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Frequency-Domain Weighted RLS Model Reduction for Complex SISO Linear System

Ping Zhou, Tianyou Chai, Qiang Liu, Hong Wang, and Chun-Yi Su

Abstract—**During the analysis and design of control systems, the complex SISO linear system that consist of several single transfer functions plus different time delay are often encountered and need to be reduced order. However, due to the complicated characteristics, it is difficult for such high-order model to obtain the reduced-order model using the existing methods. In this paper, a frequency-domain weighted recursive least squares (RLS) method is proposed for model reduction of such complex SISO linear system. The involved algorithm is derived, and the computational procedure of the model reduction is presented. At last, numerical examples are offered to verify the effectiveness of the proposed scheme for model reduction.**

I. Introduction

REDUCED-order model is often required for simplifying the design and implementation of control systems [1]. For this reason model reduction is very important in many areas of engineering sciences, especially in model-based control and optimization. The problem of model reduction can be described to find a reduced-order model to approximate the original high-order model without significant errors introduced. Over the last few decades, a great deal of attention has been paid to model reduction techniques and many important results on model reduction have been reported, which involve various efficient approaches such as dominant eigenvalue [2], Pade approximation [3], Routh approximation [4], balanced reduction [5], error minimization in the frequency-domain or in the time-domain [1, 6-8], Krylov subspace [9], and the recently proposed simple analytic rules method [10].

During the analysis and design of a control system, one usually encounters such sort of complex SISO linear system that composed of several transfer functions with different time delay. Such as the transfer function of a hybrid system that consist of several parallel connected subsystem with the same input, and the determinant of a multivariable transfer matrix with multiple input/output delays. From certain points of view, any complex stable SISO linear system can be described as the multinomial of several single transfer functions with different time delay. Because such high-order system is difficult to obtain the delay time and non-minimum phase zeros during the analysis and design of control system, it is necessary to simplify it to a single transfer function plus time delay. However almost all of the existing model reduction methods only pertains to the single transfer function, which can be considered as a particular simple case of above mentioned complex transfer multinomial. Therefore, it is hard to simplify the transfer multinomial using the existing method. For these reasons, it is necessary and helpful to develop a novel model reduction method to reduce the order of such prevalent complex SISO linear system.

Motivated by the above problems, a frequency-domain weighted recursive least squares (RLS) model reduction scheme is proposed in this paper to simplify the complex SISO linear system that described as the multinomial of several single transfer functions with different delay time. The derivation of the detailed algorithm, as well as the procedure of model reduction, is presented. At last, two illustrative simulations are offered to verify the effectiveness of proposed scheme.

II. Proposed Model Reduction Scheme

Any complex SISO linear system can be described as the following transfer function multinomial

$$\tilde{G}(s)=\frac{\sum_{i=0}^{m_1}\tilde{b}_{1,i}s^i}{\sum_{i=0}^{n_1}\tilde{a}_{1,i}s^i}e^{-\tau_1 s}+\frac{\sum_{i=0}^{m_2}\tilde{b}_{2,i}s^i}{\sum_{i=0}^{n_2}\tilde{a}_{2,i}s^i}e^{-\tau_2 s}+\cdots+\frac{\sum_{i=0}^{m_\xi}\tilde{b}_{\xi,i}s^i}{\sum_{i=0}^{n_\xi}\tilde{a}_{\xi,i}s^i}e^{-\tau_\xi s} \quad (1)$$

where $\tilde{a}_{j,i}$ and $\tilde{b}_{j,i}$ are the coefficients of each single transfer function, τ_j is the delay time of each single transfer function. In this paper, $\tilde{G}(s)$ is assumed a strictly proper and asymptotically stable transfer function multinomial. Notice that when $\xi=1$, $\tilde{G}(s)$ is simplified to a single transfer function, which is studied by most of the existing model reduction methods.

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P. Zhou, T. Y. Chai and Q. Liu are with the Key Laboratory of Integrated Automation of Process Industry (Northeastern University), Ministry of Education and Research Center of Automation, Northeastern University, Shenyang, China. (Phone: 86-24-83678332; fax: 86-24- 23895647; e-mail: zhoup1980@126.com).

