Abstract*— This paper investigates the problem of H_∞ filtering for discrete-time switched linear systems under arbitrary switching laws. New LMI-based conditions for the solvability of the problem are given via switched quadratic Lyapunov functions. By Finsler's Lemma, two sets of slack variables with special structure are introduced to provide extra degrees of freedom in optimizing the guaranteed H_∞ performance. Compared with the existing methods, the proposed method has better performances and less conservatism. An example is given to illustrate the effectiveness of the method.**

I. INTRODUCTION

A switched system is a hybrid dynamical system consisting of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems [1]. Switched systems have received a great deal of attention in recent years, see [1]-[10] and references therein. The motivation for studying switched systems comes partly from the fact that switched systems and switched multi-controller systems have numerous applications in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, and many other fields. The problems encountered in switched systems can be classified into three categories [1]. The first one is to construct a switching signal that makes the switched systems asymptotically stable. The second one is to identify certain useful classes of switching signals for which the switched system is asymptotically stable. And the third one, which is interested in this paper, is to find conditions that guarantee that the switched systems are asymptotically stable under any switching signal.

On the other hand, state estimation has been widely studied and has found many practical applications over the past decades. When *a priori* statistical information on the external disturbance signals is not known, Kalman filtering cannot be employed. To address this issue, H_∞ filtering was introduced, in which the external disturbance signal is assumed to be energy bounded and the main objective is to minimized the H_∞ norm of the filtering error system. Compared to H_2 filtering, the advantages of H_∞ filtering are twofold. First, the assumption of boundness of the noise variance is loosened. Second, the H_∞ filter tends to be more robust when there exist additional uncertainties in systems, such as quantization errors, delays, and unmodeled dynamics [11]. Broadly, there are two approaches to H_∞ filtering: one is the frequency-domain approach [12]-[14]; the other is the state-space approach [15]-[19]. Particularly, the LMI approach to robust H_∞ filtering in state-space formulation is more powerful in numerical computations and suitable for handling the optimization problems with multiple constraints [15]-[19].

This paper investigates the H_∞ filtering problem for discrete-time switched linear systems under arbitrary switching laws. New LMI-based conditions for the solvability of the problem are given via switched quadratic Lyapunov functions. More importantly, based on Finsler's Lemma, two sets of slack variables with special structure are introduced to facilitate the filtering design and to provide extra degrees of freedom in optimizing the guaranteed H_∞ performance. Compared to the existing methods, the proposed method has better performances and less conservatism.

The rest of the paper is organized as follows. Section 2 gives the problem statement. Section 3 presents a new H_∞ filtering design approach to discrete-time switched linear systems. In Section 4, the proposed method is compared with the existing methods. In Section 5, an example is given to illustrate effectiveness of the proposed method. Finally, Section 6 gives some concluding remarks.

Notations: We use standard notations throughout this paper. M^T is the transpose of the matrix M. $M > 0$ ($M < 0$) means that M is positive definite (negative definite). The symbol $*$ within a matrix represents the symmetric entries. The Hermitian part of a square matrix M is denoted by $\mathbf{He}(M) := M + M^T$. l_2 is the Lebesgue space consisting of all discrete-time vector-valued functions that are square-summable over $[0, 1, 2, \ldots, \infty)$. The l_2-norm of a causal vector signal $x(k)$ with bounded-energy is $\|x(k)\|_2 = (\sum_{k=0}^{\infty} \|x(k)\|^2)^{1/2}$.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following discrete-time switched system

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}w(k)) \\ y(k) = C_{\sigma(k)}x(k) + D_{\sigma(k)}w(k) \\ z(k) = L_{\sigma(k)}x(k) \end{cases} \quad (1)$$

where $x(k) \in \mathbf{R}^n$ is the system state, $y(k) \in \mathbf{R}^q$ is the measurement, $z(k) \in \mathbf{R}^p$ is the signal to be estimated,

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$w(k) \in \mathbf{R}^m$ is the noise input which is assumed to $l_2[0,\infty)$. The switching rule $\sigma(k)$ takes values in the finite set $\mathcal{I} = \{1,\ldots,N\}$ and it changes its value at an arbitrary discrete time. The switched system can be described by the set of modes

$$\{(A_i, B_i, C_i, D_i, L_i) | i \in \mathcal{I}\}$$

and the evolution of $\sigma(k)$ gives the switching sequence among these modes. As in [21], it is assumed here that the switching rule $\sigma(k)$ is not known a priori, but its value is real-time available.

