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Nonlinearly Parameterized Adaptive PID Control for Parallel and Series Realizations

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Abstract— **In this paper, a methodology for Lyapunov-based adaptive PID control for different nonlinearly-parameterized series and parallel PID realizations is presented using simple first and second order dominant plants. The corresponding designs are based on using only the tracking error, its derivative, its integral, and the current value of the adaptive gains in order to update the PID gains. The conventional independent parallel realization, which most existing adaptive designs have used, yields a linearly parameterized adaptive control problem. Whereas, other parallel as well as series realizations yield non-linearly parameterized adaptive systems allowing for coupled adaptation of the PID gains and further design flexibility. These coupled architectures promise to yield better adaptation and learning as they reflect the inherently coupled nature of PID tuning. Case study simulations are provided to demonstrate the capabilities of the developed algorithms.**

Index Terms— **direct adaptive control; PID control.**

I. Introduction

Proportional-Integral-Derivative (PID) controllers remain the dominant algorithm in control engineering practice due to their simplicity and fundamental capability. A long standing problem with significant interest from industry is to improve the robustness of PID controllers and reduce their sensitivity to gain tuning for system uncertainty and time-variations.

Adaptive control, e g., [6], [2], [4], [8] is a mature field with many results. However, most adaptive controllers require either a detailed process model or an approximation of that model such as neural nets in order to estimate system parameters. The problem with this approach is that for many systems the complexity of a stable adaptive controller is very high, which, limits practical usability. Furthermore, strong theoretical arbitrary stability guarantees of model-based adaptive controllers are typically violated in practice due to digital effects, saturation, and unmodeled dynamics. As a result, stable adaptive designs depend in practice on careful tuning of learning rate gains and fixed feedback gains as well as robust adaptive modifications used. Therefore, there's a great need for simpler universal controllers capturing the essence of adaptive control while retaining ease of tuning for practically stable adaptive control even if they do not possess the same degree of a priori guarantees on stability for as large class of systems in theory.

Adaptive PID control is one approach to improve the robustness and autonomy of PID controllers as well as capture the essence of adaptive control theory within a simple architecture. Numerous publications in the control community have considered this problem but with very different approaches. One approach is to use a fixed PID controller and combine it with some function approximation, e.g. neural nets, based adaptive controller. However, the complexity and usability of such controllers is no better then model-based algorithms. Other approaches use some type of heuristics to adjust the PID gains such as genetic algorithms or fuzzy logic. Whereas, fundamental adaptive PID algorithms analogous to classical direct adaptive control for full state feedback [2] without using any function approximation or heuristic methods are less common. Adaptive non-heuristic PID controllers, without use of model-based compensation or their functional approximation, are developed in [5], [1], [7], [3] with applicability to different 1^{st} and 2^{nd} order systems using different designs approaches such MRAC, MIT rule, and high gain adaptive stabilization. Such results though are only developed for the standard parallel PID realization with independent linearly parameterized adaptation. In this paper, a methodology, which generalizes these results, is developed for adaptive PID with different parallel and series realizations allowing for coupled adaptation with nonlinear parametrization. This is achieved by utilizing only the feedback tracking error, its derivative, and its integral as driving signals as well as the current gain values to adjust the adaptive gains.

The contribution of the paper is that it develops novel designs for Lyapunov-based adaptive PID control in order to update the PID gains in a generally coupled manner for different parallel and series architectures. These architectures promise to yield better adaptation then existing decoupled designs as they reflect the inherently coupled nature of PID tuning.

The paper is organized as follows. Section II develops the basic design methodology for nonlinearly-parameterized adaptive PID control. The methodology is used to develop adaptive PID controllers with different parallel and series realizations in Section III . Extensions of the basic controllers via augmentation with simple model-based adaptive terms is briefly discussed in Section IV. Case Study simulations are presented in Section V. Conclusions and future work are given in Section VI.

