2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Providing Quality of Service of Information Through Mobility

Eric W. Frew
University of Colorado, Boulder, CO, 80302

***Abstract*— This paper develops mobility control strategies for robotic agents to provide quality of service of information while carrying out active sensing tasks. Like wireless communication concepts of quality of service, a new metric is developed to encapsulate an aggregate measure of information gathering as a network service. This metric states that a given amount of information will be achieved in given time with a given probability. Conditions are given for when two of the three components can be specified independently with the third to be determined from them. Simulation results verify the ability of this new metric to bound system behavior.**

I. Introduction

Active sensing in robot networks takes advantage of robot mobility to optimize or improve information gathering activities. A key component of active sensing systems is the dependence of information-gathering on robot motion and the ability to predict in advance the effect of robot motion on the quality of information that is collected. For model-based sensing tasks (e.g. feature localization, target tracking, and diffuse target tracking) information-theoretic concepts such as mutual information, Fisher information, and entropy are used to quantify the sensitivity of the sensing task to robot motion and planning algorithms can explicitly formulate sensing objectives in terms of these criteria. Active sensing concepts have been applied to the design of trajectories for different applications ranging from passive sonar [1], vision-based geo-location [2], and active sensor networks [3].

Active sensing works when the performance of a nonlinear estimation process is a function of the system input. In many cases the optimal result is a function of the estimate of the state or parameters of interest [4]. Typical solutions invoke the certainty equivalence principle to neglect the probability distribution of these variables and the fact that the Fisher Information Matrix (FIM) itself is a random variable [3]. Active sensing does not fully address its inherent conundrum: the plans generated to optimize the uncertainty in an estimation process are dependent on the uncertain parameters they need to estimate. Thus, active sensing concepts need to be applied to expectation or minimax costs of the information criterion.

Calculation of information measures for general nonlinear estimation problems is extremely challenging. Measures such as mutual information are expectations over future random observations and process noise terms. Even in cases where state dynamics are linear and noise probability distributions are Gaussian, calculation of terms like mutual information over finite future time horizons can be challenging. Calculating information measures for general distributions is usually not possible analytically and must be approximated [5]–[7].

This work investigates the development of a quality of service (QoS) metric for information gain through active sensing. Rather than using the expectation of information, a probabilistic bound is described such that performance surpasses a specified limit some percentage of the time (or with some probability). Information is described based on a prediction of the estimate error covariance matrix. This metric is well suited for systems with unimodal probability distributions, even if they are not Gaussian. Simulations of geolocalization and target tracking demonstrate the effectiveness of the QoS concept developed in this paper.

II. Informatics and Active Sensing

Consider a discrete time nonlinear estimation problem. Let the state vector $\mathbf{x}_k$ evolve according to the state equation

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k) \tag{1}$$

where k is the discrete time, $\mathbf{u}_k$ is the vector of deterministic inputs, and $\mathbf{w}_k$ is a random disturbance vector. Furthermore, let $\mathbf{z}_k$ denote the measurement vector with

$$\mathbf{z}_k = h(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k \tag{2}$$

where $h(\mathbf{x}_k, \mathbf{u}_k)$ is the measurement function and $\mathbf{v}_k$ is random measurement noise. Finally, assume some unbiased *a priori* statistical information about $\mathbf{x}_0$ is given with probability density function $p(\mathbf{x}_0)$. Often the initial state information is assumed to have a Gaussian distribution which can be completely determined by the mean $\hat{\mathbf{x}}_0 = E[\mathbf{x}_0]$ and estimate error covariance matrix $E[(\hat{\mathbf{x}}_0 - \mathbf{x}_0)(\hat{\mathbf{x}}_0 - \mathbf{x}_0)^T] = \mathbf{P}_0$.

Ideally, optimal sensing is achieved by maximizing the information contained in the estimate of the state vector at some later time $k+T$. Different measures of information used for active sensing tasks include the entropy of the probability distribution, mutual information between the current and predicted distributions, or the expected estimate error covariance matrix $E[P_{k+T|k}]$ [3], [5], [6]. In practice calculating these terms for general nonlinear problems is challenging.

For the purposes of planning, this work uses the Extended Information Filter (EIF) framework to derive a measure of information from the matrix norm of the information matrix $\mathbf{Y}_{k|k} = \mathbf{P}_{k|k}^{-1}$ which is the inverse of the estimate error covariance matrix. If the estimation process is consistent and efficient then the information matrix accurately reflects the second moment of the probability density function of the estimate error. Otherwise it serves only as an approximation.

