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Control by Damping Injection of Electrodynamic Tether System in an Inclined Orbit

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Abstract— **Control of a satellite system with an electrodynamic tether as actuator is a time-periodic and underactuated control problem. This paper considers the tethered satellite in a Hamiltonian framework and determines a port-controlled Hamiltonian formulation that adequately describes the nonlinear dynamical system. Based on this model, a nonlinear controller is designed that will make the system asymptotically stable around its open-loop equilibrium. The control scheme handles the time-varying nature of the system in a suitable manner resulting in a large operational region. The performance of the closed loop system is treated using Floquet theory, investigating the closed loop properties for their dependency of the controller gain and orbit inclination.**

I. INTRODUCTION

The principle of electrodynamic space tethers has been studied over the last couple of decades for its potential of providing cheap propulsion for spacecrafts (see [1] for the fundamentals and [2] for a survey of the literature). A tethered satellite system (TSS) consists of two or more spacecrafts tethered with cables, also known as space tethers. The current study will consider two satellites tethered with an electrodynamic tether. An electrodynamic tether is able to collect and release free electrons from/to the ionosphere, which makes a current flow along the tether. The current will interact with the magnetic field of the Earth and give rise to a Lorentz force acting along the tether. This force can be utilized to perform orbit maneuvers.

In this work a rigid tether model has been adopted and it is assumed that the current through the tether can be controlled without limitation. In general the model is time-varying, due to the periodic changes in the magnetic field along the orbit. This time-periodic nature gives rise to a family of unstable periodic solutions, which have been investigated in [3]. The special case of an equatorial orbit, which has the advantage of being time invariant, was investigated in [4].

In this paper, the focus will be on the case of an inclined orbit, which has been investigated by several others. One proposed control strategy is to stabilize the unstable periodic solution of the tether motion. In [5] two control schemes were proposed for such stabilization using two additional actuators. The first scheme used linear feedback of the difference between a reference trajectory and the current trajectory, the other used time-delayed autosynchronization. In [6], the unstable periodic solutions were stabilized using a current through the tether as actuator. The feedback law was designed using the energy variation along the orbit to synchronize the motion with a reference trajectory. In [7], a feedback linearisation control law was designed, using the current through the tether and the tether length as control inputs, to stabilize the open-loop equilibrium. This feedback law introduced two singularities along the orbit due to the unactuated out-of-plane dynamics, which was handled by switching to an additional control law.

The main contribution of this paper is to formulate the systems as a port-controlled Hamiltonian system to establish a passive connection between input and an output from which an asymptotically stable control law is designed to stabilize the open-loop equilibrium. From the port-controlled Hamiltonian formulation the controller is interpreted as damping injection for the conservative open-loop system. Traditionally the zeros of the input function can give rise to problems in connection with the control law (see [7]). However the paper shows that these are easily handled together with the time-varying nature of the actuator due to the passive system formulation. The idea of using an energy based control method for the tether system is shown to be a natural choice since the dominating force on the system is the conservative gravity force and the perturbation force can be determined by the control input.

II. MODEL

In this section the tether model is deduced. The physical setup is first introduced. In the following sections the Lagrangian and the Hamiltonian of the system are stated and the generalized force arising from the Lorentz force will be derived. In the last section the system will be formulated as a port-controlled Hamiltonian system.

A. *Definitions and assumptions*

The TSS under consideration consist of two satellites, the main-satellite and the sub-satellite, tethered with a rigid electrodynamic tether of length l and mass m_t. The satellites are modelled as point masses with mass m_B and m_A, respectively. The mass of the main-satellite is assumed the dominating mass of the system, $m_B \gg m_A + m_t$, from which it can be assumed that the center of mass of the TSS coincides with the center of mass of the main-satellite. It is assumed that the satellites are only subject to microgravity, while the tether in addition is affected by the Lorentz force. Since no perturbation forces are affecting the main-satellite, it will follow a unperturbed Keplerian orbit, which furthermore is assumed circular with semi-major axis R_o. The model is derived with the purpose of investigating the stability of the tether w.r.t. the orbital motion, thus it will