H. Wang is with the School of Electrical and Electronic Engineering, University of Manchester, UK (e-mail: hong.wang@manchester.ac.uk)

C.-Y. Su is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, Quebec, Canada (e-mail: cysu@alcor.concordia.ca)

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Let the desired reduced model with the time delay be described by

$$G(s)=\frac{\sum_{i=0}^{m}b_i s^i}{1+\sum_{i=1}^{n}a_i s^i}e^{-\tau s}\triangleq\frac{B(s)}{A(s)}e^{-\tau s} \tag{2}$$

where $A(s)$ and $B(s)$ are the denominator and numerator polynomials of $G(s)$, respectively.

Among the above mentioned model reduction approaches, the error minimization method are used most widely because of the practical advantages. It takes account of the system responses subject to different types of inputs so that the obtained model yields zero steady-state error, preserves the stability characteristics and does not cause large errors. Besides, the control which is optimal for the reduced model is also applicable for the original high-order system. Moreover, the suitability of the reduction method of minimizing an objective function involving the approximation is assured since the method does not have any uncertainty in the reduction process [8].

Essentially, the error minimization based model reduction is to determine a set of parameters $\theta=\left[a_1\ \cdots\ a_n\ b_0\ \cdots\ b_m\right]^{\mathrm{T}}$ so that $G(s)$ can be approximated with $\tilde{G}(s)$ by minimizing certain objective, such as H_2-norm, H_∞-norm, L_2-norm or L_∞-norm [8,11-13]. In this paper, the following integral least squares performance index that proposed in [7] is used

$$J=\int_0^\infty e^2(t)\mathrm{d}t \tag{3}$$

where $e(t)$ is the Laplace inverse transform of $E(s)$, and $E(s)=(\tilde{G}(s)-G(s))U(s)=\tilde{Y}(s)-Y(s)$,$U(s)$ is the Laplace transform of input signal, $\tilde{Y}(s)$ and $Y(s)$ are the original system output and the reduced-order system output, respectively.

According to the popular Parseval theorem, the Eq.(3) is equivalent to the following frequency-domain index

$$J=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}E(s)E(-s)\mathrm{d}s \tag{4}$$

Let $s=j\omega$ .For a practical system, it is only need to take into account the problem within the selected frequency range $\omega_{\min}\sim\omega_{\max}$. Therefore the performance index showed in Eq.(4) is written as the following discrete form index

$$\begin{aligned}J&=\sum_{\varpi=1}^{M}v_\varpi\left|(\tilde{G}(j\omega_\varpi)-G(j\omega_\varpi))U(j\omega_\varpi)\right|^2\\&=\sum_{\varpi=1}^{M}v_\varpi\left|\tilde{G}(j\omega_\varpi)U(j\omega_\varpi)-G(j\omega_\varpi)U(j\omega_\varpi)\right|^2\end{aligned} \tag{5}$$

where $v_\varpi=\omega_{\varpi+1}-\omega_\varpi$ (assume that $\omega_1<\omega_2<\cdots<\omega_{M+1}$) are the weights. v_ϖ indicate that the larger the differences between the sequential two frequency points is, the heavier the weight is, and vice versa.

Eq.(5) indicates that $\theta=\left[a_1\ \cdots\ a_n\ b_0\ \cdots\ b_m\right]^{\mathrm{T}}$ are determined by matching $Y(s)$ to $\tilde{Y}(s)$ at multiple points. This multiple-point fitting approach will make modeling more robust [1].

From Eq. (2), we obtain $G(j\omega_\varpi)=B(j\omega_\varpi)e^{-\tau j\omega_\varpi}/A(j\omega_\varpi)$, then the performance index showed in Eq.(5) can be rewritten as

$$J=\sum_{\varpi=1}^{M}v_\varpi\left|\tilde{G}(j\omega_\varpi)U(j\omega_\varpi)-\frac{B(j\omega_\varpi)U(j\omega_\varpi)}{A(j\omega_\varpi)}e^{-\tau j\omega_\varpi}\right|^2 \tag{6}$$

Notice from Eq.(6) that a_i appear in the denominator and τ appears in the exponent. This causes the problem to be nonlinear. To simplify calculation, we multiply the equation error $\tilde{G}(j\omega_\varpi)U(j\omega_\varpi)-B(j\omega_\varpi)U(j\omega_\varpi)e^{-\tau j\omega_\varpi}/A(j\omega_\varpi)$ with $A(j\omega_\varpi)$ by adopting the Levy method [14], thus Eq. (6) is further rewritten as

$$J=\sum_{\varpi=1}^{M}v_\varpi\left|A(j\omega_\varpi)\tilde{G}(j\omega_\varpi)U(j\omega_\varpi)-B(j\omega_\varpi)U(j\omega_\varpi)e^{-\tau j\omega_\varpi}\right|^2 \tag{7}$$

