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# Performance Improvement in Adaptive Control of Nonlinear Systems

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*Abstract*— **This paper demonstrates how the finite-time identification procedure [1] can be used to improve the overall performance of adaptive control systems. First, we develop an adaptive compensator which guarantees exponential convergence of the estimation error provided the integral of a filtered regressor matrix is positive definite. The approach does not involve online checking of matrix invertibility and computation of matrix inverse nor switching between parameter estimation methods. The convergence rate of the parameter estimator is directly proportional to the adaptation gain and a measure of the system's excitation. The adaptive compensator is then combined with existing adaptive controllers to guarantee exponential stability of the closed-loop system. The effectiveness of the proposed method is illustrated with simulation examples.**

## I. Introduction

There are two major approaches to online parameter identification of nonlinear systems. The first is the identification of parameters as a part of state observer while the second deals with parameter identification as a part of controller. In the first approach, the observer is designed to provide state derivatives information and the parameters are estimated via estimation methods such as least squares method [14] and dynamic inversion [2]. The second trend of parameter identification is much more widespread, as it allows identification of systems with unstable dynamics. Algorithms in this area include parameter identification methods based on variable structure theory [17], [18] and those based on the notion of passivity [8].

In conventional adaptive control algorithms, the focus is on the tracking of a given reference trajectory and in most cases parameter estimation errors are not guaranteed to converge to zero due to a lack of excitation [5]. Parameter convergence is an important issue as it enhances the overall stability and robustness properties of the closed loop adaptive systems [9]. Moreover, there are control problems whereby the reference trajectory is not known *a priori* but depends on the unknown parameters of the system dynamics. For example, in adaptive extremum seeking control problems, the desired target is the operating setpoint that optimizes an uncertain cost function [3], [16].

Assuming the satisfaction of appropriate excitation conditions, asymptotic and exponential parameter convergence results are available for both linear and nonlinear systems. Some lower bounds which depends (nonlinearly) on the adaptation gain and the level of excitation in the system have been provided for some specific control and estimation algorithms [6], [12], [15]. However, it is not always easy to characterize the convergence rate.

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A parameter estimation scheme that allows exact reconstruction of the unknown parameters in finite-time was developed in [1]. The finite-time (FT) identification method has two distinguishing features. First, the true parameter estimate is obtained at any time instant a given excitation condition is satisfied, and second, the procedure allows for a direct and immediate removal of any perturbation signal injected in to the closed-loop system to aid in parameter estimation. However, the drawback of the identification algorithm is the requirement to check the invertibility of a matrix online and compute the inverse matrix when appropriate.

To avoid these concerns and enhance the applicability of the FT method in practical situations, the procedure is hereby exploited to develop a novel adaptive compensator that (almost) recovers the performance of the FT identifier. The compensator guarantees exponential convergence of the parameter estimation error at a rate dictated by the closed-loop system's excitation. It was shown how the adaptive compensator can be used to improve upon existing adaptive controllers. The modification proposed guarantees exponential stability of the parametric equilibrium provided the given PE condition is satisfied. Otherwise, the original system's closed-loop properties are preserved. The identification techniques are well suited for most adaptive mechanisms and do not require the availability of the velocity state vector. It is demonstrated, via simulation examples, that the identification procedures guarantee parameter convergence in situation where existing methods, driven by tracking or state prediction error, fail. Moreover, an algorithm is developed to remove auxiliary dither signals when convergence is achieved.

## II. Problem Description and Assumptions

The system considered is the following nonlinear parameter affine system

$$\dot{x} = f(x,u) + g(x,u)\theta \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state and $u \in \mathbb{R}^{n_u}$ is the control input. The vector $\theta \in \mathbb{R}^{n_\theta}$ is the unknown parameter vector whose entries may represent physically meaningful unknown model parameters or could be associated with any finite set of universal basis functions. It is assumed that $\theta$ is uniquely identifiable and lie within an initially known compact set $\Theta^0$. The $n_x$-dimensional vector $f(x,u)$ and the $(n_x \times n_\theta)$-$g(x,u)$ dimensional matrix are bounded and continuous in their arguments. System (1) encompasses the special class of linear systems,

$$f(x,u) = A_0 x + B_0 u$$
$$g(x,u) = [A_1 x + B_1 u,\ A_2 x + B_2 u,\ \dots\ A_{n_\theta} x + B_{n_\theta} u],$$

where $A_i$ and $B_i$ for $i = 0 \dots n_\theta$ are known matrices possibly time varying.

