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Load Transfer Control for a Crane with State Constraints

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Abstract— **For the problem of controlling a crane with variable rope length, a saturating control is developed that satisfies the constraints on the sway angle of the load and the speed of the trolley. The length of the rope can change at a piecewise constant velocity. The control law is designed based on a linear time-invariant system with a constrained input, obtained applying a feedback compensation and a coordinate transformation to the crane system. The control law has a simple form and is easily implemented; moreover it is robust against modeling errors and observation noise. The effectiveness of the control law is demonstrated by experiments and simulations.**

I. Introduction

The purpose of the load transfer control for a crane is to transfer the suspended load to a target position rapidly, suppressing the load sway. It is not easy to find a control law performing such a task keeping the input and state within the allowable ranges, only with a single input of the system, i.e., the driving force for the trolley. The problem of load transfer control becomes more difficult when the length of the suspending rope is changed to avoid obstacles or to transfer the load to a higher or lower place.

For this problem, some studies have adopted optimal control methods considering input and state constraints. Sakawa and Shindo [1] obtained optimal controls that minimized the load swing under input and state constraints and boundary conditions, using a numerical method to compute optimal controls developed based on the maximum principle. Shindo et al. [2] constructed a feedback control system using an optimal regulator where an open-loop control trajectory obtained by the method in [1] was used as the nominal trajectory. Moreover, Shirai et al. [3] applied the control technique in [2] to the control of an actual container crane. Also, Auernig and Troger [4] obtained a time optimal control in closed form that satisfied input and state constraints and boundary conditions, based on the maximum principle.

On the other hand, some studies have approximated the crane with a linear time-varying system regarding the variable rope length as a time-varying parameter, and have applied design methods for linear time-varying systems. Corriga et al. [5] derived a stabilizing control law using a time-scale transformation and a stability theorem for time-varying systems. Giua et al. [6] obtained a time-varying feedback control law that assured the asymptotic stability of the control system, using Wolovich's design method for time-varying systems. Kobayashi and Tamura [7] developed a gain-scheduling method where the closed-loop poles were placed in a fixed set of points by a state feedback with time-varying gain. Kaneshige et al. [8] applied a similar gain-scheduling method to the control of a three-dimensional overhead traveling crane. Murata et al. [9] applied a gain-scheduling method to a real crane, where the controller gain was chosen, according to the rope length, among linear-quadratic optimal regulator gains computed for various rope lengths. Nishimura et al. [10] proposed a gain-scheduling method switching the controller between H_∞ controllers designed in advance.

Bartolini et al. [11] and Lee [12] developed sliding mode controls, using a sliding surface coupling the motions of the trolley and the load, that effectively damped the load swing and were robust to modeling errors.

Yanai et al. [13] proposed a feedback control to make the load follow a given reference trajectory, based on inverse dynamics methods, often used in the control of robots.

As above, various design methods have been proposed for the control problem of a crane with variable rope length; each of them gives an effective control under the assumptions, but each control law is complicated and not very easy to be implemented.

Also, it seems to be difficult to apply these methods to the control problem of a crane which also has input and state constraints.

For the load transfer control problem for a crane with hoisting mechanism, this paper proposes a control law that satisfies the following specifications and is easy to be implemented by a computer.

1) The load is rapidly transferred to the target position.
2) The length of the suspending rope varies at a constant speed.
3) The amplitude of the load sway angle is less than a given maximum value.
4) The magnitude of the velocity of the trolley is less than a given maximum value.

Specification 4) is given from the considerations of the safety in the operation of the crane and the limitation of the driving force for the trolley.

The control law proposed in this paper has a structure similar to the one proposed by Teel [14], [15] realizing semi-global stabilization for a class of single-input partially linear composite systems, which was applied to the problem of stabilizing a ball-and-beam system. Although both control

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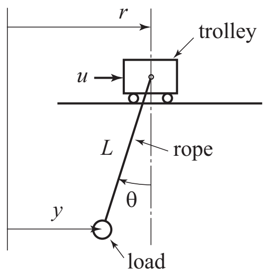

Fig. 1. Crane system.

laws can keep some of the state variables of the control system small by using a state-feedback compensation and a saturating control, the proposed one uses a state-feedback compensation designed so that the state constraint is satisfied.

