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Reliable H_∞ filter design for a class of continuous-time nonlinear systems with time-varying delay

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***Abstract*— This paper is concerned with the reliable H_∞ filtering problem against sensor failures for a class of continuous-time systems with simultaneous sector-bounded nonlinearities and varying time delay. The resulting design is such that the filtering error system is asymptotically stable and meets the prescribed H_∞ norm constraint in the nominal case as well as in the sensor failure case. A sufficient condition, which depend not only on the upper and lower bound of delay but also on the upper bound of delay derivative, for the existence of such a filter is obtained by using appropriate Lyapunov functional and linear matrix inequality (LMI) technique. What's worth mentioning is that the information about the upper bound of delay derivative is taken into consideration even if this upper bound is not smaller than 1. A numerical example is provided to demonstrate the effectiveness of the proposed designs.**

I. INTRODUCTION

During the past decades, considerable attention has been devoted to the H_∞ filtering problem since H_∞ filter admits not only the noise to be arbitrary signals with bounded energy but also the system model to have uncertainty. In particular, the LMI approach to H_∞ filtering is more powerful in numerical computations and suitable for handling the optimization problems with multiple constraints[1]. On the other hand, time delay is often one of the main sources of instability and poor performance of a control system, which is frequently encountered in many practical engineering systems such as chemical processes, electrical heating and so on. Therefore, the study of H_∞ filtering for time-delay systems has attracted much attention of many researchers in the past years, however, for systems with time-varying delays, the most existed literatures usually demand that the upper bound of the derivative of delays must be smaller than 1[2]. If the upper bound of the derivative of delays is larger than 1, the information of the derivative of delays is always discard such as [3], which is obviously unreasonable. Therefore, improved asymptotic stability conditions for time-delay systems were presented by using the information of delay derivative in [4].

Meanwhile, filtering for nonlinear system is an important research area that has attracted considerable interest. Recently, a large number of papers about the problem of nonlinear filtering have appeared, see, e.g.,$[5-6]$, and the reference therein. The filtering problem for nonlinear stochastic systems was investigated in [5, 6], and in particular, the considered filtering with variance-constrained was designed for uncertain stochastic systems with missing measurements in [6]. An H_∞ filtering design problem for uncertain stochastic time-delay systems with sector-bounded nonlinearities was presented in [5], and the similar type of nonlinear systems were also investigated in [7].

Noting that in the above mentioned works on the filtering problems, the researchers are all based on a common assumption that the sensors can provide uninterrupted signal measurements. However, contingent failures are possible for all sensors in a system in practice. A large degree of filter performances may degrade and possible hazards may happen[8]. Therefore, recently, the design of reliable controller and filter have been received increasing attention, mainly in linear systems[8-10], while the reliable controller and filter for nonlinear systems are investigated in [11-13]. In 2001, the reliable H_∞ controller design for linear systems with sensor or actuator failure via the algebraic Riccati equation (ARE) approach was considered in [8]. [9-10] studied the problem of reliable filtering problem against sensor failures for linear systems, and a method of designing adaptive reliable H_∞ filter was proposed by combining the LMI approach and adaptive method. The problem of reliable H_∞ controller design for nonlinear system was investigated in [11,12] via LMI approach. Moreover, in 2003, [13] proposed a class of reliable variable structure control laws, which were shown to be able to tolerate the outage of actuators within a prespecified subset of actuators. Unfortunately, to the best of the authors' knowledge, up to now, the reliable H_∞ filtering problem for nonlinear systems with varying time delay has not been fully investigated.

Motivated by above points, a reliable H_∞ filter is designed for a class of nonlinear systems with both sector-bounded nonlinearities and time delays. First of all, the sector-bounded nonlinearities and a general sensor failure model which covers outage cases and the possibility of partial failures are introduced. Next, the designs guarantee the asymptotic stability of the estimation errors, and the H_∞ performance of the filtering error system from the exogenous signals to the estimation errors less than a prescribed level. Then, a sufficient condition for the existence of such a reliable H_∞ filter is obtained via appropriate Lyapunov functional and LMI technique, which is dependent on the lower bound and upper bound of the time-varying delays

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and the upper bound of the delay derivative. In addition, the constraint on the upper bound of the delay derivative, which is not larger than 1, is eliminated. Finally, a numerical example is given to illustrate the effectiveness of the developed techniques.

II. PROBLEM FORMULATION

Consider a class of nonlinear continuous-time system with sector nonlinearity described as follows

$$
\begin{aligned}
&\dot{x}(t) = Ax(t) + A_d x(\varepsilon) + Ff(x(t)) + F_d f(x(\varepsilon)) + Bw(t) \\
&y(t) = Cx(t) + C_d x(\varepsilon) + Hh(x(t)) + H_d h(x(\varepsilon)) + Dw(t) \\
&z(t) = Lx(t) \\
&x(t) = \Phi(t),\ t = [-max(d(t)), 0]
\end{aligned} \tag{1}
$$

where $\varepsilon = t - d(t)$, $x(t) \in \mathbf{R}^n$ is the state vector, $w(t) \in \mathbf{R}^r$ is the disturbance input which is assumed to belong to $l_2[0,\infty)$, $z(t) \in \mathbf{R}^q$ is the regulated output and $y(t) \in \mathbf{R}^p$ is the measured output, respectively. The system matrices A, F, F_d, B, C, H, H_d, D and L are known constant matrices of appropriate dimensions. $f(\star)$ and $h(\star)$ are the vector-valued nonlinear functions.

For system (1), the state delay $d(t)$ appearing in both the dynamic and measurement equations are frequently encountered in various engineering systems such as networked control systems, long transmission lines in pneumatic systems and so on. A natural assumption on $d(t)$ can be made as follows.

Assumption 1. The positive scalar $d(t)$ is known function and denotes the time-varying delay satisfying

$$d_m \le d(t) \le d_M,\ \dot{d}(t) \le \mu, \tag{2}$$

where $0 \le d_m < d_M$ and μ are known positive constants. In particular, for the case of $\mu \ge 1$, if choosing a positive scalar $0 < \alpha < 1$, then it follows that

$$(\alpha d(t))' = \alpha \dot{d}(t) \le \alpha\mu < 1, \tag{3}$$

therefore, the information of the derivative of $d(t)$ can be used. In addition, $\Phi(t)$ is a continuous vector-valued initial function of $[-d_M, 0]$.