*Assumption 2.1:* The following assumptions are made about system (1).

1) The state of the system $x(.)$ is assumed to be accessible for measurement.
2) There is a known bounded control law $u = \alpha(.)$ and a bounded parameter update law $\dot{\hat{\theta}}$ that achieves a primary control objective.

The control objective can be to (robustly) stabilize the plant and/or to force the output to track a reference signal. Depending on the structure of the system (1), adaptive control design methods are available in the literature [7], [11].

## III. Overview of Finite-time Parameter Identification

For any given bounded control and parameter update law, the aim of the FT approach is to provide the true estimates of the plant parameters in finite-time while preserving the properties of the controlled closed-loop system.

Let $\hat{x}$ denote the state predictor for (1) with dynamics

$$\dot{\hat{x}} = f(x,u) + g(x,u)\hat{\theta} + k_w e + w\dot{\hat{\theta}}, \tag{2}$$

where $\hat{\theta}$ is a parameter estimate generated via any update law $\dot{\hat{\theta}}$, $k_w > 0$ is a design matrix, $e = x - \hat{x}$ is the prediction error and $w$ is the output of the filter

$$\dot{w} = g(x,u) - k_w w, \qquad w(t_0) = 0. \tag{3}$$

Denoting the parameter estimation error as $\tilde{\theta} = \theta - \hat{\theta}$, it follows from (1) and (2) that

$$\dot{e} = g(x,u)\tilde{\theta} - k_w\, e - w\dot{\hat{\theta}}. \tag{4}$$

The use of the filter matrix $w$ in the above development provides direct information about parameter estimation error $\tilde{\theta}$ without requiring a knowledge of the velocity vector $\dot{x}$. This is achieved by defining the auxiliary variable

$$\eta = e - w\tilde{\theta} \tag{5}$$

with $\eta$, in view of (3, 4), generated from

$$\dot{\eta} = -k_w\, \eta, \qquad \eta(t_0) = e(t_0). \tag{6}$$

Based on the dynamics (2), (3) and (6), the FT result is given by the following theorem [1].

*Theorem 3.1:* Let $Q \in \mathbb{R}^{n_\theta \times n_\theta}$ and $C \in \mathbb{R}^{n_\theta}$ be generated from the following dynamics:

$$\dot{Q} = w^T w, \qquad Q(t_0) = 0 \tag{7a}$$
$$\dot{C} = w^T(w\hat{\theta} + e - \eta), \qquad C(t_0) = 0 \tag{7b}$$

Suppose there exists a time $t_c$ and a constant $c_1 > 0$ such that $Q(t_c)$ is invertible *i.e.*

$$Q(t_c) = \int_{t_0}^{t_c} w^T(\tau)w(\tau)\, d\tau \succ c_1 I, \tag{8}$$

then

$$\theta = Q(t)^{-1} C(t) \qquad \text{for all} \quad t \geq t_c. \tag{9}$$

*Proof:* The result can be easily shown by noting that

$$Q(t)\,\theta = \int_{t_0}^{t} w^T(\tau)w(\tau)\Big[\hat{\theta}(\tau) + \tilde{\theta}(\tau)\Big]\, d\tau. \tag{10}$$

Using the fact that $w\tilde{\theta} = e - \eta$, it follows from (10) that

$$\theta = Q(t)^{-1}\int_{t_0}^{t} \dot{C}(\tau)\, d\tau = Q(t)^{-1} C(t) \tag{11}$$

and (11) holds for all $t \geq t_c$ since $Q(t) \succeq Q(t_c)$. ■

Let

$$\theta^c \triangleq Q(t_c)^{-1}\, C(t_c) \tag{12}$$

The finite-time (FT) identifier is given by

$$\hat{\theta}^c(t) = \begin{cases} \hat{\theta}(t), & \text{if } t < t_c \\ \theta^c, & \text{if } t \geq t_c. \end{cases} \tag{13}$$