Notice that Eq.(7) contains the unknown time delay τ to be estimated and it still makes the problem nonlinear. However, if τ is known, then the problem becomes to approximate a modified process $\tilde{G}_0(s)=\tilde{G}(s)e^{\tau s}$ with a rational transfer function $G_0(s)=B(s)/A(s)$. Therefore, for convenience of calculation, the following performance index is used for replacing of Eq. (7)

$$J=\sum_{\varpi=1}^{M}v_\varpi\left|A(j\omega_\varpi)\tilde{G}_0(j\omega_\varpi)U(j\omega_\varpi)-B(j\omega_\varpi)U(j\omega_\varpi)\right|^2 \tag{8}$$

The approximation to $\tilde{G}(s)$ will give an error J which is obviously related to τ . The solution to the original model reduction is obtained by minimizing this error over the possible range of τ . This is a one-dimensional search problem and can be easily solved if an estimation of the range of τ is given.

From Eq.(2), we can obtain $A(s)=1+\sum_{i=1}^{n}a_i s^i$ and $B(s)=\sum_{i=0}^{m}b_i s^i$, then substitute $A(s)$ and $B(s)$ in Eq. (8), we have

$$\begin{aligned}J=\sum_{\varpi=1}^{M}v_\varpi&\Big|[a_n(j\omega_\varpi)^n+\cdots+a_1(j\omega_\varpi)^1+1]\tilde{G}_0(j\omega_\varpi)U(j\omega_\varpi)\\&-[b_m(j\omega_\varpi)^m+\cdots+b_1(j\omega_\varpi)^1+b_0]U(j\omega_\varpi)\Big|^2\end{aligned}$$

$$
\begin{aligned}
&= \sum_{\varpi=1}^{M} v_\varpi \left| \tilde{G}_0(j\omega_\varpi)U(j\omega_\varpi) - \theta^T \mho(j\omega_\varpi) \right|^2 \\
&= \sum_{\varpi=1}^{M} v_\varpi \left| \tilde{Y}_0(j\omega_\varpi) - \theta^T \mho(j\omega_\varpi) \right|^2
\end{aligned}
$$

where

$$
\mho(j\omega_\varpi) = \begin{pmatrix} -\tilde{G}_0(j\omega_\varpi)U(j\omega_\varpi)(j\omega_\varpi) \\ \vdots \\ -\tilde{G}_0(j\omega_\varpi)U(j\omega_\varpi)(j\omega_\varpi)^n \\ U(j\omega_\varpi)(j\omega_\varpi)^0 \\ \vdots \\ U(j\omega_\varpi)(j\omega_\varpi)^m \end{pmatrix}.
$$

Moreover, the performance index can be further derived as

$$
\begin{aligned}
J &= \sum_{\varpi=1}^{M} v_\varpi \left| \tilde{Y}_0(j\omega_\varpi) - \theta^T \mho(j\omega_\varpi) \right|^2 \\
&= \sum_{\varpi=1}^{M} v_\varpi [\tilde{Y}_0(j\omega_\varpi)\overline{\tilde{Y}_0(j\omega_\varpi)} - \tilde{Y}_0(j\omega_\varpi)\overline{\theta^T \mho(j\omega_\varpi)} \\
&\quad - \theta^T \mho(j\omega_\varpi)\overline{\tilde{Y}_0(j\omega_\varpi)} + \theta^T \mho(j\omega_\varpi)\overline{\theta^T \mho(j\omega_\varpi)}]
\end{aligned} \tag{9}
$$

In the following paper, the detailed frequency-domain weighted RLS arithmetic for model reduction is derived by adopting the similar method as reference [15].

