2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

H_∞ Static Output Feedback Control for Discrete-time Switched Linear Systems with Average Dwell Time

Da-Wei Ding and Guang-Hong Yang

***Abstract*— This paper investigates the problem of H_∞ static output feedback (SOF) control for discrete-time switched linear systems with average dwell time switching. By using the multiple Lyapunov function technique, a switched SOF controller is designed such that the closed-loop system is exponentially stable and achieves a weighted L_2-gain. Sufficient conditions for SOF control are derived and formulated in terms of linear matrix inequalities (LMIs). The minimal average dwell time and the corresponding SOF controller are obtained from the LMI conditions for a given system decay degree. Additionally, based on Finsler's lemma, two sets of slack variables with special structure are introduced to provide extra freedom in the LMI optimization problem, which leads to reduction of the conservatism and improvement of the performance. A numerical example is given to illustrate the effectiveness of the proposed method.**

I. INTRODUCTION

As an important class of hybrid systems, switched systems consist of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems. In recent years, switched systems have received a great deal of attention, see [1]-[5] and references therein. The motivation for studying switched systems comes partly from the fact that switched systems and switched multi-controller systems have numerous applications in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, and many other fields. The problems encountered in switched systems can be classified into three categories [1]. The first one is to find conditions which guarantee that switched systems are asymptotically stable under any switching signal. The second one is to construct a switching signal that makes switched systems asymptotically stable. And the third one, which is of interest in this paper, is to identify certain useful classes of switching signals for which switched systems are asymptotically stable. In the study of switched systems, several approaches have been used such as multiple Lyapunov function approach [6]-[9], dwell time (average dwell time) approach [11]-[15], switched quadratic Lyapunov function approach [23] and so on. Among them, the multiple Lyapunov function technique which was proposed in [6] and later generalized in [7]-[9], has proved to be a powerful and effective tool for stability analysis and synthesis of switched systems. Meanwhile, switched systems with dwell time (or average dwell time) are also called slowly switched systems. And the average dwell time appproach is recognized to be more flexible and efficient in stability analysis of switched systems [11][13].

It is also well-known that SOF control is very useful and more realistic, since it can be easily implemented with low cost. The problem has been extensively studied in the past decades and for the SOF control problem of linear systems, there are various approaches to deal with it, see for example [16]-[22] and references therein. The problem of SOF control for discrete-time switched linear systems under arbitrary switching has been studied in [23]-[25] and sufficient existence conditions are obtained in terms of LMIs via the switched quadratic Lyapunov function approach. However, the switched quadratic Lyapunov function approach is not suitable to analyze the slowly switched systems (i.e., switched systems with average dwell time switching) due to the stricter requirements on the Lyapunov values at each switching time. Therefore, the existing methods in [23]-[25] cannot be applied to design SOF controllers for slowly switched linear systems. To the best of our knowledge, few results are available in the open literature to solve this problem.

In this paper, we investigates the problem of H_∞ SOF control for discrete-time switched linear systems with average dwell time switching. By using the multiple Lyapunov function technique combined with average dwell time approach, a switched SOF controller is designed such that the closed-loop switched system is exponentially stable and achieves a weighted L_2-gain. Sufficient conditions for SOF control are derived and formulated in terms of LMIs. And consequently the minimal average dwell time and the corresponding SOF controller gains are obtained from the LMI conditions for a given decay degree. In addition, by Finsler's lemma, two sets of slack variables with special structure are introduced to provide extra freedom in the LMI optimization problem, which lead to reducing the conservatism and improving the performance. A numerical example is given to illustrate the effectiveness of the proposed method.

The rest of the paper is organized as follows. Section 2 gives preliminaries and the problem statement. Section 3 is

This work was supported in part by the Funds for Creative Research Groups of China (No. 60821063), the State Key Program of National Natural Science of China (Grant No. 60534010), National 973 Program of China (Grant No. 2009CB320604), the Funds of National Science of China (Grant No. 60674021), the 111 Project (B08015) and the Funds of PhD program of MOE, China (Grant No. 20060145019).

Da-Wei Ding is with the College of Information Science and Engineering, Northeastern University, Shenyang, 110004, P.R. China. Email: ddaweiauto@163.com

Guang-Hong Yang is with the College of Information Science and Engineering, Northeastern University, Shenyang, 110004, P.R. China. He is also with the Key Laboratory of Integrated Automation of Process Industry, Ministry of Education, Northeastern University, Shenyang 110004, China. Corresponding author. Email: yangguanghong@ise.neu.edu.cn

the main result of the paper. First, several essential lemmas are given. Then, based on these lemmas, an SOF controller and the minimal average dwell time are obtained in terms of LMIs. Section 4 gives a numerical example to illustrate the effectiveness of the proposed method. Finally, conclusions are given in Section 5.

