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Adaptive Fault-tolerant Output-feedback Control of LTI Systems Subject to Actuator Saturation

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Abstract— **This paper is concerned with the problem of designing adaptive fault-tolerant output-feedback controllers for linear time-invariant (LTI) systems with actuator saturation. By combining the linear matrix inequality (LMI) approach for output-feedback controllers design and adaptive method, a method of designing adaptive reliable output-feedback controllers is proposed, where the controller parameter matrices are updated automatically to compensate the controller failures effects on systems based on the online estimations of eventual faults. The designs are developed in the framework of LMI approach, which can enlarge the domain of asymptotic stability of closed-loop systems in the cases of actuator saturation and actuator failures. An example is given to illustrate the efficiency of the design method.**

I. Introduction

Control systems with actuator saturation are often encountered in practice. When actuator saturation occurs, global stability of an otherwise stable linear closed-loop system can not in general be ensured. And the problem of estimating the domain of attraction for a system with a saturated linear feedback has been studied by many researchers in the last few years and various methods have appeared. Model predictive control (MPC) is an effective control algorithm for dealing with actuator saturation. Over the last few years, many formulations have been developed for the stability of MPC (see, [1], [2]). Enlargement of the domain of attraction is achieved in ([3], [4], [5], [6], [7]). Anti-windup research has been largely discussed and many constructive design algorithms have been formally proved to induce suitable stability properties (see, [8]-[14]). Many of these constructive approaches rely on sector condition and S-procedure techniques and provide LMIs for the anti-windup compensator design. In some papers, notion of invariant set and LMI-based optimization approaches were proposed to estimating the stability regions by using quadratic Lyapunov functions and the Lur'e-type Lyapunov functions. In [15] and [16], the modeling of the nonlinear behavior of the system under saturation is made by using a polytopic differential inclusion and quadratic Lyapunov functions. For determining if a given ellipsoid is contractively invariant, [17] described a condition which is based on the circle criterion or the vertex analysis.

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On the other hand, fault-tolerant design approach can be broadly classified into two types: Passive approach [18]-[22] and Active approach [23]-[28]. In the passive approach, the same controller is used throughout normal case as well as fault cases such that this passive fault tolerant controller is easily implemented. Some approaches to the design of passive fault-tolerant controllers have been addressed by several authors (see [18]-[22] and the references therein). An active fault-tolerant control system compensates for faults either by selecting a pre-computed control law or by synthesizing a new control strategy on-line. Some of these methods include a strategy involving a fast subsystem for Fault Detection and Isolation (FDI), and a supervisory system that chooses the corresponding controller for a particular type of fault.

As we all know, in practice, actuator saturation and actuator faults are the common phenomenon, and they always happen at the same time. However, noting all above results, there is no work that deals with this problem. Motivated by the above observations, this paper studies a class of linear time-invariant systems with actuator saturation and actuator faults at the same time. A general actuator fault model is considered, which covers the outage cases and the possibility of partial faults. Here, an LMI-based method is presented to deal with the fault-tolerant and saturation problem. One key difference between this paper and some existing results is that in this paper, the fault-tolerant and saturation are considered at the same time.

The paper is organized as follows. Problem statement is given in Section 2. It is followed by the adaptive fault-tolerant controller design method to enlarge the domain of asymptotic stability of closed-loop systems in the cases of actuator saturation and actuator failures in Section 3. An illustrative example is presented in Section 4 to demonstrate the proposed design methods. The paper will be concluded in Section 5.

II. Problem statement and preliminaries

Consider an LTI plant described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\sigma(u(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the plant state, $\sigma(u) \in R^m$ is the saturated control input. A, B, C are known constant matrices of appropriate dimensions.

The actuator nonlinearity with the consideration of a piecewise-linear saturation is described as

$$\sigma(u_j) = \begin{cases} u_j, & |u_j| \le u_j^{max}, \\ sign(u_j)u_j^{max}, & |u_j| > u_j^{max}, \end{cases} \quad (2)$$

for $j \in \mathbf{I}[1,m]$. Here we have slightly abused the notation by using σ to denote both the scalar valued and the vector valued saturation functions as explained in [17]. We note that it is without loss of generality to assume $u_j^{max} = 1$, as level of saturation can always be scaled to unity by scaling B and u as pointed out in [17].

To formulate the fault-tolerant control problem, the following actuator fault model from [19] and [22] is adopted in this paper:

$$u_{jq}^F(t) = (1-\rho_j^q)\sigma(u_j(t)), \quad 0 \le \underline{\rho}_j^q \le \rho_j^q \le \overline{\rho}_j^q, \\ j \in \mathbf{I}[1,m],\ q \in \mathbf{I}[1,L], \quad (3)$$

where $u_{jq}^F(t)$ represents the signal from the jth actuator that has failed in the qth fault mode, ρ_j^q is an unknown constant, the index q denotes the qth fault mode and L is the total fault modes. For every fault mode, $\underline{\rho}_j^q$ and $\overline{\rho}_j^q$ represent the lower and upper bounds of ρ_j^q, respectively. Denote

$$u_q^F(t) = [u_{1q}^F(t),\ u_{2q}^F(t),\ \cdots u_{mq}^F(t)]^T = (I-\rho^q)\sigma(u(t)) \quad (4)$$

where $\rho^q = \mathrm{diag}[\rho_1^q,\ \rho_2^q,\ \cdots \rho_m^q]$, $q \in \mathbf{I}[1,L]$. Considering the lower and upper bounds $\underline{\rho}_j^q$ and $\overline{\rho}_j^q$, the following set can be defined

$$N_{\rho^q} = \{\rho^q | \rho^q = \mathrm{diag}[\rho_1^q,\ \rho_2^q,\ \cdots \rho_m^q], \rho_j^q = \underline{\rho}_j^q \text{ or } \rho_j^q = \overline{\rho}_j^q\}. \quad (5)$$

Thus, the set N_{ρ^q} contains a maximum of 2^m elements.

