2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Fuzzy Adaptive Observer and Filter Backsteppping Control for Nonlinear Systems

Li Changying, Tong Shaocheng, Li Yongming and Li Tieshan

Abstract—**In this paper, a new fuzzy adaptive control approach is developed for a class of SISO nonlinear systems with unmeasured states. Using fuzzy logic systems to approximate the unknown nonlinear functions, a fuzzy adaptive observer based on filters is introduced for state estimation as well as system identification. Under the framework of the backstepping design, fuzzy adaptive output feedback control is constructed recursively. By theoretical analysis, all the closed-loop signals are semi-globally uniformly ultimately bounded, and the tracking errors are proved to converge to a small residual set around the origin.**

I. Introduction

FUZZY/NEURAL network control methodology has emerged in recent years as a promising way to deal with the control problems of nonlinear systems containing highly uncertain nonlinear functions. It has been shown that fuzzy logic systems/neural network can be used to approximate any nonlinear function over a convex compact region [1]. Based on this theorem, various adaptive fuzzy control approaches have been introduced for controlling nonlinear systems [3-6]. All the results mentioned above show that unknown nonlinear functions need satisfy the matching conditions. However, in practice, a large class of physical systems may be subject to some unknown nonlinear functions which do not satisfy the matching conditions.

Backstepping, which is based on the nonlinear stabilization technique of 'adding an integrator" introduced in [8], and was first used in nonlinear adaptive control in [7], leads to the discovery of a structural strict feedback condition under which the systematic construction of robust control Lyapunov function is always possible. With the development of adaptive backstepping designs in nonlinear systems, many fuzzy/neural network adaptive control schemes have been developed for unknown nonlinear systems without the requirement of matching conditions. In [9], [10], and [12] stable fuzzy/neural network adaptive backstepping controller design schemes were proposed for unknown nonlinear SISO systems. Some further results on fuzzy/neural network adaptive backstepping control approaches were reported by [3-4] for a class of MIMO nonlinear systems. However, the existing fuzzy/neural network adaptive backstepping controllers are all based on the assumption that the states of the systems are measured directly, there are few results on the fuzzy/neural network adaptive output feedback backstepping controllers. Recently, adaptive observer backstepping control approach using neural networks is proposed for a class of nonlinear systems [11], in which it utilized the separation principle to design a state observer and output feedback controller. By assuming that the observer errors are bounded, the stability of control systems is given. As pointed out in [11], the separation principle does not hold for nonlinear systems, and the stabilities of the state observer and control system can not ensure the stability of the whole closed-loop system, either.

Motivated by [11], in this paper, a new adaptive fuzzy observer based control design scheme is studied for a class of nonlinear uncertain systems by using the backstepping technique. It is proved that the proposed design scheme can achieve semi-global uniform ultimate boundedness of all the signals in the closed-loop systems, and the tracking errors converge to a small neighborhood of the origin.

II. Systems description and Problem formulation

We consider systems in the output feedback form:

$$\begin{aligned}
\dot{x}_1 &= x_2 + f_{1,0}(y) + f_1(y) \\
\dot{x}_2 &= x_3 + f_{2,0}(y) + f_2(y) \\
&\vdots \\
\dot{x}_{\rho-1} &= x_\rho + f_{\rho-1,0}(y) + f_{\rho-1}(y) \\
\dot{x}_\rho &= x_{\rho+1} + f_{\rho,0}(y) + f_\rho(y) + b_m\sigma(y)u \\
&\vdots \\
\dot{x}_{n-1} &= x_n + f_{n-1,0}(y) + f_{n-1}(y) + b_1\sigma(y)u \\
\dot{x}_n &= f_{n,0}(y) + f_n(y) + b_0\sigma(y)u \\
y &= x_1
\end{aligned} \tag{1}$$

where $x=[x_1,\cdots,x_n]\in R^n$ is the state, $u\in R$ is the control input, $y\in R$ is the output, $f_{i,0}$, $1\le i\le n$, and σ are known smooth nonlinear functions, and $f_i(y)$, $1\le i\le n$ is unknown smooth nonlinear functions, $b=[b_m,\cdots,b_0]^T \in R^{m+1}$ are vectors of known constant parameters. Only, the y is available for measurement. We rewrite (1) as

This work was supported in part by the National Natural Science Foundation of China (No.60674056, 60874056), the Outstanding Youth Funds of Liaoning Province (No.2005219001) and Educational Department of Liaoning Province (No.2006R29and No.2007T80).
Li Changying,Tong Shaocheng and Li Yongming are with the Department of Basic Mathematics, Liaoning University of Technology Jinzhou, CO 121001 China (e-mail: l_y_m_2004@163.com).
Li Tieshan is with Navigation College, Dalian Maritime University, Dalian, 116026, China. (E-mail: tieshanli@126.com)