Notations: We use standard notations throughout this paper. M^T is the transpose of the matrix M. $M > 0 (M < 0)$ means that M is positive definite (negative definite). The symbol $*$ will be used in some matrix expressions to induce a symmetric structure. The Hermitian part of a square matrix M is denoted by $\mathbf{He}(M) := M + M^T$. ℓ^2 is the Lebesgue space consisting of all discrete-time vector-valued functions that are square-summable over $[0, 1, 2, \ldots, \infty)$. The ℓ_2-norm of a causal vector signal $x(k)$ with bounded-energy is $\|x(k)\|_2 = (\sum_{k=0}^{\infty} \|x(k)\|^2)^{1/2}$. $\mathbb{N}$ represents the set of nonnegative integers.

II. Preliminaries and Problem Statement

Consider the following discrete-time switched linear system

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + B^w_{\sigma(k)}w(k) \\ z(k) &= C^z_{\sigma(k)}x(k) + D_{\sigma(k)}u(k) + D^w_{\sigma(k)}w(k) \\ y(k) &= C_{\sigma(k)}x(k) \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^p$ is the control input, $w(k) \in \mathbb{R}^m$ is the disturbance input which belongs to $\ell_2[0, \infty)$, $y(k) \in \mathbb{R}^r$ is the measurement, and $z(k) \in \mathbb{R}^q$ is the controlled output. $\sigma(k) : [0, \infty) \to \mathcal{I} = \{1, \ldots, N\}$ is the switching signal which is assumed to be a piecewise continuous function depending on time or state or both. $N > 1$ is the number of subsystems. The ith subsystem is denoted by constant matrices $A_i, B_i, B^w_i, C^z_i, D_i, D^w_i, C_i$ with the appropriate dimensions. For the switching time sequence $k_0 < k_1 < k_2 < \ldots$ of switching signal σ, the holding time $[k_l, k_{l+1})$ is called the dwell time of the currently engaged subsystem, where $l \in \mathbb{N}$.

Without loss of generality, we assume that $C_i, i \in \mathcal{I}$ are of full row rank, then there exist nonsingular transformation matrices T_i such that

$$C_i T_i = \begin{bmatrix} I & 0 \end{bmatrix} \quad (2)$$

Note that for given C_i, the corresponding T_i are generally not unique. Special T_i can be obtained as follows:

$$T_i = \begin{bmatrix} C_i^T (C_i C_i^T)^{-1} & C_i^{\perp} \end{bmatrix} \quad (3)$$

where $C_i^{\perp}$ denotes an orthogonal basis for the null space of C_i.

Definition 1: The equilibrium $x = 0$ of system (1) is said to be exponentially stable under switching signal $\sigma(k)$, if there exist constants $\mathcal{K} > 0$, $0 < \beta < 1$ such that the solution $x(k)$ of system (1) with $w = 0$ satisfies $\|x(k)\| \leq \mathcal{K}\beta^{k-k_0} \|x(k_0)\|$, $\forall k \geq k_0$.

Definition 2 [13] [15]: For $\gamma > 0$ and $0 < \alpha < 1$, system (1) is said to have a weighted L_2-gain, if under zero initial condition $x = 0$, it holds that

$$\sum_{s=k_0}^{\infty} (1-\alpha)^s z^T(s)z(s) \leq \sum_{s=k_0}^{\infty} \gamma^2 w^T(s)w(s) \quad (4)$$

for all nonzero $w(k) \in l_2[0, \infty)$.

Definition 3 [11]: For any $k_0 < k_s < k_v$, let $N_{\sigma(k)}(k_s, k_v)$ denotes the switching number of $\sigma(k)$ over (k_s, k_v). If $N_{\sigma(k)}(k_s, k_v) \leq N_0 + (k_v - k_s)/\tau_a$ for $\tau_a > 0, N_0 \geq 0$, then τ_a is called average dwell time.

In this paper, we are interested in designing a switched SOF controller

$$u(k) = K_i y(k) \quad (5)$$

where $K_i, i \in \mathcal{I}$ are to be determined. The SOF controller (5) is assumed to be switched synchronously by the switching signal σ in system (1).