From the EIF framework, the dynamics of the information matrix are

$$\mathbf{Y}_{k+1|k+1} = (\mathbf{\Gamma}_k \mathbf{Q}_k \mathbf{\Gamma}_k^T + \mathbf{\Phi}_k \mathbf{Y}_{k|k}^{-1} \mathbf{\Phi}_k^T)^{-1} + \mathbf{I}_{k+1} \quad (3)$$

where $\mathbf{I}_{k+1}$ is the measurement contribution to the information matrix

$$\mathbf{I}_{k+1} = \mathbf{H}_{k+1}^T \mathbf{R}_{k+1}^{-1} \mathbf{H}_{k+1} \quad (4)$$

with

$$\mathbf{\Gamma}_k = \nabla_{\mathbf{W}} f(\mathbf{x}_k, \mathbf{u}_k, 0), \quad (5)$$

$$\mathbf{\Phi}_k = \nabla_{\mathbf{X}} f(\mathbf{x}_k, \mathbf{u}_k, 0), \quad (6)$$

$$\mathbf{H}_k = \nabla_{\mathbf{X}} h(\mathbf{x}_k, \mathbf{u}_k), \quad (7)$$

$\mathbf{Q}_k = E[\mathbf{w}_k \mathbf{w}_k^T]$, and $\mathbf{R}_k = E[\mathbf{v}_k \mathbf{v}_k^T]$. The subscript $\mathbf{Y}_{a|b}$ denotes the information matrix at discrete time a based on all measurements collected through time b.

So far the derivation has been general to any nonlinear estimation problem. Two additional assumptions are made to apply (3) to robotic active sensing applications. In the context of the general system description of (1) and (2), the input vector is the concatenation of robot state vectors such that $\mathbf{u}_k = [\mathbf{u}_k^1; \cdots; \mathbf{u}_k^i; \cdots; \mathbf{u}_k^n]$ where $\mathbf{u}_k^i = \mathbf{x}_{robot,k}^i$ is the robot position. For the sake of generalization to different active sensing tasks, it is assumed that the components of the input vector can evolve according to another dynamic equation

$$\mathbf{u}_{k+1}^i = g_i(\mathbf{u}_k^i, \mathbf{r}_k^i) \quad (8)$$

where $\mathbf{r}_k^i$ is the input to that system. Furthermore, it is assumed that the measurement vector is the concatenation of terms from multiple robots such that $\mathbf{z}_k = [\mathbf{z}_k^1; \cdots; \mathbf{z}_k^i; \cdots; \mathbf{z}_k^n]$ where $\mathbf{z}_k^i = h_i(\mathbf{x}_k, \mathbf{u}_k^i) + \mathbf{v}_k^i$. Assuming the measurements taken by each robot are independent, the measurement contribution to the information matrix becomes

$$\mathbf{I}_{k+1} = \sum_{i=1}^{n} (\mathbf{H}_k^i)^T \mathbf{R}_{k,i}^{-1} \mathbf{H}_k^i, \quad (9)$$

$$\mathbf{H}_k^i = \nabla_{\mathbf{X}} h_i(\mathbf{x}_k, \mathbf{u}_k^i) \quad (10)$$

and $\mathbf{R}_k^i = E[\mathbf{v}_k^i \mathbf{v}_k^{i,T}]$

An objective function can be developed by taking any norm of the information matrix. Common objectives include the determinant, trace, minimum diagonal element, or minimum singular value. This work uses the determinant due to the fact that it is proportional to the square of the volume of the hyperellipsoid defined by level sets of the estimate confidence. In particular, the information measure ζ_k is defined as

$$\zeta_{k|k} = |\mathbf{Y}_{k|k}|. \quad (11)$$

The goal of *active sensing* is to exploit the dependence of the informatics of (1) - (11) on the system input $\mathbf{u}_k$ (or $\mathbf{r}_k^i \;\; \forall i$), where the dependency enters through the Jacobian matrices defined by (5) - (7). A typical active sensing problem involves choosing system inputs to maximize a prediction of the performance metric $E[\zeta_{k+T|k}(\mathbf{x}_k, \mathbf{Y}_{k|k}, \mathbf{u}_{k:k+T-1}, \mathbf{w}_{k:k+T-1}, \mathbf{z}_{k+1:k+T})]$ where $\mathbf{u}_{k:k+T-1}$ denotes the input sequence from current time k to future time $k+T-1$ and the expectation is with respect to the current state $\mathbf{x}_k$, the sequence of future process noise $\mathbf{w}_{k:k+T-1}$, and the sequence of future measurements $\mathbf{z}_{k:k+T}$.