978-1-4244-4524-0/09/$25.00 ©2009 AACC

$$\dot{x} = Ax + Ky + \sum_{i=1}^{n} B_i[f_{i,0}(y) + f_i(y)] + B\sigma(y)u \qquad (2)$$

$$y(t) = Cx(t)$$

where

$$A = \begin{bmatrix} -k_1 & & \\ \vdots & I & \\ -k_n & 0 & \dots & 0 \end{bmatrix},$$

$$K = [k_1 \ \cdots \ k_n]^T, \ B = [0 \ \cdots \ b_m \ \cdots \ b_0]^T$$

$$B_i = [0 \ \cdots \ 1 \ \cdots \ 0]^T, c = [1 \ \cdots \ 0]^T.$$

K is chosen such that A is a strict Hurwitz matrix. Thus, given a $Q > 0$, there exists a positive matrix $P > 0$ satisfying

$$A^T P + PA = -2Q \qquad (3)$$

The control objective is to design an adaptive fuzzy controller to track a given reference signal $y_r(t)$, while keeping all the signals in the closed-loop system globally bounded.

Lemma 1[1]. Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists a fuzzy logic system such as

$$\sup_{x\in\Omega} \left| f(x) - \varphi^T(x)\theta \right| \le \varepsilon \qquad (4)$$

By Lemma 1, fuzzy logic systems are universal approximation, i.e., they can approximate any smooth functions on a compact space. Due to this approximation capability, we can assume that the nonlinear terms $f_i(y)$

$$f_i(y) = \varphi_i^T(y)\theta_i + \varphi_i^T(y)\widetilde{\theta}_i + \varepsilon_i(y) \qquad (5)$$

where $1 \le i \le n$, $\widetilde{\theta}_i = \theta_i^* - \theta_i$.

Define the optimal parameter vectors θ_i^* as

$$\theta_i^* = \arg\min_{\theta\in\Omega}[\sup_{y\in U}\left| \hat{f}_i(y|\theta_i) - f_i(y) \right|] \qquad (6)$$

where Ω and U are compact regions for θ_i and y respectively. The fuzzy logic system minimum approximation errors is defined as

$$\varepsilon_i(y) = f_i(y) - \hat{f}_i(y|\theta_i^*) \qquad (7)$$

$$\delta_i(y,\theta_i) = f_i(y) - \hat{f}_i(y|\theta_i) \qquad (8)$$

Design fuzzy state observer as

$$\dot{\hat{x}} = A\hat{x} + Ky + \sum_{i=1}^{n} B_i[\hat{f}_i + f_{i,0}] + B\sigma(y)u + \sum_{i=1}^{n} \zeta_i \dot{\theta}_i \qquad (9)$$

where the $\zeta_i \in R^{n\times m_{f_i}}$ is defined by the following $n \times m_{f_i}$ matrix differential equation:

$$\dot{\zeta}_i = A\zeta_i + B_i\varphi_i^T(y) \qquad (10)$$

Let $e = x - \hat{x}$ be observer error, then from (2) and (9) we have the observer error equation.

$$\begin{aligned}\dot{e} &= Ae + \sum_{i=1}^{n} B_i(f_i(y) - \hat{f}_i(y|\theta_i)) - \sum_{i=1}^{n}\zeta_i\dot{\theta}_i \\ &= Ae + \sum_{i=1}^{n} B_i\varphi_i^T(y)\widetilde{\theta}_i - \sum_{i=1}^{n}\zeta_i\dot{\theta}_i + \varepsilon(y)\end{aligned} \qquad (11)$$

Note that $\dot{\widetilde{\theta}}_i = -\dot{\theta}_i$. Multiplying on the right by $\widetilde{\theta}_i$ and rearranging (10), we obtain

$$\sum_{i=1}^{n}\dot{\zeta}_i(t)\widetilde{\theta}_i = A\sum_{i=1}^{n}\zeta_i(t)\widetilde{\theta}_i + \sum_{i=1}^{n} B_i\varphi_i^T(y)\widetilde{\theta}_i \qquad (12)$$

define

$$\xi = e - \sum_{i=1}^{n}\zeta_i\widetilde{\theta}_i \qquad (13)$$

Using (12) and (13), (11) becomes

$$\dot{\xi} = A\xi + \varepsilon(y) \qquad (14)$$

where $\varepsilon(y) = [\varepsilon_1, \cdots, \varepsilon_n]^T$.

Assumption 1. There exist known constants $\varepsilon_{i0} > 0$ and $\delta_{i0} > 0$, such that $|\varepsilon_i(y)| \le \varepsilon_{i0}$ and $|\delta_i(y,\theta_i)| \le \delta_{i0}$, for $i = 1,2,\ldots,n$.

Denote $w_i = \varepsilon_i - \delta_i$, and we have $|w_i| \le |\varepsilon_i| + |\delta_i| \le \varepsilon_{i0} + \delta_{i0} = w_{i0}$.