Under the controller (5), the closed-loop switched system becomes

$$\begin{aligned} x(k+1) &= A_{cli}x(k) + B_{cli}w(k) \\ z(k) &= C_{cli}x(k) + D_{cli}w(k), \quad i \in \mathcal{I} \end{aligned} \quad (6)$$

where

$$\begin{aligned} A_{cli} &= A_i + B_i K_i C_i \\ B_{cli} &= B^w_i \\ C_{cli} &= C^z_i + D_i K_i C_i \\ D_{cli} &= D^w_i \end{aligned} \quad (7)$$

Then, the problem of H_∞ SOF control to be addressed in this paper is formulated as follows. Given a switched system (1) and a prescribed level of disturbance attenuation $\gamma > 0$, design a switched SOF controller (5) and find out admissible switching signals with the minimal average dwell time such that the closed-loop system (6) is exponentially stable and achieve a prescribed weighted L_2-gain.

The following multiple Lyapunov function with the form

$$V(x_k) \triangleq x_k^T P_{\sigma(k)} x_k, \quad \sigma(k) \in \mathcal{I} \quad (8)$$

will be used in the sequel.

III. Main Results

This section gives the main result of the paper. First, several lemmas are given which are essential for later development.

Lemma 1: (*Finsler's Lemma*) Let that $\xi \in \mathbb{R}^n, \mathcal{P} = \mathcal{P}^T \in \mathbb{R}^{n\times n}$, and $\mathcal{H} \in \mathbb{R}^{m\times n}$ such that $rank(\mathcal{H}) = r < n$, then the following statements are equivalent:

i) $\xi^T \mathcal{P} \xi < 0$, for all $\xi \neq 0, \mathcal{H}\xi = 0$;

ii) $\exists \mathcal{X} \in \mathbb{R}^{n\times m}$ such that $\mathcal{P} + \mathcal{X}\mathcal{H} + \mathcal{H}^T\mathcal{X}^T < 0$.

Remark 1: Note that the condition ii) remains sufficient for i) to hold even arbitrary constraints are imposed to the scaling matrices $\mathcal{X}$.

Lemma 2 [2][15]: Consider the discrete-time switched system $x_{k+1} = f_\sigma(x_k), \sigma \in \mathcal{I}$ and let $0 < \alpha < 1, \mu > 1$

be given constants. Suppose that there exists a Lyapunov function candidate $V(x) = \{V_\sigma(x)\}, \sigma \in \mathcal{I}$ satisfying the following properties:

$$\Delta V_{\sigma(k)}(x_k) \triangleq V_{\sigma(k)}(x_{k+1}) - V_{\sigma(k)}(x_k) \leq -\alpha V_{\sigma(k)}(x_k), \quad \forall k \in [k_l, k_{l+1}) \tag{9}$$

$$V_{\sigma(k_l)}(x_{k_l}) \leq \mu V_{\sigma(k_{l-1})}(x_{k_l}) \tag{10}$$

then the system is exponentially stable for any switching signal with the average dwell time

$$\tau_a \geq \tau_a^* = \text{ceil}\left[-\frac{\ln \mu}{\ln(1-\alpha)}\right] \tag{11}$$

where function ceil(v) represents rounding real number v to the nearest integer greater than or equal to v.

Lemma 3: Let $0 < \alpha < 1, \gamma > 0$ and $\mu > 1$ be given constants. If the following inequalities are satisfied

$$\Delta V_{\sigma(k)}(k) + \alpha V_{\sigma(k)}(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) < 0, \quad \forall k \in [k_l, k_{l+1}) \tag{12}$$

$$V_{\sigma(k_l)}(k_l) - \mu V_{\sigma(k_{l-1})}(k_l) \leq 0 \tag{13}$$

then the system (6) has a weighted L_2-gain for any switching signal with the average dwell time satisfying (11).

Proof: Due to the limit of the space, it is omitted. ■

Based on Lemmas 1-3, the following theorem is given to solve the H_∞ SOF control problem.