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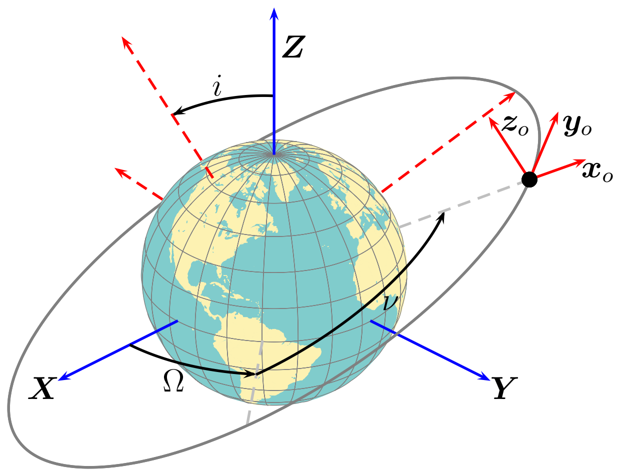

Fig. 1. Orbit description. The orbit frame $(\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{z}_o)$ occurs from an $\Omega \rightarrow i \rightarrow \nu$ rotation of the inertial frame $(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z})$.

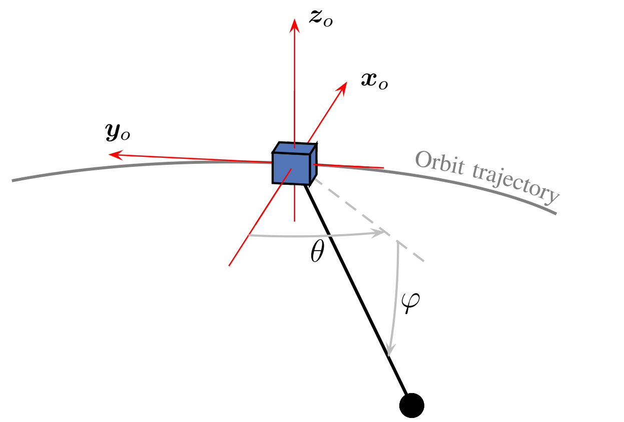

Fig. 2. In- and out-of-plane angles described w.r.t. the orbit frame $(\boldsymbol{x}_o, \boldsymbol{y}_o, \boldsymbol{z}_o)$.

only consider the influence of the Lorentz force on the attitude motion. The effect of the on the orbital motion was the subject of [8]. The motion of the tether is described in the orbit frame, defined with the $\boldsymbol{x}_o$-axis along the position vector from the Earth to the main-satellite, $\boldsymbol{y}_o$ along the velocity vector of the system and $\boldsymbol{z}_o$ normal to the orbit plane (see Fig. 1). Since the orbit is assumed circular the right ascension of the ascending node Ω, the orbit inclination i and the true anomaly ν will be adequate to describe the orbit frame w.r.t. to the inertia frame as seen in Fig. 1. The points along the tether are described using a unit vector $\boldsymbol{r}$ from the main- to the sub-satellite, from which the points along the tether can be written as $s\boldsymbol{r}$ with $s \in [0\ l]$. The tether is assumed of constant length, hence the tether motion is restricted to a sphere and the system has $n = 2$ degrees of freedom. Spherical coordinates are introduced as the generalized coordinates $\boldsymbol{q} = [\theta\ \varphi]^T$, from which $\boldsymbol{r}$ can be expressed in the orbit frame as,

$$\boldsymbol{r} = \begin{bmatrix} -\cos\theta\cos\varphi & -\sin\theta\cos\varphi & -\sin\varphi \end{bmatrix}^T, \tag{1}$$

where θ is the in-plane angle and φ the out-of-plane angle as seen in Fig. 2. The position of the main-satellite in the orbit is described by the true anomaly ν. The orbit is assumed circular thus ν is linearly increasing and it is evident to introduce ν as the non-dimensional time $\nu = \omega_o t$, which is subsequently used in the model. The current I through the tether is seen as the control input and it is assumed to be controlled without limitations. When the tether is the only actuator the system is underactuated i.e. the number of inputs m is smaller than the degrees of freedom n.

B. Lagrangian

The Lagrangian of the system can be written as the difference between kinetic and potential energy,

$$L = K - V. \tag{2}$$

From the Lagrangian the equation of motion can be found from Lagrange's equation. Defining the Jacobian of a scalar function as a column vector this can be written as,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial L}{\partial \boldsymbol{q}} = \boldsymbol{\tau}, \tag{3}$$

where $\boldsymbol{\tau} = [\tau_\theta\ \tau_\varphi]^T$ represents the generalized force acting on the system. Since the motion relative to the orbit frame is of interest, the velocities are described relative to this frame, which introduces the centrifugal and the Coriolis potential. Since the system is orbiting the Earth the main effect of the gravitational field vanishes and V includes only the Tidal force. The Lagrangian L can be written (see e.g. [3]),