III. ADAPTIVE FUZZY OUTPUT FEEDBACK CONTROL DESIGN

The detailed design procedures of fuzzy adaptive output feedback controller are described in the following steps:

Step 1: Define the tracking error for the system as

$$z_1 = y - y_r$$

Expressing x_2 in terms of its estimate as $x_2 = \hat{x}_2 + e_2$, we obtain

$$\begin{aligned}\dot{z}_1 &= \dot{y} - \dot{y}_r \\ &= x_2 + f_{1,0}(y) + f_1(y) - \dot{y}_r \\ &= \hat{x}_2 + e_2 + f_{1,0}(y) + \varphi_1^T\theta_1 + \varphi_1^T\widetilde{\theta}_1 + \varepsilon_1 - \dot{y}_r\end{aligned} \qquad (15)$$

Using (13) we obtain

$$e_2 = \xi_2 + \sum_{i=1}^{n}\zeta_{i2}\widetilde{\theta}_i \qquad (16)$$

Substituting (16) into (15) yields

$$\dot{z}_1 = \hat{x}_2 + \xi_2 + f_{1,0}(y) + \varphi_1^T\theta_1 + \varepsilon_1 + \omega_1^T\widetilde{\theta} - \dot{y}_r \qquad (17)$$

where

$$\omega_0^T = [\zeta_{21}, \zeta_{22}, \zeta_{23}, \cdots, \zeta_{2n}], \ \widetilde{\theta} = [\widetilde{\theta}_1, \cdots, \widetilde{\theta}_n]^T$$

$$\omega_1^T = [\zeta_{21} + \varphi_1^T, \zeta_{22}, \zeta_{23}, \cdots, \zeta_{2n}], \ \widetilde{\theta} = \theta^* - \theta.$$

Taking $\hat{x}_2$ as a virtual control, and define

$$z_2 = \hat{x}_2 - \alpha_1(\hat{x}_1, \theta_1, y, y_r) - \dot{y}_r \qquad (18)$$

Then we have

$$\begin{aligned}\dot{z}_1 &= \hat{x}_2 + \xi_2 + f_{1,0}(y) + \omega_1^T\tilde{\theta} + \varphi_1^T\theta_1 + \varepsilon_1 - \dot{y}_r \\ &= z_2 + \alpha_1 + f_{1,0}(y) + \varphi_1^T\theta_1 + \omega_1^T\tilde{\theta} + \xi_2 + \varepsilon_1\end{aligned} \tag{19}$$

Consider the following Lyapunov function

$$V_1 = \frac{1}{2}\xi^T P\xi + \frac{1}{2}z_1^2 + \frac{1}{2\gamma}\tilde{\theta}^T\tilde{\theta} \tag{20}$$

where $P = P^T > 0$ is a positive matrix and $\gamma > 0$ is a design constant.

The time derivative of V_1 along the solutions of (3) and (19) is

$$\begin{aligned}\dot{V}_1 &= \frac{1}{2}(\dot{\xi}^T P\xi + \xi^T P\dot{\xi}) + z_1\dot{z}_1 + \frac{1}{\gamma}\tilde{\theta}^T\dot{\tilde{\theta}} \\ &\le -\xi^T Q\xi + \xi^T P\varepsilon + z_1\xi_2 + \frac{1}{\gamma}\tilde{\theta}^T(\gamma z_1\omega_1 - \dot{\theta}) \\ &\quad + z_1[z_2 + \alpha_1 + f_{1,0}(y) + \varphi_1^T\theta_1 + \varepsilon_1]\end{aligned} \tag{21}$$

By using the inequality $2ab \le a^2 + b^2$, we have

$$\begin{aligned}\xi^T P\varepsilon + \xi_2 z_1 &\le \frac{1}{2}\|\xi\|^2 + \frac{1}{2}\|P\varepsilon\|^2 + \frac{1}{2}|\xi_2|^2 + \frac{1}{2}z_1{}^2 \\ &\le \|\xi\|^2 + \frac{1}{2}z_1{}^2 + \frac{1}{2}\|P\varepsilon\|^2\end{aligned} \tag{22}$$

Substituting (22) into (21) yields

$$\begin{aligned}\dot{V}_1 &\le -(\lambda_{\min}(Q) - 1)\|\xi\|^2 + \frac{1}{\gamma}\tilde{\theta}^T(\gamma z_1\omega_1 - \dot{\theta}) + \frac{1}{2}\|P\varepsilon\|^2 \\ &\quad + z_1[z_2 + \frac{1}{2}z_1 + \alpha_1 + f_{1,0}(y) + \varphi_1^T\theta_1 + \varepsilon_1]\end{aligned} \tag{23}$$

The intermediate control function $\alpha_1(\hat{x}_1, \theta_1, y, y_r)$ and the adaptation function τ_1 are chosen as

$$\begin{aligned}\alpha_1 &= -c_1 z_1 - \frac{1}{2}z_1 - f_{1,0}(y) \\ &\quad - \varphi_1^T(y)\theta_1 - \varepsilon_{10}\tanh(\varepsilon_{10}z_1/\kappa)\end{aligned} \tag{24}$$

$$\tau_1 = \gamma z_1\omega_1(y) - \sigma(\theta - \theta_0) \tag{25}$$

where $c_1 > 0$, $\gamma > 0$, $\sigma > 0$ and θ_0 are design parameters.