The piecewise continuous function (13) can be approximated by a smooth approximation using the logistic functions [1]. The invertibility condition (8) is equivalent to the standard persistence of excitation (PE) condition required for parameter convergence in adaptive control. The condition (8) is satisfied if the regressor matrix $g$ is PE. To show this, consider the filter dynamic (3), from which it follows that

$$w(t) = \int_{t_0}^{t} \exp^{-k_w(t-\tau)} g(\tau) d\tau \; = \; \frac{1}{s + k_w}[g(t)]. \tag{14}$$

Since $g(t)$ is PE by assumption and the transfer function $\frac{1}{s+k_w}$ is stable, minimum phase and strictly proper, we know that $w(t)$ is PE [13]. Hence, there exists $t_c$ and a $c_1$ for which (8) is satisfied. The superiority of the above design lies in the fact that the true parameter value can be computed at any time instant $t_c$ the regressor matrix becomes positive definite and subsequently stop the parameter adaptation mechanism.

## IV. Adaptive Compensation Design

Consider the nonlinear system (1) satisfying Assumption 2.1 and the state predictor

$$\dot{\hat{x}} = f(x,u) + g(x,u)\,\theta^0 + k_w(x - \hat{x}) \tag{15}$$

where $k_w > 0$ and $\theta^0$ is the nominal initial estimate of $\theta$. If we define the auxiliary variable

$$\eta = x - \hat{x} - w(\theta - \theta^0) \tag{16}$$

and select the filter dynamic as

$$\dot{w} = g(x,u) - k_w\, w, \qquad w(t_0) = 0 \tag{17}$$

then $\eta$ is generated by

$$\dot{\eta} = -k_w \eta, \qquad \eta(t_0) = e(t_0). \tag{18}$$

Based on (15) to (18), our novel adaptive compensation result is given in the following theorem.

*Theorem 4.1:* Let $Q$ and $C$ be generated from the following dynamics:

$$\begin{aligned} \dot{Q} &= w^T w, & Q(t_0) = 0 \quad (19a)\\ \dot{C} &= w^T(w\,\theta^0 + x - \hat{x} - \eta), & C(t_0) = 0 \quad (19b) \end{aligned}$$

and let $t_c$ be the time such that $Q(t_c) \succ 0$, then the adaptation law

$$\dot{\hat{\theta}} = \Gamma\,(C - Q\,\hat{\theta}), \quad \hat{\theta}(t_0) = \theta^0 \tag{20}$$

with $\Gamma = \Gamma^T \succ 0$ guarantees that $\|\tilde{\theta}\| = \|\theta - \hat{\theta}\|$ is non-increasing for $t_0 \le t \le t_c$ and converges to zero exponentially fast, starting from $t_c$. Moreover, the convergence rate is lower bounded by $\mathcal{E}(t) = \lambda_{\min}\big(\Gamma Q(t)\big)$.

*Proof:* Consider a Lyapunov function

$$V_{\tilde{\theta}} = \frac{1}{2}\tilde{\theta}^T\tilde{\theta}, \tag{21}$$

it follows from (20) that

$$\dot{V}_{\tilde{\theta}}(t) = -\,\tilde{\theta}^T(t)\,\Gamma\left(C(t) - Q(t)\hat{\theta}(t)\right). \tag{22}$$

Since $w\theta = w\,\theta^0 + x - \hat{x} - \eta$ (from (16)), then

$$C(t) = \int_{t_0}^{t} \dot{C}(\tau)\,d\tau = \int_{t_0}^{t} w^T(\tau)w(\tau)\,d\tau\;\theta = Q(t)\,\theta \tag{23}$$

and equation (22) becomes

$$\begin{aligned} \dot{V}_{\tilde{\theta}}(t) &= -\tilde{\theta}^T(t)\,\Gamma\,Q(t)\,\tilde{\theta}(t) & (24)\\ &\le -\mathcal{E}(t)\,V_{\tilde{\theta}}(t) & (25) \end{aligned}$$