The proposed control law is designed as follows. First the crane system is transformed to a linear time-invariant system through a feedback compensation, a change of variables, and some approximations. This linear system consists of a stable and an unstable second-order system (the unstable one has the poles $\{0, 0\}$) , and allows the designer to treat the amplitude constraint on the load sway angle as the input constraint, which is much easier to treat. Then the proposed control law is obtained by constructing a saturating control law that asymptotically stabilizes the linear system. This control law can also consider the limitation of the velocity of the trolley by choosing a design parameter. The effectiveness of the control law is examined by experiments and simulations.

II. Mathematical model of the controlled object and problem statement

Fig.1 shows the crane system considered in this study. The load is suspended by the rope whose supporting point is on the trolley. Suppose that the position of the trolley and the length of the rope can be changed by respective driving units. In the modeling the mass of the rope, the air resistance, and the friction at the supporting point are neglected, and the load is regarded as a point mass.

Let $\theta(t)$, $r(t)$, $L(t)$, and $u(t)$ be, respectively, the angular displacement of the pendulum (the rope and the load), the position of the trolley, the length of the rope, and the force applied to the trolley, at time t. The symbol g denotes the acceleration of gravity.

The equations of motion for the crane system are represented by

$$\ddot{\theta} = -2\frac{\dot{L}}{L}\dot{\theta} - \frac{g}{L}\sin\theta + \frac{\ddot{r}}{L}\cos\theta \tag{1}$$

$$\ddot{r} = \mu \tag{2}$$

where the one for the trolley (2) has been linearized using the driving force for the trolley u, and where μ is the new input after the linearization. Suppose that θ is constrained as

$$|\theta(t)| \leq a, \quad \forall t \geq 0 \tag{3}$$

where $a > 0$ is the maximum allowable amplitude of θ. Also, assume that $L(t)$ is a linear function of time t and $L(t) > 0$.

The problem is to find a control law that rapidly transfer the trolley to the target position and asymptotically stabilize the crane system there under the constraint (3).

Without loss of generality, let the target position of the trolley be $r = 0$.

III. Design method

A. Reduction to a problem with constrained input

Suppose $|\theta(t)|$ is sufficiently small, and introduce the following variable:

$$y = r - L\theta \tag{4}$$

which represents the distance from the nominal position of the trolley ($r = 0$) to the load (see Fig.1). Since $L(t)$ is a linear function of t, there holds

$$\ddot{L} \equiv 0. \tag{5}$$

Thus, the following relation holds:

$$\begin{aligned} \ddot{y} &= \ddot{r} - 2\dot{L}\dot{\theta} - L\ddot{\theta} \\ &\approx \ddot{r} - 2\dot{L}\dot{\theta} - L\left(-2\frac{\dot{L}}{L}\dot{\theta} - \frac{g}{L}\theta + \frac{\ddot{r}}{L}\right) \\ &= g\theta \end{aligned} \tag{6}$$

where $\ddot{\theta}$ was approximated as (see (1))

$$\ddot{\theta} \approx -2\frac{\dot{L}}{L}\dot{\theta} - \frac{g}{L}\theta + \frac{\ddot{r}}{L}. \tag{7}$$

For the mathematical model of the crane system, (6) will be used in place of (2).

Let $v(t)$ be a new input of the crane system. Also, let $v(s)$ and $\theta(s)$ be the Laplace transforms of $v(t)$ and $\theta(t)$, respectively, and $G(s)$ the transfer function from $v(s)$ to $\theta(s)$.

The input μ in (2) is designed so that $G(s)$ has the form

$$G(s) = \frac{\theta(s)}{v(s)} = \frac{1}{(1 + Ts)^2} \tag{8}$$

where $T > 0$ is the design parameter. The relation (8) can be written in the time domain as

$$\ddot{\theta} = -\frac{1}{T^2}\theta - \frac{2}{T}\dot{\theta} + \frac{1}{T^2}v. \tag{9}$$

The input μ making (9) hold is obtained from (1), (2), and (9) as

$$\mu = \frac{L}{\cos\theta}\left(\frac{2\dot{L}\dot{\theta}}{L} + \frac{g}{L}\sin\theta - \frac{1}{T^2}\theta - \frac{2}{T}\dot{\theta} + \frac{1}{T^2}v\right). \tag{10}$$

Then the following relation holds for the 1-norm of $G(s)$, denoted $\|G(s)\|_1$:

$$\|G(s)\|_1 := \int_0^\infty |g(t)| dt = 1 \tag{11}$$

where $g(t)$ is the impulse response of $G(s)$.