For convenience in the following sections, for all possible fault modes L, the following uniform actuator fault model is exploited:

$$u^F(t) = (I-\rho)\sigma(u(t)),\ \rho \in \{\rho^1\ \cdots \rho^L\} \quad (6)$$

and ρ can be described by $\rho = \mathrm{diag}[\rho_1,\ \rho_2,\ \cdots \rho_m]$.

The following definition and lemmas will be used in the sequel.

Definition 1: For a matrix $C_{cl} \in R^{m\times n}$, denote the jth row of C_{cl} as C_{clj}, define

$$\wp(C_{cl}) = \{x \in R^n : \quad |C_{clj}x| \le 1, \quad j \in \mathbf{I}[1,m]\},$$

then $\wp(C_{cl})$ is the region in the state space where saturation does not occur.

For $x(0) = x_0 \in R^n$, denote the state trajectory of system (4) as $\psi(t,x_0)$. Then the *domain of attraction* of the origin is

$$\ell := \{x_0 \in R^n : \quad lim_{t\to\infty}\psi(t,x_0) = 0\}.$$

Lemma 1: If there exists a symmetric matrix Θ with

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix}$$

and Θ_{11}, $\Theta_{22} \in R^{Nn\times Nn}$ such that the following inequalities hold:

$$\Theta_{22jj} \le 0,\ \ j \in \mathbf{I}[1,N],$$
$$\Theta_{11} + \Theta_{12}\Delta(\delta) + (\Theta_{12}\Delta(\delta))^T + \Delta(\delta)\Theta_{22}\Delta(\delta) \ge 0,\ \delta \in \Delta_v$$
$$\begin{bmatrix} Q & E \\ E^T & F \end{bmatrix} + G^T\Theta G < 0,\ \rho \in \{\rho^1\ \cdots\ \rho^L\},\ \rho^q \in N_{\rho^q}$$

then inequality

$$W(\delta) = Q + \textstyle\sum_{j=1}^N \delta_j E_j + (\sum_{j=1}^N \delta_j E_j)^T \\ + \textstyle\sum_{j=1}^N \sum_{p=1}^N \delta_j\delta_p F_{jp} < 0$$

holds for all $\delta_j \in [\underline{\delta}_j\ \ \overline{\delta}_j]$, where $Q = Q^T \in R^{n\times n}$ and $F_{pj} = F_{pj}^T \in R^{n\times n}$, $E_j \in R^{n\times n}$

$$\Delta(\delta) = \mathrm{diag}[\delta_1 I_{n\times n} \quad \cdots \quad \delta_N I_{n\times n}], \quad E = [E_1\ E_2\ \cdots\ E_N],$$

$$F = \begin{bmatrix} F_{11} & \cdots & F_{1N} \\ \cdots & \cdots & \cdots \\ F_{N1} & \cdots & F_{NN} \end{bmatrix}, G = \begin{bmatrix} \begin{bmatrix} I_{n\times n} \\ \cdots \\ I_{n\times n} \end{bmatrix} & 0 \\ 0 & I_{Nn\times Nn} \end{bmatrix}.$$

Let $\mathbf{D}$ be a set of $m\times m$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^m elements in $\mathbf{D}$ and we denote its elements as D_i, $i \in \mathbf{I}[0,2^m-1]$, where for $i = z_1 2^{m-1} + Z_2 2^{m-2} + \cdots + z_m$ with $z_j \in \{0,1\}$, the diagonal elements of D_i are $\{1-z_1, 1-z_2, \cdots, 1-z_m\}$. Denote $D_i^- = I - D_i$. It is easy to see that $D_i^- \in \mathbf{D}$. Then we have

Lemma 2: ([29]) Let u, $v \in R^m$ with $u = [u_1, u_2, ..., u_m]^T$ and $v = [v_1, v_2, ..., v_m]^T$. Suppose that $|v_j| \le 1$ for all $j \in \mathbf{I}[1,m]$. Then,

$$\sigma(u) \in co\{D_i u + D_i^- v : i \in \mathbf{I}[0,2^m-1]\}, \quad (7)$$

where co denotes the convex hull. Then, the following problem will be considered in this paper.

Problem 1: Find an adaptive controller such that in both normal operation and fault cases, the domain of asymptotic stability is enlarged as possible for closed-loop system with actuator saturation.

Remark 1: For the above problem to be solved, it is necessary for the pair $(A, B(I-\rho))$ to be stabilizable for each $\rho \in \{\rho^1 \cdots \rho^L\}$.