Using the following property with regard to function $\tanh(\cdot)$

$$z_1\varepsilon_1 - z_1\varepsilon_{10}\tanh(\varepsilon_{10}z_1/\kappa) \le 0.2785\kappa = \kappa' \tag{26}$$

where $\kappa > 0$ is an arbitrary small constant.

Substituting (24) and (26) into (23) results in

$$\begin{aligned}\dot{V}_1 &\le -(\lambda_{\min}(Q) - 1)\|\xi\|^2 - c_1 z_1^2 + z_1 z_2 + \frac{1}{2}\|P\varepsilon\|^2 \\ &\quad + \frac{1}{\gamma}\tilde{\theta}^T(\tau_1 - \dot{\theta}) + \frac{\sigma}{\gamma}\tilde{\theta}^T(\theta - \theta_0) + \kappa'\end{aligned} \tag{27}$$

Step 2 : Differentiating z_2 yields:

$$\begin{aligned}z_2 &= \dot{\hat{x}}_2 - \dot{\alpha}_1(\dot{\hat{x}}_1, \theta_1, y, y_r) - \ddot{y}_r \\ &= \hat{x}_3 + H_2 + w_2 - \omega_2^T\tilde{\theta} - \varpi_2^T\dot{\theta} - \frac{\partial\alpha_1}{\partial y}(\xi_2 + \delta_1) - \ddot{y}_r\end{aligned} \tag{28}$$

where

$$\begin{aligned}H_2 &= k_2 e_1 + f_{2,0}(y) + \varphi_2^T\theta_2 - \frac{\partial\alpha_1}{\partial y}(\hat{x}_2 + \varphi_1^T\theta_1) \\ &\quad - \frac{\partial\alpha_1}{\partial\hat{x}_1}(\hat{x}_2 + k_1 e_1 + \varphi_1^T\theta_1) - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r\end{aligned}$$

$$\omega_2^T = \frac{\partial\alpha_1}{\partial y}[\zeta_{21}, (\zeta_{22} - (\frac{\partial\alpha_1}{\partial y})^{-1}\varphi_2^T), \cdots, \zeta_{2n}]$$

$$\begin{aligned}\varpi_2^T &= [\frac{\partial\alpha_1}{\partial\theta_1} - \zeta_{21} + \frac{\partial\alpha_1}{\partial\hat{x}_1}\zeta_{11}, -\zeta_{22} + \frac{\partial\alpha_1}{\partial\hat{x}_1}\zeta_{12}, \\ &\quad \cdots, -\zeta_{2n} + \frac{\partial\alpha_1}{\partial\hat{x}_1}\zeta_{1n}]\end{aligned}$$

Taking $\hat{x}_3$ as a virtual control, and introducing the variable

$$z_3 = \hat{x}_3 - \alpha_2(\hat{x}_1, \hat{x}_2, \theta_1, \theta_2, \bar{\zeta}^{(2)}, y, y_r, \dot{y}_r) - \ddot{y}_r \tag{29}$$

where $\bar{\zeta}^{(2)} = [\zeta_{11}, \zeta_{21}, \zeta_{12}, \zeta_{22}, \cdots, \zeta_{1n}, \zeta_{2n}]$.

Then we have

$$\begin{aligned}\dot{z}_2 &= z_3 + \alpha_2 + H_2 - \omega_2^T\tilde{\theta} \\ &\quad - \varpi_2^T\dot{\theta} + w_2 - \frac{\partial\alpha_1}{\partial y}(\xi_2 + \delta_1)\end{aligned} \tag{30}$$

Consider the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{31}$$

The time derivative of V_2 along the solutions of (30) is

$$\begin{aligned}\dot{V}_2 &\le -(\lambda_{\min}(Q) - 1)\|\xi\|^2 - c_1 z_1^2 + z_1 z_2 + \frac{1}{2}\|P\varepsilon\|^2 \\ &\quad + \frac{\sigma}{\gamma}\tilde{\theta}^T(\theta - \theta_0) + \kappa' + \frac{1}{\gamma}\tilde{\theta}^T(\tau_1 - \gamma z_2\omega_2 - \dot{\theta}) \\ &\quad + z_2[z_3 + \alpha_2 + H_2 - \varpi_2^T\dot{\theta} + w_2 - \frac{\partial\alpha_1}{\partial y}(\xi_2 + \delta_1)]\end{aligned} \tag{32}$$

By using the inequality $2ab \le a^2 + b^2$, we have

$$-\frac{\partial\alpha_1}{\partial y}z_2(\xi_2 + \delta_1) \le \|\xi\|^2 + (\frac{\partial\alpha_1}{\partial y}z_2)^2 + \frac{1}{2}\delta_1^2 \tag{33}$$