This implies non-increase of $\|\tilde{\theta}\|$ for $t \ge t_0$ and the exponential claim follows from the fact that $\Gamma\,Q(t) = \Gamma\int_{t_0}^{t} w(\tau)^T w(\tau) d\tau$ is positive definite for all $t \ge t_c$. The convergence rate is shown by noting that for all $t \ge t_c$

$$\dot{V}_{\tilde{\theta}}(t) = -\tilde{\theta}^T(t)\,\Gamma\,\left(Q(t_c) + \int_{t_c}^{t} w(\tau)^T\,w(\tau)\,d\tau\right)\tilde{\theta}(t), \tag{26}$$

$$\le -\,\tilde{\theta}^T(t)\,\Gamma\,Q(t_c)\,\tilde{\theta}(t) \le -\mathcal{E}(t_c)\,V(t) \tag{27}$$

which implies

$$\|\tilde{\theta}(t)\| \le \exp^{-\mathcal{E}(t_c)(t-t_0)}\,\|\tilde{\theta}(t_0)\|, \quad \forall t \ge t_c \tag{28}$$

■

Both the FT identification (9) and the adaptive compensator (20) use the static relationship developed between the unknown parameter $\theta$ and some measurable matrix signals $C$, *i.e*, $Q\theta = C$. However, instead of computing the parameter values at a known finite-time by inverting matrix $Q$, the adaptive compensator is driven by the estimation error $Q\tilde{\theta} = C - Q\hat{\theta}$.

## V. Incorporating Adaptive Compensator for Performance Improvement

It is assumed that the given control law $u$ and stabilizing update law (herein denoted as $\dot{\hat{\theta}}^s$) result in closed-loop error system

$$\begin{aligned} \dot{Z} &= AZ + \Phi^T\tilde{\theta}^s & (29a)\\ \dot{\hat{\theta}}^s &= -\Gamma\Phi Z & (29b) \end{aligned}$$

where the matrix $A$ is such that $A + A^T < -2\,k_A\,I < 0$, $\Phi$ is a bounded matrix function of the regressor vectors, $\tilde{\theta}^s = \theta - \hat{\theta}^s$ and $Z = [z_1, z_2, \ldots z_{n_x}]^T$ is a vector function of the tracking error with $z_1 = y - y^r$. This implies that the adaptive controller guarantees uniform boundedness of the estimation error $\tilde{\theta}^s$ and asymptotic convergence of the tracking error $Z$ dynamics. Such adaptive controllers are very common in the literature. Examples include linearized control laws [11] and controllers designed via backstepping [7], [10].

Given the stabilizing adaptation law $\dot{\hat{\theta}}^s$, we propose the following update law which is a combination of the stabilizing update law (29b) and the adaptive compensator (20)

$$\dot{\hat{\theta}} = \Gamma\,\left(\Phi Z + C - Q\,\hat{\theta}\right). \tag{30}$$

Since $C(t) = Q(t)\,\theta$, the resulting error equations becomes

$$\begin{bmatrix} \dot{Z} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} A & \Phi^T \\ -\Gamma\Phi & -\Gamma\,Q \end{bmatrix} \begin{bmatrix} Z \\ \tilde{\theta} \end{bmatrix}. \tag{31}$$

Considering the Lyapunov function $V = \frac{1}{2}\left(z^T z + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}\right)$, and differentiating along (31) we have

$$\dot{V} = \frac{1}{2}z^T(A + A^T)z - \tilde{\theta}^T Q\tilde{\theta} \le -k_A\,z^T z - \tilde{\theta}^T Q\,\tilde{\theta} \tag{32}$$

Hence $\tilde{\theta} \to 0$ exponentially for $t \ge t_c$ and the initial asymptotic convergence of $Z$ is strengthened to exponential convergence.

For feedback linearizable systems

$$\begin{aligned} \dot{x}_i &= x_{i+1} \quad 1 \le i \le n-1\\ \dot{x}_n &= f_1(x) + f_2(x)u + \theta^T\,g_n(x)\\ y &= x_1 \end{aligned}$$

the PE condition $Q(t_c) \succ 0$ translates to a priori verifiable sufficient condition on the reference setpoint. It requires the rows of the regressor vector $g_n(x)$ to be linearly independent along a desired trajectory $x^r(t)$ on any finite interval $t \in [t_1, t_2)$, $t_1 < t_2 < \infty$. This condition is less restrictive than the one given in [4] for the same class of system. This is because the linear independence requirement herein is only required over a finite interval and it can be satisfied by a non-periodic reference trajectory while the asymptotic stability result in [4] relies on a T-periodic reference setpoint. Moreover exponential, rather than asymptotic stability of the parametric equilibrium is achieved.