III. Main results

A. An improved condition for set invariance

The dynamics with actuator faults (6) and saturation is described by

$$\dot{x}(t) = Ax(t) + B(I-\rho)\sigma(u(t)) \\ y(t) = Cx(t) \quad (8)$$

The controller structure is chosen as

$$\dot{\xi}(t) = f(\xi(t),\ y),\ \ \xi(t) \in R^n \\ u(t) = C_K(\hat{\rho}(t))\xi(t) \quad (9)$$

with

$$u(t) = C_K(\hat{\rho}(t))\xi(t) = (C_{K0} + C_{Ka}(\hat{\rho}(t)) + C_{Kb}(\hat{\rho}(t)))\xi(t) \quad (10)$$

where $\hat{\rho}(t)$ is the estimation of ρ, $C_{Ka}(\hat{\rho}(t)) = \sum_{j=1}^m C_{Kaj}\hat{\rho}_j(t)$ and $C_{Kb}(\hat{\rho}(t)) = \sum_{j=1}^m C_{Kbj}\hat{\rho}_j(t)$.

By lemma 1, the saturated linear feedback, with $\xi(t) \in \wp(H(\hat{\rho}(t)))$, can be expressed as

$$\sigma(C_K(\hat{\rho}(t))\xi(t)) = \textstyle\sum_{i=0}^{2^m-1} \eta_i [D_i C_K(\hat{\rho}(t)) + D_i^- H(\hat{\rho}(t))]\xi(t) \quad (11)$$

for some scalars $0 \le \eta_i \le 1$, $i \in \mathbf{I}[0, 2^m-1]$, such that $\sum_{i=0}^{2^m-1} \eta_i = 1$, and the following equality holds

$$
\begin{aligned}
(I-\rho)\sigma(u(t)) &= \textstyle\sum_{i=0}^{2^m-1} \eta_i[(I-\rho)D_iC_{K0} + D_iC_{Ka}(\rho) \\
&\quad -\rho D_iC_{Ka}(\hat{\rho}) + (I-\hat{\rho}(t))D_iC_{Kb}(\hat{\rho}(t)) + D_iC_{Ka}(\tilde{\rho}(t)) \\
&\quad +\tilde{\rho}D_iC_{Kb}(\hat{\rho}(t)) + (I-\rho)D_i^-H_{K0} + D_i^-H_{Ka}(\rho) \\
&\quad -\rho D_i^-H_{Ka}(\hat{\rho}) + (I-\hat{\rho}(t))D_i^-H_{Kb}(\hat{\rho}(t)) \\
&\quad +D_i^-H_{Ka}(\tilde{\rho}(t)) + \tilde{\rho}D_i^-H_{Kb}(\hat{\rho}(t))]\xi(t)
\end{aligned} \tag{12}
$$

where $\tilde{\rho}(t) = \hat{\rho}(t) - \rho$. It should be noted that though $C_{Ka}(\hat{\rho}(t))$ and $C_{Kb}(\hat{\rho}(t))$ have the same forms, we deal with them in different ways in (12), which gives more freedom and less conservativeness in Theorem 1.

Definition 2: Let $P \in R^{n\times n}$ be a positive-define matrix. Denote

$$
\begin{aligned}
\varepsilon(P,\delta) &= \{x \in R^n: \quad x^TPx \le \delta\}. \\
\varepsilon^-(P,\delta) &= \{x \in R^n: \quad x^TPx < \delta\}. \\
\varepsilon^*(P,\delta) &= \{x \in R^n: \quad x^TPx + \textstyle\sum_{j=1}^m \frac{\tilde{\rho}_j^2(t)}{l_j} \le \delta\}.
\end{aligned}
$$

Let $V(t) = x^TPx + \sum_{j=1}^m \frac{\tilde{\rho}_j^2(t)}{l_j}$. If $\dot{V}(t) < 0$ for all $x \in \varepsilon^*(P,\delta)\backslash\{0\}$, the domain $\varepsilon^*(P,\delta)$ is contractively invariant. Clearly, if $\varepsilon^*(P,\delta)$ is contractively invariant, then it is inside the domain of attraction.

We note that the scalars η_i's are functions of ξ and $\hat{\rho}$ and their values are available in real-time. These scalars in a way reflect the severity of control saturation. In general, there are multiple choices of η_i's satisfying the same constraint, leading to nonunique representation of (11). In the following lemma, we provide one choice of such η_i's, which are Lipschitzian functions in ξ and $\hat{\rho}$ and thus are particularly useful in our controller design.

Lemma 3: ([16]) Let $\xi(t) \in \wp(H(\hat{\rho}(t)))$. For each $j \in \mathbf{I}[1,m]$, let

$$
\lambda_j(\xi(t),\hat{\rho}(t)) = \begin{cases} 1, & \text{if } C_K(\hat{\rho}(t))_j\xi(t) = H(\hat{\rho}(t))_j\xi(t) \\ \frac{\sigma(C_K(\hat{\rho}(t))_j\xi(t)) - H(\hat{\rho}(t))_j\xi(t)}{(C_K(\hat{\rho}(t))_j - H(\hat{\rho}(t))_j)\xi(t)}, & \text{otherwise} \end{cases}
$$

and for each $i \in \mathbf{I}[0, 2^m-1]$, let $z_j \in \{0,1\}$ be such that $i = z_1 2^{m-1} + z_2 2^{m-2} + \ldots + z_m$, and define

$$
\begin{aligned}
\eta_i(\xi(t),\, \hat{\rho}(t)) = \prod_{j=1}^m [z_j(1-\lambda_j(\xi(t),\, \hat{\rho}(t))) \\
+ (1-z_j)\lambda_j(\xi(t),\, \hat{\rho}(t))]
\end{aligned} \tag{13}
$$

Then, η_i's are functions Lipschitz in ξ and $\hat{\rho}$, such that, $\sum_{i=0}^{2^m-1} \eta_i = 1$, $0 \le \eta_i \le 1$, $i \in \mathbf{I}[0, 2^m-1]$. Moreover, they satisfy relation (11).