In order to simplify the active sensing problem, this work ignores the dependence of the expectation of the information measure on the future process noise and measurement sequences. The prediction of the estimate of the system is

$$\hat{\mathbf{x}}_{k+t+1|k} = f(\hat{\mathbf{x}}_{k+t|k}, \mathbf{u}_{k+t}, 0) \quad (12)$$

and the Jacobians $\mathbf{\Gamma}_{k+t}$, $\mathbf{\Phi}_{k+t}$, and $\mathbf{H}_{k+t}$ are all calculated based on $\hat{\mathbf{x}}_{k+t|k}$. It should be noted that the impact of the noise terms in the expectation of the information measure has a second order effect since the information matrix still contains terms that describe how the process noise ($\mathbf{\Gamma}_k \mathbf{Q}_k \mathbf{\Gamma}_k^T$) and measurements ($\mathbf{I}_{k+1}$) contribute information.

Two active sensing problems can now be defined based on the information matrix norm $|\zeta_{k+T|k}|$. The first will be referred to as the *fixed time, maximum information (FTMI) problem*. Given an estimate of the current state $\mathbf{x}_k$ and a fixed time horizon T, determine the sequence of inputs[1]

$$\mathbf{u}_{k:k+T-1}^* = \arg\max_{\mathbf{u}(\cdot)} E\left[\zeta_{k+T|k}(\mathbf{x}_k, \mathbf{Y}_{k|k}, \mathbf{u}_{k:k+T-1})\right]. \quad (13)$$

The second problem is referred to as *fixed information, minimum time (FIMT)*. Given an estimate of the current state $\mathbf{x}_k$ and a predefined level of information ζ_0, determine the sequence of inputs

$$\mathbf{u}_{k:k+T-1}^* = \arg\min_{\mathbf{u}(\cdot)} T \quad (14)$$

$$\text{subject to } E\left[\zeta_{k+T|k}(\mathbf{x}_k, \mathbf{Y}_{k|k}, \mathbf{u}_{k:k+T-1})\right] \geq \zeta_0. \quad (15)$$

III. Quality of Service of Information

While the two problem formulations described in the preceding section are commonly used to perform active sensing, they are both incomplete in terms of long term performance guarantees and practical usage. The FTMI formulation is the most common active sensing problem and the least useful in practical terms. For many applications more information is only better up to a point. For example, successful navigation through a cluttered environment can be achieved by different robots given different levels of map accuracy. Planning to gather the optimal amount of information wastes resources.

The time-optimal FIMT problem is more practical since it attempts to achieve a given level of information as quickly as possible. This problem is not nearly as common in the literature as the FTMI problem. It's solution is still limited since it only optimizes the time to reach the expected information level. There are no guarantees that the system actually achieves that level. One would expect that half the time the FIMT solution does not actually achieve the specified level.

[1]For simplicity the optimization will always be written as a function of the input vector $\mathbf{u}$ with the understanding that it may actually be solved by selection of the inputs $\mathbf{r}^i$ when appropriate.

In order to address the shortcomings of both the FTMI and FIMT active sensing problems, this paper presents a new *quality of service* objective. The stochastic nature of the active sensing problem necessitates a probabilistic framework, however better bounds on performance can be given that can be utilized by higher level processes.

The concept of quality of service of information is drawn from networked communication metrics using the same term. In networking, QoS describes the aggregate performance of the network by combining absolute and probabilistic measures. A common QoS statement of network performance would state that "XX percentage of data packets of size YY will be delivered with delay less than ZZ seconds."

This work creates a similar measure for information gathering activities by robotic agents. In particular, quality of service of information is defined by three different parameters:

Definition 1: **Quality of Service of Information** At least $P_{qos}\%$ percent of the time (or with probability p_{qos} or greater, or with confidence c_{qos} or greater) the active sensing problem will be solved such that ζ_{qos} or more information is obtained in T_{qos} seconds or less.

Unlike the objectives for the FTMI and FIMT problems, the notion of QoS puts performance bounds on the solution of the active sensing problem. These bounds are important because they allow higher level processes to reason about the outcome of active sensing problem. This capability is important for resource allocation and scheduling of multiple robots in complex environments with multiple active sensing tasks. For example, if a task scheduler can count on a given active sensing task being achieved by some future time, it can schedule the robot devoted to the initial task for that time. Neither the FTMI nor FIMT problem formulations allows for this higher level reasoning.