Define the indicator functions

$$\mu(k) = [\mu_1(k), \ldots, \mu_N(k)]^T \tag{2}$$

with

$$\mu_i(k) = \begin{cases} 1, & \sigma(k) = i \\ 0, & \text{otherwise} \end{cases}$$

then (1) can be written in the form ($\sum_p$):

$$\begin{cases} x(k+1) = \sum_{i=1}^{N} \mu_i(k) A_i x(k) + \sum_{i=1}^{N} \mu_i(k) B_i w(k) \\ y(k) = \sum_{i=1}^{N} \mu_i(k) C_i x(k) + \sum_{i=1}^{N} \mu_i(k) D_i w(k) \\ z(k) = \sum_{i=1}^{N} \mu_i(k) L_i x(k) \end{cases} \tag{3}$$

Here, we are interested in designing a filter described by ($\sum_f$):

$$\begin{aligned} \hat{x}(k+1) &= \sum_{i=1}^{N} \mu_i(k) \hat{A}_i \hat{x}(k) + \sum_{i=1}^{N} \mu_i(k) \hat{B}_i y(k) \\ \hat{z}(k) &= \sum_{i=1}^{N} \mu_i(k) \hat{L}_i \hat{x}(k) + \sum_{i=1}^{N} \mu_i(k) \hat{D}_i y(k) \end{aligned} \tag{4}$$

where $\hat{x}(k) \in \mathbf{R}^n$ and $\hat{z}(k) \in \mathbf{R}^p$ and the matrices $\hat{A}_i, \hat{B}_i, \hat{L}_i$ and $\hat{D}_i$ are to be determined.
Combining (3) with (4), we obtain the following filter error system ($\sum_e$):

$$\begin{aligned} \xi(k+1) &= \sum_{i=1}^{N} \mu_i(k) A_{cl,i} \xi(k) + \sum_{i=1}^{N} \mu_i(k) B_{cl,i} w(k) \\ \tilde{z}(k) &= \sum_{i=1}^{N} \mu_i(k) C_{cl,i} \xi(k) + \sum_{i=1}^{N} \mu_i(k) D_{cl,i} w(k) \end{aligned} \tag{5}$$

where $\xi(k) = [x(k)^T, \hat{x}(k)^T]^T$, $\tilde{z}(k) = z(k) - \hat{z}(k)$, and

$$\begin{aligned} A_{cl,i} &= \begin{bmatrix} A_i & 0 \\ \hat{B}_i C_i & \hat{A}_i \end{bmatrix}, B_{cl,i} = \begin{bmatrix} B_i \\ \hat{B}_i D_i \end{bmatrix} \\ C_{cl,i} &= \begin{bmatrix} L_i - \hat{D}_i C_i & -\hat{L}_i \end{bmatrix}, D_{cl,i} = -\hat{D}_i D_i \end{aligned} \tag{6}$$

The H_∞ filtering problem in this paper can be formulated as follows: given a discrete-time switched system $\sum_p$, design a filter $\sum_f$ such that the filter error system $\sum_e$ is asymptotically stable and satisfies the H_∞ performance

$$\|\tilde{z}(k)\|_2 < \gamma \|w(k)\|_2 \tag{7}$$

for all nonzero $w(k) \in l_2[0,\infty)$ and a given positive constant γ.

In the following, we investigate the filtering problem using the switched quadratic Lyapunov function defined as follows [21]

$$V(k, \xi(k)) = \xi(k)^T P_{\sigma(k)} \xi(k) = \xi(k)^T (\sum_{i=1}^{N} \mu_i(k) P_i) \xi(k) \tag{8}$$

where $P_i, i = 1,\ldots,N$ are symmetric positive-definite matrices. For the filter error system $\sum_e$ without disturbances, if such a positive-definite Lyapunov function exists and

$$\Delta V(k, \xi(k)) = V(k+1, \xi(k+1)) - V(k, \xi(k)) \tag{9}$$

is negative definite along all possible trajectories of the system, then it is asymptotically stable.