Substituting (33) into (32) yields

$$\begin{aligned}\dot{V}_2 &\le -(\lambda_{\min}(Q) - 2)\|\xi\|^2 - c_1 z_1^2 + z_1 z_2 + \frac{1}{2}\|P\varepsilon\|^2 + \frac{1}{2}\delta_1^2 \\ &\quad + \kappa' + \frac{1}{\gamma}\tilde{\theta}^T(\tau_1 - \gamma z_2\omega_2 - \dot{\theta}) + \frac{\sigma}{\gamma}\tilde{\theta}^T(\theta - \theta_0) \\ &\quad + z_2[z_3 + \frac{\partial\alpha_1}{\partial y}z_2 + \alpha_2 + H_2 - \varpi_2^T\dot{\theta} + w_2]\end{aligned} \tag{34}$$

Choosing intermediate control function α_2 and the adaptation function τ_2 as

$$\begin{aligned}\alpha_2 &= -z_1 - c_2 z_2 - \frac{\partial\alpha_1}{\partial y}z_2 - H_2 \\ &\quad + \varpi_2^T\tau_2 - w_{20}\tanh(w_{20}z_2/\kappa)\end{aligned} \tag{35}$$

$$\tau_2 = \tau_1 - \gamma z_2\omega_2(y) \tag{36}$$

where $c_2 > 0$ is a design parameter constant.

Using the following property with regard to function $\tanh(\cdot)$

$$z_2 w_2 - z_2 w_{20} \tanh(w_{20} z_2/\kappa) \le \kappa' \quad (37)$$

From (35)-(37), we have

$$\begin{aligned}\dot{V}_2 \le& -(\lambda_{\min}(Q)-2)\|\xi\|^2 - \sum_{k=1}^{2} c_k z_k^2 \\ &+ z_2 z_3 + (\frac{1}{\gamma}\tilde{\theta}^T + z_2 \varpi_2^T)(\tau_2 - \dot{\theta}) \\ &+ \frac{\sigma}{\gamma}\tilde{\theta}^T(\theta-\theta_0) + \frac{1}{2}\delta_1^2 + \frac{1}{2}\|P\varepsilon\|^2 + 2\kappa'\end{aligned} \quad (38)$$

Step i **.** $(3 \le i \le \rho - 1)$: A similar procedure is employed recursively at each step. By defining

$$z_i = \hat{x}_i - \alpha_{i-1}(\hat{x}_1,\cdots,\hat{x}_{i-1},\theta_1,\cdots,\theta_{i-1}, \bar{\zeta}^{(i-1)}, y, y_r,\cdots,y_r^{(i-1)}) - y_r^{(i)}$$

where

$$\bar{\zeta}^{(i-1)} = [\zeta_{11},\cdots,\zeta_{i-11},\zeta_{12},\cdots,\zeta_{i-12},\cdots,\zeta_{1n},\cdots,\zeta_{i-1n}]$$

Then we have

$$\begin{aligned}z_i =& \hat{x}_{i+1} + H_i + w_i - \omega_i^T\tilde{\theta} \\ &- \varpi_i^T\dot{\theta} - \frac{\partial\alpha_{i-1}}{\partial y}(\xi_2+\delta_1) - y_r^{(i)}\end{aligned} \quad (39)$$

where

$$\begin{aligned}H_i =& k_i e_1 + f_{i,0}(y) + \varphi_i^T\theta_i - \sum_{j=1}^{n}\frac{\partial\alpha_{i-1}}{\partial\zeta_j}\dot{\zeta}_j - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}} y_r^{(k)} \\ &- \frac{\partial\alpha_{i-1}}{\partial y}(\hat{x}_2 + \varphi_1^T\theta_1) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}(\hat{x}_{j+1} + k_j e_1 + \varphi_j^T\theta_j)\end{aligned}$$

$$\begin{aligned}\varpi_i^T =& [\frac{\partial\alpha_{i-1}}{\partial\theta_1} - \zeta_{i1} + \frac{\partial\alpha_{i-1}}{\partial\hat{x}_1}\zeta_{11} + \cdots + \frac{\partial\alpha_{i-1}}{\partial\hat{x}_{i-1}}\zeta_{i-11}, \\ &\frac{\partial\alpha_{i-1}}{\partial\theta_2} - \zeta_{i2} + \frac{\partial\alpha_{i-1}}{\partial\hat{x}_1}\zeta_{12} + \cdots + \frac{\partial\alpha_{i-1}}{\partial\hat{x}_{i-1}}\zeta_{i-12}, \\ &\cdots, \frac{\partial\alpha_{i-1}}{\partial\theta_{i-1}} - \zeta_{ii-1} + \frac{\partial\alpha_{i-1}}{\partial\hat{x}_1}\zeta_{1i-1} + \cdots + \frac{\partial\alpha_{i-1}}{\partial\hat{x}_{i-1}}\zeta_{i-1i-1}, \\ &\cdots, -\zeta_{in} + \frac{\partial\alpha_1}{\partial\hat{x}_1}\zeta_{1n} + \cdots + \frac{\partial\alpha_{i-1}}{\partial\hat{x}_{i-1}}\zeta_{i-1n}]\end{aligned}$$

$$\omega_i^T = \frac{\partial\alpha_{i-1}}{\partial y}[\zeta_{21},\zeta_{22},\cdots,(\zeta_{2i} - (\frac{\partial\alpha_{i-1}}{\partial y})^{-1}\varphi_i^T),\cdots,\zeta_{2n}]$$

