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Further Tracking Results for Input-constrained Minimum-Phase Systems

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Abstract. **Minimum phase systems are the only systems that allow (in disturbance-free unconstrained case) the achievement of perfect tracking in presence of arbitrary reference signals. The question is whether this still holds when the control input is subject to input saturation constraint. Surprisingly, most works on global output tracking in presence of input saturation focused on nonminimum phase systems. Then, perfect tracking is only achievable for constant references. In this paper, it is shown that more powerful tracking results are achievable for minimum phase systems. Specifically, perfect tracking is guaranteed for arbitrary type reference signals that satisfy a well defined strict compatibility condition. When, the reference signal is just compatible in the mean then the tracking quality depends on the reference variation rate. For periodic reference signals (not necessarily compatible with constraint), all the closed-loop system signals are shown to be periodic with the same period.**

Keywords*:* Minimum phase systems, input saturation constraint, output reference tracking, input-output stability, incremental stability**.**

I. INTRODUCTION

In this paper, the focus is made on global output reference tracking for stable linear systems in presence of input constraint. It is well known that such an issue is closely related to the system phase nature. In the unconstrained case, global tracking of arbitrary type reference signals is achievable only for minimum phase systems. The question is whether this still holds in the case of input saturation constraint. Surprisingly, early relevant results concerned nonminimum phase systems, e.g. ([2], [3]). It was shown, using saturated versions of (adaptive) pole placement regulators, that perfect global tracking is only possible for constant reference signals that are strictly compatible with the input limitation. The problem of perfect global tracking of not necessarily constant references was dealt with in [8] considering minimum-phase stable systems controlled by saturated (adaptive) model reference regulator. It was shown that the tracking error converges globally to zero whatever the nature of the reference signal provided this is strictly compatible with the constraint. While such result constitutes a theoretical progress, its practical applicability is limited. In practical applications, the following tracking issues are important: (i) Does the regulator still show a tracking capability when facing reference signals that are only compatible in the mean (but not strictly)? (ii) How behaves the regulator in presence of reference signals with no compatibility feature? These issues are addressed in the present paper considering input-constrained *minimum-phase* systems controlled by saturated model reference regulators. In addition to perfect tracking of strictly compatible reference signals, it is shown that average tracking performances are ensured when the reference signal is just compatible in the *mean*. Then, the tracking error is *proportional* in the mean to the *mean rate* of the reference sequence. Furthermore, in the case of just periodic reference signals (not necessarily compatible), all closed loop signals are in steady-state periodic and oscillate with the same frequency as the reference.

The paper is organized as follows: Section 2 is devoted to formulating the control problem and designing the regulator; key technical lemmas are presented in Section 3 and used in Section 4 to establish the regulator tracking performances; a conclusion and reference list end the paper.

II. CONTROL PROBLEM FORMULATION AND REGULATOR DESIGN

We are considering discrete-time SISO linear systems [1]:

$$A(q^{-1})\, y(t) = q^{-d} B(q^{-1})\, u(t) \qquad (t \in IN) \tag{1a}$$

in presence of the input constraint:

$$|u(t)| \le u_M \tag{1b}$$

with:

$$A(q^{-1}) = 1 + a_1 q^{-1} + .. + a_{na} q^{-na} \tag{2a}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + .. + b_{nb} q^{-nb} \qquad (b_0 \neq 0) \tag{2b}$$

where u(t) and $y(t)$ are the system input and output (respectively); u_M denotes the maximal allowed control value; q^{-1} is the backward-shift operator; (na, nb, d) are integers and (a_i, b_i) are real numbers. The polynomials $z^{na} A(z^{-1})$ and $z^{nb} B(z^{-1})$ are Hurwitz i.e. the system is BIBO stable and minimum-phase. The stability assumption is required to make the system controllable in presence of the control limitation (1b). The minimum phase requirement is necessary because we seek perfect global tracking in presence of arbitrary reference signals $\{y^*(t)\}$ that are compatible with the constraint (1b) (in a sense made precise later).