By using the functions $\eta_i(\xi(t),\, \hat{\rho}(t))'$s, the output feedback controller (9) can be parameterized as

$$
\begin{aligned}
\dot{\xi}(t) &= (\textstyle\sum_{i=0}^{2^m-1} \eta_i A_{Ki}(\hat{\rho}))\xi(t) + (\textstyle\sum_{i=0}^{2^m-1} \eta_i B_{Ki}(\hat{\rho}))y(t) \\
u(t) &= (I-\rho)\sigma(C_K(\hat{\rho})\xi(t))
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
A_{Ki}(\hat{\rho}) &= A_{Ki0} + A_{Kia}(\hat{\rho}) + A_{Kib}(\hat{\rho}) \\
B_{Ki}(\hat{\rho}) &= B_{Ki0} + B_{Kia}(\hat{\rho}) + B_{Kib}(\hat{\rho}) \\
C_K(\hat{\rho}) &= C_{K0} + C_{Ka}(\hat{\rho}) + C_{Kb}(\hat{\rho}) \\
B_{Kia}(\hat{\rho}) &= \textstyle\sum_{j=1}^m \hat{\rho}_j B_{Kiaj},\ B_{Kib}(\hat{\rho}) = \sum_{j=1}^m \hat{\rho}_j B_{Kibj} \\
C_{Ka}(\hat{\rho}) &= \textstyle\sum_{j=1}^m \hat{\rho}_j C_{Kaj},\ C_{Kb}(\hat{\rho}) = \sum_{j=1}^m \hat{\rho}_j C_{Kbj} \\
A_{Kia}(\hat{\rho}) &= \textstyle\sum_{j=1}^m \hat{\rho}_j A_{Kiaj} \\
A_{Kib}(\hat{\rho}) &= \textstyle\sum_{j=1}^m \sum_{s=1}^m \hat{\rho}_j\hat{\rho}_s A_{Kibjs} + \sum_{j=1}^m \hat{\rho}_j A_{Kibj}
\end{aligned}
$$

Motivated by the quasi-LPV structure of both the plant and the controller, we consider the following auxiliary LPV system, if $\varepsilon(P,\,\delta) \subset \wp([0\, H(\hat{\rho})])$ is an invariant set.

$$
\dot{x}_e(t) = A_e(\eta)x_e(t) = \textstyle\sum_{i=0}^{2^m-1} \eta_i(A_{ei}x_e(t)), \quad \eta \in \Gamma \tag{15}
$$

where $x_e = [x^T(t)\ \xi^T(t)]^T$, $\eta = [\eta_0,\, \eta_1,\, \cdots,\, \eta_{2^m-1}]$, and

$$
\begin{aligned}
\Gamma &= \{\eta \in R^{2^m}: \textstyle\sum_{i=0}^{2^m-1} \eta_i = 1,\, 0 \le \eta_i \le 1,\ i \in I[0,\, 2^m-1]\} \\
A_{ei} &= \begin{bmatrix} A & B_2(I-\rho)[D_iC_K(\hat{\rho}) + D_i^-H(\hat{\rho})] \\ B_{Ki}(\hat{\rho})C & A_{Ki}(\hat{\rho}) \end{bmatrix}
\end{aligned}
$$

The following theorem establishes conditions on the output-feedback controller coefficient matrices under which the LPV system (15) is asymptotically stable with Lyapunov function. Denote $\Delta_{\hat{\rho}} = \{\hat{\rho} = (\hat{\rho}_1 \cdots \hat{\rho}_m) : \hat{\rho}_j \in \{\min_q\{\underline{\rho}_j^q\},\ \max_q\{\overline{\rho}_j^q\}\},\ q \in \mathbf{I}[1,L]\}$ and $B^j = [0 \cdots b^j \cdots 0]$ with $B = [b^1 \cdots b^m]$.

Theorem 1: $\varepsilon^*(P,\delta)$ is a contractively invariant set for normal and actuator failure cases, if there exist matrices $0 < N_1 < Y_1$, A_{Ki0}, A_{Kiaj}, A_{Kibjs}, B_{Ki0}, B_{Kiaj}, B_{Kibj}, C_{K0}, C_{Kaj}, C_{Kbj}, H_{K0}, H_{Kaj}, H_{Kbj}, $j \in \mathbf{I}[1,m]$, $s \in \mathbf{I}[1,m]$ and symmetric matrixes Θ^i, $i \in \mathbf{I}[0, 2^m-1]$ with

$$
\Theta^i = \begin{bmatrix} \Theta_{11}^i & \Theta_{12}^i \\ \Theta_{12}^{iT} & \Theta_{22}^i \end{bmatrix}
$$

and Θ_{11}^i, $\Theta_{22}^i \in R^{m(2n)\times m(2n)}$ such that the following inequalities hold for all $D_i \in \mathbf{D}$ and $\varepsilon^*(P,\delta) \subset \wp([0\ H(\hat{\rho})])$, $i.e.$, $|[0\ H(\hat{\rho})]_j x_e| \le 1$ for all $x_e \in \varepsilon^*(P,\delta)$, $j \in \mathbf{I}[1,m]$.