## VI. Dither Signal Update

Perturbation signal is usually added to the desired reference setpoint or trajectory to guarantee the convergence of system parameters to their true values. To reduce the variability of the closed-loop system, the added PE signal must be systematically removed in a way that sustains parameter convergence.

Suppose the dither signal $d(t)$ is selected as a linear combination of sinusoidal functions as detailed in [1], Section III.D. Let $a^0$ be the vector of the selected dither amplitude and let $T > 0$ be the first instant for which $d(T) = 0$, the amplitude of the excitation signal is updated as follows:

$$a(t) = \begin{cases} a^0, & t \in [0,\, T) \\ \exp^{-\gamma \bar{\mathcal{E}}\, T}\; a(j-1)T, & t \in [jT,\, (j+1)T), \\ & j \geq 1 \end{cases} \tag{33}$$

where the gain $\gamma > 0$ is a design parameter, $a(0) = a^0$ and

$$\mathcal{E}(0) = 0, \qquad \mathcal{E}(\tau) = \lambda_{\min}\big(Q(\tau)\big)$$
$$\bar{\mathcal{E}} = \max\big\{\mathcal{E}(jT),\;\; \mathcal{E}((j-1)T)\big\}.$$

It follows from (33) that the reference setpoint will be subject to PE with constant amplitude $a^0$ if $t \in [0,\, T)$. After which the trajectory of $a(t)$ will be dictated by the filtered regressor matrix $Q$. The amplitude vector $a(t)$ will start to decay exponentially when $Q(t)$ becomes positive definite. Note that parameter convergence will be achieved regardless of the value of the gain $\gamma$ selected as the only requirement for convergence is $Q(t) \succ 0$.

*Remark 6.1:* The other major approach used in traditional adaptive control is parameter estimation based design. A well designed estimation based adaptive control method achieves modularity of the controller-identifier pair. For nonlinear systems, the controller module must possess strong parametric robustness properties while the identifier module must guarantee certain boundedness properties independent of the control module. Assuming the existence of a bounded controller that is robust with respect to $(\tilde{\theta}, \dot{\hat{\theta}})$, the adaptive compensator (20) serves as a suitable identifier for modular adaptive control design.

## VII. Simulation Examples

We consider the following nonlinear system in parametric strict feedback form [10], [1]

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta_1 x_1 \\ \dot{x}_2 &= x_3 + \theta_2 x_1 \\ \dot{x}_3 &= \theta_3 x_1^3 + \theta_4 x_2 + \theta_5 x_3 + (1 + x_1^2)u \\ y &= x_1, \end{aligned} \tag{34}$$

where $\theta^T = [\theta_1,\, \ldots,\, \theta_5]$ are unknown parameters.

The simulation is first performed for the nominal system (34) and then for the system under additive disturbance:

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta_1 x_1 + [1\;\; 0]\vartheta \\ \dot{x}_2 &= x_3 + \theta_2 x_1 + [1\;\; x_1]\vartheta \\ \dot{x}_3 &= \theta_3 x_1^3 + \theta_4 x_2 + \theta_5 x_3 + (1 + x_1^2)u + [0\;\; 1]\vartheta \\ y &= x_1, \end{aligned} \tag{35}$$

where the disturbance vector is selected as $\vartheta = [0.1\sin(2\pi t/5),\; 0.1\sin(2\pi t/5) + 0.2\cos(\pi t)x_1,\; 0.2\cos(\pi t)]^T$. The actual parameter vector was $\theta = [-1,\, -2,\, 1,\, 2,\, 3\,]$ and the tracking signal remains a constant $y_r = 1$, which is sufficiently rich of order one.