II. Methodology

The basic problem is to design PID controllers with adaptive gains based on Lyapunov stability without use of detailed model-based compensation and parameter estimation. In particular, the designs are based on updating the PID gains directly using only the error, its derivative, its

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integral, and the current value of the adaptive gains. This motivated by the fact that fixed PID controllers are non-model based and in principal only require basic knowledge about the controlled plant but are very sensitive to tuning. However, as the problem of tuning fixed PID controllers is coupled, then a similar approach to adaptive PID is taken, which yields a nonlinearly parameterized adaptive control problem. In order to verify Lyapunov stability, minimal plant models for first and second order dominant systems are used in order to obtain the adaptive PID designs as a starting step. The range of applicability of the developed algorithms in terms of provably stabilizable classes of systems is not the focus of this paper.

Consider the following class of plants consisting of a chain of integrators:

$$a\, y^{(n)} = u \tag{1}$$

Where $y^{(n)}$ is the n^{th} derivative of the targeted output y, where n is the chosen dominant order of the system. Whereas, unknown constant parameter a is the high frequency gain. The main assumptions for the designs are given by:

Assumption 2.1: The dominant order $n \leq 2$ is a known constant.

Assumption 2.2: The signals $y, y^{(1)}$ are available.

Assumption 2.3: The sign of the scalar a is known and constant, without loss of generality $a > 0$ is assumed.

Assumption 2.4: The reference trajectory r and its first n derivatives $r^{(1)}, \ldots, r^{(n)}$ are known, bounded and, piecewise continuous.

The procedure starts with creating a filtered version of the actual tracking error to create a first order equivalent error system as in sliding mode and the robust part of adaptive controllers in [6]. Define the following error variables:

$$\begin{aligned} z &= -(d/dt + K_{pp})^{n-1} e \\ z_I &= \int z\, dt \end{aligned}$$

Where $K_{pp} > 0$ is a chosen scalar, $e = r - y$ is the tracking error for a desired reference r. Let $\dot{z} = y^{(n)} - w_{ff}$ and denote the feedforward gain regressor w_{ff} where $w_{ff} = f(y, \dot{y}, r, r^{(1)}, \ldots, r^{(n)})$ only, which are all available signals. Note that for $n = 1$, we have $w_{ff} = \dot{r}$, whereas $w_{ff} = \ddot{r} + K_{pp}\dot{e}$ for $n = 2$. Consider the following control law:

$$u = -K_{pv}\, z - K_{iv} z_I + (K_{ff} + \hat{K}_{ff}) w_{ff} + u_a \tag{2}$$

Where $K_{pv} > 0$ is a fixed proportional gain, $K_{iv} > 0$ is a fixed integral gain. Whereas, K_{ff} and $\hat{K}_{ff}$ are fixed and adaptive feedforward gains respectively. The adaptive PID control term is $u_a(\hat{K}, e, \dot{e}, \int e)$ with $\hat{K} \in \mathbb{R}^3$ being a vector of adaptive PID gains. Substituting Equation (2) into Equation (1) yields:

$$a\, \dot{z} = -K_{pv} z - K_{iv} z_I + \tilde{K}_{ff}\, w_{ff} + u_a$$

Where $\tilde{K}_{ff} = \hat{K}_{ff} - a + K_{ff}$ is the feedforward gain estimation error.

Consider the following Lyapunov function:

$$V = a z^2 + K_{iv} z_I^2 + \gamma_{ff}^{-1} \tilde{K}_{ff}^2 + \hat{K}^{'} \Gamma^{-1} \hat{K} \tag{3}$$

Where $\gamma_{ff} > 0$ is the adaptation gain for the feedforward gain $\hat{K}_{ff}$ and $\Gamma = \Gamma^{'} > 0$ is a symmetric positive definite adaptation gain matrix for the update of the adaptive PID gains $\hat{K}$, which will be enforced to be diagonal for simplicity. The above Lyapunov function is in the typical form found in adaptive control with the exception that the adaptive PID gains $\hat{K}$ rather then some gain or parameter estimation error is used.