$$
\begin{aligned}
&\Theta_{22jj}^i \le 0,\ j \in \mathbf{I}[1,m], i \in \mathbf{I}[0, 2^m-1] \\
&\Theta_{11}^i + \Theta_{12}^i\Delta(\hat{\rho}) + (\Theta_{12}^i\Delta(\hat{\rho}))^T + \Delta(\hat{\rho})\Theta_{22}^i\Delta(\hat{\rho}) \ge 0,\ \hat{\rho} \in \Delta_{\hat{\rho}} \\
&\begin{bmatrix} Q_i & R_i \\ R_i^T & S_i \end{bmatrix} + G^T\Theta^iG < 0,\ i \in \mathbf{I}[0, 2^m-1], \\
&\rho \in \{\rho^1 \cdots \rho^L\},\ \rho^q \in N_{\rho^q}
\end{aligned} \tag{16}
$$

where

$$
\begin{aligned}
R_i &= \begin{bmatrix} R_{i1} & R_{i2} & \cdots & R_{im} \end{bmatrix} \\
Q_i &= \begin{bmatrix} Y_1A - N_1B_{Ki0}C + (Y_1A - N_1B_{Ki0}C)^T & T_{1i} \\ * & T_{2i} \end{bmatrix} \\
R_{ij} &= \begin{bmatrix} -N_1B_{Kibj}C - N_1B_{Kiaj}C & T_{3i} \\ N_1B_{Kibj}C + N_1B_{Kiaj}CS\begin{bmatrix} 0 \\ C^{\perp} \end{bmatrix} & T_{4i} \end{bmatrix} \\
S_i &= [S_{ijs}],\ j,\ s \in \mathbf{I}[1,m],\quad S_{ijs} = \begin{bmatrix} 0 & T_{5i} \\ T_{6i} & T_{7i} \end{bmatrix}
\end{aligned}
$$

with

$$
\begin{aligned}
T_{1i} &= Y_1B[(I-\rho)(D_iC_{K0}+D_i^-H_{K0})+D_iC_{Ka}(\rho)\\
&+D_i^-H_{Ka}(\rho)]-N_1A_{Ki0}-N_1A_{Kia}(\rho)\\
&+\begin{bmatrix} 0 \\ C^\perp \end{bmatrix}^T S^T[-Y_1B_2(D_iC_{Ka}(\rho)+D_i^-H_{Ka}(\rho))\\
&+N_1A_{Kia}(\rho)]+(-N_1A+N_1B_{Ki0}C\\
&+N_1B_{Kia}(\rho)C-[N_1B_{Kia}(\rho)CS]\begin{bmatrix} 0 \\ C^\perp \end{bmatrix})^T\\
T_{2i} &= -N_1B[(I-\rho)(D_iC_{K0}+D_i^-H_{K0})+D_iC_{Ka}(\rho)\\
&+D_i^-H_{Ka}(\rho)]+(-N_1B[(I-\rho)(D_iC_{K0}+D_i^-H_{K0})\\
&+D_iC_{Ka}(\rho)+D_i^-H_{Ka}(\rho)])^T+N_1A_{Ki0}\\
&+N_1A_{Kia}(\rho)+(N_1A_{Ki0}+N_1A_{Kia}(\rho))^T\\
T_{3i} &= Y_1B[-\rho(D_iC_{Kaj}+D_i^-H_{Kaj})+D_iC_{Kbj}+D_i^-H_{Kbj}]\\
&-N_1A_{Kibj}+\begin{bmatrix} 0 \\ C^\perp \end{bmatrix}^T S^T[Y_1B((D_iC_{Kaj}+D_i^-H_{Kaj})\\
&-\rho(D_iC_{Kbj}+D_i^-H_{Kbj}))-N_1A_{Kiaj}]\\
T_{4i} &= N_1B\rho(D_iC_{Kaj}+D_i^-H_{Kaj})\\
&-N_1B(D_iC_{Kbj}+D_i^-H_{Kbj})+N_1A_{Kibj}\\
T_{5i} &= -Y_1B^j(D_iC_{Kbs}+D_i^-H_{Kbs})-N_1A_{Kibjs}\\
&+\begin{bmatrix} 0 \\ C^\perp \end{bmatrix}^T S^TY_1B^j(D_iC_{Kbs}+D_i^-H_{Kbs})\\
T_{6i} &= (-Y_1B^s(D_iC_{Kbj}+D_i^-H_{Kbj})-N_1A_{Kibsj}\\
&+\begin{bmatrix} 0 \\ C^\perp \end{bmatrix}^T S^TY_1B^s(D_iC_{Kbj}+D_i^-H_{Kbj}))^T\\
T_{7i} &= N_1B^j(D_iC_{Kbs}+D_i^-H_{Kbs})+N_1A_{Kibjs}\\
&+[N_1B^j(D_iC_{Kbs}+D_i^-H_{Kbs})+N_1A_{Kibjs}]^T\\
G &= \begin{bmatrix} \begin{bmatrix} I_{(2n)\times(2n)} \\ \cdots \\ I_{(2n)\times(2n)} \end{bmatrix} & 0 \\ 0 & I_{m(2n)\times m(2n)} \end{bmatrix},\\
\Delta(\hat{\rho}) &= \mathrm{diag}[\hat{\rho}_1 I_{(2n)\times(2n)} \ \cdots \ \hat{\rho}_m I_{(2n)\times(2n)}].
\end{aligned}
$$