Given this notion of quality of service, the next issue is solving the trajectory optimization problem to achieve it. Assume the desired QoS is given by the tuple $q = \{p_{qos}, \zeta_{qos}, T_{qos}\}$ and that this QoS demand can be achieved by the system. In order to solve the trajectory optimization problem two new deterministic optimization problems are defined that are related to the original FTMI and FIMT problems.

For the moment assume the system state is not a random variable, but is still uncertain. Let the true state $\mathbf{x}_k \in X_k$ belong to a set $X_k \subset \Re^n$. Rather than optimize over the expectation of the true state, two new optimization problems are defined based on this restricted set X_k. In particular, the first will be referred to as the *deterministic fixed time, maximum information (D-FTMI)* problem. Given the set X_k that is known to contain the current state and a fixed time horizon T, determine the sequence of inputs

$$\mathbf{u}^*_{k:k+T-1} = \arg\max_{\mathbf{u}(\cdot)} \min_{\mathbf{x}_k \in X_k} \zeta_{k+T|k}(\mathbf{x}_k, \mathbf{Y}_{k|k}, \mathbf{u}_{k:k+T-1}) \tag{16}$$

with

$$\zeta^*_{DFTMI} = \max_{\mathbf{u}(\cdot)} \min_{\mathbf{x}_k \in X_k} \zeta_{k+T|k}(\mathbf{x}_k, \mathbf{Y}_{k|k}, \mathbf{u}_{k:k+T-1}). \tag{17}$$

Likewise, the second is the *deterministic fixed information, minimum time (D-FIMT)* problem. Given the set X_k and information level ζ_0, determine the sequence of inputs

$$\mathbf{u}^*_{k:k+T-1} = \arg\max_{\mathbf{u}(\cdot)} T \tag{18}$$

$$\text{subject to } \min_{\mathbf{x}_k \in X_k} \zeta_{k+T|k}(\mathbf{x}_k, \mathbf{Y}_{k|k}, \mathbf{u}_{k:k+T-1}) \geq \zeta_0 \tag{19}$$

with

$$T^*_{DFIMT} = \max_{\mathbf{u}(\cdot)} T. \tag{20}$$

The advantage of these two new problem formulations is that performance bounds can now be extracted. Assuming the true target state is actually in the set X_k, the maximin objective functions ensure that the achieved value function must be as good as or better than the initial solution. Careful selection of the region X_k will enable trajectory design to satisfy QoS demands based on the DFTMI and DFIMT problems.

Let $p_k(\mathbf{x}_k)$ be the probability density function for the current state estimate. The region $X_{qos,k}$ is selected such that

$$\int_{X_{qos,k}} p_k(\mathbf{x}) d\mathbf{x} = p_{qos}. \tag{21}$$

Thus, by (21) the true target state has a probability of p_{qos} of being in $X_{qos,k}$. The above definition places no restrictions on the number of disjoint regions comprising $X_{qos,k}$ or on the inclusion of any specific point, e.g the state estimate $\hat{\mathbf{x}}_k$, in the region. Note, an infinite number of regions satisfy (21).

Given that the region $X_{qos,k}$ was selected such that $\mathbf{x}_k \in X_{qos,k}$ with probability p_{qos}, the DFTMI and DFIMT problems can be used to provide a trajectory that satisfies the QoS demand by taking $X_k = X_{qos,k}$. Let ζ^* be the value for the solution $\mathbf{u}^*_{DFTMI}$ to the DFTMI problem when taking $T = T_{qos}$. If the true state is in X_k and the QoS demand is feasible, then $\zeta^* \geq \zeta_{qos}$. Since the true target state is in X_k with probability p_{qos}, the input sequence $\mathbf{u}^*_{DFTMI}$ satisfies the QoS demand. Alternatively, let T^* be the time for the solution $\mathbf{u}^*_{DFIMT}$ to the DFIMT problem when taking $\zeta_0 = \zeta_{qos}$. If the true state is in X_k and the QoS demand is feasible, then $T^* \leq T_{qos}$ and the result of the DFIMT problem also satisfies the QoS demand. Thus, either problem formulation can be used to design a trajectory to provide the QoS.

IV. Feasible Quality of Service Demands

The previous section showed how the DFTMI and DFIMT problems can be used to derive trajectories that satisfy feasible QoS demands. In practice it may not be possible to specify an arbitrary QoS demand that is feasible. However, this section discusses the fact that any two of the three components of the QoS specification can be arbitrarily set for certain classes of active sensing problems.