Let $\mathcal{R}$ be the set of all solutions of (9), $[\theta(t)\ \dot{\theta}(t)]'$, $\forall t \geq 0$, reachable from the origin by some input v satisfying $|v(t)| \leq a$.

Thanks to (11), condition (3) is satisfied if the following two conditions hold (see Appendix I).

$$[\theta(0)\ \dot{\theta}(0)]' \in \mathcal{R} \tag{12}$$

$$|v(t)| \leq a, \quad \forall t \geq 0 \tag{13}$$

The mathematical model of the crane system compensated by (10) is written as

$$\ddot{\theta} = -\frac{1}{T^2}\theta - \frac{2}{T}\dot{\theta} + \frac{1}{T^2}v \tag{14}$$

$$\ddot{y} = g\theta. \tag{15}$$

Note that the above equations represent a linear time-invariant model, in spite of the fact that $L(t)$ varies as a linear function of t.

(14) is exactly linearized by (10), so it does not involve any approximations; (15) involves the approximations $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ (see (6)).

Let the state be

$$x = \begin{bmatrix} \theta & \dot{\theta} & y & \dot{y} \end{bmatrix}'.$$

Then (14) and (15) are represented in state equation form as

$$\dot{x} = Ax + Bv \tag{16}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{1}{T^2} & -\frac{2}{T} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ g & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{T^2} \\ 0 \\ 0 \end{bmatrix}.$$

A solution of the problem will be obtained by solving the following problem with constrained input which can be solved more easily: find a control law that asymptotically stabilizes the system (16) under conditions (12) and (13).

Since this control law satisfies the constraint (3), it is also a solution of the original problem.

B. Stabilization by partial state feedback

The problem in Section III-A will be solved by reducing the problem to a much easier one, by decomposing the system (16) into a stable and an unstable subsystem by a change of coordinates, where a control law is to be found that asymptotically stabilizes the unstable subsystem, a second-order system, under the constraint of the input v.

Introduce the change of coordinates

$$w = Sx \tag{17}$$

where

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ T^2 & 0 & \frac{1}{g} & \frac{2T}{g} \\ 2T & T^2 & 0 & \frac{1}{g} \end{bmatrix}.$$

By (17) the system (16) is transformed as

$$\dot{w} = \tilde{A}w + \tilde{B}v \tag{18}$$

where

$$\tilde{A} = SAS^{-1}, \quad \tilde{B} = SB$$

$$\tilde{A} = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ -\frac{1}{T^2} & -\frac{2}{T} & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right], \quad \tilde{B} = \left[\begin{array}{c} 0 \\ \frac{1}{T^2} \\ \hline 0 \\ 1 \end{array}\right].$$

The state w is partitioned conformably with the partition of $\tilde{A}$ and $\tilde{B}$ as

$$w = \begin{bmatrix} w_s \\ w_u \end{bmatrix}$$

where

$$w_s = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad w_u = \begin{bmatrix} w_3 \\ w_4 \end{bmatrix}.$$

The idea of the design is as follows: obtain a control law $v(t) = f(w_u(t))$ that asymptotically stabilizes the w_u subsystem under the constraint $|v(t)| \leq a$, and apply it to the whole system. Then $w_u(t)$ approaches 0, and so does $v(t)$. Since the w_s subsystem is asymptotically stable, $w_s(t)$ also approaches 0.

From (18), the w_u subsystem is

$$\dot{w}_u = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} w_u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v. \tag{19}$$

To steer the state w_u to the origin rapidly, time optimal control under condition (13) is adopted. The time optimal control that moves the state w_u from any initial state to the origin is given by [16]

$$v = -a\,\mathrm{sgn}(\Gamma(w_u)) \tag{20}$$

where

$$\Gamma(w_u) = \begin{cases} \xi(w_u) = \frac{w_3}{a} + \frac{1}{2a^2}w_4|w_4| & \text{if} \quad \xi(w_u) \neq 0 \\ w_4 & \text{if} \quad \xi(w_u) = 0 \end{cases}.$$

When this control is applied to the system (18), the state of the w_u subsystem is first moved to 0 in minimum time; and after that v is set to be 0, so the state of the w_s subsystem also approaches 0.

Specifically, the resulting control input μ is obtained by substituting the v in (20) into (10); of course, the initial values $[\theta(0), \dot{\theta}(0)]'$ should satisfy (12).