Since $A(q^{-1})$ and q^{-d} are coprime, there exist unique polynomials of the form:

$$R(q^{-1}) = 1 + r_1 q^{-1} + r_2 q^{-2} + \ ... \ + r_{d-1} q^{-d+1} \tag{3a}$$

[1] Throughout the paper, *IN* denotes the set of integer numbers and *IR* the set of real numbers.

$$S(q^{-1}) = s_0 + s_1 q^{-1} + s_2 q^{-2} + \ldots + s_{n-1} q^{-n+1} \quad (3b)$$

such that:

$$A(q^{-1})R(q^{-1}) + q^{-d}S(q^{-1}) = C(q^{-1})\Lambda(q^{-1}) \quad (4)$$

where $n = \max(na, nb)$ and $C(q^{-1})$ is any Hurwitz polynomial of the form:

$$C(q^{-1}) = 1 + c_1 q^{-1} + c_2 q^{-2} + \ldots + c_n q^{-n}$$

$$\Lambda(q^{-1}) = 1 + \lambda_1 q^{-1} + \lambda_2 q^{-2} + \ldots + \lambda_{d-1} q^{-d+1}$$

The saturated model-reference regulator we are proposing consists in generating an auxiliary control signal $\{v(t)\}$ according to the following control law:

$$v(t) = \frac{\Lambda(q^{-1}) - R(q^{-1})}{\Lambda(q^{-1})} u(t) - \frac{S(q^{-1})}{\Lambda(q^{-1})B(q^{-1})} y(t) + \frac{C(q^{-1})}{B(q^{-1})} y^*(t+d) \quad (5)$$

and letting the control action $u(t)$ be:

$$u(t) = sat(v(t)) \overset{def}{=} sign(v(t)) min\{u_M, |v(t)|\} \quad (6)$$

Remarks 2.1.

1) The saturated regulator (5a-b) coincides with the standard linear model-reference regulator:

$$B(q^{-1})\, R(q^{-1})\, u(t) + S(q^{-1}) y(t) = C(q^{-1})\Lambda(q^{-1}) y^*(t+d)$$

(7) (6)

whenever the control signal stops saturating for a long time. Then, one gets $C(q^{-1})\Lambda(q^{-1})e(t+d) = 0$ with $e(t) \overset{def}{=} y(t) - y^*(t)$ is the tracking error. The error $e(t)$ then vanishes exponentially fast because the polynomial $C(q^{-1})\Lambda(q^{-1})$ is Hurwitz. The latter determines the regulation dynamics of the closed-loop system.

2) The reference signal $y^*(t)$ may be the output of a model reference system, e.g. $y^*(t) = (1/A_m(q^{-1}))u_m(t)$ where $u_m(t)$ denotes the ideal output reference and $A_m(q^{-1})$ is a Hurwitz polynomial such that $A_m(1) = 1$. The transfer function $1/A_m(q^{-1})$ is then referred to tracking dynamics of the closed-loop system. The point is that the regulation dynamics (defined by $C(q^{-1})\Lambda(q^{-1})$) and the tracking dynamics are presently independently chosen.

3) The above regulator design differs from the state-space design proposed in [8]. In the latter the tracking dynamics are identical to the regulation dynamics ($A_m(q^{-1}) = C(q^{-1})\Lambda(q^{-1})$).

III. Technical Tools and Preliminary results

In this section, we recall a number of technical tools borrowed from the theory of input-output stability, e.g. (Vidyasagar, 2002). In particular, Lemma 3.1 will play a central role in the analysis of the next section.

A. Preliminary notions

Throughout, Ω denotes the linear space of all causal real sequences (i.e. $s : IN \to IR$). The l_p norm of $s \in \Omega$ is denoted $\|s\|_p$ ($1 \le p < \infty$). For any integer $T > 0$, s_T designates the truncated sequence i.e. $s_T(t) = s(t)$ for $0 \le t \le T$ and $s_T(t) = 0$ *for* $t > T$.

Definition 3.1 (*Sectoricity*). *A dynamic nonlinear map* ϕ *:* $IN \times IR \to IR$ *belongs to the sector* $[a,b]$ *(with* $a < b$ *) if:*

$$az^2 \le z\phi(t,z) \le bz^2, \quad \text{for all } (t,z) \in IN \times IR \quad (8)$$

The set of such functions is denoted $S[a,b]$ □

Definition 3.2 *(stability). Let* $H : \Omega \to \Omega$ *be any dynamic nonlinear operator.*

1) H *is* l_p*-stable (for some* $1 \le p \le \infty$*) if there exists a real* γ *such that:* $\|Hu\|_p \le \gamma \|u\|_p$*, for all* $u \in l_p$*. The smallest* γ *is called* l_p*-gain of* H *and denoted* $\gamma_p(H)$.