Take the partial derivative of J with respect to θ, we have

$$
\begin{aligned}
\frac{\partial J}{\partial \theta} &= \sum_{\varpi=1}^{M} v_\varpi [-\tilde{Y}_0(j\omega_\varpi)\overline{\mho(j\omega_\varpi)} - \mho(j\omega_\varpi)\overline{\tilde{Y}_0(j\omega_\varpi)} \\
&\quad + \mho(j\omega_\varpi)\overline{\mho^T(j\omega_\varpi)}\theta + \overline{\mho(j\omega_\varpi)}\mho^T(j\omega_\varpi)\theta]
\end{aligned} \tag{10}
$$

Let $\frac{\partial J}{\partial \theta} = 0$, we get

$$
\begin{aligned}
&\sum_{\varpi=1}^{M} v_\varpi [\mho(j\omega_\varpi)\overline{\mho^T(j\omega_\varpi)} + \overline{\mho(j\omega_\varpi)}\mho^T(j\omega_\varpi)]\theta \\
&= \sum_{\varpi=1}^{M} v_\varpi [\tilde{Y}_0(j\omega_\varpi)\overline{\mho(j\omega_\varpi)} + \mho(j\omega_\varpi)\overline{\tilde{Y}_0(j\omega_\varpi)}]
\end{aligned} \tag{11}
$$

Define $P_k^{-1} = \sum_{\varpi=1}^{k} v_\varpi [\mho(j\omega_\varpi)\overline{\mho^T(j\omega_\varpi)} + \overline{\mho(j\omega_\varpi)}\mho^T(j\omega_\varpi)]$, then we can further obtain

$$
\begin{aligned}
P_k^{-1} &= P_{k-1}^{-1} + v_k [\mho(j\omega_k)\overline{\mho^T(j\omega_k)} + \overline{\mho(j\omega_k)}\mho^T(j\omega_k)] \\
&= P_{k-1}^{-1} + v_k [2\,\mathrm{Re}(\mho(j\omega_k))\,\mathrm{Re}(\mho^T(j\omega_k)) \\
&\quad + 2\,\mathrm{Im}(\mho(j\omega_k))\,\mathrm{Im}(\mho^T(j\omega_k))]
\end{aligned} \tag{12}
$$

From Eq. (11) and Eq. (12), we have

$$
\begin{aligned}
P_k^{-1}\theta_k &= \sum_{\varpi=1}^{k} v_\varpi [\tilde{Y}_0(j\omega_\varpi)\overline{\mho(j\omega_\varpi)} + \mho(j\omega_\varpi)\overline{\tilde{Y}_0(j\omega_\varpi)}] \\
&= P_{k-1}^{-1}\theta_{k-1} + v_k [\tilde{Y}_0(j\omega_k)\overline{\mho(j\omega_k)} + \mho(j\omega_k)\overline{\tilde{Y}_0(j\omega_k)}]
\end{aligned} \tag{13}
$$

It follows that

$$
\begin{aligned}
\theta_k = \theta_{k-1} + P_k \{ v_k [\tilde{Y}_0(j\omega_k\overline{\mho(j\omega_k)} + \mho(j\omega_k)\overline{\tilde{Y}_0(j\omega_k)}] \\
- v_k [\mho(j\omega_k)\overline{\mho^T(j\omega_k)} + \overline{\mho(j\omega_k)}\mho^T(j\omega_k)] \} \theta_{k-1}
\end{aligned} \tag{14}
$$

Define

$$
Q_{k-1}^{-1} = P_{k-1}^{-1} + 2v_k\,\mathrm{Re}(\mho(j\omega_k))\,\mathrm{Re}(\mho^T(j\omega_k)) \tag{15}
$$

From Eq.(12) and Eq.(15), we get

$$
P_k^{-1} = Q_{k-1}^{-1} + 2v_k\,\mathrm{Im}(\mho(j\omega_k))\,\mathrm{Im}(\mho^T(j\omega_k)) \tag{16}
$$

From Eq.(15) and Eq.(16), Q_{k-1} and P_k can be obtained as follows by using the famous matrix inversion lemma

$$
Q_{k-1} = \left[I - \frac{2v_k P_{k-1}\,\mathrm{Re}(\mho(j\omega_k))\,\mathrm{Re}(\mho^{\mathrm{T}}(j\omega_k))}{1 + 2v_k\,\mathrm{Re}(\mho^{\mathrm{T}}(j\omega_k))P_{k-1}\,\mathrm{Re}(\mho(j\omega_k))} \right] P_{k-1} \tag{17}
$$

$$
P_k = \left[I - \frac{2v_k Q_{k-1}\,\mathrm{Im}(\mho(j\omega_k))\,\mathrm{Im}(\mho^{\mathrm{T}}(j\omega_k))}{1 + 2v_k\,\mathrm{Im}(\mho^{\mathrm{T}}(j\omega_r))Q_{k-1}\,\mathrm{Im}(\mho(j\omega_k))} \right] Q_{k-1} \tag{18}
$$