Since the control (20) does not have any robustness against modeling errors and observation noise, a saturating control approximating (20) is used in practice. The algorithm of the saturating control will be given in Section IV.

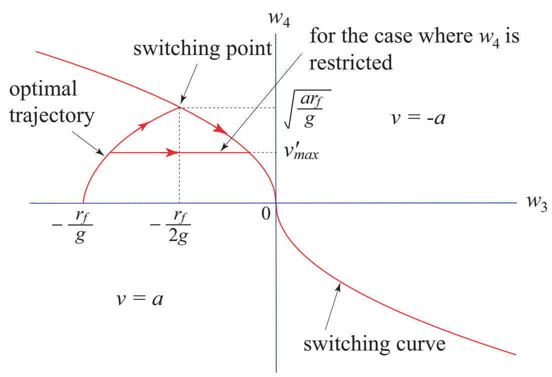

Fig. 2. Switching curve and optimal trajectories; $r_f > 0$.

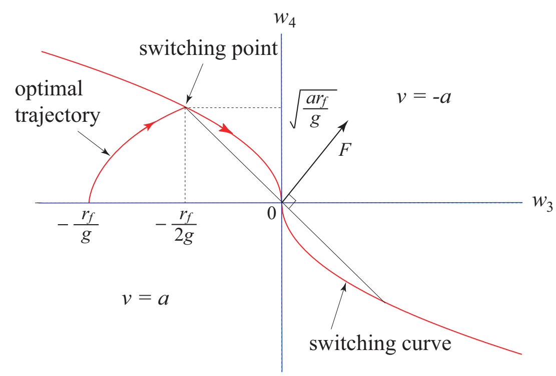

Fig. 3. Switching curve and vector F; $r_f > 0$.

C. Control law lowering the maximum value of $|\dot{r}(t)|$

Often there is a request to limit the magnitude of the velocity of the trolley due to safety concerns and the limitation on the driving force for the trolley. It is not easy to limit the maximum value of $|\dot{r}|$ directly, so instead limiting the variable w_4 in the above control law will be considered. This can easily be done by adding the following logic to the control law (20) (see Fig.2).

$$\text{set } v = 0 \quad \text{if} \quad \xi(w_u) \neq 0 \text{ and } \operatorname{sgn}(v)w_4 = v'_{max} \tag{21}$$

where v_{max} is a given positive number and v'_{max} is defined by

$$v'_{max} = \frac{v_{max}}{g}. \tag{22}$$

By the modification with (21),

$$|w_4(t)| \leq v'_{max} \tag{23}$$

is guaranteed, provided that $|w_4(0)| \leq v'_{max}$.

Let

$$L_0 = \max_t L(t), \quad L_v = \max_t |\dot{L}(t)|$$

and design T as

$$T = \sqrt{\frac{L_0}{g}}. \tag{24}$$

Then an upper bound of $|\dot{r}(t)|$ can be obtained as (see Appendix II)

$$|\dot{r}(t)| \leq v_{max} + \left\{ \left(2 + \frac{2}{e}\right) \sqrt{L_0 g} + L_v \right\} a. \tag{25}$$

It is seen from this relation that to decrease the upper bound, the values of v_{max}, a, L_0, and L_v should be lowered.

D. Application to the case where $\dot{L}(t)$ changes

It has been assumed that $L(t)$ is a linear function of t, i.e., $\dot{L}(t)$ is constant, but $\dot{L}(t)$ often varies with time in the operation of cranes. Although $\dot{L}(t)$ varies continuously in practice, we assume $\dot{L}(t)$ is piecewise-constant to make the following analysis easier.

If $\dot{L}$ changes in a stepwise pattern from A_1 to A_2 at time t_1, then $\ddot{L}(t_1)$ is given by

$$\ddot{L}(t_1) = \delta(t - t_1)(A_2 - A_1). \tag{26}$$

Moreover, $\ddot{y}$ can be computed as

$$\begin{aligned} \ddot{y} &= \ddot{r} - 2\dot{L}\dot{\theta} - L\ddot{\theta} - \ddot{L}\theta \\ &\approx \ddot{r} - 2\dot{L}\dot{\theta} - L\left(-2\frac{\dot{L}}{L}\dot{\theta} - \frac{g}{L}\theta + \frac{\ddot{r}}{L}\right) - \ddot{L}\theta \\ &= g\theta - \ddot{L}\theta. \end{aligned} \tag{27}$$

It follows from (26) and (27) that the impulsive disturbance $\delta(t - t_1)(A_1 - A_2)\theta(t)$ enters $\ddot{y}$ at time t_1; thereby $\dot{y}(t)$ jumps by $(A_1 - A_2)\theta(t_1)$ at time t_1. However, even if such disturbances occur, the state approaches the equilibrium point because the closed-loop system is asymptotically stable.