2) H *is* l_p*-incrementally stable if it is* l_p*-stable and there is a real* $\tilde{\gamma}$ *such that* $\|Hu_1 - Hu_2\|_p \le \tilde{\gamma}\|u_1 - u_2\|_p$*, whatever* $u_1, u_2 \in l_p$*. The smallest* $\tilde{\gamma}$ *is called incremental* l_p*-gain and is denoted* $\tilde{\gamma}_p(H)$ □

The next definitions make precise the sense of reference signal compatibility and sequence smallness in the mean.

Definition 3.3 (Giri et al, 1988). *Let* α *be any real number and* $s \in \Omega$ *any real sequence.* s *is said to be* α*-small in the mean (briefly* α*-SM), if:* $\limsup_{k\to\infty} \frac{1}{k} \sum_{t=h+1}^{h+k} s(t) \le \alpha$ *(for all* $h \in IN$*). For a given* α*, the set of all* α*-SM sequences is denoted* $SM(\alpha)$*. The mean size of a bounded sequence* s *is the smallest real* α *such that* $|s| \in SM(\alpha)$. □

Remark 3.1. In the above definition and throughout, the notations $s \in SM(\alpha)$ and $\{s(t)\} \in SM(\alpha)$ are indifferently used.

Definition 3.4. *Consider the system (2.1a) and a bounded sequence* $y^* \in \Omega$*. Let* $u^* \in \Omega$ *be any signal obtained from* y^* *solving the following difference equation:*

$$B(q^{-1})u^*(t) = A(q^{-1})y^*(t+d), \quad t \in IN \quad (9)$$

with any initial conditions $u^*(-i) \in [-u_M, u_M]$*,* $i = 1, \ldots, nb$.

1) y^* *is said to be strictly compatible with the constraint if, there is a real* $0 < \delta < 1$*, such that* $\left|u^*(t)\right| \le (1-\delta)u_M$ *(for all sufficiently large t).*

2) A not strictly-compatible sequence y^* *is characterized by its MCD denoted* $1/\mu$ *where* μ *is the smallest real* μ *such that* $\left|u^* - sat(u^*)\right| \in SM(\mu)$ □

Remark 3.2. The signal u^* generated by (9) is referred to control signal induced by the reference y^*. If y^* is strictly-compatible, the resulting u^* stops pressing on the allowed control boundaries $\pm u_M$ after a finite transient period. A not strictly-compatible reference y^* leads to an input reference that not only presses, infinitely often, on the allowed control boundaries but goes beyond them. □

B. Technical Lemmas

Lemma 3.1 *(Properties of the saturation function).*
The function $sat(.)$ *has the following properties:*

1) $\left|v - sat(v)\right| = |v| - \left|sat(v)\right|$, *(for all* v*)*

2) $v - sat(v) \ne 0 \;\Rightarrow\; sat(v) = u_M sign(v)$, *(for all* v*)*

3) For all real numbers v_1, v_2*, there exists* $\xi \in [0\,1]$ *such that:*

$$sat(v_1) - sat(v_2) = \xi(v_1 - v_2)$$
(10)

Proof. See e.g. (Chaoui et al, 1998, 2001).

Lemma 3.2. *Let* $s, s_1, s_2 \in \Omega$, $p \in \{1,2\}$ *and* $\mu, \mu_1, \mu_2, k_1, k_2$ *be any real numbers.*

1) If $\left|s_1\right| \in SM(\mu_1)$ *and* $\left|s_2\right| \in SM(\mu_2)$ *then:*

$$\left|k_1 s_1 + k_2 s_2\right|^p \in SM(p\left|k_1^p\right|\mu_1^p + p\left|k_2^p\right|\mu_1^p).$$

2) If H *is an* l_p*-stable operator then, for all* $s \in \Omega$ *:*

$$|s|^p \in S(\mu) \;\Rightarrow\; |Hs|^p \in SM\left((\gamma_p(H))^p \mu\right) \quad □$$

Proof. See e.g. (Chaoui et al, 1998).