and also $\hat{\rho}_j(t)$ is determined according to the adaptive law

$$
\begin{aligned}
\dot{\hat{\rho}}_j &= \mathrm{Proj}_{[\min_q\{\underline{\rho}_j^q\},\ \max_q\{\overline{\rho}_j^q\}]}\{L_{1j}\}\\
&= \begin{cases} 0, \text{ if} & \begin{array}{c}\hat{\rho}_j=\min_q\{\underline{\rho}_j^q\} \text{ and } L_{1j}\le 0\\ \text{or } \hat{\rho}_j=\max_q\{\overline{\rho}_j^q\} \text{ and } L_{1j}\ge 0\end{array}\\ & L_{1j}, \ \text{otherwise} \end{cases}
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
L_{1j} &= l_j\sum_{i=0}^{2^m-1}\eta_i\{\xi^TO_1[A_{Kiaj}-BD_iC_{Kaj}\\
&-B^jD_iC_{Kb}(\hat{\rho})-BD_i^-H_{Kaj}-B^jD_i^-H_{Kb}(\hat{\rho})]\xi\\
&+\begin{bmatrix} y \\ 0 \end{bmatrix}^T S^T[M_1(BD_iC_{Kaj}+B^jD_iC_{Kb}(\hat{\rho})\\
&+BD_i^-H_{Kaj}+B^jD_i^-H_{Kb}(\hat{\rho}))-O_1A_{Kiaj}]\xi\\
&+\xi^TO_1B_{Kiaj}CS\begin{bmatrix} y \\ 0 \end{bmatrix}\},
\end{aligned}
$$

$M_1=\delta Y_1$, $O_1=\delta N_1$. $l_j>0(j\in\mathbf{I}[1,m])$ and $\delta>0$ are the adaptive law gains to be chosen according to practical applications.

Proof: For the limitation of space, the proof is omitted.

If we take the following output-feedback controller with fixed parameter matrices A_{Ki0}, B_{Ki0}, C_{K0}, $i\in\mathbf{I}[0,2^m-1]$

$$
\begin{aligned}
\dot{\xi}(t) &= (\textstyle\sum_{i=0}^{2^m-1}\eta_iA_{Ki0})\xi(t)+(\sum_{i=0}^{2^m-1}\eta_iB_{Ki0})y(t)\\
u(t) &= (I-\rho)\sigma(C_{K0}\xi(t))
\end{aligned} \tag{18}
$$

then combing (18) with (1), it follows:

$$
\dot{x}_{e1}(t)=A_{e1}(\eta)x_{e1}(t) \tag{19}
$$

$$
A_{e1}(\eta)=\textstyle\sum_{i=0}^{2^m-1}\eta_i(A_{e1i}x_{e1}(t)),\quad \eta\in\Gamma \tag{20}
$$

where $x_{e1}=[x^T(t)\ \ \xi^T(t)]^T$,

$$
A_{e1i}=\begin{bmatrix} A & B_2(I-\rho)[D_iC_{K0}+D_i^-H_0] \\ B_{Ki0}C_2 & A_{Ki0} \end{bmatrix}
$$

Based on system (19), the following lemma is presented.

Lemma 4: Consider the closed-loop system described by (19), we have that the following statements are equivalent:

(i) there exist a symmetric matrix $X>0$ and controller $\mathbf{K}$ described by (18) such that

$$
A_{e1i}^TX+XA_{e1i}<0
$$

holds for $\rho\in\{\rho^1\ \cdots\ \rho^L\}$, $\rho^q\in N_{\rho^q}$

(ii) there exist symmetric matrices Y_1 and N_1 with $0<N_1<Y_1$, and a controller described by (18) with $A_{Ki0}=A_{Kei0}$, $B_{Ki0}=B_{Kei0}$, $C_{K0}=C_{Ke0}$, $H_0=H_{e0}$, $i\in\mathbf{I}[0,2^m-1]$ such that

$$
\begin{bmatrix} Y_1A-N_1B_{Ki0}C+(Y_1A-N_1B_{Ki0}C)^T & T_0 \\ * & T_1 \end{bmatrix}<0 \tag{21}
$$

with

$$
\begin{aligned}
T_0 &= Y_1B_2(I-\rho)[D_iC_{K0}+D_i^-H_0]-N_1A_{Ki0}+(-N_1A+N_1B_{Ki0}C)^T\\
T_1 &= -N_1B_2(I-\rho)[D_iC_{K0}+D_i^-H_0]+N_1A_{Ki0}\\
&+(-N_1B_2(I-\rho)[D_iC_{K0}+D_i^-H_0]+N_1A_{Ki0})^T
\end{aligned}
$$

Proof: The proof is similar to the proof of Lemma 2 in [31]. To avoid overlap, it is omitted.

Next, a theorem is given to show that the condition in Theorem 1 for the adaptive controller design is more relaxed than that in Lemma 4 for the traditional controller design with fixed parameter matrices.

Theorem 2: If condition (i) or (ii) in Lemma 4 holds, then the condition of Theorem 1 holds.