IV. Experimental Results

In the experiment, from the initial state where the crane is at rest, the trolley is transferred by a distance of r_f and then the system is asymptotically stabilized there.

The control law (20) is not for practical use because when modeling errors or observation noise exists, chattering occurs. Thus the following saturating control approximating (20) is used:

$$v_1 = -\text{sat}(kFw_u, a), \quad k > 0 \tag{28}$$

where k is the design parameter that relates to the magnitude of the region where v_1 does not saturate, and where the saturating function $\text{sat}(\cdot, \cdot)$ is defined by

$$\text{sat}(\xi, a) = \operatorname{sgn}(\xi) \min\{|\xi|, a\}.$$

The control law (28) approximates the following relay control

$$v_0 = -a \operatorname{sgn}(Fw_u) \tag{29}$$

with

$$F = \begin{bmatrix} \sqrt{\frac{a|r_f|}{g}} & \frac{|r_f|}{2g} \end{bmatrix} \tag{30}$$

which gives the same switching point as (20) when the trolley is transferred by a distance of r_f (see Fig.3).

Finally, by combining v_1 with (21) the following algorithm to compute v is obtained:

$$v = \begin{cases} 0 & \text{if} \quad \operatorname{sgn}(v_1)w_4 > v'_{max} - \epsilon \\ v_1 & \text{otherwise} \end{cases} \tag{31}$$

where ϵ is a small positive number. Note that (21) is modified as above because (21) is also not robust against disturbances, noise, and numerical errors.

Specifically, the resulting control input μ in (2) is obtained by substituting the v in (31) into (10).

The algorithm for computing v is summarized as follows:

Off-line part:

1) Give a, v_{max}, and r_f.
2) Compute T and F using, respectively, (24) and (30).
3) Design k and ϵ.

On-line part:

1) Obtain r, $\dot{r}$, θ, $\dot{\theta}$, L, and $\dot{L}$.
2) Construct x as
$$x = [\ \theta \quad \dot{\theta} \quad y \quad \dot{y}\ ]'$$
where from (4)
$$y = r - L\theta, \quad \dot{y} = \dot{r} - \dot{L}\theta - L\dot{\theta}.$$
3) Compute w_u by
$$w_u = S_u x$$
where S_u is the matrix consisting of the third and fourth rows of S, i.e.,
$$S_u = \begin{bmatrix} T^2 & 0 & \dfrac{1}{g} & \dfrac{2T}{g} \\ 2T & T^2 & 0 & \dfrac{1}{g} \end{bmatrix}.$$
4) Compute v using (31).

Fig.4 shows a view of the experimental system. The cart position r, the sway angle θ, and the rope length L were measured by potentiometers. The rates of change of these signals were estimated using an observer, an approximate differentiator, and the input-output relation of the transfer function, respectively. A size C battery was used as the load which was hoisted by a pulley of 45 [mm] in diameter. The cart and the pulley were driven by geared DC motors whose power outputs were 3.1 [W] and 3.6 [W], respectively. These DC motors were compensated in advance by rate feedback and a first-order-lag filter to have a robust input-output property; the resulting transfer functions of the drive units (from the input voltage of the motor driver to the output, i.e., r or L) had the form of an integrator plus a first-order lag. Moreover, when constructing the control laws, nonlinear forces such as the reaction forces from the load and frictional forces were ignored and treated as disturbances.