Computing $\dot{V}$ yields:

$$\begin{aligned} \dot{V} &= -K_{pv} z^2 + z\, u_a + \tilde{K}_{ff}(z\, w_{ff} + \gamma_{ff}^{-1} \dot{\hat{K}}_{ff}) \\ &\quad + \hat{K}^{'} \Gamma^{-1} \dot{\hat{K}} \end{aligned}$$

This yields the typical choice for the feedforward adaptation, see for instance [6]:

$$\dot{\hat{K}}_{ff} = -\gamma_{ff}\, w_{ff}\, z \tag{4}$$

Therefore, we are left with:

$$\dot{V} = -2K_{pv} z^2 + 2z\, u_a + 2\hat{K}^{'} \Gamma^{-1} \dot{\hat{K}}$$

Recall that for a C^3, i.e., three times continuously differentiable function, using the mean value theorem suggests that $\exists\, \alpha \in [0, 1]$:

$$\begin{aligned} f(y + x, t) &= f(x, t) + y^{'} \nabla f(x, t) + \frac{1}{2} y^{'} \nabla^2 f(x, t) y \\ &\quad + \frac{1}{6} y^{\prime} \nabla^3 f(x + \alpha y, t) y\, y \end{aligned}$$

Using the above exact 3^{rd} order expansion for u_a yields:

$$\begin{aligned} u_a(\hat{K}, t) &= u_a(0, t) + \hat{K}^{'} \nabla u_a(0, t) + \frac{1}{2} \hat{K}^{'} \nabla^2 u_a(0, t) \hat{K} \\ &\quad + \frac{1}{6} \hat{K}^{'} \nabla^3 u_a(\alpha \hat{K}) \hat{K}\, \hat{K} \end{aligned}$$

Where $u_a(\hat{K}, e, \dot{e}, \int e)$ is denoted by $u_a(\hat{K}, t)$. Where $\nabla u_a(0, t)$ is the gradient of the adaptive PID control term u_a with respect to $\hat{K}$ and evaluated at $\hat{K} = 0$. Whereas, $\nabla^2 u_a(0, t)$ is the hessian of u_a with respect to $\hat{K}$ and evaluated at $\hat{K} = 0$. Whereas, $\nabla^3 u_a(\alpha \hat{K}, t)$ is the third derivative tensor of order 3 with argument $\alpha \hat{K}$. Note that $\nabla^3 u_a$ generally requires dealing with tensor algebra but the associated term reduces to a very simple quantity for this particular problem.

Enforcing the condition $u_a(0, t) = 0$ on the adaptive PID to be designed, which simply suggests that the adaptive control vanishes when all adaptive gains are zero, and comparing the obtained expression for u_a with $\dot{V}$ suggests the following update law for the PID gains:

$$\begin{aligned} \dot{\hat{K}} &= -\Gamma \left(\nabla u_a(0, t) + \frac{1}{2} \nabla^2 u_a(0, t) \hat{K} \right) z \\ &\quad - \Gamma \left(\frac{1}{6} \nabla^3 u_a(\alpha \hat{K}, t) \hat{K} \hat{K} \right) z \end{aligned} \tag{5}$$

Substituting Equation (5) into the expression for $\dot{V}$ yields:

$$\dot{V} \;=\; -2K_{pv}z^2 \leq 0$$

Which proves Lyapunov stability of the system. This yields the following closed loop error dynamics, in addition to Equation (5):

$$\begin{aligned} a\,\dot{z} &= -K_{pv}z - K_{iv}z_I + \tilde{K}_{ff}\,w_{ff} + u_a(\hat{K},t) \\ \dot{\tilde{K}}_{ff} &= -\,\gamma_{ff}\,w_{ff}\,z \end{aligned} \tag{6}$$

A formal statement is made next:

Theorem 1: Under assumptions (2.1-2.4) for plant given by Equation (1) and controller given by Equations (2), (5), and (4) $\forall\ \gamma_{ff} > 0$, $\Gamma = \Gamma^{'} > 0$ and $K_{pv}, K_{iv} > 0$ and an adaptive PID feedback $u_a(\hat{K}, e, \dot{e}, \int e)$ the closed loop system given by Equations (6) and (5) is Globally Lyapunov Stable and $e \to 0$ asymptotically.