Proof: If condition (i) or (ii) in Lemma 4 holds, then it is easy to see that the condition in Theorem 1 is feasible with $A_{Kiaj}=A_{Kibj}=A_{Kibjs}=B_{Kiaj}=B_{Kibj}=C_{Kaj}=C_{Kbj}=H_{Kaj}=H_{Kbj}=0$, $i\in\mathbf{I}[0,2^m-1]$, $j\in\mathbf{I}[1,m]$, $s\in\mathbf{I}[1,m]$. The proof is completed.

B. Controller design

From Theorem 1, we can obtain various controller gains and domains satisfying the set invariance condition. So, how to choose the "largest" one of them becomes an interesting problem. In this section, we will give a method to find the "largest" domain.

The following definition will be used in the sequel.

Definition 3: Define X_R is a prescribed bounded convex set. $X_R=\varepsilon(R,1)=\{x\in R^{n\times n}:\quad x^TRx\le 1\}$, $\quad R>0$ or $X_R=co\{x_1,x_2,...,x_l\}$. For a set $S\in R^n$, $\alpha_R(S)=sup\{\alpha>0:\quad \alpha X_R\subset S\}$.

In Theorem 1, a condition for the set $\varepsilon^*(P,\delta)$ to be inside the domain of attraction is given. With the above shape reference sets, we can choose from all the $\varepsilon^*(P,\delta)$'s that satisfy the condition of Theorem 1 such that the quantity $\alpha_R(\varepsilon^*(P,\delta))$ is maximized. The problem can be formulated as follows

$$\begin{aligned} &\sup\ \alpha \\ s.t.\quad &(a)\quad \alpha X_R \subset \varepsilon^*(P,\delta), \\ &(b)\quad (16), \\ &(c)\quad \varepsilon^*(P,\delta) \subset \wp([0\ H(\hat{\rho})]). \end{aligned} \tag{22}$$

By Definition 2, we know that (a) and (c) cannot be shown as LMIs directly. Then we give the following proposition.

Proposition 1: Obviously, $\varepsilon^*(P,\delta) \subset \varepsilon(P,\delta)$, which implies that (c) holds if (c1) holds, where

$$(c1)\quad \varepsilon(P,\delta) \subset \wp([0\ H(\hat{\rho})]), \tag{23}$$

By Definition 2, we have

$$x_e^T P x_e + \sum\nolimits_{j=1}^m \frac{\tilde{\rho}_j^2(t)}{l_j} \le \delta \Leftrightarrow x_e^T \frac{P}{\delta} x_e + \sum\nolimits_{j=1}^m \frac{\tilde{\rho}_j^2(t)}{\delta l_j} \le 1.$$

Let $F(t) = \sum_{j=1}^m \frac{\tilde{\rho}_j^2(t)}{\delta l_j}$. Then, by (17) and (3), it follows that $\tilde{\rho}_j(t) \le \max_j\{\overline{\rho}_j^q\} - \min_j\{\underline{\rho}_j^q\}$. We can choose l_j and δ sufficiently large so that $F(t)$ is sufficiently small. Then the conclusion can be drawn as follows:

For system (8) and controller (9) there must exist $\delta > 0$ and $l_i > 0$ such that the closed-loop system (15) is asymptotically stable in domain $\varepsilon^-(P,\delta)$ if (b) and (c1) hold.

Then we can get the "largest" domain of asymptotic stability by solving the following optimization problem

$$\begin{aligned} &\sup\ \alpha \\ s.t.\quad &(a1)\quad \alpha X_R \subset \varepsilon(P,\delta), \\ &(b),\ \ (c1). \end{aligned} \tag{24}$$

If the given shape reference set X_R is a polyhedron as defined in Definition 1, then Constraint (a1) is equivalent to

$$\alpha^2 x_q^T (\frac{P}{\delta}) x_q \le 1 \Leftrightarrow \begin{bmatrix} 1/\alpha^2 & x_q^T(\frac{P}{\delta}) \\ (\frac{P}{\delta})x_q & (\frac{P}{\delta}) \end{bmatrix} \ge 0, \tag{25}$$

for all $q \in \mathbf{I}[1,l]$. If X_R is a ellipsoid $\varepsilon(R,1)$, then (a1) is equivalent to

$$\frac{R}{\alpha^2} \ge \frac{P}{\delta} \Leftrightarrow \begin{bmatrix} (1/\alpha^2)R & (\frac{P}{\delta}) \\ (\frac{P}{\delta}) & (\frac{P}{\delta}) \end{bmatrix} \ge 0. \tag{26}$$

Condition (c1) is equivalent to

$$\delta[0\ h(\hat{\rho})]_j P^{-1} [0\ h(\hat{\rho})]_j^T \le 1 \Leftrightarrow \begin{bmatrix} 1 & [0\ h(\hat{\rho})]_j \\ * & (\frac{P}{\delta}) \end{bmatrix} \ge 0. \tag{27}$$

for all $j \in \mathbf{I}[1,m]$, where $[0\ h(\hat{\rho})]_j$ be the jth row of $[0\ H(\hat{\rho})]$. We have that (26) is equivalent to the following inequalities.

$$(c2)\quad \begin{bmatrix} -1 & -[0\ H_{K0s}] \\ * & -X \end{bmatrix} + \sum_{j=1}^m \hat{\rho}_j \begin{bmatrix} 0 & [0\ -H_{Kajs} - H_{Kbjs}] \\ * & 0 \end{bmatrix} \le 0, \quad \hat{\rho} \in \Delta_{\hat{\rho}}$$

where H_{Kajs} is the sth row of H_{Kaj}, $s \in \mathbf{I}[1,m]$.