A. Case 1

First, consider the case where p_{qos} and T_{qos} are given. The claim is that both variables can be arbitrarily set and there will be a value for the final parameter ζ_{qos} that yields a full, feasible QoS specification. For this case, the claim is proven by showing how to calculate ζ_{qos}.

Let $X_k = X_{qos,k}$ be defined by (21) based on the first parameter p_{qos}. Further, let $T = T_{qos}$ be the time specified for the DFTMI problem. Solve (17) to yield the information level ζ^*_{DFTMI}. Although it is possible that $\zeta^*_{DFTMI} < \zeta_k$, i.e. that information is lost, it still represents the best possible information level that can be guaranteed for a target in X_k. Since the level is feasible, the QoS demand $q_1 = \{p_{qos}, \zeta^*_{DFTMI}, t_{qos}\}$ is feasible.

B. Case 2

Next, consider the case where p_{qos} and ζ_{qos} are given. The logic of Case 1 cannot be applied directly here since it is possible that the DFIMT based on ζ_{qos} is infeasible. This could happen if $\zeta_{qos} > \zeta_k$ and more information is lost due to process noise and state dynamics than is gained by the measurement contributions. Therefore to prove this case we make the additional assumption that there always exists a trajectory such that more information can be gained through measurements than is lost due to process noise and state dynamics. Localization of static targets using range and bearing sensors is an example application that satisfies this assumption. In this example, there is no process noise, the state dynamics do not change the information matrix ($\mathbf{\Phi_k}$ is the identity matrix), and the measurements always contribute information ($\mathbf{I}_k$ is positive definite).

Assuming information can always be gained, Let $X_k = X_{qos,k}$ be defined by (21) based on the first parameter p_{qos}. Further, let $\zeta_0 = \zeta_{qos}$ be the information level specified for the DFIMT problem. Solve (20) to yield the optimal time T^*_{DFIMT}. The feasible QoS demand is then $q_2 = \{p_{qos}, \zeta_{qos}, T^*_{DFIMT}\}$ is feasible.

C. Case 3

Finally, consider the case where ζ_{qos} and T_{qos} are given. As with Case 2, it is possible that the information level ζ_{qos} cannot be reached in any amount of time. Thus another assumption is needed on the amount of information that can be extracted from the measurements.

In particular, consider a target tracking application and let $r_i = \|\mathbf{p}_i - \mathbf{p}_{tar}\|$ be the distance between the target position $\mathbf{p}_{tar}$ and the robot position $\mathbf{p}_i$. Assume that the measurement contribution to the information matrix $\mathbf{I}_k(r_i)$ is a function of the distance between the robot and target such that $|\mathbf{I}_k| \rightarrow \infty$ as $r_i \rightarrow 0$. For example, this assumption applies when a bearing sensor is used. Further, assume the robot can move with maximum speed $v_i > 0$. Also, assume the information lost due to process noise and state dynamics is bounded from above and that the value of the probability density function is greater than zero everywhere.

In order to show that ζ_{qos} and T_{qos} can be achieved as part of a QoS demand for target tracking applications, it suffices to show that there exists some region X_{qos} such that an arbitrarily large amount of information can be achieved in an arbitrarily short amount of time. For a target in that region, any other worse pair of demands could also be achieved.

Based on the assumptions on the measurement contribution to information, arbitrarily large ζ_k can be achieved by colocating the robot with the target position. Thus the region X_{qos} should be defined as the set of locations that the robot can reach. Given the time limit T_{qos} on reaching the QoS level, the region X_{qos} corresponds t the forward reachable set of the robot. The probability p_{qos} to complete the QoS demand is the solution to (21) with $X_k = X_{qos,k}$. For a target located far away from the robot location, there can still be some small finite probability, e.g. if a normal distribution is used to describe the target position estimate, that the target is in the region $X_{qos,k}$. In this case the region would not contain the target estimate (i.e. the mean of the distribution).