Taking $\hat{x}_{i+1}$ as a virtual control, and introducing the variable

$$z_{i+1} = \hat{x}_{i+1} - \alpha_i - y_r^i$$

Then we have

$$\begin{aligned}\dot{z}_i =& z_{i+1} + \alpha_i + H_i - \omega_i^T\tilde{\theta} \\ &- \varpi_i^T\dot{\theta} + w_i - \frac{\partial\alpha_{i-1}}{\partial y}(\xi_2+\delta_1)\end{aligned} \quad (40)$$

Consider the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \quad (41)$$

The time derivative of V_i along the solutions of (40) is

$$\begin{aligned}\dot{V}_i =& \dot{V}_{i-1} + z_i[z_{i+1} + \alpha_i + H_i - \omega_i^T\tilde{\theta} \\ &- \varpi_i^T\dot{\theta} + w_i - \frac{\partial\alpha_{i-1}}{\partial y}(\xi_2+\delta_1)]\end{aligned} \quad (42)$$

By mathematical induction, we have

$$\begin{aligned}\dot{V}_{i-1} \le& -(\lambda_{\min}(Q)-i+1)\|\xi\|^2 - \sum_{k=1}^{i-1} c_k z_k^2 + z_{i-1}z_i \\ &+ \frac{1}{2}\|P\varepsilon\|^2 + \frac{i-2}{2}\delta_1^2 + \frac{\sigma}{\gamma}\tilde{\theta}^T(\theta-\theta_0) \\ &+ (\frac{1}{\gamma}\tilde{\theta}^T + \sum_{j=2}^{i-2} z_j\varpi_j^T)(\tau_{i-1} - \dot{\theta}) + (i-1)\kappa'\end{aligned} \quad (43)$$

Substituting (43) into (42) results in

$$\begin{aligned}\dot{V}_i \le& -(\lambda_{\min}(Q)-i+1)\|\xi\|^2 - \sum_{k=1}^{i-1} c_k z_k^2 + z_{i-1}z_i \\ &+ \frac{1}{2}\|P\varepsilon\|^2 + \frac{i-2}{2}\delta_1^2 + \frac{\sigma}{\gamma}\tilde{\theta}^T(\theta-\theta_0) \\ &+ (\frac{1}{\gamma}\tilde{\theta}^T + \sum_{j=2}^{i-1} z_j\varpi_j^T)(\tau_{i-1} - \dot{\theta}) + (i-1)\kappa' \\ &+ z_i[z_{i+1} + \alpha_i + H_i - \omega_i^T\tilde{\theta} \\ &- \varpi_i^T\dot{\theta} + w_i - \frac{\partial\alpha_{i-1}}{\partial y}(\xi_2+\delta_1)]\end{aligned} \quad (44)$$

By using the inequalities

$$-\frac{\partial\alpha_{i-1}}{\partial y}z_i[\xi_2+\delta_1] \le \|\xi\|^2 + (\frac{\partial\alpha_{i-1}}{\partial y})^2 z_i^2 + \frac{1}{2}\delta_1^2 \quad (45)$$

$$z_i w_i - z_i w_{i0}\tanh(w_{i0}z_i/\kappa) \le \kappa' \quad (46)$$

Choose intermediate control function α_i and adaptation functions τ_i as

$$\begin{aligned}\alpha_i =& -z_{i-1} - c_i z_i - (\frac{\partial\alpha_{i-1}}{\partial y})^2 z_i - H_i + \varpi_i^T\tau_i \\ &+ \gamma\omega_i\sum_{j=2}^{i-1} z_j\varpi_j^T - w_{i0}\tanh(w_{i0}z_i/\kappa)\end{aligned} \quad (47)$$

$$\tau_i = \tau_{i-1} - \gamma z_i\omega_i(y) \quad (48)$$

where $c_i > 0$ is a design constant.

From (45)-(48) we can obtain

$$\begin{aligned}\dot{V}_i \le& -(\lambda_{\min}(Q)-i)\|\xi\|^2 - \sum_{k=1}^{i} c_k z_k^2 \\ &+ z_i z_{i+1} + (\frac{1}{\gamma}\tilde{\theta}^T + \sum_{j=2}^{i} z_j\varpi_j^T)(\tau_i - \dot{\theta}) \\ &+ \frac{\sigma}{\gamma}\tilde{\theta}^T(\theta-\theta_0) + \frac{i-1}{2}\delta_1^2 + \frac{1}{2}\|P\varepsilon\|^2 + i\kappa'\end{aligned} \quad (49)$$