Remark 1. In many exist works, such as [2], the upper bound of delay derivative μ should be smaller than 1. Though the results in [3] can be applied to the case of $\mu \ge 1$, these stability conditions were independent on the upper bound of the delay derivative μ. However, the case that the delay derivative is larger than or equal to 1 is universal[4]. Therefore, in order to use the information of the derivative of $d(t)$, (3) is considered which also has been studied in [4].

To ensure the achievement of filter design objective, the following basic assumptions are also assumed to be valid.

Assumption 2. The vector-valued nonlinear functions $f(\star)$ and $h(\star)$ are assumed to satisfy the following sector-bounded conditions:

$$
\left\{
\begin{array}{l}
\left[\ f(x) - f(y) - M_1\delta\ \right]^T \left[\ f(x) - f(y) - M_2\delta\ \right] \le 0 \\
\left[\ h(x) - h(y) - N_1\delta\ \right]^T \left[\ h(x) - h(y) - N_2\delta\ \right] \le 0
\end{array}
\right. \tag{4}
$$

where $\delta = x - y$, $\forall x, y \in \mathbf{R}^n$, and $M_1, M_2 \in \mathbf{R}^{n\times n}$ and $N_1, N_2 \in \mathbf{R}^{p\times n}$ are known constant matrices. In what follows, for presentations implicity and without loss of generality,we always assume that:

$$f(0) = 0,\ h(0) = 0. \tag{5}$$

Remark 2. It is obvious that, the conditions in Assumption 2 are more general than the usual sigmoid functions and the recently commonly used Lipschitz conditions, see e.g. [5]. And M_1, N_1 and M_2, N_2 are lower and upper slope bound, respectively.

The sensor outage cases are considered here

$$
\begin{aligned}
&y_{ij}^F(t) = (1-\rho_i^j)y_i(t), 0 \le \underline{\rho_i^j} \le \rho_i^j \le \overline{\rho_i^j} \le 1, \\
&i = 1, \cdots p, j = 1, \cdots L,
\end{aligned} \tag{6}
$$

where ρ_i^j is an unknown constant. Here, the index j denotes the jth failure mode, L denotes the total number of the failure modes, and $y_{ij}^F(t)$ represents the measured signal from the ith sensor that has failed in the jth failure mode. For every faulty mode, $\underline{\rho_i^j}$ and $\overline{\rho_i^j}$ represent the lower and upper bounds of ρ_i^j, respectively. Note that, when $\underline{\rho_i^j} = \overline{\rho_i^j} = 0$, there is no failure for the ith sensor y_i in the jth failure mode. When $\underline{\rho_i^j} = \overline{\rho_i^j} = 1$, the ith sensor y_i is outage in the jth failure mode. When $0 < \underline{\rho_i^j} < \overline{\rho_i^j} < 1$, it corresponds to the case of partial failure of y_i. Denote

$$y_j^F(t) = \left[\ y_{1j}^F(t), y_{2j}^F(t) \cdots y_{pj}^F(t)\ \right]^T = (I - \rho^j)y(t) \tag{7}$$

where $\rho^j = diag\left[\ \rho_1^j, \rho_2^j, \cdots \rho_p^j\ \right]$, $j = 1, \cdots L$. The scaling factors ρ^j satisfy

$$
\begin{aligned}
N_{\rho^j} = \{\rho^j | \rho^j = diag\left[\ \rho_1^j, \rho_2^j, \cdots \rho_p^j\ \right] \in \mathbf{R}^p, \\
0 \le \underline{\rho_i^j} \le \rho_i^j \le \overline{\rho_i^j} \le 1,\ i = 1, 2, \cdots p\}.
\end{aligned} \tag{8}
$$

For convenience in the following sections, for all possible failure modes, we use a uniform sensor failure model

$$y^F(t) = (I - \rho)y(t), \rho \in \{\rho^1, \rho^2, \cdots \rho^L\}. \tag{9}$$

Then, the system (1) with sensor failure (9) is described by

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + A_d x(\varepsilon) + Ff(x(t)) + F_d f(x(\varepsilon)) + Bw(t) \\
y^F(t) &= (I - \rho)(Cx(t) + C_d x(\varepsilon) + Hh(x(t)) + H_d h(x(\varepsilon)) \\
&\quad + Dw(t)) \\
z(t) &= Lx(t)
\end{aligned} \tag{10}
$$

The reliable filter is of the form

$$
\begin{aligned}
\dot{\bar{x}}(t) &= A_F \bar{x}(t) + B_F y^F(t) \\
\bar{z}(t) &= C_F \bar{x}(t)
\end{aligned} \tag{11}
$$

where $\bar{x}(t) \in \mathbf{R}^n$ is the filter state, $\bar{z}(t) \in \mathbf{R}^q$ is the estimation of $z(t)$, A_F, B_F and C_F are the filter parameter matrices to be designed. Here, we assume that the filter is of the same order as the system model.

Applying the filter (11) to the system (10), we obtain the filtering error system

$$\begin{aligned}\dot{\xi}(t) &= \bar{A}\xi(t)+\bar{A}_d\xi(\varepsilon)+\bar{A}_f f(K\xi(t))+\bar{A}_{fd}f(K\xi(\varepsilon))\\ &\quad+\bar{A}_h h(K\xi(t))+\bar{A}_{hd}h(K\xi(\varepsilon))+\bar{B}w(t)\\ e(t) &= \bar{C}\xi(t)\end{aligned} \tag{12}$$

where $\xi(t)=\begin{bmatrix} x(t)\\ \bar{x}(t)\end{bmatrix}$, $K=\begin{bmatrix} I & 0\end{bmatrix}$, $e(t)=z(t)-\bar{z}(t)$ is the estimation error, and $\bar{A}=\begin{bmatrix} A & 0\\ B_F(I-\rho)C & A_F\end{bmatrix}$, $\bar{A}_d=\begin{bmatrix} A_d & 0\\ B_F(I-\rho)C_d & 0\end{bmatrix}$, $\bar{A}_f=\begin{bmatrix} F\\ 0\end{bmatrix}$, $\bar{A}_{fd}=\begin{bmatrix} F_d\\ 0\end{bmatrix}$, $\bar{A}_h=\begin{bmatrix} 0\\ B_F(I-\rho)H\end{bmatrix}$, $\bar{A}_{hd}=\begin{bmatrix} 0\\ B_F(I-\rho)H_d\end{bmatrix}$, $\bar{B}=\begin{bmatrix} B\\ B_F(I-\rho)D\end{bmatrix}$, $\bar{C}=\begin{bmatrix} L & -C_F\end{bmatrix}$.