V. Examples

This section considers motion planning for a nonholonomic kinematic vehicle performing bearings-only geolocalization of a stationary target and tracking of an uncertain moving target. In these examples the state variable $\mathbf{x}_k$ is the target position (or position and velocity for a moving target) while the input $\mathbf{u}_k$ is the robot position which is driven by the vehicle turning rate (which serves as the actual input $\mathbf{r}_k$). For the stationary target case the state transition matrix $\mathbf{\Phi}_k$ is the identity matrix. For the case of a moving target with state $\mathbf{x} = [x, y, \dot{x}, \dot{y}]^T$ and sample time T_s, a constant velocity model is used with noisy acceleration so

$$\mathbf{x}_{k+1} = \mathbf{\Phi}_k \mathbf{x}_{k+1} + \mathbf{\Gamma}_k \mathbf{w}_k \tag{22}$$

with

$$\mathbf{\Phi}_k = \begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

and

$$\mathbf{\Gamma}_k = \begin{bmatrix} T_s^2/2 & 0 \\ 0 & T_s^2/2 \\ T_s & 0 \\ 0 & T_s \end{bmatrix} \tag{24}$$

and $\mathbf{Q}_k = 0.1$ m^2/s^4.

Robot motion is modeled as the discrete version of a simple unicycle such that:

$$\mathbf{x}^i_{r,k+1} = \mathbf{x}^i_{r,k} + \begin{bmatrix} u_1 \cdot T_s \cdot \mathrm{sinc}(\phi) \cos(\psi_k + \phi) \\ u_1 \cdot T_s \cdot \mathrm{sinc}(\phi) \sin(\psi_k + \phi) \\ u_2 \cdot T_s \end{bmatrix} \tag{25}$$

where $\mathbf{x}^i_k = [x_k, y_k, \phi_k]^T$ is the robot state, $u_1 = 20.0$ m/s is the vehicle speed which is kept constant, and u_2 is the vehicle turning rate constrained by $\omega_{max} = 0.5$ rad/sec, T_s is the sample time, $\phi = 0.5 \cdot u_2 \cdot T_s$, and $\mathrm{sinc(x)}$ is the sine cardinal function. The notation is changed slightly here such that $\mathbf{u}_k = \mathbf{x}_v$ and $\mathbf{r}_k = \mathbf{u}_k$ in the original formulation.

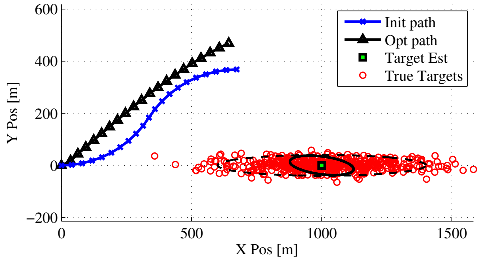

Fig. 1. Robot paths while performing bearings-only localization

This work considers bearing-only sensors. The measurement obtained by a robot at state $\mathbf{x}_k^i$ is:

$$\mathbf{z}_k^i = \beta_k^i = \arctan\left(\frac{y_k - y_{r,k}^i}{x_k - x_{r,k}^i}\right) + \mathbf{v} \tag{26}$$

where $\mathbf{v}$ is zero-mean Gaussian noise with covariance

$$E[\mathbf{v}\mathbf{v}^T] = \mathbf{R} = \frac{2\pi}{180}\text{rad}. \tag{27}$$

For a stationary target this measurement model yields the state-dependent measurement matrix

$$\mathbf{H}_k^i = \left[\begin{array}{cc} \frac{1}{r_k}\sin\beta_k^i & -\frac{1}{r_k}\cos\beta_k^i \end{array}\right] \tag{28}$$

and for the moving target

$$\mathbf{H}_k^i = \left[\begin{array}{cccc} \frac{1}{r_k}\sin\beta_k^i & -\frac{1}{r_k}\cos\beta_k^i & 0 & 0 \end{array}\right]. \tag{29}$$

The target set X_k is determined from the 2-σ level curves of the initial estimate error distribution, which is assumed to be Gaussian. Thus,

$$X_k = \left\{\mathbf{x} \in \Re^d | (\mathbf{x} - \hat{\mathbf{x}})^T \cdot \mathbf{P}_k^{-1} \cdot (\mathbf{x} - \hat{\mathbf{x}}) \leq 4\right\} \tag{30}$$

where $\mathbf{P}_k$ and the dimension d are problem-dependent.

For both examples presented below, estimation of the target state is performed using a standard Sigma Point filter [8]. Details of the Sigma Point filter are beyond this paper and are available in Ref [8].