The following lemma is useful throughout the paper.

Lemma 1 [23]: (*Finsler's Lemma*) Let that $\xi \in \mathbf{R}^n, \mathcal{P} = \mathcal{P}^T \in \mathbf{R}^{n\times n}$, and $\mathcal{H} \in \mathbf{R}^{m\times n}$ such that $rank(\mathcal{H}) = r < n$, then the following statements are equivalent:
i) $\xi^T \mathcal{P} \xi < 0$, for all $\xi \neq 0, \mathcal{H}\xi = 0$;
ii) $\exists \mathcal{X} \in \mathbf{R}^{n\times m}$ such that $\mathcal{P} + \mathcal{X}\mathcal{H} + \mathcal{H}^T \mathcal{X}^T < 0$.

Note that the condition ii) remain sufficient for i) to hold even arbitrary constraints are imposed to the scaling matrices $\mathcal{X}$.

III. H_∞ FILTERING DESIGN

In this section, we will present a new H_∞ filtering approach to discrete-time switched linear systems $\sum_p$. Firstly, based on switched quadratic Lyapunov function and Finsler's lemma, the following lemma is obtained.

Lemma 2: Given a constant $\gamma > 0$, the filter error system $\sum_e$ is asymptotically stable with H_∞ performance γ, if there exist symmetric positive definite matrices P_i, and matrices $G_i, F_i, i \in \mathcal{I}$ such that the following inequalities are satisfied

$$\begin{bmatrix} P_j - G_i - G_i^T & 0 & G_i A_{cl,i} - F_i^T & G_i B_{cl,i} \\ * & -I & C_{cl,i} & D_{cl,i} \\ * & * & -P_i + \mathbf{He}\{F_i A_{cl,i}\} & F_i B_{cl,i} \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \ \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{10}$$

Proof: First, we establish the stability of filter error system $\sum_e$. When $w(k) = 0$, the first equality of (5) becomes

$$\xi(k+1) = A_{cl,i} \xi(k), \quad i \in \mathcal{I} \tag{11}$$

which can be written in the form

$$\begin{bmatrix} -I & A_{cl,i} \end{bmatrix} \begin{bmatrix} \xi(k+1) \\ \xi(k) \end{bmatrix} = 0, \quad i \in \mathcal{I} \tag{12}$$

By simple congruence transformation on (10), we obtain

$$\begin{bmatrix} -I & 0 & C_{cl,i} & D_{cl,i} \\ * & P_j - G_i - G_i^T & G_i A_{cl,i} - F_i^T & G_i B_{cl,i} \\ * & * & -P_i + \mathbf{He}\{F_i A_{cl,i}\} & F_i B_{cl,i} \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \ \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{13}$$

From (13), we have

$$\begin{bmatrix} P_j - G_i - G_i^T & G_i A_{cl,i} - F_i^T \\ * & -P_i + F_i A_{cl,i} + A_{cl,i}^T F_i^T \end{bmatrix} < 0 \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (14)$$

The inequalities (14) are rewritten as

$$\begin{bmatrix} P_j & 0 \\ 0 & -P_i \end{bmatrix} + \mathbf{He}\{\begin{bmatrix} G_i \\ F_i \end{bmatrix} \begin{bmatrix} -I & A_{cl,i} \end{bmatrix}\} < 0 \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (15)$$

Based on Finsler's lemma, (15) are equivalent to

$$\begin{bmatrix} \xi(k+1) \\ \xi(k) \end{bmatrix}^T \begin{bmatrix} P_j & 0 \\ 0 & -P_i \end{bmatrix} \begin{bmatrix} \xi(k+1) \\ \xi(k) \end{bmatrix} < 0, \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (16)$$

that is

$$\xi(k+1)^T P_j \xi(k+1) - \xi(k)^T P_i \xi(k) < 0 \quad (17)$$

Multiply (17) by $\mu_i(k)$ and $\mu_i(k+1) = \mu_j(k)$ and sum them over the indices i and j ranging from 1 to N. As $\sum_{i=1}^N \mu_i(k) = \sum_{j=1}^N \mu_j(k) = 1$, we have

$$V(k+1, \xi(k+1)) - V(k, \xi(k)) < 0, \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (18)$$

which establishes the stability of the system $\sum_e$.