Integrating Eq. (14), Eq.(17) and Eq.(18), the final frequency -domain weighted RLS formulae for model reduction is obtained as follows

$$
\begin{cases}
\theta_k = \theta_{k-1} + P_k \{ v_k \left[\tilde{Y}_0(j\omega_k)\overline{\mho(j\omega_k)} + \overline{\tilde{Y}_0(j\omega_k)}\mho(j\omega_k) \right] \\
\quad - v_k \left[\mho(j\omega_k)\overline{\mho^T(j\omega_k)} + \overline{\mho(j\omega_k)}\mho^T(j\omega_k) \right] \theta_{k-1} \} \\
Q_{k-1} = \left[I - \frac{2v_k P_{k-1}\,\mathrm{Re}(\mho(j\omega_k))\,\mathrm{Re}(\mho^{\mathrm{T}}(j\omega_k))}{1 + 2v_k\,\mathrm{Re}(\mho^{\mathrm{T}}(j\omega_k))P_{k-1}\,\mathrm{Re}(\mho(j\omega_k))} \right] P_{k-1} \\
P_k = \left[I - \frac{2v_k Q_{k-1}\,\mathrm{Im}(\mho(j\omega_k))\,\mathrm{Im}(\mho^{\mathrm{T}}(j\omega_k))}{1 + 2v_k\,\mathrm{Im}(\mho^{\mathrm{T}}(j\omega_r))Q_{k-1}\,\mathrm{Im}(\mho(j\omega_k))} \right] Q_{k-1}
\end{cases} \tag{19}
$$

Some initial parameters can be determined by using the tentative method that $P_0 = \sigma I$ and $\theta_0 = \varepsilon$, where σ is a sufficiently large positive number, and ε is a sufficiently real vector.

The above recursive formulae will be repeated until the given maximum recursion number or the following maximum estimating error of θ meets

$$
\max_{\forall i} \left| \frac{\theta_k(i) - \theta_{k-1}(i)}{\theta_{k-1}(i)} \right| < \eta \tag{20}
$$

where η is a user-specified threshold, $\theta_k(i)$ denotes the i-th element of θ_k.

In general, model reduction and system identification are service for system control. Therefore, the selection of exciting input signal $U(s)$ in model reduction should to consider the used control strategy. If the control strategy emphasizes in accuracy of step response, $U(s)$ can choose the unit step input. If the control strategy requires a finer response for high frequency input, $U(s)$ can choose the impulse input.

III. COMPUTATIONAL PROCEDURE

The computational procedure required for the frequency-domain weighted RLS model reduction for the above mentioned complex linear system that consist of several single transfer functions in parallel is summarized as follows

Step 1: According to the actual requirement, choose $N, \tau_0, \Delta\tau$, and obtain $\tau_i = \tau_0 + (i-1)\Delta\tau,\ i = 1, \cdots, N$.

Step 2: Determine the frequency range $(\omega_{\min} \sim \omega_{\max})$ Hz, such as $(10^{-3} \sim 10^{3})$ Hz, and average it into M parts in the logarithm coordinate, then the k -th frequency ω_k can be calculated by $\omega_k = 10^{(-\omega_{\min} + (\omega_{\max} - \omega_{\min})k/M)}$.

Step 3: Choose the exciting input signal $U(s)$ according to the actual requirement.

Step 4: For each τ_i, find a rational approximation solution $G_0(s)$ to modified model $\tilde{G}_0(s) = \tilde{G}(s)e^{\tau_i s}$ with the proposed weighted RLS model reduction formulae.

Step 5: For each rational approximation solution $G_0(s)$ obtained in step 4, calculate $G(s) = G_0(s)e^{-t_i s}$, and then evaluate the corresponding approximation error e in Eq. (21)

$$e = \frac{1}{N}\sum_{k=1}^{N}\left|\tilde{G}(j\omega_k)U(j\omega_k) - G(j\omega_k)U(j\omega_k)\right|^2 \tag{21}$$

Step 6: Take as the solution $G(s)$ that yields the minimum error e in Eq. (21).

IV. ILLUSTRATIVE EXAMPLES

In this section, two simulation examples are presented to demonstrate the effectiveness of the proposed frequency-domain weighted RLS based model reduction scheme.