Step ρ **:** In the ρ th design step, the actual control input u will appears. We consider the Lyapunov function as

$$V_\rho = V_{\rho-1} + \frac{1}{2}z_\rho^2 \quad (50)$$

In the ρ th design step, by setting $i=\rho$, the control u and adaptation functions θ are described by

$$u=\frac{1}{b_m\sigma(y)}[-z_{\rho-1}-c_\rho z_\rho-(\frac{\partial\alpha_{\rho-1}}{\partial y})^2 z_\rho$$
$$-H_\rho+\varpi_\rho^T\tau_\rho+\gamma\omega_\rho\sum_{j=2}^{\rho-1}z_j\varpi_j^T \quad (51)$$
$$-w_{\rho 0}\tanh(w_{\rho 0}z_\rho/\kappa)+y_r^{(\rho)}]$$

$$\tau_i=\tau_{i-1}-\gamma z_i\omega_i(y)\ ,\ 2\le i\le\rho \quad (52)$$

$$\dot{\theta}=\tau_\rho \quad (53)$$

In the similar derivation procedure in Step i, we can obtain that the time derivative of V_ρ

$$\dot{V}_\rho\le-(\lambda_{\min}(Q)-\rho)\|\xi\|^2-\sum_{k=1}^{\rho}c_k z_k^2+\frac{1}{2}\|P\varepsilon\|^2$$
$$+\frac{\rho-1}{2}\delta_1^2+\frac{\sigma}{\gamma}\tilde{\theta}^T(\theta-\theta_0)+\rho\kappa' \quad (54)$$

By the inequality and based on Assumption 1, (54) can be rewritten as

$$\dot{V}_\rho\le-(\lambda_{\min}(Q)-\rho)\|\xi\|^2-\sum_{k=1}^{\rho}c_k z_k^2+\frac{1}{2}\|P\|^2\sum_{i=1}^{\rho}\varepsilon_{i0}^2$$
$$+\frac{\rho-1}{2}\delta_1^2-\frac{\sigma}{2\gamma}\tilde{\theta}^T\tilde{\theta}+\frac{\sigma}{2\gamma}\left\|\theta^*-\theta_0\right\|^2+\rho\kappa' \quad (55)$$

Let

$$c=\min\{2(\lambda_{\min}(Q)-\rho)/\lambda_{\min}(P),2c_i,\sigma;i=1,2,\cdots,\rho\}$$

$$\lambda=\frac{\sigma}{2\gamma}\sum_{k=1}^{\rho}\left\|\theta^*-\theta_0\right\|^2+\frac{1}{2}\|P\|^2\sum_{i=1}^{\rho}|\varepsilon_{i0}|^2+\frac{1}{2}\delta_1^2+\rho\kappa'$$

Then (55) becomes

$$\dot{V}\le-cV+\lambda \quad (56)$$

The above design and analysis procedure is summarized in the following theorem.

Theorem 1: Suppose the bounding Assumptions 1 holds. Then the fuzzy adaptive output tracking design described by the state observer (9), control law (51) and parameter adaptive laws (52) and (53) guarantee that the closed–loop system is semi-globally uniformly ultimately bounded and the output tracking error converges to a small neighborhood of the origin.

Proof: From (56) we have

$$V(t)\le V(t_0)e^{-c(t-t_0)}+\frac{\lambda}{c} \quad (57)$$

If choose a positive matrix Q, such that $\lambda_{\min}(Q)-\rho>0$, then from (57), it can be shown the signals $x(t)$, $\hat{x}(t)$, $e(t)$, $\theta(t)$ and $u(t)$ are semi-globally uniformly ultimately bounded, and that $|y(t)-y_r(t)|\le\sqrt{2V(t_0)}e^{-\frac{c}{2}(t-t_0)}+\sqrt{\frac{2\lambda}{c}}$.

In order to achieve the tracking error convergences to a small neighborhood around zero, the parameters c_i, σ and Q should be chosen appropriately.

IV. SIMULATION

Consider In this section, the proposed adaptive fuzzy control approach is applied to the following the example to verify its effectiveness

$$\dot{x}_1(t)=x_2(t)+(x_1-x_1^3)/(1+x_1^4)$$
$$\dot{x}_2(t)=u(t)-e^{-x_1^2}\sin(5x_1) \quad (58)$$
$$y(t)=x_1(t)$$

The given tracking reference signal is $y_r=\sin(t)$.

Choosing fuzzy membership functions as

$$\mu_{F_1^l}(x_1)=\exp[-\frac{(x_1-3+l)^2}{16}],\quad l=1,\cdots,5.$$

$$\mu_{F_2^l}(x_1)=\exp[-\frac{(x_1-3+l)^2}{4}],\quad l=1,\cdots,5.$$

Design parameters in controller and in adaptive laws are chosen as

$$k_1=2,\ k_2=2,\ \gamma_1=\gamma_2=0.1,\ \sigma=0.2,$$
$$\varepsilon_{10}=\varepsilon_{20}=0.01,\ c_1=10,\ c_2=10.$$

If the initial conditions are chosen as

$$x_1(0)=0,\ x_2(0)=0,\ \hat{x}_1(0)=0,$$
$$\hat{x}_2(0)=0,\ \theta_1(0)=\theta_2(0)=[0,0,0,0,0].$$

Then we obtain the simulation results, which are shown by Figure1-Figure 3.