Theorem 1: Let $\alpha > 0, \gamma > 0$ and $\mu > 1$ be given constants. If there exist symmetric matrices $P_i \in \mathbb{R}^{n\times n}$, scalar λ and matrices $G_i \in \mathbb{R}^{n\times n}$, $F_i \in \mathbb{R}^{n\times n}$, $L_i \in \mathbb{R}^{m\times n}$, $\forall i \in \mathcal{I}$ with the following structure

$$G_i = \begin{bmatrix} G_{i11} & 0 \\ G_{i21} & G_{i22} \end{bmatrix}, \quad F_i = \begin{bmatrix} \lambda G_{i11} & 0 \\ F_{i21} & F_{i22} \end{bmatrix}, \quad L_i = \begin{bmatrix} L_{i1} & 0 \end{bmatrix} \tag{14}$$

satisfying the following inequalities

$$\begin{bmatrix} \Xi_{11} & * & * & * \\ 0 & -I & * & * \\ \Xi_{31} & T_i^{-1}B_i^w & \Xi_{33} & * \\ \Xi_{41} & D_i^w & \Xi_{43} & -\gamma^2 I \end{bmatrix} < 0 \tag{15}$$

$$P_i - \mu P_j \leq 0 \tag{16}$$

where

$$\begin{aligned}
\Xi_{11} &= T_i^{-1}P_iT_i^{-T} - G_i - G_i^T \\
\Xi_{31} &= T_i^{-1}A_iT_iG_i + T_i^{-1}B_iL_i - F_i^T \\
\Xi_{33} &= \mathbf{He}\{(T_i^{-1}A_iT_iF_i + \lambda T_i^{-1}B_iL_i)\} \\
&\quad - (1-\alpha)T_i^{-1}P_iT_i^{-T} \\
\Xi_{41} &= C_i^zT_iG_i + D_iL_i \\
\Xi_{43} &= C_i^zT_iF_i + \lambda D_iL_i
\end{aligned}$$

and T_i are given by (3), then the closed-loop system (6) is exponentially stable and has a weighted L_2-gain for any switching signal with the average dwell time satisfying (11). Moreover, if (15)-(24) are feasible, then the switched SOF controller can be given by

$$K_i = L_{i1}G_{i11}^{-1} \tag{17}$$

Proof: Firstly, we establish the exponential stability of system (6). Pre- and post-multiplying

$$\begin{bmatrix} T_i & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & T_i & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \tag{18}$$

and its transpose to (15) obtains

$$\begin{bmatrix} \Lambda_{11} & * & * & * \\ 0 & -I & * & * \\ \Lambda_{31} & B_i^w & \Lambda_{33} & * \\ \Lambda_{41} & D_i^w & \Lambda_{43} & -\gamma^2 I \end{bmatrix} < 0 \tag{19}$$

where

$$\begin{aligned}
\Lambda_{11} &= P_i - T_iG_iT_i^T - T_iG_i^TT_i^T \\
\Lambda_{31} &= A_iT_iG_iT_i^T + B_iL_iT_i^T - T_iF_i^TT_i^T \\
\Lambda_{33} &= \mathbf{He}\{A_iT_iF_iT_i^T + \lambda B_iL_iT_i^T\} - (1-\alpha)P_i \\
\Lambda_{41} &= C_i^zT_iG_iT_i^T + D_iL_iT_i^T \\
\Lambda_{43} &= C_i^zT_iF_iT_i^T + \lambda D_iL_iT_i^T
\end{aligned}$$

It follows from (7), (14) and (17) that

$$\begin{aligned}
&A_iT_iG_iT_i^T + B_iL_iT_i^T \\
&= A_iT_iG_iT_i^T + B_i\begin{bmatrix} L_{i1} & 0 \end{bmatrix}T_i^T \\
&= A_iT_iG_iT_i^T + B_i\begin{bmatrix} K_iG_{i11} & 0 \end{bmatrix}T_i^T \\
&= A_iT_iG_iT_i^T + B_i\begin{bmatrix} K_i & 0 \end{bmatrix}\begin{bmatrix} G_{i11} & 0 \\ G_{i21} & G_{i22} \end{bmatrix}T_i^T \\
&= A_iT_iG_iT_i^T + B_iK_i\begin{bmatrix} I & 0 \end{bmatrix}G_iT_i^T \\
&= A_iT_iG_iT_i^T + B_iK_iC_iT_iG_iT_i^T \\
&= (A_i + B_iK_iC_i)T_iG_iT_i^T \\
&= A_{cli}T_iG_iT_i^T
\end{aligned} \tag{20}$$