Example 1. Consider the following complex linear system that consist of two second-order systems plus time delay, it is desired to obtain first-order reduced model (FORM) and second-order reduced model (SORM) using the proposed model reduction arithmetic.

$$\tilde{G}(s) = \frac{0.00545e^{-10s}}{s^2 + 0.1394s + 0.0044} - \frac{0.010326e^{-4s}}{s^2 + 0.1293s + 0.0042}$$

First, some key parameters are determined as

$$\begin{cases} \sigma = 10^6, \eta = 10^{-5}, \tau_0 = 4, N = 100, \Delta\tau = 0.06 \\ \varepsilon = \begin{cases} 10^{-5} I_{5\times1}, \text{SORD} \\ 10^{-5} I_{3\times1}, \text{FORD} \end{cases} \\ (\omega_{\min} \sim \omega_{\max}) = (10^{-3} \sim 10^{3})\text{Hz}, M = 600 \end{cases}$$

Then choose the unit step input as the exciting input signal, which means that $U(s) = 1/s$. Finally, the FORM and the SORM are obtained as Eq. (22) and Eq. (23), respectively.

$$G(s) = \frac{3.6737\times10^{-4}s - 1.2197}{21.9737s + 1}e^{-6.4s} \tag{22}$$

$$G(s) = \frac{-1.2142\times10^{-7}s^2 + 6.6537\times10^{-5}s - 1.22}{115.0258\,s^2 + 23.8051s + 1}e^{-4s} \tag{23}$$

Fig.1, Fig. 2 and Fig.3 show the response effect of the original model and the reduced-order models due to the unit square wave input, the unit ramp input and the unit sine input, respectively. It can be seen that no matter what the input is, the obtained lower-order model can approximate to the original model very closely. From the comparisons of approximation effect of the FORM and the SORM, it can conclude that the SORM has better accuracy. This is because the SORM can cover a wider range of dynamics of the original model. Moreover, it is easy to obtain that the steady-state responses of the original model, the FORM and the SORM are $\tilde{G}(0) = -1.2199$, $G(0) = -1.2197$ and $G(0) = -1.22$, respectively. This means that the reduced-order models almost have yielded zero steady-state error. Therefore, the reduced model obtained by using the proposed model reduction method not only can cause limited large dynamic approximation errors, but also can preserve the stability characteristics.

Example 2. Consider the following more complex linear system that consist of three second-order systems plus time delay, it is still desired to obtain FORM and SORM.

$$\tilde{G}(s) = -\frac{0.3543}{(13.5s+1)(2.61s+1)}e^{-3.14s} - \frac{0.5519}{(16.55s+1)(2.55s+1)}e^{-2.345s} - \frac{0.3853\times(51.6s+1)}{(15.5s+1)(6.6s+1)}$$

Firstly, choose $U(s)=1/s$, and determine the parameters as follows

$$\begin{cases} \sigma = 10^6, \eta = 10^{-5}, \tau_0 = 0, N = 100, \Delta\tau = 0.03 \\ \varepsilon = \begin{cases} 10^{-5} I_{5\times1}, \text{SORM} \\ 10^{-5} I_{3\times1}, \text{FORM} \end{cases} \\ (\omega_{\min} \sim \omega_{\max}) = (10^{-3} \sim 10^{3})\text{Hz}, M = 600 \end{cases}$$

Then, by using the proposed model reduction method, the FORM and the SORM are obtained as Eq. (24) and Eq. (25), respectively.

$$G(s) = \frac{-3.0549\times10^{-4}s - 1.2915}{5.6437s + 1}e^{-0s} \tag{24}$$

$$G(s) = \frac{-4.3753s^2 - 7.06s - 1.2916}{43.5119s^2 + 10.7588s + 1}e^{-0.36s} \tag{25}$$