Proof:
Using the Lyapunov function V given by Equation (3) and system equations (6) and (5) then global Lyapunov stability is shown by computing $\dot{V} = -2K_{pv}z^2 \leq 0$, see above analysis. Furthermore, by applying typical Barbalat Lemma arguments concludes uniform continuity of $\dot{V}$ since $V \geq 0$ and $\ddot{V}$ is bounded, therefore $z \to 0$ asymptotically and thus $e \to 0$ asymptotically. ■

Note that adaptation may be turned off and stability of the system is clearly maintained. The above statement does not specify the adaptive PID control law $u_a(\hat{K}, e, \dot{e}, \int e)$. Next, adaptive PID controllers will be designed based on Equations (2)-(5) for different PID realizations.

A. Remarks

- The role played by the fixed PID is different here from that in model-based adaptive control, where the fixed part is assumed to be designed for the ideal response and model based adaptive terms need to cancel the apparent dynamics in order to realize this ideal closed loop response. Note, however, that this is why a conservative or too aggressive choice for the fixed part does significantly affect the performance of adaptive controllers in practice. In here, these feedback gains are not assumed to be well designed and are expected to be better tuned by the adaptive PID.
- Note that the assumption that both y and $\dot{y}$ are available is a prerequisite to PID control, even if only y is measured as $\dot{y}$ is usually obtained through some type of filtered differentiation in practice.
- Note that the constant α that appears in the update law given by Equation (5) from the application of the mean value theorem is unknown. However, since the control law u_a is a polynomial of at most order three in the three PID gains, then $\nabla^3 u_a$ is independent of $\hat{K}$ and thus, α will not be needed.
- Note that the coupled adaptation laws, as will be seen in next sections, are easily implementable as they are simply a set of coupled differential equations as opposed to a set of decoupled differential equations as commonly found in adaptive control.
- Note that the adaptation law of Equation (5) can be used with not only linear PID controllers but also nonlinear PID controllers, with absolute values, polynomials and exponential functions of the error, its derivative, or its integral.
- The developed control can be combined with MRAC for full state feedback control and almost strictly positive real plants, by observing the similarity in the underlying problem structure in these cases, this will not be shown here for space limitations.

III. Parallel and Series PID Designs

PID controllers can be represented in different parallel and series realizations. Due to space limitations, the design for only 3 realizations will shown and contrasted next.

A. Design for Standard Parallel PID

The standard parallel realization yields a traditional linearly parameterized adaptive control problem where each gain is multiplied by it's "regressor" which are the error, its derivative, and its integral. Most, if not all, existing adaptive PID designs fall under this form such as [1], [7], [5], [3] with applicability to different 1^{st} and 2^{nd} order systems using different designs approaches such MRAC, MIT rule, and high gain adaptive stabilization. The design presented in this section may be viewed as a generalization of the ones in [1], [7], [5], [3] though derived differently.

The basic adaptive controller using the conventional parallel PID design is given by:

$$u_a \;=\; \hat{K}_P\,e + \hat{K}_D\,\dot{e} + \hat{K}_I \int e\,dt \tag{7}$$

Where $\hat{K}_P$ is the adaptive proportional gain, $\hat{K}_D$, is the adaptive derivative gain, $\hat{K}_I$ is the adaptive integral gain.

Using Equations (7) and (5) the adaptive gains are updated using the following adaptation laws:

$$\dot{\hat{K}}_P \;=\; -\gamma_P\,e\,z \tag{8}$$

$$\dot{\hat{K}}_D \;=\; -\gamma_D\,\dot{e}\,z \tag{9}$$

$$\dot{\hat{K}}_I \;=\; -\gamma_I \int e\,dt\,z \tag{10}$$

Where $\gamma_P, \gamma_D, \gamma_I > 0$ are the adaptation gains for proportional, derivative, and integral gains, respectively such that $\Gamma = diag(\gamma_P, \gamma_D, \gamma_I)$. Therefore, the overall design is given by Equations (2), (4), (7) and, (8)-(10).

B. Design for Parallel PID with Overall Integral Gain

The adaptive controller using the parallel design with overall integral gain is given by:

$$u_a \;=\; \hat{K}_I\left(\int e\,dt + \hat{K}_{Pi}\,e + \hat{K}_{Di}\,\dot{e}\right) \tag{11}$$

Where $\hat{K}_I$ is the adaptive integral gain, $\hat{K}_{Di}$, is the adaptive derivative gain scaled by the integral gain, $\hat{K}_{Pi}$ is the adaptive proportional gain scaled by the integral gain.