For convenience in the following sections, we denote the filtering error system without sensor failures, i.e., $\rho=0$, as follows:

$$\begin{aligned}\dot{\xi}(t) &= \tilde{A}\xi(t)+\tilde{A}_d\xi(\varepsilon)+\tilde{A}_f f(K\xi(t))+\tilde{A}_{fd}f(K\xi(\varepsilon))\\ &\quad+\tilde{A}_h h(K\xi(t))+\tilde{A}_{hd}h(K\xi(\varepsilon))+\tilde{B}w(t)\\ e(t) &= \tilde{C}\xi(t)\end{aligned} \tag{13}$$

where $\tilde{A}=\begin{bmatrix} A & 0\\ B_FC & A_F\end{bmatrix}$, $\tilde{A}_d=\begin{bmatrix} A_d & 0\\ B_FC_d & 0\end{bmatrix}$, $\tilde{A}_f=\begin{bmatrix} F\\ 0\end{bmatrix}$, $\tilde{A}_{fd}=\begin{bmatrix} F_d\\ 0\end{bmatrix}$, $\tilde{A}_h=\begin{bmatrix} 0\\ B_FH\end{bmatrix}$, $\tilde{A}_{hd}=\begin{bmatrix} 0\\ B_FH_d\end{bmatrix}$, $\tilde{B}=\begin{bmatrix} B\\ B_FD\end{bmatrix}$, $\tilde{C}=\begin{bmatrix} L & -C_F\end{bmatrix}$.

Our objectives are to develop a filter of the form (11) such that the filtering error systems (12) and (13) satisfy the following requirements:

1) While there is no exogenous disturbance, that is $w(t)=0$, the filtering error systems (12) and (13) are asymptotically stable.
2) For given constants $\gamma_f>\gamma_n>0$, find the filter (11) such that
   i. The filtering error system (12) in the nominal case, i.e., (13), is with an H_∞ performance index no larger than γ_n;
   ii. The filtering error system (12) in the sensor failures case, i.e., $\rho\in\{\rho^1,\rho^2,\cdots\rho^L\}$ with $\rho^j\in N_{\rho^j}, j=1,\cdots,L$, is with an H_∞ performance index no larger than γ_f;

The filter of form (11) satisfying above objectives is said to be an reliable H_∞ filter for the system (1) with (6) and guarantee that the filtering error systems (12) and (13) are asymptotically stable at the same time.

Now, we first provide some important lemmas which will be useful in the derivation of our main results.

Lemma 1.(S-procedure)[14] Let $T_0(x),T_1(x),\cdots,T_p(x)$ be quadratic function of $x\in\mathbf{R}^n$

$$T_i(x)=x^T\Psi_i x,\ i=0,1,\cdots,p \tag{14}$$

with $\Psi_i=\Psi_i^T$. Then the implication

$$T_1(x)\le 0,\cdots,T_p(x)\le 0 \Longrightarrow T_0(x)<0 \tag{15}$$

holds if there exist nonnegative scalar $\tau_1,\cdots\tau_p$ such that

$$\Psi_0-\sum_{i=1}^{p}\tau_i\Psi_i<0 \tag{16}$$

Lemma 2. Assume that $f(\star)$ is a vector-valued nonlinear function and $M_1,M_2\in R^{n\times n}$ are known constant matrices, then we have

$$\begin{bmatrix} f(x)-M_1x\end{bmatrix}^T\begin{bmatrix} f(x)-M_2x\end{bmatrix}\le 0,\ \forall x\in R^n, \tag{17}$$

which implies

$$\begin{bmatrix} x^T & f^T(x)\end{bmatrix}\begin{bmatrix}\hat{M}_1 & \hat{M}_2\\ \hat{M}_2^{\mathrm{T}} & I\end{bmatrix}\begin{bmatrix} x\\ f(x)\end{bmatrix}\le 0 \tag{18}$$

with $\hat{M}_1=(M_1^TM_2+M_2^TM_1)/2, \hat{M}_2=-(M_1^T+M_2^T)/2$.

Proof. Due to the limit of the space, the proof is omitted.

Lemma 3.[15] For any positive symmetric matrix $M\in\mathbf{R}^{n\times n}$, a scalar $\tau>0$, vector function $\varpi:[0,r]\to\mathbf{R}^n$ such that the integrations concerned are well defined, then

$$(\int_0^\tau\varpi(s)ds)^TM(\int_0^\tau\varpi(s)ds)\le\tau\int_0^\tau\varpi^T(s)M\varpi(s)ds. \tag{19}$$

III. MAIN RESULT

To facilitate the presentation, we denote $\hat{M}_1=(M_1^TM_2+M_2^TM_1)/2, \hat{M}_2=-(M_1^T+M_2^T)/2, \hat{N}_1=(N_1^TN_2+N_2^TN_1)/2, \hat{N}_2=-(N_1^T+N_2^T)/2$, $\zeta=1-\mu$, $\beta=1-\alpha$, $\lambda=1-\alpha\mu$, $\eta=d_M-d_m$, $\kappa_1=\frac{1}{d_M}$, $\kappa_2=\frac{1}{d_m}$, $\nu_1=\kappa_2+\kappa_1+\frac{\kappa_1}{\alpha}$, $\nu_2=\frac{4}{\eta}+\kappa_1+\frac{\kappa_1}{\alpha}$, $\nu_3=\kappa_2+\frac{1}{\eta}$, $\nu_4=\frac{\kappa_1}{\alpha}+\frac{\kappa_1}{\beta}$, $\varepsilon_1=t-d_M, \varepsilon_2=t-d_m, \varepsilon_3=t-\alpha d(t), U_1=\hat{B}_F(I-\rho)$, and the notation $He\{M\}:=M+M^T$ is also used.

Before continuing with the solution to the synthesis problem, we present the following theorem which guarantees that the filtering error system (12) is asymptotically stable and has H_∞ performance criterions at the same time.