In the same way, we can obtain

$$A_iT_iF_iT_i^T + \lambda B_iL_iT_i^T = A_{cli}T_iF_iT_i^T \tag{21}$$

$$C_i^zT_iG_iT_i^T + D_iL_iT_i^T = C_{cli}T_iG_iT_i^T \tag{22}$$

$$C_i^zT_iF_iT_i^T + \lambda D_iL_iT_i^T = C_{cli}T_iF_iT_i^T \tag{23}$$

Substituting (20)-(23) into (19) obtains

$$\begin{bmatrix} \Upsilon_{11} & * & * & * \\ 0 & -I & * & * \\ \Upsilon_{31} & B_{cl,i} & \Upsilon_{33} & * \\ C_{cli}T_iG_iT_i^T & D_{cli} & C_{cli}T_iF_iT_i^T & -\gamma^2 I \end{bmatrix} < 0 \tag{24}$$

where

$$\begin{aligned}
\Upsilon_{11} &= P_i - T_iG_iT_i^T - T_iG_i^TT_i^T \\
\Upsilon_{31} &= A_{cli}T_iG_iT_i^T - T_iF_i^TT_i^T \\
\Upsilon_{33} &= \mathbf{He}\{A_{cli}T_iF_iT_i^T\} - (1-\alpha)P_i
\end{aligned}$$

Pre- and post-multiplying

$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

and its transpose to (24) obtains

$$\begin{bmatrix} \Upsilon_{11} & * & * & * \\ \Upsilon_{31} & \Upsilon_{33} & * & * \\ 0 & B_{cl,i}^T & -I & * \\ C_{cli}T_iG_iT_i^T & C_{cli}T_iF_iT_i^T & D_{cli} & -\gamma^2 I \end{bmatrix} < 0 \quad (25)$$

From (25) we have

$$\begin{bmatrix} \Upsilon_{11} & * \\ \Upsilon_{31} & \Upsilon_{33} \end{bmatrix} < 0 \quad (26)$$

which can be rewritten as follows

$$\begin{bmatrix} P_i & 0 \\ 0 & -(1-\alpha)P_i \end{bmatrix} + \mathbf{He}\{\begin{bmatrix} T_iG_i^TT_i^T \\ T_iF_i^TT_i^T \end{bmatrix}\begin{bmatrix} -I & A_{cli}^T \end{bmatrix}\} < 0 \quad (27)$$

Consider the dual system of (6) with $w = 0$

$$x(k+1) = A_{cli}^T x(k) \quad (28)$$

and rewrite it in the form

$$\begin{bmatrix} -I & A_{cli}^T \end{bmatrix}\begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} = 0 \quad (29)$$

Based on Finsler's lemma, if (27) holds then the following inequality holds

$$\begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix}^T \begin{bmatrix} P_i & 0 \\ 0 & -(1-\alpha)P_i \end{bmatrix}\begin{bmatrix} x(k+1) \\ x(k) \end{bmatrix} < 0 \quad (30)$$

which is equivalent to

$$x(k+1)^TP_ix(k+1) - x(k)^TP_ix(k) < -\alpha x(k)^TP_ix(k) \quad (31)$$

then (9) is satisfied. In addition, it follows from (24) that (10) is satisfied. From Lemma 2, the closed-loop system without disturbances is exponentially stable for any switching signal with the average dwell time satisfying (11).

Now we consider the weighted L_2-gain of system (6). The inequality (24) can be rewritten as follows

$$\mathcal{P} + \mathcal{X}\mathcal{H} + \mathcal{H}^T\mathcal{X}^T < 0 \quad (32)$$

where

$$\mathcal{P} = \begin{bmatrix} P_i & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & -(1-\alpha)P_i & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\mathcal{X} = \begin{bmatrix} T_iG_i^TT_i^T & 0 \\ 0 & I \\ T_iF_i^TT_i^T & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathcal{H} = \begin{bmatrix} -I & 0 & A_{cli}^T & C_{cli}^T \\ 0 & -I & B_{cli}^T & D_{cli}^T \end{bmatrix} \quad (33)$$

Consider the dual system of (6)

$$\begin{aligned} x(k+1) &= A_{cli}^T x(k) + C_{cli}^T w(k) \\ z(k) &= B_{cli}^T x(k) + D_{cli}^T w(k) \end{aligned} \quad (34)$$

and define the augmented signal ξ as

$$\xi = \begin{bmatrix} x(k+1) \\ z(k) \\ x(k) \\ w(k) \end{bmatrix} \quad (35)$$

then, (34) can be rewritten in the form

$$\mathcal{H}\xi = 0 \quad (36)$$

By Finsler's lemma, if (32) holds then the following inequality holds

$$\xi^T\mathcal{P}\xi < 0 \quad (37)$$

Substituting (33) into (37), we have

$$\begin{aligned} x(k+1)^TP_ix(k+1) - (1-\alpha)x(k)^TP_ix(k) \\ +z(k)^Tz(k) - \gamma^2 w(k)^Tw(k) < 0 \end{aligned} \quad (38)$$

which is nothing but (12). Additionally, (13) is satisfied due to (24). Based on Lemma 3, the system has a weighted L_2-gain γ. And thus the proof is completed. ■