Using Equations (11) and (5) the adaptive gains are updated using the following adaptation laws:

$$\dot{\hat{K}}_{Pi} = -\gamma_{Pi}\,\frac{\hat{K}_I}{2}\,e\,z \qquad (12)$$

$$\dot{\hat{K}}_{Di} = -\gamma_{Di}\,\frac{\hat{K}_I}{2}\,\dot{e}\,z \qquad (13)$$

$$\dot{\hat{K}}_{I} = -\gamma_I\left(\int e\,dt + \frac{\hat{K}_{Pi}}{2}\,e + \frac{\hat{K}_{Di}}{2}\,\dot{e}\right)z \qquad (14)$$

Where $\gamma_{Pi}, \gamma_{Di}, \gamma_I > 0$ are the adaptation gains for scaled proportional, scaled derivative, and integral gains, respectively such that $\Gamma = diag(\gamma_{Pi}, \gamma_{Di}, \gamma_I)$. Therefore, the overall design is given by Equations (2), (4), (11) and, (12)-(14).

C. Design for Standard Series PID

The adaptive controller using the conventional series design is given by:

$$\begin{aligned} u_a &= \hat{K}_{iv}\left(\int \dot{e}\,dt + \hat{K}_{pp}\int e\,dt\right) \\ &\quad + \hat{K}_{pv}\left(\dot{e} + \hat{K}_{pp}e\right) \end{aligned} \qquad (15)$$

Where $\hat{K}_{pp}$ is the adaptive outer proportional loop gain, $\hat{K}_{pv}$ is the adaptive inner proportional loop gain, $\hat{K}_{iv}$ is the adaptive integral loop gain.

Using Equations (15) and (5) the adaptive gains are updated using the following adaptation laws:

$$\dot{\hat{K}}_{pp} = -\gamma_{pp}\left(\frac{\hat{K}_{pv}}{2}\,e + \frac{\hat{K}_{iv}}{2}\int e\,dt\right)z \qquad (16)$$

$$\dot{\hat{K}}_{pv} = -\gamma_{pv}\left(\dot{e} + \frac{\hat{K}_{pp}}{2}\,e\right)z \qquad (17)$$

$$\dot{\hat{K}}_{iv} = -\gamma_{iv}\left(\int \dot{e}\,dt + \frac{\hat{K}_{pp}}{2}\int e\,dt\right)z \qquad (18)$$

Where $\gamma_{pp}, \gamma_{pv}, \gamma_{iv} > 0$ are the adaptation gains for outer proportional, inner proportional, and integral gains, respectively such that $\Gamma = diag(\gamma_{pp}, \gamma_{pv}, \gamma_{iv})$. Therefore, the overall design is given by Equations (2), (4), (15) and, (16)-(18).

Other possible realizations include parallel PID with overall proportional or derivative gains as well as series PID with overall derivative or integral gains.

IV. Augmentation with Model-based Designs

The above design may be simply augmented with well known model-based designs, which could be useful if some important non-linear terms are structurally known and need to be canceled. Consider systems in the SISO companion form given below:

$$a\,y^{(n)} = -W(y, y^{(n-1)})'\,\theta + \beta(y, y^{(n-1)})u \qquad (19)$$

The systems given by Equation (19) are a generalization of those in Equation (1) where a and y are as defined in Section II, which are simply first and second order nonlinear systems . Whereas, vector $\theta = [\theta_1, \theta_2, \ldots, \theta_m]'$ is a vector of plant parameters. An additional assumption is made for these systems:

Assumption 4.1: $W(y, y^{(n-1)}) = [w_1, w_2, \ldots, w_m]$ with $w_i(y, y^{(n-1)})\ \forall i = 1\ldots n$ and $\beta(y, y^{(n-1)})$ are known smooth functions in $\mathbb{R}$ and $\beta(y, y^{(n-1)}) \neq 0\ \forall y, y^{(n-1)}$.