Lemma 3.3. *Consider the feedback system of Fig.1 involving a proper Hurwitz transfer function* $G(z)$ *in closed-loop with the nonlinear map* $\psi : IR \to IR$, $z \to z - sat(z)$*. Then, one has the following properties:*

1) The feedback is l_∞*-stable if* $\gamma_a < 2$ *with:*

$$\gamma_a = \gamma_\infty\left(\frac{G(z)}{1 + G(z)/2}\right) \tag{11}$$

Then one has:

$$\left\|e_2\right\|_\infty \le \frac{\gamma_a}{1 - \frac{\gamma_a}{2}}\left\|u_1\right\|_\infty + \frac{1 + \frac{\gamma_a}{2}}{1 - \frac{\gamma_a}{2}}\left\|u_2\right\|_\infty \tag{12}$$

2) The feedback is l_2*-stable if:*

$$\inf_{0 \le \omega < 2\pi} \mathrm{Re}\left(G(e^{j\omega})\right) > -1 \tag{13}$$

Under condition (13), the dynamic nonlinear map $w \to e_2$ *is* l_2*-incrementally stable, with* $w \overset{def}{=} u_2 + G(z)u_1$

□

Proof. See e.g. [9]

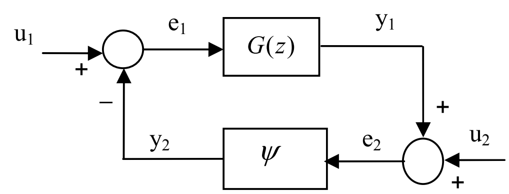

Fig 1. Feedback system in Lemma 3.1

IV. ANALYSIS OF THE SATURATED REGULATOR PERFORMANCES

In this section, the tracking capability of the saturated regulator defined by (5a-b) is analyzed using the technical tools presented in Section 3. Quite interesting results are established for different classes of reference sequences. First, we investigate the properties of operator $y^* \to v$.

Proposition 4.1. *Consider the constrained system (1a-b) in closed-loop with the saturated regulator (5a-b). Then, the nonlinear dynamic map* $y^* \to v$ *has the feedback structure of Fig 1 with:*

$$u_1 = 0,\; e_2 = v,\; y_2 = v - u \tag{18a}$$

$$u_2(t) = u^*(t) + \varepsilon(t) \tag{18b}$$

$$G(z) = \frac{A(z^{-1})\Lambda(z^{-1}) - C(z^{-1})}{C(z^{-1})} \tag{19a}$$

$$\psi : IR \to IR,\; z \to z - sat(z) \tag{19b}$$

where $u^*(t)$ *is as in Definition 3.4 and* $\varepsilon(t)$ *is an exponentially vanishing term* □

Proof. See [9].

Proposition 4.2. *Consider the constrained system (1a-b) in closed-loop with the saturated regulator (5a-b). Then, one has the following properties:*

1) If the reference sequence y^* *is bounded, all signals of the closed-system are bounded.*

2) If the reference sequence y^* *is periodic, all signals of the closed-loop system are periodic (in steady state) with the same period as* y^* □

Proof. See [9].

Theorem 4.1 *(Tracking strictly compatible reference signals). Consider the constrained system (1a-b) in closed-loop with the saturated regulator (5a-b). Let the reference sequence y^* be strictly compatible with the constraint (1b) and let $C(z^{-1})$ be chosen so that the following two conditions are satisfied:*

$$\gamma_b < 1 \qquad (33a)$$

$$\frac{1+\gamma_b}{1-\gamma_b} < 1+\delta \qquad (33b)$$

with

$$\gamma_b \overset{def}{=} \gamma_\infty\left(\frac{A(z^{-1})A(q^{-1})-C(z^{-1})}{A(z^{-1})A(q^{-1})+C(z^{-1})}\right) \qquad (34)$$

where δ is as in Definition 3.4 (Part 2). Then, there exists an integer $T \in IN$, such that for all $t \geq T$:

1) $|v(t)| < u_M$ *and, consequently,* $u(t) = v(t)$

2) $C(q^{-1})\left(y(\mathrm{t+d}) - y^*(t+d)\right) = 0$ *and, consequently,* $\lim_{t\to\infty}\left(y(t) - y^*(t)\right) = 0$ □