Using the adaptive controller presented in [10], we modify the given stabilizing update law by adding the adaptive compensator (20) to it. The modification significantly improve upon the performance of the standard adaptation mechanism as shown in Figures 1 and 2. All the parameters converged to their values and we recover the performance of the finite-time identifier (13). Figures 3 and 4 depict the performance of the output and the input trajectories. While the transient behaviour of the output and input trajectories is slightly improved for the nominal system under the proposed adaptive compensator, a significant improvement is obtained for the system subject to additive disturbances.

## VIII. Conclusions

To enhance the applicability of the finite-time (FT) procedure [1] in practical situations, a novel adaptive compensator that (almost) recovers the performance of the FT identifier is proposed in this paper. The compensator guarantees exponential convergence of the parameter estimation error at a rate dictated by the adaptation gain and the closed-loop system's excitation. It is shown that the adaptive compensator can be used to improve upon existing adaptive controllers. The application reported in Section V is just an example, the adaptive compensator can easily be incorporated into other adaptive control algorithms.

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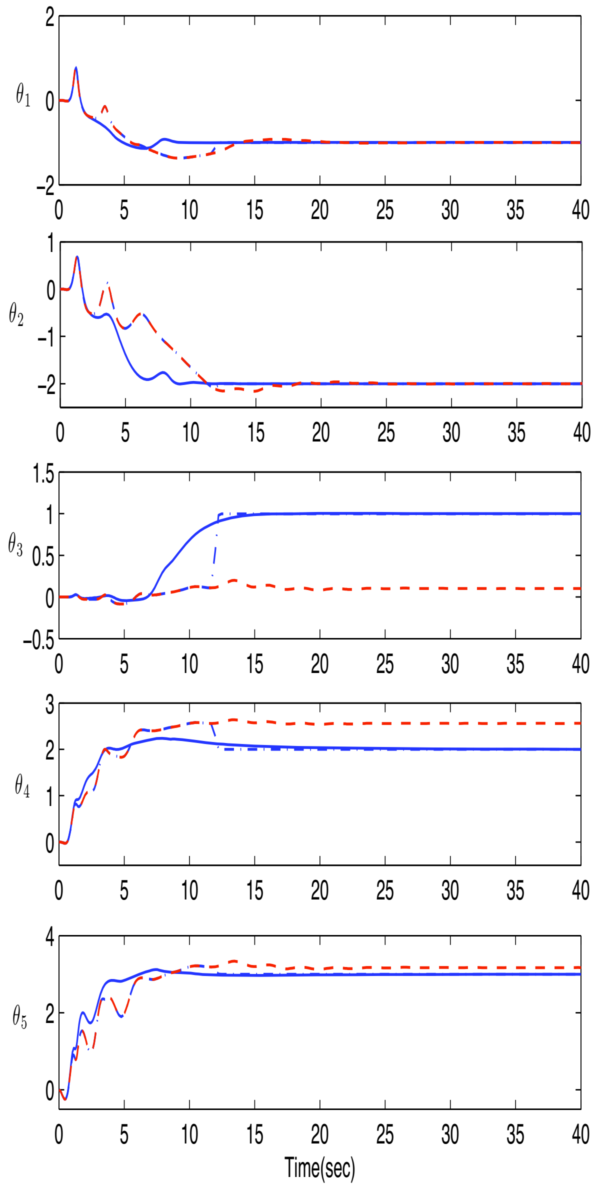


Fig. 1. Trajectories of parameter estimates. Solid(-): compensated estimates; dashdot(-.): FT estimates; dashed(- -): standard estimates [10].