Then, we consider the H_∞ performance. The inequalities (10) can be written in the following form

$$\mathcal{P}_{ij} + \mathcal{X}_i \mathcal{H}_i + \mathcal{H}_i^T \mathcal{X}_i^T < 0, \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (19)$$

where

$$\mathcal{P}_{ij} = \begin{bmatrix} P_j & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & -P_i & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \quad \mathcal{X}_i = \begin{bmatrix} G_i & 0 \\ 0 & I \\ F_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{H}_i = \begin{bmatrix} -I & 0 & A_{cl,i} & B_{cl,i} \\ 0 & -I & C_{cl,i} & D_{cl,i} \end{bmatrix} \quad (20)$$

Define the augmented signal

$$\eta(k) = \begin{bmatrix} \xi(k+1) \\ \tilde{z}(k) \\ \xi(k) \\ w(k) \end{bmatrix} \quad (21)$$

then the system $\sum_e$ can be written as follows

$$\mathcal{H}_i \eta(k) = 0 \quad (22)$$

Based on Finsler's lemma, if (19) hold then the following inequalities hold

$$\eta(k)^T \mathcal{P}_{ij} \eta(k) < 0, \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (23)$$

Substituting (20) and (21) into (23) obtains

$$\xi(k+1)^T P_j \xi(k+1) - \xi(k)^T P_i \xi(k) < \gamma^2 w(k)^T w(k) - \tilde{z}(k)^T \tilde{z}(k), \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (24)$$

Multiplying (24) by $\mu_i(k)$ and $\mu_i(k+1) = \mu_j(k)$ and summing them over the indices i and j ranging from 1 to N, we have

$$V(k+1, \xi(k+1)) - V(k, \xi(k)) < \gamma^2 w(k)^T w(k) - \tilde{z}(k)^T \tilde{z}(k), \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (25)$$

Summing over the index k ranging from 0 to ∞, it follows that $0 < V(\infty) < \gamma^2 \|w(k)\|_2^2 - \|\tilde{z}(k)\|_2^2$ for all nonzero $w(k) \in \ell_2$ and $\xi(0) = 0$. Therefore $\|\tilde{z}(k)\|_2 < \gamma \|w(k)\|_2$. And thus we complete the proof. ∎

Remark 1: In Lemma 2, by the aid of Finsler's lemma, two sets of slack variables $G_i, F_i, i \in \mathcal{I}$ are introduced to separate Lyapunov matrices P_i from system matrices $A_{cl,i}$ and to provide extra degrees of freedom in optimizing the guaranteed H_∞ performance. This leads to performance improvement and reduction of conservatism in filtering design. In addition, switched quadratic Lyapunov functions instead of common quadratic Lyapunov functions are used to reduce the conservatism further. Note that the method of introducing two slack variables to reduce the conservativeness of robust stability and filtering problems for linear systems was first introduced in [24] and [17], respectively.

Now we partition Lyapunov matrices $P_i, i \in \mathcal{I}$ in the following blocked matrices

$$P_i = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i12}^T & P_{i22} \end{bmatrix} \quad (26)$$

where $P_{i11} = P_{i11}^T > 0$, $P_{i22} = P_{i22}^T > 0$, and the dimensions of P_{i11} and P_{i22} are consistent with those of A_i and $\hat{A}_i$, respectively. Motivated by [17], let slack variables G_i and $F_i, i \in \mathcal{I}$ have the following structure

$$G_i = \begin{bmatrix} G_{i11} & G_{i2} \\ G_{i21} & G_{i2} \end{bmatrix}, F_i = \begin{bmatrix} F_{i11} & \lambda_1 G_{i2} \\ F_{i21} & \lambda_2 G_{i2} \end{bmatrix} \quad (27)$$

where λ_1, λ_2 are scalar parameters and the dimensions of G_{i11}, F_{i11} and G_{i2} are consistent with those of A_i and $\hat{A}_i$ respectively.

Based on Lemma 2 and (26), (27), we have the following theorem to solve the H_∞ filtering problem.