Theorem 1. Given scalars $\gamma_n>0$, $\gamma_f>0$ and the known constant matrices $\hat{M}_1,\hat{M}_2,\hat{N}_1,\hat{N}_2$. If there exist matrices $P_n=P_n^T>0, P_f=P_f^T>0$, $Q_{in}=Q_{in}^T\ge 0, Q_{if}=Q_{if}^T\ge 0$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ $\bar{A}$, $\bar{B}$, $\bar{C}$ and nonnegative scalars $\tau_i, i=1,2,\cdots,4$ such that

$$\begin{bmatrix}\mathscr{M}_{11n} & *\\ \mathscr{M}_{12n} & \mathscr{M}_{13n}\end{bmatrix}<0 \tag{20}$$

holds for $\rho=0$ and

$$\begin{bmatrix}\mathscr{M}_{11f} & *\\ \mathscr{M}_{12f} & \mathscr{M}_{13f}\end{bmatrix}<0 \tag{21}$$

holds for $\rho\in\{\rho^1,\rho^2,\cdots\rho^L\}$ with $\rho^j\in N_{\rho^j}, j=1,\cdots,L$, where

$$\mathscr{M}_{11n}=\begin{bmatrix}\Omega_{11} & * & * & *\\ \Omega_{12} & -\tau_1 I & * & *\\ \Omega_{13} & 0 & -\tau_3 I & *\\ \Theta_{11n} & 0 & 0 & \Theta_{12n}\end{bmatrix},$$

$$\mathscr{M}_{12n}=\begin{bmatrix}\Theta_{13n} & 0 & 0 & \Theta_{14n} & 0 & 0\\ P_n\tilde{A} & P_n\tilde{A}_f & P_n\tilde{A}_h & P_n\tilde{A}_d & P_n\tilde{A}_{fd} & P_n\tilde{A}_{hd}\\ \tilde{C} & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\mathscr{M}_{13n}=diag\{\Omega_{16},\Omega_{17},\Omega_{18},\Omega_{19},-I\},$$

$$\mathscr{M}_{11f}=\begin{bmatrix}\Pi_{11} & * & * & * \\ \Pi_{12} & -\tau_1 I & * & * \\ \Pi_{13} & 0 & -\tau_3 I & * \\ \Theta_{11f} & 0 & 0 & \Theta_{12f}\end{bmatrix},$$
$$\mathscr{M}_{12f}=\begin{bmatrix}\Theta_{13f} & 0 & 0 & \Theta_{14f} & 0 & 0 \\ P_f\bar{A} & P_f\bar{A}_f & P_f\bar{A}_h & P_f\bar{A}_d & P_f\bar{A}_{fd} & P_f\bar{A}_{hd} \\ \bar{C} & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$
$$\mathscr{M}_{13f}=diag\{\Pi_{16},\Pi_{17},\Pi_{18},\Pi_{19},-I\},$$

$$\Theta_{11n}=\begin{bmatrix}\Omega_{14} \\ \tilde{A}_{fd}^T P_n \\ \tilde{A}_{hd}^T P_n\end{bmatrix},\Theta_{12n}=\begin{bmatrix}\Omega_{15} & * & * \\ -\tau_2\hat{M}_2^T K & -\tau_2 I & * \\ -\tau_4\hat{N}_2^T K & 0 & -\tau_4 I\end{bmatrix},$$
$$\Theta_{13n}=\begin{bmatrix}\kappa_2 P_n \\ 0 \\ \frac{\kappa_1}{\alpha}P_n \\ \tilde{B}^T P_n\end{bmatrix},\Theta_{14n}=\begin{bmatrix}\frac{1}{\eta}P_n \\ \frac{3}{\eta}P_n \\ \frac{\kappa_1}{\beta}P_n \\ 0\end{bmatrix},\Theta_{11f}=\begin{bmatrix}\Pi_{14} \\ \bar{A}_{fd}^T P_f \\ \bar{A}_{hd}^T P_f\end{bmatrix},$$
$$\Theta_{12f}=\begin{bmatrix}\Pi_{15} & * & * \\ -\tau_2\hat{M}_2^T K & -\tau_2 I & * \\ -\tau_4\hat{N}_2^T K & 0 & -\tau_4 I\end{bmatrix},\Theta_{13f}=\begin{bmatrix}\kappa_2 P_f \\ 0 \\ \frac{\kappa_1}{\alpha}P_f \\ \bar{B}^T P_f\end{bmatrix},$$
$$\Theta_{14f}^T=\begin{bmatrix}\frac{1}{\eta}P_f & \frac{3}{\eta}P_f & \frac{\kappa_1}{\beta}P_f & 0\end{bmatrix}$$

with $\Omega_{11}=\sum_{i=1}^{4}Q_{in}+P_n\tilde{A}+\tilde{A}^T P_n-\nu_1 P_n-\tau_1 K^T\hat{M}_1 K-\tau_3 K^T\hat{N}_1 K$, $\Omega_{12}=\tilde{A}_f^T P_n-\tau_1\hat{M}_2^T K$, $\Omega_{13}=\tilde{A}_h^T P_n-\tau_3\hat{N}_2^T K$, $\Omega_{14}=\tilde{A}_d^T P_n+\kappa_1 P_n$, $\Omega_{15}=-\zeta Q_{3n}-\nu_2 P_n-\tau_2 K^T\hat{M}_1 K-\tau_4 K^T\hat{N}_1 K$, $\Omega_{16}=-Q_{1n}-\nu_3 P_n$, $\Omega_{17}=-Q_{2n}-\frac{3}{\eta}P_n$, $\Omega_{18}=-\lambda Q_{4n}-\nu_4 P_n$, $\Omega_{19}=\begin{bmatrix}-\gamma_n^2 I & * \\ P_n\bar{B} & -\frac{\kappa_1}{3}P_n\end{bmatrix}$ and $\Pi_{11}=\sum_{i=1}^{4}Q_{if}+P_f\bar{A}+\bar{A}^T P_f-\nu_1 P_f-\tau_1 K^T\hat{M}_1 K-\tau_3 K^T\hat{N}_1 K$, $\Pi_{12}=\bar{A}_f^T P_f-\tau_1\hat{M}_2^T K$, $\Pi_{13}=\bar{A}_h^T P_f-\tau_3\hat{N}_2^T K$, $\Pi_{14}=\bar{A}_d^T P_f+\kappa_1 P_f$, $\Pi_{15}=-\zeta Q_{3f}-\nu_2 P_f-\tau_2 K^T\hat{M}_1 K-\tau_4 K^T\hat{N}_1 K$, $\Pi_{16}=-Q_{1f}-\nu_3 P_f$, $\Pi_{17}=-Q_{2f}-\frac{3}{\eta}P_f$, $\Pi_{18}=-\lambda Q_{4f}-\nu_4 P_f$, $\Pi_{19}=\begin{bmatrix}-\gamma_f^2 I & * \\ P_f\bar{B} & -\frac{\kappa_1}{3}P_f\end{bmatrix}$. Then, the filtering error systems (12) and (13) are asymptotically stable and satisfy H_∞ performance constraint simultaneously.