Proof. Part 1: Using Proposition 4.1, the nonlinear dynamic map $y^* \to v$ is given the equivalent feedback structure of Fig 1. Then, applying Lemma 3.3 (Part 1), the feedback is l_∞-stable provided that $\gamma_a < 2$ with:

$$\gamma_a = \gamma_\infty\left(\frac{G(z)}{1+G(z)/2}\right) = \gamma_\infty\left(2\frac{A(z^{-1})A(q^{-1})-C(z^{-1})}{A(z^{-1})A(q^{-1})+C(z^{-1})}\right) \qquad (35)$$

using (19a). But, from (34) one has $\gamma_a = 2\gamma_b$. Therefore, (35) does hold because $\gamma_b < 1$. Consequently, the system (23)-(24) (represented by the feedback of Fig 1) is actually l_∞-stable. Furthermore, Lemma 3.3 (Part 1) gives, due to inequality (12):

$$\|v\|_\infty \leq \frac{1+\gamma_b}{1-\gamma_b}\left\|u^*(t)+\varepsilon(t)\right\|_\infty \qquad \text{(using (12) and (18a-b))}$$

$$\leq \left(1-\delta^2\right)u_M + \left(1+\delta\right)\|\varepsilon(t)\|_\infty \qquad \text{(using (33b))} \qquad (36)$$

Using the system causality and the fact that the time origin (i.e. $t=0$) is not physically fixed, one gets from (36) that, for all $\tau > 0$:

$$\max_{\tau\leq t<\infty}|v(t)| \leq \left(1-\delta^2\right) u_M + \left(1+\delta\right)\max_{\tau\leq t<\infty}|\varepsilon(t)| \qquad (37)$$

As $\varepsilon(t)$ is exponentially vanishing, there exists a $T > 0$ such that:

$$\max_{T\leq t<\infty}|\varepsilon(t)| < \frac{\delta^2 u_M}{2(1+\delta)}$$

which, together with (37), implies:

$$\max_{T\leq t<\infty}|v(t)| \leq \left(1-\frac{\delta^2}{2}\right)u_M < u_M$$

This proves Part 1 of Theorem 4.1.

Part 2: From Part 1, the saturated control law (5a-b) reduces to the linear control law (6) for $t \geq T$. In view of (1a), $A(q^{-1})y(\mathrm{t+d})$ can be substituted to $B(q^{-1})u(\mathrm{t})$ in the left side of (6). Doing so, one gets for $t \geq T$:

$$R(q^{-1})A(q^{-1})y(t+d) + S(q^{-1})y(\mathrm{t}) = \mathrm{C}(q^{-1})y^*(t+d)$$

which proves Part 2 using (4) ■

Remark 4.2. 1) Note that conditions (34a-b) can be fulfilled choosing $C(z^{-1})$ sufficiently close to $A(z^{-1})$. Indeed, it is seen from (34) that $\lim_{C(z^{-1})\to A(z^{-1})} \gamma_b = 0$

2) In fact, the above conditions define a neighborhood of the controlled system poles (i.e. the zeros of $A(z^{-1})$) within which must be placed the regulation poles of the closed-loop system (i.e. the zeros of $C(z^{-1})$). When this pole assignment is respected, the regulator stops saturating after a transient period and the control law (5a-b) coincides with the standard linear law (6). Furthermore, as $y(\mathrm{t}) - y^*(t)$ vanishes asymptotically, it follows comparing (1a) and (9) that $u(\mathrm{t}) - u^*(t)$ vanishes in turn □

The next theorem describes the tracking performances for non-strictly compatible references. Let y^* be any bounded reference and $1/\mu$ its MCD. Due to Definition 3.4, μ is the smallest real satisfying:

$$\left\{\left(u^*(t) - sat(u^*(t))\right)^2\right\} \in SM(\mu) \qquad (38)$$

where u^* is the input reference defined by (3.2). For convenience, the following notations will be used throughout:

$$\tilde{s}(t) \overset{def}{=} s(t) - sat(u^*(t)) \qquad (39)$$

where $s(t)$ is any of the three sequences $v(t), u(t)$ and $u^*(t)$.