A. Bearings-Only Target Localization

The first example considers a stationary target with initial estimate located at [1000, 0] meters with covariance matrix

$$\mathbf{P}_k = \left[\begin{array}{cc} 200^2 & 0 \\ 0 & 20^2 \end{array}\right]. \tag{31}$$

The robot starts at the origin. Two paths are compared, each comprised of twenty 2 second segments for a total duration of 40 seconds (see Fig. 1). The first is an s-shaped curve while the second is the result of the DFTMI optimization using the first as initial guess. The quality of service demand for the paths is determined by computing the maximin cost for each. The target set has probability $p_{tar} = 0.86$ of containing the true target state. Combined, the predicted quality of service for the two paths are $q1 = \{0.86, 1.9 \cdot 10^{-7}, 40\}$ and $q2 = \{0.86, 2.1 \cdot 10^{-7}, 40\}$.

To evaluate the accuracy of the quality of service prediction, simulations were run drawing 500 samples from the initial estimate error distribution to serve as the true target state (Fig. 1). The Sigma Point filter was used to derive state estimates and estimate error information (covariance) matrices. For sake of comparison, the determinant of each information matrix was scaled by the predicted value ζ_{qos}. The results from all 500 runs are plotted for each path in Fig. 2 and Fig. 3.

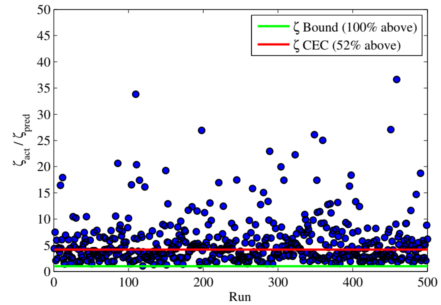

Fig. 2. Robot paths while performing bearings-only localization

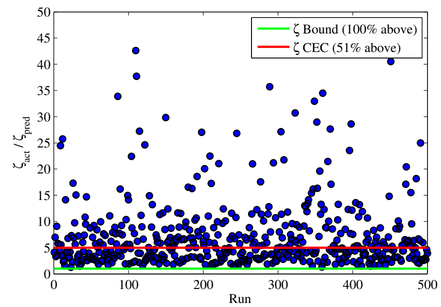

Fig. 3. Robot paths while performing bearings-only localization

For both paths, the information contained in the estimates was always greater than predicted (noted by a ratio greater than one). The main reason for this is the fact that the filter generates Sigma Points and updates its estimates based on the state estimate, not the true value. Thus, even when the true target state was furthest from the robot (corresponding to the worst case prediction) the measurement contribution to the information was based on the closer estimate.

The second (red) lines at ratios of 4.1 and 5.0, on Fig. 2 and Fig. 3, denote the predicted information value obtained when invoking the certainty equivalence principle (CEP) to calculate the information based only on the state estimate itself. The plot shows that this value would be a poor approximation since it is achieved only 52% and 51% of the time, respectively.

B. Bearings-Only Tracking

The second example considers a moving target with initial position at [1000, 0] meters with initial velocity $[10/\sqrt{(2)}, 10/\sqrt{(2)}]$ meters per second and covariance matrix

$$\mathbf{P}_k = \left[\begin{array}{cccc} 200^2 & 0 & 0 & 0 \\ 0 & 20^2 & 0 & 0 \\ 0 & 0 & 4^2 & 0 \\ 0 & 0 & 0 & 4^2 \end{array}\right]. \tag{32}$$