Proof. Let us choose a *Lyapunov* functional candidate

$$V(t)=V_1(t)+V_2(t)+V_3(t) \tag{22}$$

where

$$\begin{aligned}V_1(t)&=\xi^T(t)P_f\xi(t)+\textstyle\int_{\varepsilon_2}^t\xi^T(s)Q_{1f}\xi(s)ds+\int_{\varepsilon_1}^t\xi^T(s)\\&\quad Q_{2f}\xi(s)ds,\\V_2(t)&=\textstyle\int_{\varepsilon}^t\xi^T(s)Q_{3f}\xi(s)ds+\int_{\varepsilon_3}^t\xi^T(s)Q_{4f}\xi(s)ds,\\V_3(t)&=\textstyle\int_{-d_m}^0\int_{t+\theta}^t\dot{\xi}^T(s)P_f\dot{\xi}(s)dsd\theta+\int_{-d_M}^{-d_m}\int_{t+\theta}^t\dot{\xi}^T(s)\\&\quad P_f\dot{\xi}(s)dsd\theta+2\textstyle\int_{-d_M}^0\int_{t+\theta}^t\dot{\xi}^T(s)P_f\dot{\xi}(s)dsd\theta,\end{aligned} \tag{23}$$

where $P_f=P_f^T>0$ and $Q_{if}=Q_{if}^T\geq 0,(i=1,\cdots,4)$ are to be determined.

Then, after some manipulation including applying Lemma 1, Lemma 2 and Lemma 3, we can obtain (20) and (21). Due to the limit of the space, the detail is omitted. This completes the proof.

Similar to the common method, which can be found in many papers concerning the reliable controlling and filtering problems, see [10,11], is to set

$$P_n=P_f=P \tag{24}$$

before converting all the inequalities to LMIs. Consequently, based on LMI technique, we give the sufficient condition for the existence of the reliable H_∞ filter as following theorem.

Theorem 2. For prescribed $\gamma>0$ and known constant matrices $\hat{M}_1,\hat{M}_2,\hat{N}_1,\hat{N}_2$, assume that there exist matrices $0<P_{1n}=P_{1n}^T\in\mathbf{R}^{n\times n}$, $P_{2n}\in\mathbf{R}^{n\times n}$, $0<P_{3n}=P_{3n}^T\in\mathbf{R}^{n\times n}$, $0<P_{1f}=P_{1f}^T\in\mathbf{R}^{n\times n}$, $P_{2f}\in\mathbf{R}^{n\times n}$, $0<P_{3f}=P_{3f}^T\in\mathbf{R}^{n\times n}$, $X\in\mathbf{R}^{n\times n}$, $Y\in\mathbf{R}^{n\times n}$, $\hat{A}_F\in\mathbf{R}^{n\times n}$, $\hat{B}_F\in\mathbf{R}^{n\times p}$, $\hat{C}_F\in\mathbf{R}^{q\times n}$ and nonnegative scalars $\tau_i,i=1,2,\cdots,4$ such that

$$\begin{bmatrix}\Sigma_{n21} & * \\ \Sigma_{n22} & \Sigma_{n23}\end{bmatrix}<0,\begin{bmatrix}Q_{i1n} & * \\ Q_{i2n} & Q_{i3n}\end{bmatrix}\geq 0 \tag{25}$$

holds for $\rho=0$, and

$$\begin{bmatrix}\Sigma_{f31} & * \\ \Sigma_{f32} & \Sigma_{f33}\end{bmatrix}<0,\begin{bmatrix}Q_{i1f} & Q_{i2f}^T \\ * & Q_{i3f}\end{bmatrix}\geq 0 \tag{26}$$

holds for $\rho\in\{\rho^1,\rho^2,\cdots\rho^L\}$ with $\rho^j\in N_{\rho^j},j=1,\cdots,L$, where $\Sigma_{n21}=\begin{bmatrix}\Lambda_{11n} & * & * & * \\ \Lambda_{12} & \Lambda_{13} & * & * \\ \Lambda_{14} & 0 & \Lambda_{15n} & * \\ \Lambda_{16} & 0 & \Lambda_{17} & \Lambda_{18n}\end{bmatrix},\Sigma_{n22}^T=\begin{bmatrix}\mathscr{N}_{11}^T \\ \mathscr{N}_{12}^T\end{bmatrix}$,

$\Sigma_{n23}=diag\{-Q_{13n}-\nu_3 R,\Lambda_{31n},\Lambda_{32n},\Lambda_{33n},-I\}$,

$$\Sigma_{f21}=\begin{bmatrix}\Lambda_{11f} & * & * & * \\ \Lambda_{12} & \Lambda_{13} & * & * \\ \Lambda_{14} & 0 & \Lambda_{15f} & * \\ \Lambda_{16} & 0 & \Lambda_{17} & \Lambda_{18f}\end{bmatrix},\Sigma_{f22}^T=\begin{bmatrix}\mathscr{N}_{11}^T \\ \mathscr{N}_{13}^T\end{bmatrix},$$