Using standard model based design procedures based on that in [6] the control in Equation (2) is updated to:

$$\begin{aligned} u &= \frac{u_d}{\beta(x)} \\ u_d &= -K_{pv}\,z - K_{iv}z_I + (K_{ff} + \hat{K}_{ff})w_{ff} + u_a \\ &\quad + W'\hat{\theta} \\ \dot{\hat{\theta}} &= -\Gamma_\theta W(y, \dot{y})'\,z \end{aligned} \qquad (20)$$

Where the matrix $\Gamma_\theta = \Gamma_\theta' > 0$ is the adaptation gain matrix for parameter vector θ. The above result simply states that we can add model based adaptive terms $W(y, \dot{y})'\hat{\theta}$ to the non-model based control law of Equation (2) with u_a and the PID adaptation is given by any of the designs in Section III or IV. This yields the following error dynamics, in addition to Equation (5):

$$\begin{aligned} a\,\dot{z} &= -K_{pv}z - K_{iv}z_I + \tilde{K}_{ff}\,w_{ff} + u_a(\hat{K}, t) \\ &\quad + W'\tilde{\theta} \\ \dot{\hat{K}}_{ff} &= -\gamma_{ff}\,w_{ff}\,z \\ \dot{\hat{\theta}} &= -\Gamma_\theta W(y, \dot{y})'\,z \end{aligned} \qquad (21)$$

Where $\tilde{\theta} = \hat{\theta} - \theta$. The Lyapunov function , Equation (3) of Section II is updated by using the following Lyapunov function:

$$V_c = az^2 + K_{iv}z_I^2 + \gamma_{ff}^{-1}\tilde{K}_{ff}^2 + \hat{K}'\Gamma^{-1}\hat{K} + \tilde{\theta}'\Gamma_\theta^{-1}\tilde{\theta} \qquad (22)$$

For completeness, the formal result is stated below.

Theorem 2: Under assumptions (2.1-2.4) and assumption 5.1 for plant given by Equation (19) and controller given by Equations (20), (5), and (4) $\forall\ \gamma_{ff} > 0$, $\Gamma = \Gamma' > 0$, $\Gamma_\theta = \Gamma_\theta' > 0$ and $K_{pv}, K_{iv} > 0$ and an adaptive PID feedback $u_a(\hat{K}, e, \dot{e}, \int e)$ the closed loop system given by Equations (21), and (5) is Globally Lyapunov Stable and $e \to 0$ asymptotically .

Proof:

Using the Lyapunov function V_c given by Equation (22) and system equations (21), and (5), then global Lyapunov stability is shown by computing $\dot{V}_c = -2K_{pv}z^2 \leq 0$. Furthermore, by applying typical Barbalat Lemma arguments concludes uniform continuity of $\dot{V}_c$ since $V_c \geq 0$ and $\ddot{V}_c$ is bounded, therefore $z \to 0$ asymptotically and thus $e \to 0$ asymptotically. ■

Similarly, the result may be extended to more general classes of systems as long as the model-based control terms do not significantly increase the complexity and practicality of the controller.

V. Simulations

In this section a case study simulation will be used to demonstrate the developed methodology. Consider the following plant transfer function:

$$\frac{y(s)}{u(s)} = \frac{8e-5\left(\left(\frac{s}{2\pi 60}\right)^2+\frac{0.12s}{2\pi 60}+1\right)}{s^2\left(\left(\frac{s}{2\pi 100}\right)^2+\frac{0.2s}{2\pi 100}+1\right)\left(\frac{s}{2\pi 8000}+1\right)}$$

The plant used for simulations is of order 5 and relative degree 3. However, the system is a 2^{nd} order dominant system and thus the designs of Section II with $n = 2$ will be used with the plant treated as a double integrator. The chosen designs of Section III are compared . In these simulations, the fixed part of the adaptive PID controller of Equation (2) uses nonoptimally tuned gain values $K_{ff} = 3e-5$, $K_{pp} = 25$, $K_{pv} = 0.004$, and $K_{iv} = 0.4$ in order to evaluate the ability of the adaptive algorithms to automatically optimize the PID controller's tuning. Therefore, it is expected that the performance will be improved over the nonadaptive controller but the main focus is on the behavior of different adaptive PID realizations.