Letting $F_i = 0$, Theorem 1 reduces to the following corollary:

Corollary 1: If there exist symmetric matrices $P_i \in \mathbb{R}^{n\times n}$, and matrices $G_i \in \mathbb{R}^{n\times n}$, $L_i \in \mathbb{R}^{m\times n}$, $i \in \mathcal{I}$ with the following structure

$$G_i = \begin{bmatrix} G_{i11} & 0 \\ G_{i21} & G_{i22} \end{bmatrix}, \quad L_i = \begin{bmatrix} L_{i1} & 0 \end{bmatrix} \quad (39)$$

satisfying the following inequalities

$$\begin{bmatrix} \Gamma_{11} & * & * & * \\ 0 & -I & * & * \\ \Gamma_{31} & T_i^{-1}B_i^w & -T_i^{-1}P_iT_i^{-T} & * \\ \Gamma_{41} & D_i^w & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (40)$$

$$P_i - \mu P_j \le 0 \quad (41)$$

where

$$\begin{aligned} \Gamma_{11} &= T_i^{-1}P_iT_i^{-T} - G_i - G_i^T \\ \Gamma_{31} &= T_i^{-1}A_iT_iG_i + T_i^{-1}B_iL_i \\ \Gamma_{41} &= C_i^zT_iG_i + D_iL_i \end{aligned}$$

and T_i are given by (3), then the closed-loop system (6) is exponentially stable and has a weighted L_2-gain for any switching signal with the average dwell time satisfying (11). Moreover, if (40)-(49) are feasible, then the switched SOF controller can be given by $K_i = L_{i1}G_{i11}^{-1}$.

Proof: The proof of this corollary can be done using the same technique and arguments as in the proof of Theorem 1. Thus it is omitted here. ■

Remark 2: When λ in F_i is set to be fixed parameter, the condition in Theorem 1 becomes convex and can be solved by LMI Control Toolbox [27]. In Theorem 1, by using the multiple Lyapunov function technique combined with Finsler's lemma, two sets of slack variables G_i, F_i with special structure are introduced to provide extra free dimensions in the solution space. This directly leads to reduction of the conservativeness of the solutions and improvement of the performance. Compared to Theorem 1, Corollary 1 is more conservative since only one set of variables are introduced.

IV. Numerical Example

In this section, an example is given to illustrate the effectiveness of the proposed method.

Consider a discrete-time switched linear system consisting of three subsystems described as follows

$$A_1 = \begin{bmatrix} -0.5871 & -0.8441 & -0.0092 \\ -0.6865 & -0.5090 & -0.8561 \\ 0.0974 & 0.4523 & -0.2280 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1930 & -0.4204 \\ -0.7359 & 0.0346 \\ 0.5073 & -0.9077 \end{bmatrix}, B_1^w = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$C_1^z = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix},$$

$$D_1^w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix};$$

$$A_2 = \begin{bmatrix} 0.1089 & 0.2458 & -0.9035 \\ 0.3998 & -0.9213 & -0.4161 \\ 0.6745 & -0.5750 & 0.7138 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.4164 & 0.0244 \\ 0.8297 & -0.4366 \\ -0.0900 & -0.8416 \end{bmatrix}, B_2^w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},$$

$$C_2^z = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix},$$

$$D_2^w = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix};$$

$$A_3 = \begin{bmatrix} 0.3049 & 0.4247 & 0.8979 \\ 0.8848 & 0.2485 & -0.4161 \\ 0.6981 & 0.1034 & 0.2403 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.2458 & 0.7409 \\ 0.2501 & 0.1580 \\ 0.1709 & 0.7205 \end{bmatrix}, B_3^w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C_3^z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, D_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix},$$

$$D_3^w = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, C_3 = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}.$$