$\Sigma_{f23}=diag\{-Q_{13f}-\nu_3 R,\Lambda_{31f},\Lambda_{32f},\Lambda_{33f},-I\}$,

$$\Lambda_{11n}=\begin{bmatrix}\Upsilon_{20n} & * \\ \Upsilon_{21n} & \Upsilon_{22n}\end{bmatrix},\Lambda_{12}=\begin{bmatrix}\Upsilon_{23} & \Upsilon_{24} \\ -\tau_3\hat{N}_2^T & \Upsilon_{25}\end{bmatrix},$$
$$\Lambda_{13}=diag\{-\tau_1 I,-\tau_3 I\},\Lambda_{14}=\begin{bmatrix}A_d^T S+\kappa_1 S & \Upsilon_{26} \\ A_d^T S+\kappa_1 S & \Upsilon_{28}\end{bmatrix},$$
$$\Lambda_{15n}=\begin{bmatrix}\Upsilon_{27n} & * \\ \Upsilon_{29n} & \Xi_{20n}\end{bmatrix},\Lambda_{16}=\begin{bmatrix}F_d^T S & F_d^T R \\ 0 & (U_1 H_d)^T \\ \kappa_2 S & \kappa_2 S\end{bmatrix},$$
$$\Lambda_{17}=\begin{bmatrix}-\tau_2\hat{M}_2^T & -\tau_2\hat{M}_2^T \\ -\tau_4\hat{N}_2^T & -\tau_4\hat{N}_2^T \\ \frac{1}{\eta}S & \frac{1}{\eta}S\end{bmatrix},\Lambda_{18n}=diag\{-\tau_2 I,-\tau_4 I,\Xi_{21n}\},$$
$$\mathscr{N}_{11}=\begin{bmatrix}\kappa_2\Lambda_{21} & 0 & \frac{1}{\eta}\Lambda_{21} & 0 \\ 0 & 0 & \frac{3}{\eta}\Lambda_{22} & 0 \\ \frac{\kappa_1}{\alpha} & 0 & \frac{\kappa_1}{\beta}\Lambda_{22} & 0 \\ \Lambda_{23} & 0 & 0 & 0 \\ \Lambda_{24} & \Lambda_{25} & \Lambda_{26} & \Lambda_{27} \\ \Lambda_{28} & 0 & 0 & 0\end{bmatrix},\Lambda_{21}=\begin{bmatrix}S & R\end{bmatrix},$$
$$\mathscr{N}_{12}^T=\begin{bmatrix}-Q_{12n}^T-\nu_3 S & 0 & 0 & 0 & 0 & 0\end{bmatrix},\Lambda_{22}=\begin{bmatrix}S & S \\ S & R\end{bmatrix},$$
$$\mathscr{N}_{13}^T=\begin{bmatrix}-Q_{12f}^T-\nu_3 S & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$
$$\Lambda_{23}=\begin{bmatrix}B^T S & (RB+U_1 D)^T\end{bmatrix},\Lambda_{24}=\begin{bmatrix}SA & SA \\ \Xi_{22} & RA+U_1 C\end{bmatrix},$$
$$\Lambda_{25}=\begin{bmatrix}SF & 0 \\ RF & \hat{B}_F H\end{bmatrix},\Lambda_{26}=\begin{bmatrix}SA_d & SF_d \\ \Xi_{23} & RA_d+U_1 C_d\end{bmatrix},$$
$$\Lambda_{27}=\begin{bmatrix}SF_d & 0 \\ RF_d & U_1 H_d\end{bmatrix},\Lambda_{31n}=\begin{bmatrix}\Xi_{24n} & * \\ \Xi_{25n} & -Q_{23n}-\frac{3}{\eta}R\end{bmatrix},$$

$\Lambda_{28} = \left[\begin{array}{cc} L-\hat{C}_F & L \end{array}\right], \Lambda_{32n} = \left[\begin{array}{cc} \Xi_{26n} & * \\ \Xi_{27n} & -\lambda Q_{43n} - \nu_4 R \end{array}\right]$,

$\Lambda_{33n} = \left[\begin{array}{ccc} -\gamma_n^2 I & * & * \\ SB & -\frac{\kappa_1}{3}S & * \\ RB+\hat{B}_F D & -\frac{\kappa_1}{3}S & -\frac{\kappa_1}{3}R \end{array}\right]$,

$\Lambda_{11f} = \left[\begin{array}{cc} \Upsilon_{20f} & * \\ \Upsilon_{21f} & \Upsilon_{22f} \end{array}\right], \Lambda_{18f} = diag\{-\tau_2 I, -\tau_4 I, \Xi_{21f}\}$,

$\Lambda_{15f} = \left[\begin{array}{cc} \Upsilon_{27f} & * \\ \Upsilon_{29f} & \Xi_{20f} \end{array}\right], \Lambda_{31f} = \left[\begin{array}{cc} \Xi_{24f} & * \\ \Xi_{25f} & -Q_{23f} - \frac{3}{\eta}R \end{array}\right]$,

$\Lambda_{32f} = \left[\begin{array}{cc} \Xi_{26f} & * \\ \Xi_{27f} & -\lambda Q_{43f} - \nu_4 R \end{array}\right]$,