Proposition 4.3. *Consider the constrained system (2.1a-b) in closed-loop with the regulator (2.6a-b). Suppose that $C(z^{-1})$ satisfies the condition:*

$$\gamma_c < 1 \quad \textit{with} \quad \gamma_c \overset{def}{=} \gamma_1\left(\frac{A(z^{-1})-C(z^{-1})}{A(z^{-1})+C(z^{-1})}\right) \qquad (41)$$

Then, $|v-u| \in SM(K^*\mu)$ *and* $\left|y-y^*\right| \in SM(K^*\mu)$ *for some constant* $K^* > 0$ *(independent of μ)*□

Proof. Combining (1a) and (5) in a way to eliminate $y(t)$ one gets:

$$v(t) = -\frac{A(q^{-1})-C(q^{-1})}{C(q^{-1})}\left(v(t)-u(t)\right) + u^*(t) + \varepsilon(t)$$

Substracting $u^*(t)$ from both sides yields:

$$\tilde{v}(t) = -\frac{A(q^{-1}) - C(q^{-1})}{C(q^{-1})}\left(\tilde{v}(t) - \tilde{u}(t)\right) + \tilde{u}^*(t+d) + \varepsilon(t) \quad (42a)$$

Let ϕ denotes the nonlinear dynamic map:

$$\phi : IR \to IR\,; \quad \tilde{v}(t) \to \tilde{v}(t) - \tilde{u}(t) \quad (42b)$$

It is readily seen that the system (42a-b) fits the feedback scheme of Fig 2. Now, let us show that $\phi \in S[0\ \ 1]$. This amounts to prove that, for all t :

$$0 \le \tilde{v}(t)(\tilde{v}(t) - \tilde{u}(t)) \le (\tilde{v}(t))^2 \quad (44)$$

First, it is readily seen that (44) holds if $|v(t)| \le u_M$ (as then $\tilde{v}(t) - \tilde{u}(t) = v(t) - u(t) = 0$). So, let us consider the case where $|v(t)| > u_M$. Then, one has $u(t) = u_M sign(v(t))$ which, together with the fact that $|sat(u^*(t))| \le u_M$ implies, successively:

$$sign\left(v(t) - sat(u^*(t))\right) = sign\left(v(t) - u(t)\right) \quad (45)$$

$$|v(t) - u(t)| = |v(t)| - |u(t)| = |v(t)| - u_M \le |v(t)| - |sat(u^*(t))|$$
$$\le |v(t) - sat(u^*(t))| \quad (46)$$

Noticing that $\tilde{v}(t) - \tilde{u}(t) = v(t) - u(t)$ and $\tilde{v}(t) = v(t) - sat(u^*(t))$ it follows from (45)-(46) that $sign\left(\tilde{v}(t) - \tilde{u}(t)\right) = sign(\tilde{v}(t))$ and $|\tilde{v}(t) - \tilde{u}(t)| \le |\tilde{v}(t)|$. These clearly imply that (44) holds. Hence, the last statement holds in all cases and so we actually have that $\phi \in S[0\ \ 1]$.

Now, applying Lemma 3.3 (part 1), it follows that the feedback of Fig 2 is l_1-stable if:

$$\gamma_a < 2 \quad (47)$$

with $\gamma_a = \gamma_1\left(\frac{G(s)}{1 + G(s)/2}\right)$ and $G(z) = \frac{A(z^{-1}) - C(z^{-1})}{C(z^{-1})}$. It is readily checked that $\gamma_a = 2\gamma_1\left((A(z^{-1}) - C(z^{-1}))/(A(z^{-1}) + C(z^{-1}))\right)$. Then, the condition $\gamma_a < 2$ is nothing other than assumption (41). Therefore, the feedback of Fig 2 is actually l_1-stable. Applying Lemma 3.2 (Part 2), it follows from (4.13) that $|\tilde{v} - \tilde{u}| = |v - u| \in SM(K_1\mu)$, where $K_1 > 0$ is the l_1-gain of the map $\tilde{u}^*(t+d) + \varepsilon(t) \to \tilde{v} - \tilde{u}$. This establishes the first part of the proposition.