Note that A_1-A_3 are all unstable. Let $\mu = 2$, $\alpha = 0.5$, then we obtain $\tau_a^* = 1$. By using Theorem 1, the following control gains are obtained

$$K_1 = \begin{bmatrix} -1.1519 \\ -0.8379 \end{bmatrix}, K_2 = \begin{bmatrix} 0.4797 \\ 0.2214 \end{bmatrix}, K_3 = \begin{bmatrix} 0.5627 \\ -0.7362 \end{bmatrix}$$

and the optimal weighted L_2-gain $\gamma_{min} = 4.0200$. The closed-loop state response with initial states chosen as $x(0) = [-4\ 3\ 5]^T$ and the disturbance chosen as $w(k) = 1/(20k+1)$ is shown in Fig. 2. It is clear that the switched system has been stabilized by the SOF Controller under the switching signal shown in Fig. 1. In addition, the condition in Corollary 1 gives the following control gains

$$K_1 = \begin{bmatrix} -1.1467 \\ -0.6949 \end{bmatrix}, K_2 = \begin{bmatrix} 0.4487 \\ 0.2693 \end{bmatrix}, K_3 = \begin{bmatrix} 0.6375 \\ -0.7068 \end{bmatrix}$$

and the optimal weighted L_2-gain $\gamma_{min} = 4.2310 > 4.0200$. This shows that Theorem 1 is less conservative than Corollary 1 due to the fact that slack variables F_{i21}, F_{i22} in F_i provides extra freedom in the LMI optimization problem in Theorem 1.

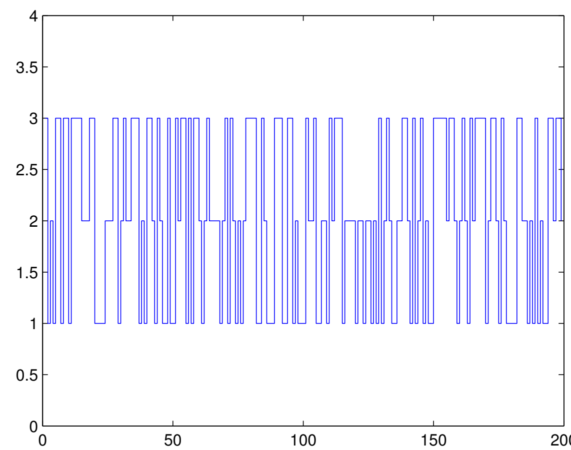

Fig. 1. Switching signal

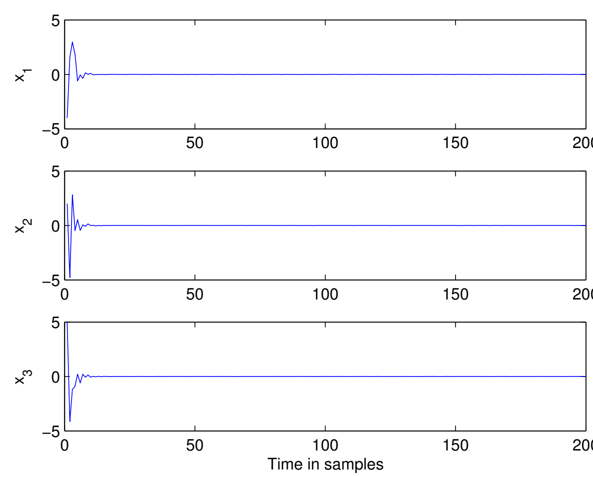

Fig. 2. Response of the closed-loop system

V. CONCLUSIONS

This paper considers the problem of H_∞ SOF control for discrete-time switched linear systems with average dwell time switching. By the aid of the multiple Lyapunov function technique combined with Finsler's lemma, a switched SOF controller has been designed. An example has also been given to illustrate the effectiveness of the proposed method.

REFERENCES

[1] D. Liberzon and A. S. Morse, "Basic problems in stability and design of switched systems," *IEEE Contrl. Syst. Mag.*, vol. 19, no. 5, pp.59-70, 1999.

[2] D. Liberzon, *Switching in Systems and Control*, Birkhauser, Boston, 2003.

[3] R. A. Decarlo, M. S. Branicky, S. Pettersson, and B. Lennartson, "Perspectives and results on the stability and stabilizability of hybrid systems," Proceedings of the IEEE, Special issue on Hybrid Systems, P. J. Antsaklis Ed., vol. 88, no. 7, pp. 1069-1082, 2000.

[4] Z. Sun and S. S. Ge, "Analysis and synthesis of switched linear control systems," *Automatica*, vol. 41, pp. 181-195, 2005.

[5] Z. Sun and S. S. Ge, *Switched Linear Systems: Control and Design*, Springer-Verlag, 2005.