$\Lambda_{33f} = \left[\begin{array}{ccc} -\gamma_f^2 I & * & * \\ SB & -\frac{\kappa_1}{3}S & * \\ RB+U_1 D & -\frac{\kappa_1}{3}S & -\frac{\kappa_1}{3}R \end{array}\right]$, with $\mathscr{R}_1 = SA + A^T S - \nu_1 S - \tau_1 \hat{M}_1 - \tau_3 \hat{N}_1, \mathscr{R}_2 = RA + U_1 C + \hat{A}_F + A^T S - \nu_1 S - \tau_1 \hat{M}_1 - \tau_3 \hat{N}_1, \mathscr{R}_3 = He\{RA + U_1 C\} - \nu_1 R - \tau_1 \hat{M}_1 - \tau_3 \hat{N}_1, \mathscr{R}_4 = -\tau_2 \hat{M}_1 - \tau_4 \hat{N}_1 - \nu_2 S, \mathscr{R}_5 = -\tau_2 \hat{M}_1 - \tau_4 \hat{N}_1 - \nu_2 S, \mathscr{R}_6 = -\tau_2 \hat{M}_1 - \tau_4 \hat{N}_1 - \nu_2 R$, $\Upsilon_{20n} = \sum_{i=1}^{4} Q_{i1n} + \mathscr{R}_1$, $\Upsilon_{21n} = \sum_{i=1}^{4} Q_{i2n} + \mathscr{R}_2$, $\Upsilon_{22n} = \sum_{i=1}^{4} Q_{i3n} + \mathscr{R}_3$, $\Upsilon_{23} = F^T S - \tau_1 \hat{M}_2^T$, $\Upsilon_{24} = F^T R - \tau_1 \hat{M}_2^T$, $\Upsilon_{25} = (U_1 H)^T - \tau_3 \hat{N}_2^T$, $\Upsilon_{26} = A_d^T R + (U_1 C_d)^T + \kappa_1 S$, $\Upsilon_{27n} = -\zeta Q_{31n} + \mathscr{R}_4$, $\Upsilon_{28} = A_d^T R + (U_1 C_d)^T + \kappa_1 R$, $\Upsilon_{29n} = -\zeta Q_{32n} + \mathscr{R}_5$, $\Xi_{20n} = -\zeta Q_{33n} + \mathscr{R}_6$, $\Xi_{21n} = -Q_{11n} - \nu_3 S$, $\Xi_{22} = RA + U_1 C + \hat{A}_F$, $\Xi_{23} = RA_d + U_1 C_d$, $\Xi_{24n} = -Q_{21n} - \frac{3}{\eta}S$, $\Xi_{25n} = -Q_{22n} - \frac{3}{\eta}S$, $\Xi_{26n} = -\lambda Q_{41n} - \nu_4 S$, $\Xi_{27n} = -\lambda Q_{42n} - \nu_4 S$, $\Upsilon_{20f} = \sum_{i=1}^{4} Q_{i1f} + \mathscr{R}_1$, $\Upsilon_{21f} = \sum_{i=1}^{4} Q_{i2f} + \mathscr{R}_2$, $\Upsilon_{22f} = \sum_{i=1}^{4} Q_{i3f} + \mathscr{R}_3, \Upsilon_{27f} = -\zeta Q_{31f} + \mathscr{R}_4, \Upsilon_{29f} = -\zeta Q_{32f} + \mathscr{R}_5$, $\Xi_{20f} = -\zeta Q_{33f} + \mathscr{R}_6$, $\Xi_{21n} = -Q_{11f} - \nu_3 S$, $\Xi_{24f} = -Q_{21f} - \frac{3}{\eta}S$, $\Xi_{25f} = -Q_{22f} - \frac{3}{\eta}S$, $\Xi_{26f} = -\lambda Q_{41f} - \nu_4 S, \Xi_{27f} = -\lambda Q_{42f} - \nu_4 S$.

Moreover, if there exist solutions of this inequality, the reliable filter can be given by

$$A_F = (S-R)^{-1}\hat{A}_F, B_F = (S-R)^{-1}\hat{B}_F, C_F = \hat{C}_F. \quad (27)$$

Proof. Due to the limit of the space, the proof is omitted.

It is noted that the conditions in Theorem 2 are LMI conditions with respecting to the scalar γ_n and γ_f, which denote the reliable H_∞ performance bounds for the nominal and the sensor failure cases of the filtering error system (12), respectively. Therefore, γ_n and γ_f can be minimized by using convex optimization algorithms.

Remark 3. The sufficient conditions expressed in LMIs are presented in Theorem 2, which there exist sensor failures. When the sensor failures are not considered, i.e., $\rho = 0$, the problem reduces to standard H_∞ filter design, which (25) should be satisfied.

IV. NUMERICAL SIMULATION

To illustrate the validity and effectiveness of the reliable H_∞ filter, a numerical simulation is carried out to provide a comparison among the approaches proposed in this paper.

Consider the following nonlinear continuous-time time-delay system (1) with the following parameters:

$$A = \left[\begin{array}{ccc} -0.08 & -4.52 & 1.76 \\ 1.15 & 1.67 & -1.93 \\ 1.93 & 2.27 & -3.10 \end{array}\right], D = \left[\begin{array}{cc} 1 & 0 \\ 2 & 1 \end{array}\right],$$

$$A_d = \left[\begin{array}{ccc} 0.1 & -0.1 & 0 \\ 0.1 & -0.2 & 0 \\ 0 & -0.1 & -0.1 \end{array}\right], B = \left[\begin{array}{cc} -0.01 & 0.03 \\ -0.02 & -0.05 \\ 0.03 & 0.04 \end{array}\right],$$

$$F = F_d = \left[\begin{array}{ccc} 0.1 & 0.1 & 0.3 \\ 0.2 & 0.2 & 0 \\ 0.3 & 0.1 & 0.1 \end{array}\right], C = \left[\begin{array}{ccc} 0 & 0.1 & -0.2 \\ 0.1 & -0.1 & 0.2 \end{array}\right],$$

$$C_d = \left[\begin{array}{ccc} 0 & -0.2 & 0.1 \\ 0.1 & 0 & 0.2 \end{array}\right], H = H_d = \left[\begin{array}{ccc} 0.3 & -0.1 & 0 \\ 0.2 & 0 & 0.1 \end{array}\right],$$

$$L = \left[\begin{array}{ccc} -1 & 0 & 1 \end{array}\right], M_1 = N_1 = \left[\begin{array}{ccc} 0.01 & 0.01 & -0.01 \\ 0.01 & 0.02 & 0.04 \\ -0.02 & 0.01 & 0.02 \end{array}\right],$$

$$M_2 = N_2 = \left[\begin{array}{ccc} -0.01 & 0.01 & -0.01 \\ -0.03 & -0.02 & -0.02 \\ 0.02 & -0.03 & -0.04 \end{array}\right], f(x(t)) = h(x(t))$$

$$= \left[\begin{array}{c} 0.02x_1(t)sin^2(x_1(t)) - 0.01(x_1(t) - x_2(t) + x_3(t)) \\ -0.01(x_1(t) - x_3(t)) \\ -0.01(x_2(t) + x_3(t)) \end{array}\right],$$

and for convenience, we denote $\varsigma(t) = x(\varepsilon)$, then we have

$$f(x(\varepsilon)) = h(x(\varepsilon)) = f(\varsigma(t)) = h(\varsigma(t)) = \left[\begin{array}{c} 0.02\varsigma_1(t)sin^2(\varsigma_1(t)) - 0.01(\varsigma_1(t) - \varsigma_2(t) + \varsigma_3(t)) \\ -0.01(\varsigma_1(t) - \varsigma_3(t)) \\ -0.01(\varsigma_2(t) + \varsigma_3(t)) \end{array}\right],$$

where $f(\star)$ and $h(\star)$ satisfy (4). On the other hand, the minimum bound and upper bound delay time $d(t)$ are given by $d_m = 0.5$ and $d_M = 1$, respectively, and let $\alpha = 0.05$.