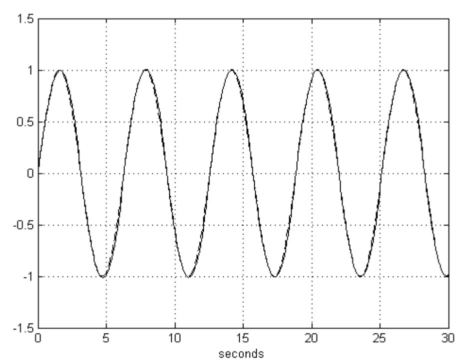

Fig.1. The trajectories of x_1 "-" and y_r "- -".

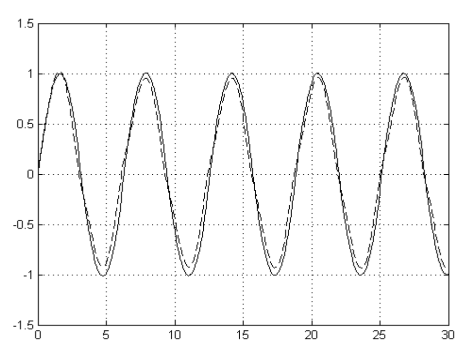

Fig.2. The trajectories of x_1 "-" and $\hat{x}_1$ "- -".

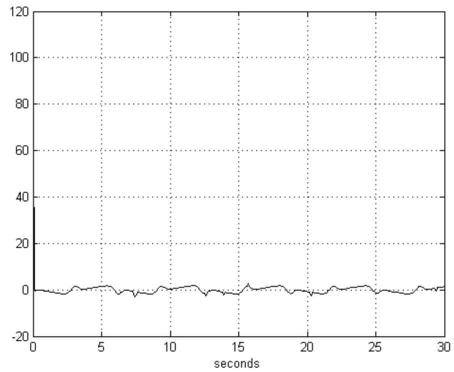

Fig.3. The trajectory of u

V. CONCLUSION

In this paper, a new fuzzy adaptive control approach is developed for a class of SISO nonlinear systems with unmeasured states. The main contribution of this paper is that by designing a fuzzy adaptive state observer based on K-filters, the application of adaptive fuzzy backstepping control is extended to a new class of nonlinear systems with states unmeasured. In addition, the stability of the closed-loop has been proved by using Lyapunov method, i.e., the proposed adaptive fuzzy control scheme can guarantee the all signals of closed-loop are boundedness and the tracking error of the system converges to a small neighborhood of the origin.

REFERENCES

[1] L. X. Wang, *Adaptive fuzzy systems and control: Design and stability analysis,* Prentice-Hall, Englewood Cliffs, NJ, 1994.

[2] T. Y. Chai, S. C. Tong, "Fuzzy direct adaptive control for a class of nonlinear systems," *Fuzzy Sets and Systems* vol.103, pp.379-389, 1999.

[3] B. Chen, X. P. Liu, "Fuzzy approximate disturbance decoupling of MIMO nonlinear systems by backstepping and application to chemical processes, " *IEEE Trans. Fuzzy Systems* vol.13, pp.832-847, 2005.

[4] B. Chen, X. P. Liu, S. C. Tong, "Fuzzy approximate disturbance decoupling of MIMO nonlinear systems by backstepping approach," *Fuzzy Sets and Systems,* vol.158, pp.1097-1125, 2007.

[5] X. P. Liu, G. X. Gu, and K. M. Zhou. "Robust stabilization of MIMO nonlinear systems by bacstepping," *Automatica,* vol.35, no.2, pp.987–992, 1999.

[6] B. S. Chen, C. H. Lee, Y. C. Chang, "H^{∞} tracking design of uncertain nonlinear SISO systems: adaptive fuzzy approach," *IEEE Trans. Fuzzy Systems* ,vol.4 , pp.32-43, 1996.

[7] I. Kanelakopoulos, P. V. Kokotovic, and A. S. Morse. "Systematic design of adaptive controller for feedback linearizable systems," *IEEE Trans. Automatic Control,* vol.36, pp.1241–1253, 1991.

[8] M. Kristic, I. Kanellakopoulos, P.V. Kokotovic, *Nonlinear and adaptive control design,* New York, Wiley, 1995.

[9] S. C. Tong, Y. M. Li, "Direct Adaptive Fuzzy Backstepping Control for a Class Nonlinear Systems," *International Journal of Innovative Computing, Information and Control,* vol.3, no.3, pp.887-896, 2007.

[10] M. Wang, B. Chen, "Direct Adaptive Fuzzy Tracking control for a class of perturbed strict-feedback nonlinear systems," *FuzzySets and Systems*, vol.158, pp.2655-2670, 2007.

[11] J. Y. Choi, J. A. Farrell, "Adaptive Observer Backstepping Control Using Neural Networks," *IEEE Trans. Neural Networks,* vol.12, no.5, pp.1103-1112, 2001.

[12] Y. S. Yang, G. Feng, J. S. Ren, "A combined backstepping and small-gain approach to robust adaptive fuzzy control for strict-feedback nonlinear systems," *IEEE Trans. Systems Man Cybernet.-Part A:* Systems and Humans, vol.34, pp.406–420, 2004.