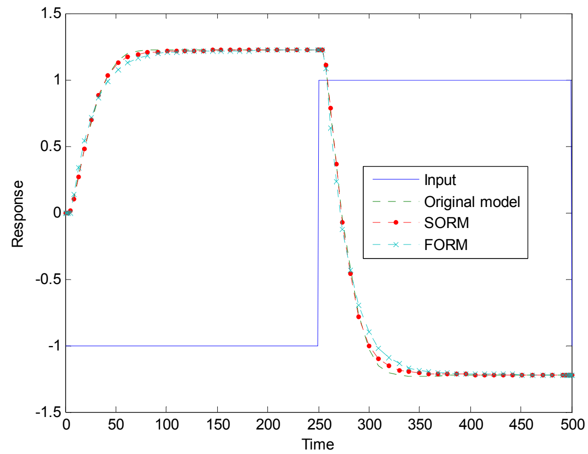

Fig. 1 Unit square wave response of original and reduced-order models

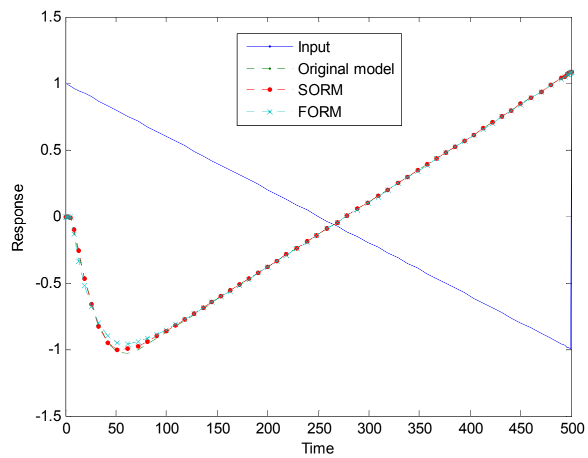

Fig. 2 Unit ramp response of original and reduced-order models

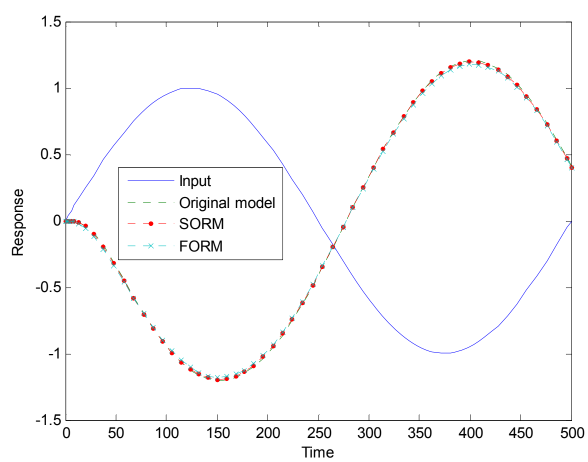

Fig. 3 Unit sine response of original and reduced-order models

Fig. (4) shows the unit step response error of the reduced-order models. It can be seen that the absolute response errors of the FORM and the SORM are all tiny, which means that no matter how complex the original system is, the obtained reduced-order models with the proposed model reduction scheme all have fine approximation to the original model. This example further illustrates the effectiveness of the proposed method for model reduction of the complex SISO linear system.

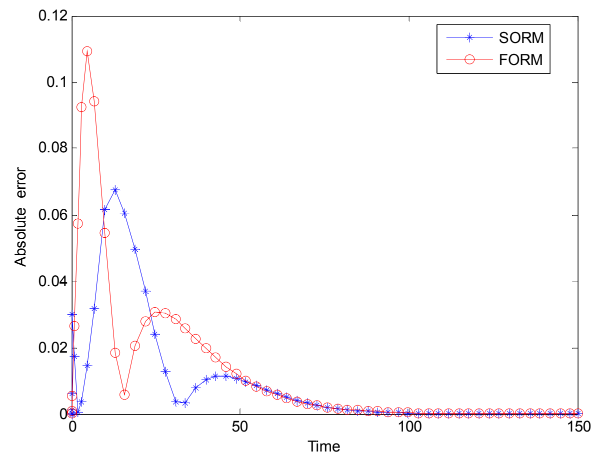

Fig. 4 Unit step response error of reduced-order models

V. CONCLUSION

A new frequency-domain weighted RLS method has been presented and validated in this paper for model reduction of SISO linear dynymic system that described as the multinomial of several single transfer functions with different delay time. The proposed method shows some good performances in the following aspects:

- The proposed method not only can cause less dynamic error, but also can yield zero steady-state error, thus can preserve the stability characteristics.
- The proposed method can be used to simplify any SISO linear system to a single transfer function plus time delay with any order. This is because all complex SISO linear system can be described as the form of Eq. (1), and any desired reduced-model also can be described as the form of Eq. (2).
- The quadratic performance index used in this paper (as shown in Eq. (8) or Eq. (9)) includes the exciting input signal, which expresses the requirement for optimal performance index subjected to the selected input signal. Therefore, it reflects the thought of control-oriented identification that involved in the proposed model reduction method.

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