$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\Lambda\left(\dot{\varphi}^2 + \cos^2\varphi\left((1+\dot{\theta})^2 + 3\cos^2\theta\right)\right), \tag{4}$$

where $(\dot{\ })$ denotes differentiation w.r.t. ν and $\Lambda = \frac{1}{3}\omega_o^2 l^2(3m_A + m_t)$. The term $2\dot{\theta}\cos^2\varphi$ represents the Coriolis potential and $\cos^2\varphi$ the centrifugal potential. From (3) it is seen that $\boldsymbol{\tau}$ can be scaled by Λ^{-1}, which leaves a parameterless Lagrangian.

C. Hamiltonian

The generalized momenta can be found as $\boldsymbol{p} = \frac{\partial L}{\partial \dot{\boldsymbol{q}}} = [p_\theta\ p_\varphi]^T$ from which,

$$p_\theta = \left(1 + \dot{\theta}\right)\cos^2\varphi \tag{5a}$$

$$p_\varphi = \dot{\varphi}. \tag{5b}$$

The Hamiltonian H is given as,

$$\begin{aligned} H(\boldsymbol{q}, \boldsymbol{p}) &= \boldsymbol{p}^T\dot{\boldsymbol{q}} - L\left(\boldsymbol{q}, \dot{\boldsymbol{q}}\left(\boldsymbol{p}, \boldsymbol{q}\right)\right) \\ &= \frac{1}{2}\left(p_\varphi^2 + \frac{p_\theta^2}{\cos^2\varphi} - 2p_\theta - 3\cos^2\theta\cos^2\varphi\right) + 2. \end{aligned} \tag{6}$$

The constant 2 is added, without loss of generality, to get a positive semi definite Hamiltonian. The singularities at $\varphi = \pm\frac{\pi}{2}$ are coursed by the use of spherical coordinates. Using H the equation of motion can be written using Hamilton's equation,

$$\dot{\boldsymbol{q}} = \frac{\partial H}{\partial \boldsymbol{p}}, \tag{7a}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{q}} + \boldsymbol{Q}, \tag{7b}$$

where $\boldsymbol{Q} = \Lambda^{-1}\boldsymbol{\tau}$. The equations result in the following four coupled first order differential equations,

$$\dot{\theta} = \frac{p_\theta}{\cos^2\varphi} - 1, \tag{8a}$$

$$\dot{\varphi} = p_\varphi, \tag{8b}$$

$$\dot{p}_\theta = -\frac{3}{2}\cos^2\varphi\sin 2\theta + \tau_\theta, \tag{8c}$$

$$\dot{p}_\varphi = -\frac{p_\theta^2}{\cos^2\varphi}\tan\varphi - \frac{3}{2}\cos^2\theta\sin 2\varphi + \tau_\varphi. \tag{8d}$$

From Hamilton's equation (8) it is obvious that the equilibria of the unforced system ($\boldsymbol{Q} = \boldsymbol{0}$) is placed at the extrema of H. The open-loop equilibrium between the Earth and the main-satellite is described as $p_\theta^* = 1$ and $p_\varphi^* = \theta^* = \varphi^* = 0$.

D. Generalized forces

The Lorentz force on a tether section of unit length is,

$$\bar{\boldsymbol{F}}_e = I\boldsymbol{r} \times \boldsymbol{B}, \tag{9}$$

where $\boldsymbol{B}$ is the magnetic field of the Earth. To find the generalized force $\boldsymbol{\tau}$ associated with the generalized coordinates, the Lorentz force per unit length is projected onto the generalized coordinates and integrated along the tether,

$$\tau_i = \int_0^l \bar{\boldsymbol{F}}_e \cdot \frac{\partial\,(s\boldsymbol{r})}{\partial q_i}\,ds, \quad \text{for } i = 1, 2. \tag{10}$$