Theorem 1: The filter error system $\sum_e$ is asymptotically stable with H_∞ performance γ, if for some scalars λ_1, λ_2, there exist symmetric matrices P_{i11}, P_{i22} and matrices $P_{i12}, G_{i11}, G_{i21}, G_{i2}, F_{i11}, F_{i21}, T_{i1}, T_{i2}, T_{i3}, T_{i4}, i \in \mathcal{I}$ such that

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & G_{i11}B_i + T_{i2}D_i \\ * & \Xi_{22} & 0 & \Xi_{24} & \Xi_{25} & G_{i21}B_i + T_{i2}D_i \\ * & * & -I & \Xi_{34} & -T_{i3} & -T_{i4}D_i \\ * & * & * & \Xi_{44} & \Xi_{45} & F_{i11}B_i + \lambda_1 T_{i2}D_i \\ * & * & * & * & \Xi_{55} & F_{i21}B_i + \lambda_2 T_{i2}D_i \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (28)$$

where

$$\begin{aligned}
\Xi_{11} &= P_{j11} - G_{i11} - G_{i11}^T \\
\Xi_{12} &= P_{j12} - G_{i2} - G_{i21}^T \\
\Xi_{14} &= G_{i11}A_i + T_{i2}C_i - F_{i11}^T \\
\Xi_{15} &= S_{i1} - F_{i21}^T \\
\Xi_{22} &= P_{j22} - G_{i2} - G_{i2}^T \\
\Xi_{24} &= G_{i21}A_i + T_{i2}C_i - \lambda_1 G_{i2}^T \\
\Xi_{25} &= S_{i1} - \lambda_2 G_{i2}^T \\
\Xi_{34} &= L_i - T_{i4}C_i \\
\Xi_{44} &= -P_{i11} + \mathbf{He}\{F_{i11}A_i + \lambda_1 T_{i2}C_i\} \\
\Xi_{45} &= -P_{i12} + \lambda_1 T_{i1} + A_i^T F_{i21}^T + \lambda_2 C_i^T T_{i2}^T \\
\Xi_{55} &= -P_{i22} + \lambda_2 T_{i1} + \lambda_2 T_{i1}^T
\end{aligned}$$

The filter is given by

$$\hat{A}_i = G_{i2}^{-1}T_{i1}, \quad \hat{B}_i = G_{i2}^{-1}T_{i2}, \quad \hat{L}_i = T_{i3}, \quad \hat{D}_i = T_{i4}$$

Proof: Let $G_{i2}\hat{A}_i = T_{i1}, G_{i2}\hat{B}_i = T_{i2}, \hat{L}_i = T_{i3}$ and $\hat{D}_i = T_{i4}$. Substituting (27) into (10) immediately obtains (28). ■

Remark 2: The special structure of (27) was firstly introduced in [17] to design robust filtering for uncertain linear systems. It simplified the filtering design and reduced the conservatism.

Letting $F_i = 0$, Theorem 1 reduces to the following corollary.

Corollary 1: The filter error system $\sum_e$ is asymptotically stable with H_∞ performance γ, if there exist symmetric matrices P_{i11}, P_{i22} and matrices $P_{i12}, G_{i11}, G_{i21}, G_{i2}, T_{i1}, T_{i2}, T_{i3}, T_{i4}$, $i \in \mathcal{I}$ satisfying the following LMIs

$$\begin{bmatrix}
\Omega_{11} & \Omega_{12} & 0 & \Omega_{14} & T_{i1} & \Omega_{16} \\
* & \Omega_{22} & 0 & \Omega_{24} & T_{i1} & \Omega_{26} \\
* & * & -I & \Omega_{34} & -T_{i3} & -T_{i4}D_i \\
* & * & * & -P_{i11} & -P_{i12} & 0 \\
* & * & * & * & -P_{i22} & 0 \\
* & * & * & * & * & -\gamma^2 I
\end{bmatrix} < 0 \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{29}$$

where

$$\begin{aligned}
\Omega_{11} &= P_{j11} - G_{i11} - G_{i11}^T \\
\Omega_{12} &= P_{j12} - G_{i2} - G_{i21}^T \\
\Omega_{14} &= G_{i11}A_i + T_{i2}C_i \\
\Omega_{16} &= G_{i11}B_i + T_{i2}D_i \\
\Omega_{22} &= P_{j22} - G_{i2} - G_{i2}^T \\
\Omega_{24} &= G_{i21}A_i + T_{i2}C_i \\
\Omega_{26} &= G_{i21}B_i + T_{i2}D_i \\
\Omega_{34} &= L_i - Ti4C_i
\end{aligned}$$