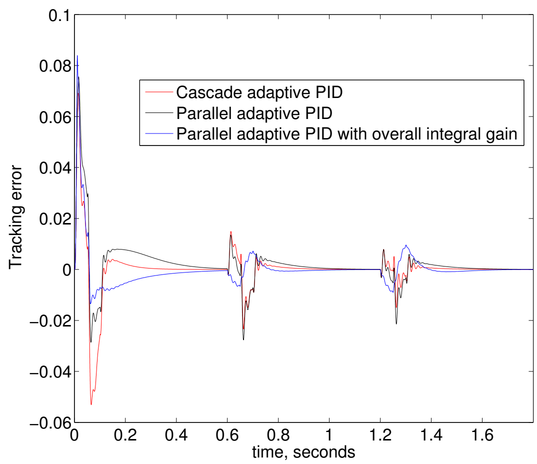

Fig. 1. Tracking error for three different adaptive PID realizations ; overall response.

Figure 1 shows the tracking error with the system commanded to follow three filtered step commands with 0.5 seconds between them. The three adaptive controllers demonstrate different transient behavior. In the simulations shown the same equivalent relative adaptation gains are used for all adaptive controllers given by Equations (8)- (10), Equations (12)-(14), and Equations (16)-(18). This means γ_I and γ_{ff} are the same, $\gamma_{Pi} = \gamma_P/\gamma_I$, and $\gamma_{Di} = \gamma_D/\gamma_I$, which reflects the difference between the two realizations with the integral gain being an overall gain in the nonlinearly parameterized design of Equations (12)-(14). Similarly, the same linear-equivalent adaptation gains are used for the cascade adaptive controller. Figure 2 shows that, for the chosen adaptation gains, the cascade design displays much better settling and transient response then the other methods. For reference, the response of this adaptive controller is compared to that of the fixed PID used, i.e., the same fixed part of the adaptive controllers with adaptation gain matrix $\Gamma = 0$. The performance improvement is evident. Note that

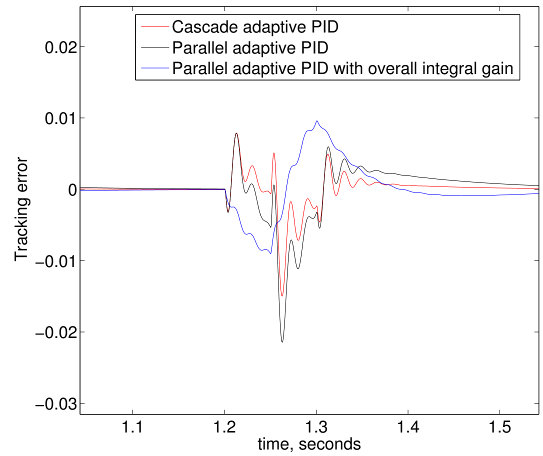

Fig. 2. Tracking error for three different adaptive PID realizations ; end of simulation.

the fixed part is intentionally detuned to compare the learning capability of the adaptive PID controllers.

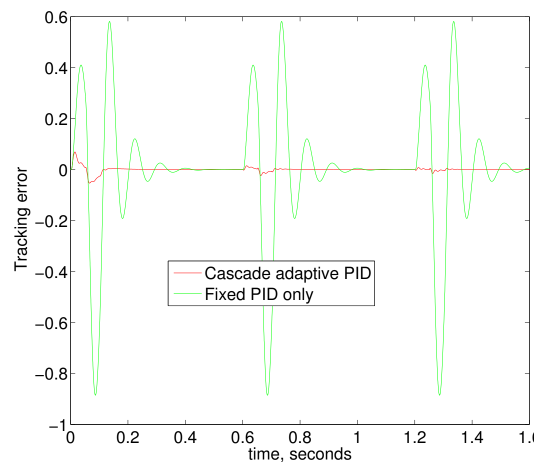

Fig. 3. Tracking error for an adaptive cascade PID and the nonadaptive PID part.

The behavior of the control signal and adaptive gains for the three adaptive methods is shown in Figure 4, 5, and 6. It is noteworthy that the control signal is larger with the overall integral gain method, during the first command. Note that if all gains are converted to the same equivalent representation, e.g. parallel K_D, K_I, K_P the values they converge too are different with different methods, which explains the difference in tracking response shape in Figure 1. For example, comparing the parallel and cascade adaptive PID controllers at the end of the simulation shown, we get final proportional gain of 4 versus 8.8, and derivative gains of 0.075 versus 0.08, and integral gain of -12 versus 85.