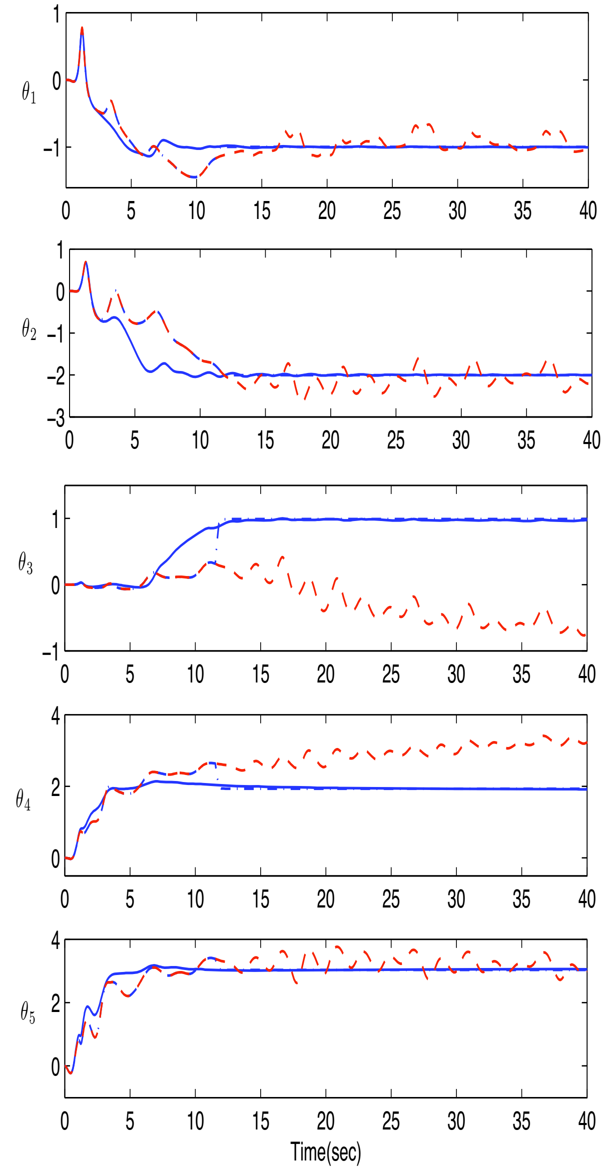


Fig. 2. Trajectories of parameter estimates under additive disturbances. Solid(-): compensated estimates; dashdot(-.): FT estimates; dashed(- -): standard estimates [10].


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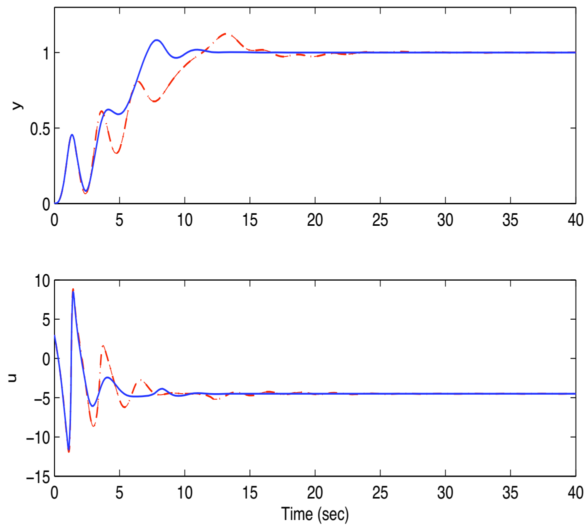


Fig. 3. Trajectories of system's output and input for different adaptation laws. Solid(-): compensated estimates; dashdot(-.): FT estimates; dashed(- -): standard estimates [10].

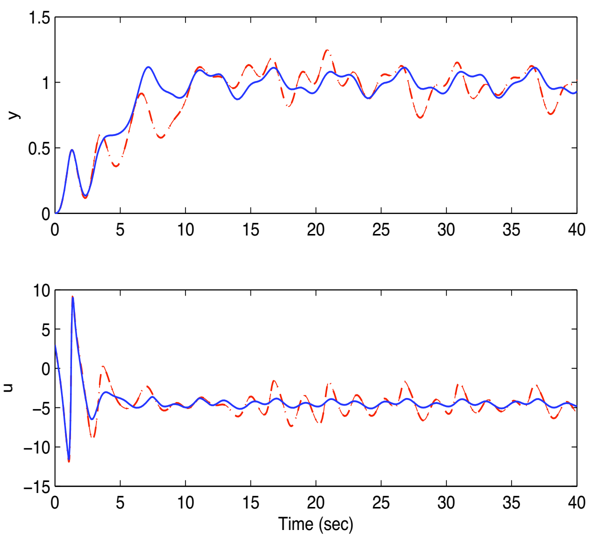


Fig. 4. Trajectories of system's output and input under additive disturbances for different adaptation laws. Solid(-): compensated estimates; dashdot(-.): FT estimates; dashed(- -): standard estimates [10].