The maximum allowable amplitude of θ, i.e., a, was set as
$$a = 0.1\,[\text{rad}]$$
and v_{max} in (22) as
$$v_{max} = 0.1\,[\text{m/s}].$$
$L(t)$ was varied by using a servo system with the reference signal $L_r(t)$ shown in Fig.5 by the dashed-dotted line. The transfer function of the drive system was designed as
$$\frac{L(s)}{L_r(s)} = \frac{1}{(T_L s + 1)^2} \tag{32}$$

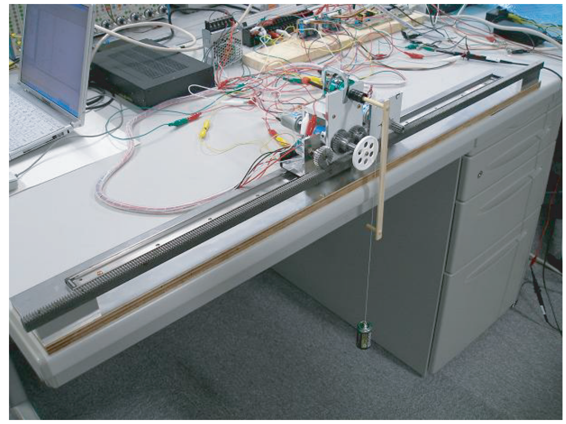

Fig. 4. View of the experimental system.

where the time constant T_L was chosen as small as 0.1 [s] so that $L(t)$ approximated $L_r(t)$. From Fig.5, we obtained $L_0 = \max_t L(t) = 0.651$ [m], and from (24),
$$T = \sqrt{\frac{L_0}{g}} = 0.258\,[\text{s}]$$
where g was set to be 9.81 [m/s^2].

r_f was given to be 0.4 [m] and k was chosen as
$$k = \frac{40}{\|F\|} = 596.74.$$
The parameter ϵ was designed as $\epsilon = 0.05v'_{max}$ so that we have $v'_{max} - \epsilon = 0.95v'_{max}$ in (31).

Figs.5 and 6 show the results of the experiment and those of the corresponding simulation based on the nonlinear plant model. It can be seen from Fig.6 that r is controlled to approach the target position of 0 [m], satisfying the constraint of θ, $|\theta(t)| \leq 0.1$ [rad]. The constraint of θ is assured to be satisfied because conditions (12) and (13) hold ((12) holds since $[\theta(0), \dot{\theta}(0)]' = 0$). Also, there holds $|\dot{r}(t)| \leq 0.1$ [m/s]; theoretically, it is guaranteed from (25) that
$$|\dot{r}(t)| \leq 0.816\,[\text{m/s}].$$
In general, (25) gives a conservative upper bound. We see from various simulations that under ordinary operational conditions for the crane we usually have
$$|\dot{r}(t)| \leq v_{max}.$$
Fig.6 also shows the control v and the input voltage V_{in} to the PWM (pulse-width modulation) driver of the DC motor for the cart; a constant voltage of 1.75 [V] is added to the input voltage to compensate the dead zone of the geared DC motor. The high-frequency content in the waveform of θ is due to the vibration of members (a coupling and a wooden link) in the measurement unit of the sway angle. Overall, the experimental results are in good agreement with the numerical ones in spite of sensor noise, estimation errors of the state, and neglected nonlinearities.

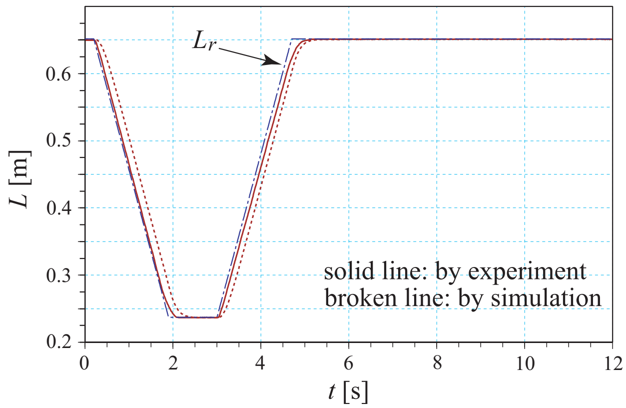

Fig. 5. $L_r(t)$ and $L(t)$; $T_L = 0.1$ s.