The filter is given by

$$\hat{A}_i = G_{i2}^{-1}T_{i1}, \quad \hat{B}_i = G_{i2}^{-1}T_{i2}, \quad \hat{L}_i = T_{i3}, \quad \hat{D}_i = T_{i4}.$$

Remark 3: Corollary 1 can also be deduced form Theorem 1 in [20] without considering uncertainties. Due to the fact that only one set of slack variables are introduced, Corollary 1 is more conservative than Theorem 1.

IV. Comparison with the existing methods

Using the methods in [18], [19] and [22], we can easily obtain the following lemmas to solve the H_∞ filtering problem, respectively.

Lemma 3 [18]: The filter error system $\sum_e$ is asymptotically stable with H_∞ performance γ, if there symmetric matrices Y, Z and matrices Q, G, F satisfying the following LMIs:

$$\begin{bmatrix}
Z & Z & ZA_i & ZA_i & ZB_i & 0 \\
* & Y & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} & 0 \\
* & * & Z & Z & 0 & L_i^T - G^T \\
* & * & * & Y & 0 & L_i^T \\
* & * & * & * & I & 0 \\
* & * & * & * & * & \gamma^2 I
\end{bmatrix} > 0, \ \forall i \in \mathcal{I} \tag{30}$$

where

$$\begin{aligned}
\Lambda_{23} &= YA_i + FC_i + Q \\
\Lambda_{24} &= YA_i + FC_i \\
\Lambda_{25} &= YB_i + FD_i
\end{aligned}$$

and the filter is given by

$$\begin{aligned}
\hat{A}_i &= -Y^{-1}Q(I - Y^{-1}Z)^{-1}, \\
\hat{B}_i &= -Y^{-1}F, \\
\hat{L}_i &= G(I - Y^{-1}Z)^{-1}, \\
\hat{D}_i &= 0.
\end{aligned}$$

Lemma 4 [19]: The filter error system $\sum_e$ is asymptotically stable with H_∞ performance γ, if there symmetric matrices R_{i11}, R_{i22} and matrices $R_{i12}, M_i, E_i, H_i, i \in \mathcal{I}$ and X, Y, S satisfying the following LMIs:

$$\begin{bmatrix}
-R_{j11} & -R_{j12}^T & A_iX & A_i & B_i & 0 \\
* & -R_{j22} & M_i & \Upsilon_{24} & \Upsilon_{25} & 0 \\
* & * & \Upsilon_{33} & \Upsilon_{34} & 0 & \Upsilon_{36} \\
* & * & * & \Upsilon_{44} & 0 & L_i^T \\
* & * & * & * & -\gamma^2 I & 0 \\
* & * & * & * & * & -I
\end{bmatrix} < 0 \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \tag{31}$$

where

$$\begin{aligned}
\Upsilon_{24} &= YA_i + E_iC_i \\
\Upsilon_{25} &= YB_i + E_iD_i \\
\Upsilon_{33} &= R_{i11} - (X + X^T) \\
\Upsilon_{34} &= R_{i12}^T - (I + S^T) \\
\Upsilon_{36} &= (L_iX - H_i)^T \\
\Upsilon_{44} &= R_{i22} - (Y + Y^T)
\end{aligned}$$

and the filter is given by

$$\begin{aligned}\hat{A}_i &= V^{-1}[M_i - YA_iX - E_iC_iX]U^{-1}\\ \hat{B}_i &= V^{-1}E_i\\ \hat{L}_i &= H_iU^{-1}\\ \hat{D}_i &= 0\\ VU &= S - YX\end{aligned}$$