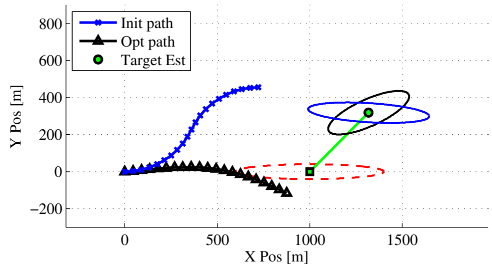

Fig. 4. Robot paths while performing bearings-only tracking

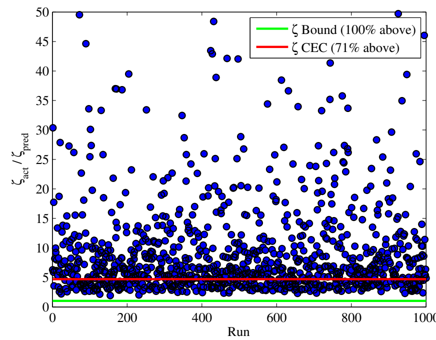

Fig. 5. Robot paths while performing bearings-only localization

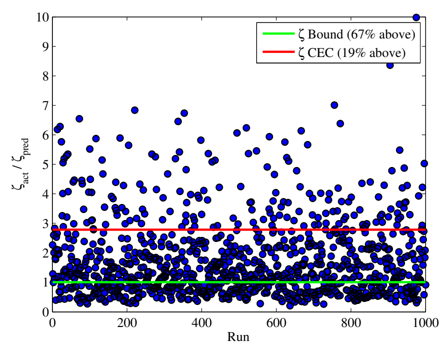

Fig. 6. Robot paths while performing bearings-only localization

The robot starts at the origin and two paths are compared, each comprised of twenty 2 second segments for a total duration of 40 seconds (see Fig. 4). Like the first example, the first path is an s-shaped curve while the second is the result of the DFTMI optimization using the first as initial guess. Here, the target set has probability $p_{tar} = 0.60$ of containing the true target state. Combined, the predicted quality of service for the two paths are $q1 = \{0.60, 5.6 \cdot 10^{-11}, 40\}$ and $q2 = \{0.60, 4.1 \cdot 10^{-11}, 40\}$.

To evaluate the accuracy of the quality of service prediction, simulations were run drawing 1000 samples from the initial estimate error distribution. The Sigma Point filter was used to derive state estimates and estimate error information (covariance) matrices. The results from all runs are plotted for both each path in Fig. 5 and Fig. 6.

For the optimized path (Fig. 5) all estimates contain more information than predicted. By comparison, for the initial path the QoS requirement is met only 67% of the time. This is still greater than the QoS demand of 60%. The red lines again correspond to the CEP solutions with ratios of 4.7 and 2.8, on Fig. 5 and Fig. 6. In this case the information gain predicted by the CEP solution is obtained 71% and 19% of the time, respectively. These results suggest that for the moving target case, where process noise decreases information, optimizing the robot trajectory is much more important than the stationary target case where information is never lost.

VI. Conclusion

This work presented a quality of service metric for active sensing tasks that provides a performance bound in terms of information collected over a given amount of time that will be satisfied with some probability. Unlike other information metrics, this quality of service demand is intended to predict performance so that higher order planning algorithms can be integrated into active sensing applications.

Derivation of the quality of service metric was based on a deterministic, bounded formulation of information gathering. In general, the three parameters in the quality of service metric cannot be specified independently. In some cases any two of the three parameters can be specified with the third being dependent on them. In the case that the probability of success and total time are given, a method for determining an achievable information level was given.

Simulation results verified the quality of service concept for geolocalization and bearings only tracking. In all cases the metric correctly bounded the success rate of the estimation process given robot paths. As expected, the quality of service demand was conservative. The main reason was the fact that prediction of the estimate error covariance did not account for the fact that the estimate itself, which is used to linearize the estimation process to determine information gain, changes over time. Unexpectedly, the metric was much more conservative for paths that were optimized for information gain.

References

[1] J. M. Passerieux and D. V. Cappel, "Optimal observer maneuver for bearings-only tracking," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, no. 3, pp. 777–788, 1998.

[2] J. Ousingsawat and M. E. Campbell, "On-line estimation and path planning for multiple vehicles in an uncertain environment," *International Journal of Robust and Nonlinear Control*, vol. 14, no. 8, pp. 741–66, May 2004.

[3] T. H. Chung, V. Gupta, J. W. Burdick, and R. M. Murray, "On a decentralized active sensing strategy using mobile sensor platforms in a network," in *43rd IEEE Conference on Decision and Control*, vol. 2. Nassau, Bahamas: IEEE, Dec 2004, pp. 1914–19.

[4] A. C. Atkinson and A. N. Donev, *Optimum Experiment Designs*. London: Clarendon Press, 1992.

[5] A. Ryan, "Information-theoretic tracking control based on particle filter estimate," in *AIAA Guidance, Navigation and Control Conference and Exhibit*, August 2008.

[6] M. L. Hernandez, "Optimal sensor trajectories in bearings-only tracking," in *Proceedings of the Seventh International Conference on Information Fusion*, Stockholm, Sweden, June 2004.

[7] J. Taylor, R.M., B. Flanagan, and J. Uber, "Computing the recursive posterior cramer-rao bound for a nonlinear nonstationary system," vol. vol.6, Hong Kong, China, 2003, pp. 673 – 6.

[8] S. Julier, J. Uhlmann, and H. F. Durrant-Whyte, "New method for the nonlinear transformation of means and covariances in filters and estimators," *IEEE Transactions on Automatic Control*, vol. 45, no. 3, pp. 477–482, 2000.