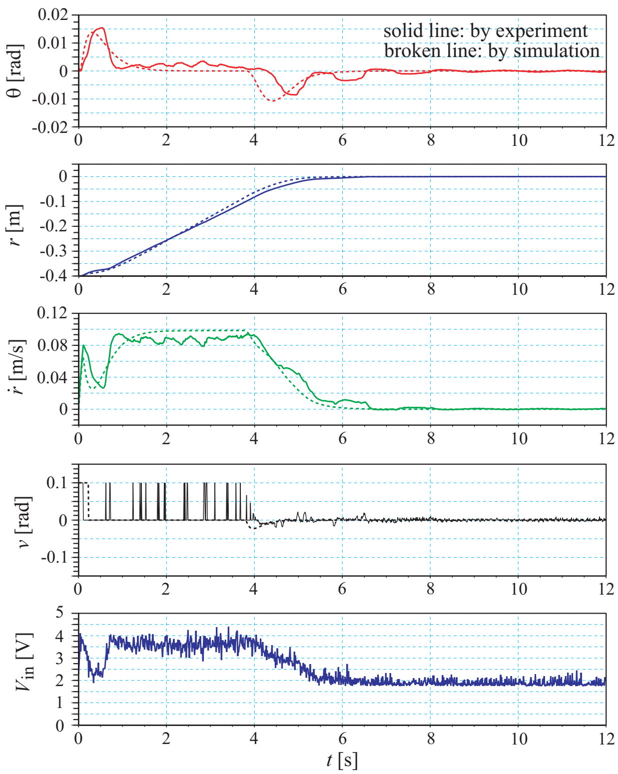

Fig. 6. Experimental (solid lines) and numerical (broken lines) results.

The control law described in this section is the proposed one; it is robust against modeling errors, disturbances, and observation noise.

If the switching point of the corresponding optimal control is largely changed by a change of the target position or a large disturbance, the gain vector F needs to be redesigned for the new r_f.

V. Conclusion

For the load transfer control of a crane with variable rope length, a control law has been proposed that can consider the constraints on the load sway angle and the velocity of the trolley. The control is computed through a simple algorithm and is easily implemented. Also, it is robust against modeling errors and observation noise owing to the use of a saturating control. Since the control law is applicable to the case where the rope length varies at a piece-wise constant speed, it is possible to operate the crane in such a way that the operator hoists or lowers the load watching the obstacles during the control period.

Appendix I
Proof of the Fact That If (12) and (13) Hold, Then (3) Is Satisfied.

Conditions (12) and (13) can be replaced by

$$\left[\begin{array}{cc} r(-\infty) & \dot{r}(-\infty) \end{array}\right]' = 0, \quad |v(t)| \le a, \quad \forall t > -\infty.$$

From these, $r(t)$ is computed as

$$r(t) = \int_{-\infty}^{t} g(t-\tau)v(\tau)d\tau.$$

Use of the change of variable $\eta = t - \tau$ yields

$$r(t) = \int_{0}^{\infty} g(\eta)v(t-\eta)d\eta.$$

From this and (11), the following inequality is obtained.

$$|r(t)| \le \int_0^{\infty} |g(\eta)| \cdot |v(t-\eta)| d\eta \le a$$

Appendix II
An Upper Bound of $|\dot{r}(t)|$

Picking up the expression for w_4 from (17) gives

$$w_4 = 2T\theta + T^2\dot{\theta} + \frac{\dot{y}}{g}. \tag{33}$$

Substituting the relation $\dot{y} = \dot{r} - \dot{L}\theta - L\dot{\theta}$ into (33) and rearranging it yield

$$\dot{r} = gw_4 - (2Tg - \dot{L})\theta - (T^2 g - L)\dot{\theta}. \tag{34}$$

When $[\theta(0)\ \dot{\theta}(0)]' \in \mathcal{R}$, for any v satisfying $|v| \le a$, we have

$$|\theta(t)| \le \|G(s)\|_1 a = a \tag{35}$$

$$|\dot{\theta}(t)| \le \|sG(s)\|_1 a = \frac{2a}{Te} \tag{36}$$

where we used the fact that the transfer function from v to θ is given by (8). From (34), (35), (36), and (23), we obtain

$$|\dot{r}(t)| \le v_{max} + \max_t(|2Tg - \dot{L}|)a + \max_t\left(\frac{2|T^2 g - L|}{Te}\right)a. \tag{37}$$

Letting

$$L_v = \max_t |\dot{L}|$$

and using (24), we have

$$\max_t(|2Tg - \dot{L}|) \le 2Tg + L_v = 2\sqrt{L_0 g} + L_v$$

$$\max_t\left(\frac{2|T^2 g - L|}{Te}\right) \le \frac{2T^2 g}{Te} = \frac{2\sqrt{L_0 g}}{e}.$$

Substitution of these two into (37) yields

$$|\dot{r}(t)| \le v_{max} + \left\{\left(2 + \frac{2}{e}\right)\sqrt{L_0 g} + L_v\right\}a. \tag{38}$$

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