Lemma 5 [22]: The filter error system $\sum_e$ is asymptotically stable with H_∞ performance γ, if there symmetric matrices S_i, N_i and matrices $M_i, H_i, E_i, Q_i, R_i, i \in \mathcal{I}$ and X, Y, W, satisfying the following LMIs:

$$\begin{bmatrix} \Gamma_{11} & * & * & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * & * & * \\ 0 & 0 & \gamma I & * & * & * \\ A_iX & A_i & B_i & S_j & * & * \\ Q_i & \Gamma_{52} & E_iD_i + YB_i & M_j & N_j & * \\ \Gamma_{61} & \Gamma_{62} & -R_iD_i & 0 & 0 & \gamma I \end{bmatrix} > 0 \quad \forall (i,j) \in \mathcal{I} \times \mathcal{I} \quad (32)$$

where

$$\begin{aligned}\Gamma_{11} &= X + X^T - S_i\\ \Gamma_{21} &= I + W - M_i\\ \Gamma_{22} &= Y + Y^T - N_i\\ \Gamma_{52} &= YA_i + E_iC_i\\ \Gamma_{61} &= L_iX - H_i\\ \Gamma_{62} &= L_i - R_iC_i\end{aligned}$$

and the filter is given by

$$\begin{aligned}\hat{A}_i &= V^{-1}[Q_i - YA_iX]U^{-1}\\ \hat{B}_i &= V^{-1}E_i\\ \hat{L}_i &= (H_i - R_iC_iX)U^{-1}\\ \hat{D}_i &= R_i\\ VU &= W - YX\end{aligned}$$

Remark 4: Lemma 3 is Theorem 5 in [18], which can be used to design H_∞ filtering for discrete-time switched systems. Lemma 4 is deduced from Theorem 3 in [19] without considering time delay. And Lemma 5 is deduced from Theorem 5 in [22]. The deduction of these lemmas is straightforward, so the proofs are omitted here.

Remark 5: Note that Lemma 3 is based on common quadratic Lyapunov functions, while Lemmas 4-5, Corollary 1, and Theorem 1 are based on switched quadratic Lyapunov functions. So the latter are less conservative than the former. In Lemmas 4-5, only one slack variable is introduced to facilitate the filtering design. In Corollary 1, one set of slack variables G_i are introduced. However, in Theorem 1, two set of slack variables G_i and F_i are introduced, which enlarge the solution space for the H_∞ optimization and thus can reduce the conservatism of filtering design and improve the H_∞ performance. Therefore, Theorem 1 is the least conservative among all the above methods. A numerical example will be given to compare the conservatism of these methods in the following section.

V. EXAMPLE

In this section, an example is presented to illustrate the effectiveness of the proposed method.

Consider the switched system $\sum_p$ with $N = 3$ and

$$\begin{aligned}
&A_1 = \begin{bmatrix} 0.6 & 0.15 \\ 0.3 & -0.4 \end{bmatrix}, B_1 = \begin{bmatrix} -0.4 \\ 0.5 \end{bmatrix},\\
&C_1 = \begin{bmatrix} 0.35 & -0.5 \end{bmatrix}, D_1 = 0.04, L_1 = \begin{bmatrix} 2.4 & -1.3 \end{bmatrix},\\
&A_2 = \begin{bmatrix} 0.3 & -0.1 \\ 0.2 & -0.16 \end{bmatrix}, B_2 = \begin{bmatrix} 1.5 \\ 0.1 \end{bmatrix},\\
&C_2 = \begin{bmatrix} 1.2 & 0.7 \end{bmatrix}, D_2 = 0.15, L_2 = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix},\\
&A_3 = \begin{bmatrix} 0.5 & -0.2 \\ 1 & 0.3 \end{bmatrix}, B_3 = \begin{bmatrix} -0.2 \\ 0.5 \end{bmatrix},\\
&C_3 = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix}, D_3 = -0.025, L_3 = \begin{bmatrix} 0.4 & 0.33 \end{bmatrix}.
\end{aligned}$$