If X_R is a polyhedron, then from (23) and (26), the optimization problem (23) is equivalent to

$$\begin{aligned} &\inf\quad \gamma \\ s.t.\quad &(a2)\ \begin{bmatrix} \gamma & x_q^T X \\ X x_q & X \end{bmatrix} \ge 0,\quad q \in \mathbf{I}[1,\ l], \\ &(b),\ \ (c2), \end{aligned} \tag{28}$$

where $\gamma = 1/\alpha^2$.

If X_R is an ellipsoid, we need only to replace (a2) with

$$(a3)\quad \begin{bmatrix} \gamma R & X \\ X & X \end{bmatrix} \ge 0. \tag{29}$$

It should be noted that condition (16) is not convex. But when $C_{K0}, C_{Kaj}, C_{Kbj}, H_{K0}, H_{Kaj}, H_{Kbj}$ are given, they become LMIs.

From Theorem 1, we have the following algorithm to design the adaptive output feedback controller.

Algorithm 1:

Step 1: Suppose that all states of system (1) can be measured. Minimize the index γ to design the state-feedback controller. Then, the matrices C_{K0}, C_{Kaj}, C_{Kbj}, H_{K0}, H_{Kaj}, H_{Kbj} can be given.

Step 2: Solve the following optimization problem

$$\begin{aligned} &\inf\quad \gamma \\ s.t.\quad &(a2),\ \ (b),\ \ (c2) \end{aligned} \tag{30}$$

Then the resulting A_{Ki0}, A_{Kiaj}, A_{Kibjs}, B_{Ki0}, B_{Kiaj}, B_{Kibj}, C_{K0}, C_{Kaj}, C_{Kbj}, $i \in \mathbf{I}[0, 2^m - 1]$, $j \in \mathbf{I}[1,m]$, $s \in \mathbf{I}[1,m]$ will form the dynamic output feedback controller gains.

Remark 2: Step 1 is to determine matrices C_{K0}, C_{Kaj}, C_{Kbj}, H_{K0}, H_{Kaj}, H_{Kbj}, which solves the corresponding adaptive controller design problem via state feedback. This procedure is adopted from [30], and convex conditions are described in [30]. To avoid overlap, the conditions appear in Step 1 will be omitted.

From Lemma 4, we have the following algorithm to design the fault-tolerant controller with fixed gains.

Algorithm 2:

Step 1: Suppose that all states of system (1) can be measured. Minimize the index γ to design the state-feedback controller. Then, the matrices C_{K0}, H_{K0} can be given.

Step 2: Solve the following optimization problem

$$\begin{aligned} &\inf\quad \gamma \\ s.t.\quad &(a2),\ \ (21),\ \ (c2) \end{aligned} \tag{31}$$

Then the resulting A_{Ki0}, B_{Ki0}, C_{K0}, $i \in \mathbf{I}[0, 2^m - 1]$ will form the dynamic output feedback controller gains.

Remark 3: Step 1 is to determine matrices C_{K0}, H_{K0}, which solves the corresponding controller design problem via state feedback [30].

Remark 4: In Step 1, for some cases, the magnitude of the designed gains C_{K0} (C_{Kaj} and C_{Kbj}) may be too large to be applied in Step 2. For solving the problem, by adding the following constraints, where Q and Y_{K0} are variables in conditions of Step 1.

$$Q > \alpha I,\quad Y_{K0} Y_{K0}^T < \beta I, \tag{32}$$

then the magnitude of C_{K0} can be reduced. In fact, by $C_{K0} = Y_{K0}Q^{-1}$ and (32), it follows that

$$\| C_{K0} \| < \sqrt{\beta}/\alpha.$$

The similar method can be used for the gains C_{Kaj} and C_{Kbj}.

IV. Examples

Example 1. Consider the system of form (8) with

$$A = \begin{bmatrix} 0.01 & 0.1 \\ 0.1 & 0.01 \end{bmatrix}, \quad B = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \; C = [1\;0]$$

and the following two possible fault modes:
Fault mode 1: Both of the two actuators are normal, that is,

$$\rho_1^1 = \rho_2^1 = 0$$

Fault mode 2: The first actuator is outage and the second one may be normal or loss of effectiveness, described by

$$\rho_1^2 = 1, \;\; 0 \le \rho_2^2 \le a,$$

where $a = 0.5$ denotes the maximal loss of effectiveness for the second actuator. Let

$$R = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$$

After implementing Algorithm 2, we have that $\gamma^* = 1.9669$. When Algorithm 1 is used to design adaptive output-feedback controller, the optimal index is given as $\gamma^* = 0.7648$. Obviously, the optimal index γ is smaller for Algorithm 1. The phenomenon indicates the superiority of our adaptive method.

V. Conclusions

In this paper, an adaptive fault-tolerant controllers design method has been presented for linear time-invariant systems with actuator saturation. The designs were developed in the framework of linear matrix inequality (LMI) approach, which can enlarge the domain of asymptotic stability of closed-loop systems in the cases of actuator saturation and actuator failures. An example has been given to illustrate the efficiency of the design method.

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