To prove the second part, notice that the control law (2.6a) can be rewritten:

$$B(q^{-1})R(q^{-1})u(t) + S(q^{-1})y(t)$$
$$= A(q^{-1})C(q^{-1})y^*(t+d) + A(q^{-1})B(q^{-1})\left(u(t) - v(t)\right)$$

In view of (2.1a), $B(q^{-1})u(t)$ can be substituted to $A(q^{-1})y(t+d)$ in the left side of (4.21). Doing so, one gets for $t \ge nb + d - 1$:

$$R(q^{-1})A(q^{-1})y(t+d) + S(q^{-1})y(t)$$
$$= A(q^{-1})C(q^{-1})y^*(t+d) + A(q^{-1})B(q^{-1})\left(u(t) - v(t)\right)$$

Using (4), this yields:

$$A(q^{-1})C(q^{-1})y(t+d) = A(q^{-1})C(q^{-1})y^*(t+d)$$
$$+ A(q^{-1})B(q^{-1})\left(u(t) - v(t)\right)$$

or, equivalently:

$$y(t+d) - y^*(t+d) = \frac{B(q^{-1})}{C(q^{-1})}\left(u(t) - v(t)\right) + \varepsilon(t) \quad (48)$$

where $\varepsilon(t)$ is exponentially vanishing. As $B(q^{-1})/C(q^{-1})$ is l_1-stable (because $C(q^{-1})$ is Hurwitz) and $|v - u| \in SM(K_1\mu)$ it follows, applying Lemma 3.2 (Part 2) to (4.22), that $|y - y^*| \in SM\left(K_1K_2\mu\right)$ with K_2 the l_1-gain of the transfer function $B(q^{-1})/C(q^{-1})$. Proposition 3 is proved with $K^* = \max(K_1, K_1K_2)$ ■

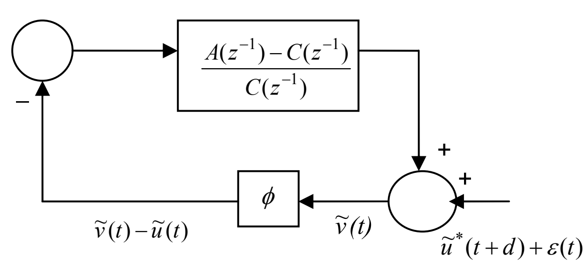

Fig 2. Feedback representation of the system (43a-b)

Remark 4.3. 1) Condition (41) defines (just as did conditions (33a-b)) a neighborhood of the controlled system poles in which must be assigned the closed-loop system regulation poles.

2) The average tracking result of Theorem 4.2 guarantees that the mean size of $|v - u|$ and $|y - y^*|$ are inversely

proportional to $1/\mu$ (the MCD of the reference). The larger the MCD, the better the average tracking quality. The particular case where $\mu = 0$ is interesting because then the average tracking error is null. Then $v(t)$ violates the saturation limit infinitely often (Remark 3.3, part 2). Note that the MCD concept is an original feature of the present work.

3) It is worthy noticing that $\gamma_b = \gamma_c = \|h\|_1$ with $h(t)$ the impulse response of the transfer function $(A(z^{-1}) - C(z^{-1})/(A(z^{-1}) + C(z^{-1}))$ (e.g. [7]). Hence, condition (41) allows a much broader choice of $C(q^{-1})$ than (33a-b). This is not surprising as the average tracking performance in Theorem 4.2 is less strong than the perfect global tracking in Theorem 4.1. However, the former involves a much wider class of admissible reference signals. Typically, any bounded reference $y^*(t)$ is admissible in Theorem 4.2. ■

V. SIMULATIONS

The simulation results are omitted for space limitations. They will be presented in the conference.

VI. CONCLUSION

We have considered the problem of controlling constrained discrete-time minimum-phase linear systems. The originality of the work is the design of the specific regulator (5a-b) that is shown to provide powerful tracking properties. Specifically, in presence of reference signals that are strictly compatible with the constraint (Definition 4.4), the regulator ensures perfect output reference tracking (Theorem 4.1). In presence of arbitrary bounded (but not necessarily strictly-compatible) RS, the average tracking quality depends on the reference MCD $1/\mu$ (Definition 3.4). The larger the MCD is, the better the tracking quality (Theorem 4.2). Finally, in the case of periodic reference signals (not necessarily compatible with the constraint), it is shown that all signals of the closed-loop system are in turn periodic with the same period as the driving reference (Proposition 4.2). It is the first time that such a high level of tracking performances is achieved in present of input constraint.

VII. REFERENCES.

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