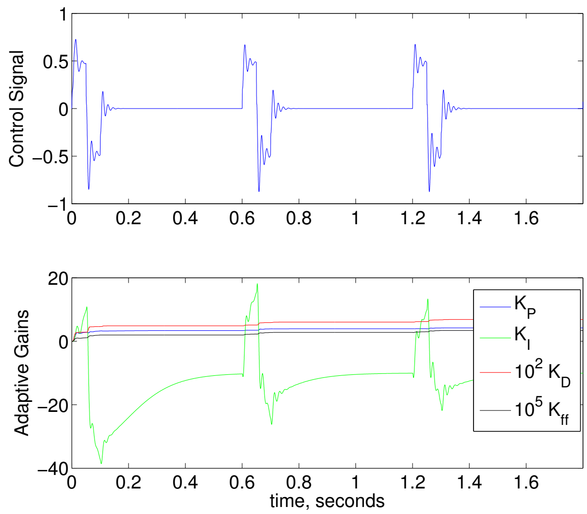

Fig. 4. Adaptive Standard Parallel PID : (a) Control signal, (b) Adaptive gains.

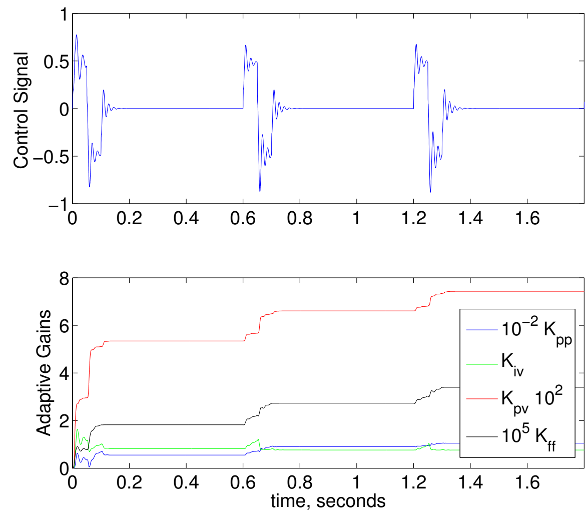

Fig. 6. Adaptive cascade PID: (a) Control signal, (b) Adaptive gains.

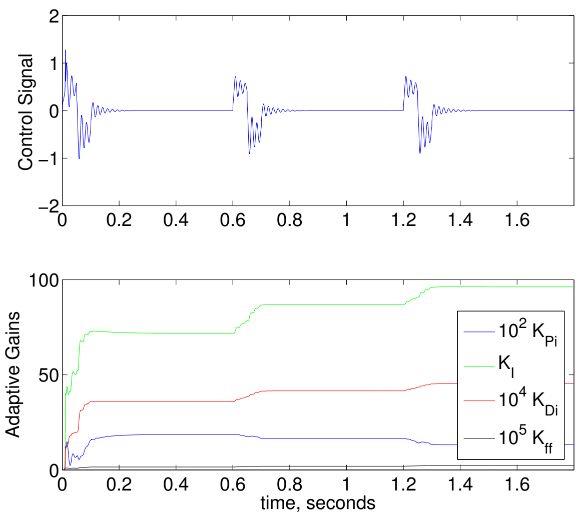

Fig. 5. Adaptive Parallel PID with overall Integral gain: (a) Control signal, (b) Adaptive gains.

VI. Conclusions

A methodology for Lyapunov-based adaptive PID control for different nonlinearly-parameterized series and parallel PID realizations has been presented. Lyapunov stability and asymptotic tracking are proven using the exact third order expansion of the control law using the mean value theorem. The corresponding developed designs are based on using only the tracking error, its derivative, its integral, and the current value of the adaptive gains in order to update the PID gains. The developed nonlinearly parameterized adaptive PID controllers allow for coupled adaptation of the PID gains and further design flexibility as verified by case study simulations. Future work will focus on further analysis of when advantages of different realizations as well as extending the algorithms' capabilities including utilizing robust adaptive modifications and formal robustness analysis for more general classes of plants.

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