The purpose here is to design an H_∞ filter for the switched system above. By Theorem 1 with $\lambda_1 = -0.4, \lambda_2 = -0.3$, a filter in the form $\sum_f$ is obtained as follows:

$$\begin{aligned}
&\hat{A}_1 = \begin{bmatrix} 0.0532 & 0.1207 \\ -0.0493 & -0.1086 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} -0.2074 \\ -1.0160 \end{bmatrix},\\
&\hat{L}_1 = \begin{bmatrix} 0.0346 & 0.5214 \end{bmatrix}, \hat{D}_1 = 3.1807\\
&\hat{A}_2 = \begin{bmatrix} -0.1187 & 0.1008 \\ -0.0231 & -0.2330 \end{bmatrix}, \hat{B}_2 = \begin{bmatrix} 0.1687 \\ -0.1748 \end{bmatrix},\\
&\hat{L}_2 = \begin{bmatrix} 0.1713 & 0.1188 \end{bmatrix}, \hat{D}_2 = 0.3466,\\
&\hat{A}_3 = \begin{bmatrix} -0.1541 & -0.3065 \\ -0.1099 & -0.2219 \end{bmatrix}, \hat{B}_3 = \begin{bmatrix} -16.1838 \\ -10.2806 \end{bmatrix},\\
&\hat{L}_3 = \begin{bmatrix} 0.0346 & 0.5214 \end{bmatrix}, \hat{D}_3 = 3.1807.
\end{aligned}$$

and the optimal H_∞ norm $\gamma_{min} = 1.4022$. The switching signal is generated randomly and shown in Fig. 1. Given the initial conditions $x(0) = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, and the noise signal is chosen as $w(k) = 1/(10k + 1)$, which belongs to $l_2[0, \infty)$. Then, the state responses of the plant and the filter are shown in Fig. 2 and Fig. 3, respectively. And the filter error response is shown in Fig. 4. From Figs. 2-4, we know that the H_∞ filter meets the specified requirements and works well.

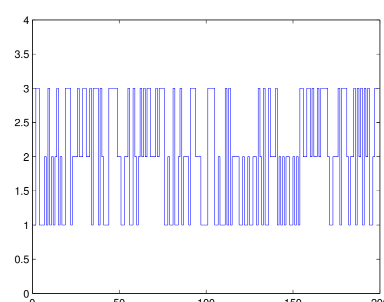

Fig. 1. Switching signal

According to Lemmas 3-5, Corollary 1, and Theorem 1, the optimal H_∞ performances, γ_{min}, are listed in Table 1. It is clear that Theorem 1 is the least conservative.

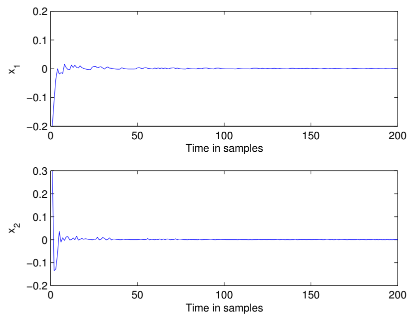

Fig. 2. State response of $x(k)$

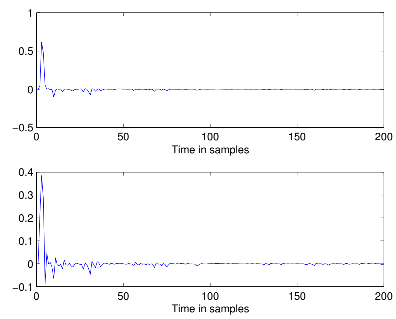

Fig. 3. State response of $\hat{x}(k)$

Table 3 H_∞ performance index

Methods	Lem. 3	Lem. 4	Lem. 5	Cor. 1	Thm. 1
γ_{min}	4.2332	3.4347	2.1276	1.7705	1.4022

VI. Conclusion

This paper is concerned with the problem of H_∞ filtering for a class of discrete-time switched linear systems. New LMI-based conditions for the solvability of the problem have been given via switched quadratic Lyapunov functions combined with Finsler's lemma. Compared to the existing methods, the proposed one has better performances and less conservatism. An example has also been given to illustrate the effectiveness of the method.

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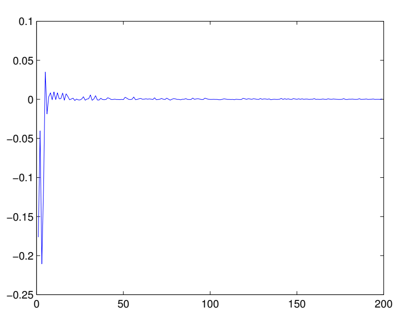

Fig. 4. Filter error response $\tilde{z}(k)$

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