[6] P. Peleties and R. A. Decarlo, "Asymptotic stability of m-switched systems using Lyapunov-like functions". In *Proceedings of American control conference*, pp. 1679-1684, 1991.

[7] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems,"*IEEE Trans. Autmat. Contr.*, vol. 43, no. 4, pp. 475-482, 1998.

[8] H. Ye, A. N. Michel and L. Hou, " Stability theory for hybrid dynamical systems," *IEEE Trans. Autmat. Contr.*, vol. 43, no. 4, pp. 461-474, 1998.

[9] J. Zhao and David J. Hill, " On stability, L_2-gain and H_∞ control for switched systems,"*Automatica*, Vol. 44, pp. 1220-1232 , 2008.

[10] A. S. Morse, "Supervisory control of families of linear set-point controllers, part I: exact matching," *IEEE Trans. Autmat. Contr.*, vol. 41, pp. 1413-1431, 1996.

[11] J. P. Hespanha and A. S. Morse, " Stability of switched systems with average dwell-time," Proceeding of the 38th Conference on Decision and Control, Phoenix, AZ, pp. 2655-2660, 1999.

[12] J. P. Hespanha, "Uniform stability of switched linear systems: extensions of LaSalle's invariance principle,"*IEEE Trans. Autmat. Contr.*, vol. 49, no. 4, pp. 470-482, 2004.

[13] G. Zhai, B. Hu, K. Yasuda and A. N. Michel, "Disturbance attenuation properties of time-controlled switched systems", *Journal of Franklin Institute*, 338, pp. 765-779.

[14] X.-M. Sun, J. Zhao and David J. Hill, "Stability and L_2-gain analysis for switched delay systems: A delay-dependent method," *Automatica*, Vol. 42, pp. 1769-1774, 2006.

[15] L. Zhang, E. K. Boukas and P. Shi, "Exponential H_∞ filtering for uncertain discrete-time switched linear systems with average dwell time: A μ-dependent approach", *Int. J. Robust Nonlinear control*, vol. 18, pp. 1188-1207, 2008.

[16] V. L. Syrmos, C. T. Abdallah, P. Dorato and K. Grigoriadis, "Static output feedback–A survey," *Automatica*, Vol. 33, no. 2, pp. 125-137, 1997.

[17] J. C. Geromel, C. C. deSouza and R. E. Skelton, "Static output feedback controllers: Stability and convexity," *IEEE Trans. Autmat. Contr.*, vol. 43, pp. 120-125, 1998.

[18] C. A. R. Crusius and A. Trofino, "Sufficient LMI conditions for output feedback control problems," *IEEE Trans. Autmat. Contr.*, vol. 44, pp. 1053-1057, 1999.

[19] M. C. de Oliveira, J. C. Geromel and J. Bernussou, "Extended H_2 and H_∞ characterizations and controller parameterizations for discrete-time systems," *Int. J. Control*, vol. 75, no. 9, pp.1131-1134, 2002.

[20] K. H. Lee, J. H. Lee and W. H. Kwon , "Sufficient LMI conditions for H_∞ output feedback stabilization of linear discrete-time systems," *IEEE Transactions on Automatic Control,* vol. 51, no. 4, pp. 675-680, 2006.

[21] J. Dong and G.-H. Yang,"Static output feedback control synthesis for linear systems with time-invariant parametric uncertainties," *IEEE Trans. Autmat. Contr.*, vol. 52, no. 10, pp. 1930-1936, 2007.

[22] J. Dong and G.-H. Yang, "Robust static output feedback control for linear discrete-time systems with time-varying uncertainties", *Systems Control Letters*, Vol. 57, no. 2, pp. 123-131, 2008.

[23] J. Daafouz, P. Riedinger and C. Iung, "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach," *IEEE Trans. Automat. Contr.*, Vol. 47, pp. 1883-1887, 2002.

[24] G. I. Bara and M. Boutyeb, "Switched output feedback stabilization of discrete-time switched systems," presented at the *Conf. Decision Control*, San Diego, USA, Dec. 12-15, 2006.

[25] G. I. Bara, "Robust switched output feedback control for discrete-time switched linear systems," presented at the *Conf. Decision Control*, New Orleans, USA, Dec. 12-14, 2007.

[26] S. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*. Philadelphia, PA: SIAM, 1994.

[27] P. Gahinet, A. Nemirovski, A. Laub and M. Chilali, *The LMI Control Toolbox*. Natick, MA: Mathworks, 1995.