Here, the following four possible sensor failure modes are considered:

- Nominal mode 1: Both of the two sensors are nominal, that is, $\rho^1 = diag[\rho_1^1, \rho_2^1] = diag[0,0]$.
- Sensor failure mode 2: The first sensor is nominal and the second is outage, that is, $\rho^2 = diag[\rho_1^2, \rho_2^2] = diag[0,1]$.
- Sensor failure mode 3: The first sensor is outage and the second is nominal, that is, $\rho^3 = diag[\rho_1^3, \rho_2^3] = diag[1,0]$.
- Sensor failure mode 4: The two sensors are partial failure, that is, $\rho^4 = diag[\rho_1^4, \rho_2^4] = diag[0.4, 0.5]$.

Then, for various μ, we can obtain the optimal reliable H_∞ performances and the filter parameters with $a = 10$ and $b = 1$. Due to the limit of the space, the filter gain matrices are omitted.

For comparison, Table 1 gives out the H_∞ norm performances for the nominal and the sensor failure cases by the above two approaches with various μ. Table 1 shows that the standard filter has the best performance in the nominal case. However, the optimal H_∞ performance of the standard filter is serious deteriorative in the sensor failure case, while the reliable H_∞ filter performs well.

Table 1 Comparison of γ by different methods with various μ

μ	Design methods		γ	
			Nominal	Sensor Failures
$\mu = 0.5$	Thm. 2	RF	1.3331	2.5784
	Rem. 3	SF	1.0531	8.5237
$\mu = 1$	Thm. 2	RF	1.3447	2.5864
	Rem. 3	SF	1.0657	8.4632
$\mu = 1.5$	Thm. 2	RF	1.3288	2.5764
	Rem. 3	SF	1.0703	8.3846

• RF: Reliable Filter • SF: Standard Filter.

In order to show the effectiveness of our method more clearly, a simulation is also reformed. In the following simulation, let the system initial state be $x_0 = \left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right]$ and the filter initial state be $\bar{x}_0 = \left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right]$. And we assume the disturbance input $w^T(t) = \left[\begin{array}{cc} w_1^T(t) & w_2^T(t) \end{array}\right]$ as following:

$$w_1(t) = w_2(t) = \begin{cases} -0.2cos(t) & 15 \le t \le 20 \\ 0 & otherwise, \end{cases} \quad (28)$$

and the delay d(t) is assume as

$$d(t) = 0.5|sin(3t)| + 0.5. \quad (29)$$

It obviously that $d_m = 0.5$, $d_M = 1$ and $\mu = 1.5$. Fig.1 and Fig.2 show the estimation error $e(t)$ response of the filters designed by the proposed methods for the reliable filter and the standard filter in the nominal case and in the sensor failure case, respectively. We can also compute $\|e(t)\|_2 = \int_0^\infty e^T(s)e(s)ds$ and $\|w(t)\|_2 = \int_0^\infty w^T(s)w(s)ds$, respectively, and denote $\vartheta = \frac{\|e(t)\|_2}{\|w(t)\|_2}$, then we can obtain the following tables.

Table 2 Comparison of Reliable H_∞ filter with standard H_∞ filter in the nominal case under the disturbance w(t)

w(t)	RF		SF	
$\|w(t)\|_2$	$\|e(t)\|_2$	ϑ_1	$\|e(t)\|_2$	ϑ_2
0.4844	1.0590	2.1862	0.9498	1.9608

• RF: Reliable Filter • SF: Standard Filter.

Table 3 Comparison of Reliable H_∞ filter with standard H_∞ filter in the sensor failure case under the disturbance w(t)

w(t)	RF		SF	
$\|w(t)\|_2$	$\|e(t)\|_2$	ϑ_3	$\|e(t)\|_2$	ϑ_4
0.4844	1.2072	2.4922	2.1351	4.4077

• RF: Reliable Filter • SF: Standard Filter.

From Table 2 and Table 3, we can find, the reliable and standard filter's performance indexes in the nominal case are smaller than the reliable filter and the standard filter's performance indexes in the sensor failure case, which implies that the reliable filter and the standard filter perform well under the condition that there isn't considered the sensor failures. ϑ_4 is bigger than ϑ_3, which illustrates the standard filter designed by Remark 3 is sensitive to the sensor failures. Furthermore, from Fig.1 and Fig.2, we can easy find that the standard filter performs well, however, the standard filter is serious deteriorative in the sensor failure case. This phenomenon shows the effectiveness of our design methods.

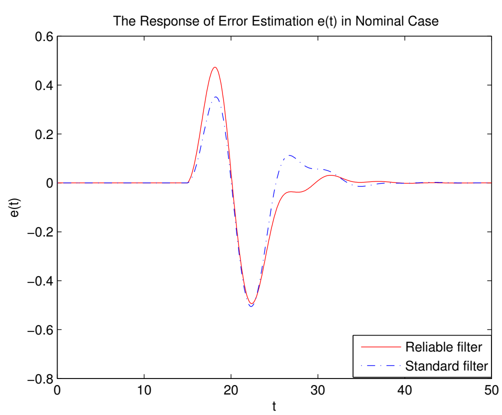

Fig. 1. Comparison between the Relable H_∞ filter and the standard H_∞ filter in the nominal case

V. CONCLUSION

A reliable H_∞ filter design method for a class of continuous-time systems with both sector-bounded nonlinearities and time delays is presented against sensor failures. The information about the upper bound of delay derivative is taken into consideration even if this upper bound is not smaller than 1. A delay-dependent sufficient condition for the existence of the filter to meet H_∞ performance is presented via LMI, and the explicit expression of the desired filter is also developed. And the convex optimization algorithm is given to obtain the solution. A numerical example verifies the effectiveness of the proposed methods.

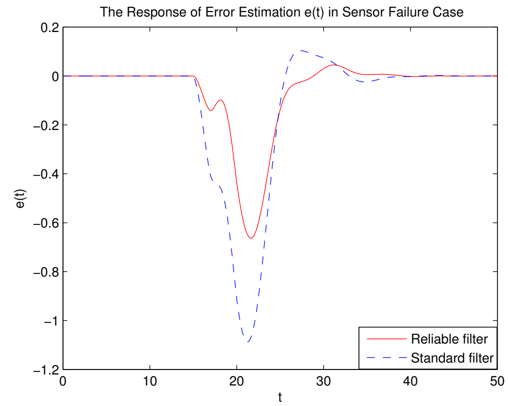

Fig. 2. Comparison between the reliable H_∞ filter and standard H_∞ filter in sensor failure case

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