A dipole model is a simple and widely used approximation of the magnetic field of the Earth. To avoid unnecessary complexity, the dipole moment is aligned with the rotational axis of the Earth. This results in a model independent of the rotation of the Earth. The B-field can be written in the orbit frame as,

$$\boldsymbol{B} = \frac{\mu_m}{R_o^3}\begin{bmatrix} -2\sin\nu\sin i \\ \cos\nu\sin i \\ \cos i \end{bmatrix}, \tag{11}$$

where μ_m is the strength of the B-field. Using (11) the generalized force is,

$$\boldsymbol{Q} = \boldsymbol{b}(\boldsymbol{q}, \nu)u, \tag{12}$$

where u is a dimensionless quantity proportional to the input current, which in turn can be written as,

$$u = \frac{3}{2}\frac{1}{3m_A + m_t}\frac{\mu_m}{\mu}I. \tag{13}$$

Here μ is the standard gravitational parameter of the Earth. The vector $\boldsymbol{b}(\boldsymbol{q}, \nu) = [b_\theta(\boldsymbol{q}, \nu)\ b_\varphi(\boldsymbol{q}, \nu)]^T$ will be denoted as the input function and is of great importance for the control design. It is essential for the controllability of the system and it will appear to be an important part of establishing a passive input-output connection for the system.

The input function can be written as,

$$b_\theta(\boldsymbol{q}, \nu) = \cos^2\varphi\tan\varphi\sin i\,(\cos\nu\sin\theta - 2\sin\nu\cos\theta) - \cos^2\varphi\cos i, \tag{14a}$$

$$b_\varphi(\boldsymbol{q}, \nu) = \sin i\,(\cos\theta\cos\nu + 2\sin\theta\sin\nu)\,. \tag{14b}$$

$\boldsymbol{b}(\boldsymbol{q}, \nu)$ is in general quite complicated reflecting the fact that the magnetic field varies along the orbit (from which the time dependency occurs) and that the Lorentz force depends upon the tether orientation relative to the B-field. In the special case of an equatorial orbit ($i = 0°$) the input function becomes time invariant, but at the same time b_φ vanish and the out-of-plane motion will become unactuated. This case was treated in [4].

In the case of an inclined orbit $b_\varphi(\boldsymbol{q}, \nu)$ will have two zeros along the orbit, determined by $2\tan\theta = -\cot\nu$. For the open-loop equilibrium these are placed at $\nu = \pm\frac{\pi}{2}$. The zeros of $b_\theta(\boldsymbol{q}, \nu)$ occurs in a more complicated scheme. It can be seen from (14a) that for non-polar orbits ($i \neq 90°$) zeros cannot occur for small out-of-plane angels (more specifically for $2\,|\tan\varphi| < |\cot i|$), hence no zeros occur for the open-loop equilibrium or any other equilibrium in the orbit plane. For a polar orbit, b_θ vanish if $\sin 2\varphi = 0$ or $\cot\nu = 2\cos\theta$, i.e. the in-plane motion is unactuated at the open-loop equilibrium.

A critical situation where the system is uncontrollable can occur if $b_\theta = b_\varphi = 0$ for a period of time. This situation will occur if the tether and the magnetic field are parallel, and no Lorentz force can be generated along the tether. We will not treat this situation in this work.

E. Port-controlled Hamiltonian system description

Introducing a state vector $\boldsymbol{x} = \left[\boldsymbol{q}^T\ \boldsymbol{p}^T\right]^T$ the system is rewritten as,

$$\dot{\boldsymbol{x}} = \boldsymbol{J}\frac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x}, \nu)u \tag{15a}$$

$$y = \boldsymbol{g}^T(\boldsymbol{x}, \nu)\frac{\partial H}{\partial \boldsymbol{x}}, \tag{15b}$$

where

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \quad \text{and} \quad \boldsymbol{g} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{b}(\boldsymbol{q}, \nu) \end{bmatrix}. \tag{16}$$

This is a standard formulation of a mechanical system where only the momentum states are actuated. The output function (15b) is chosen to establish a passive input output connection. This formulation is called a port-controlled Hamiltonian description (see [9, p. 73]) and can, for single input systems, in general be written as,

$$\dot{\boldsymbol{x}} = (\boldsymbol{J}(\boldsymbol{x}, \nu) - \boldsymbol{R}(\boldsymbol{x}, \nu))\frac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{g}(\boldsymbol{x}, \nu)u \tag{17a}$$

$$y = \boldsymbol{g}^T(\boldsymbol{x}, \nu)\frac{\partial H}{\partial \boldsymbol{x}}, \tag{17b}$$

where $\boldsymbol{J} \in \mathbb{R}^{2n\times 2n}$ is the interconnection matrix and $\boldsymbol{R} \in \mathbb{R}^{2n\times 2n}$ is the damping matrix. It is assumed that the interconnection matrix is skew-symmetric $\boldsymbol{J} = -\boldsymbol{J}^T$ and that the damping matrix is symmetric and positive semi-definite, $\boldsymbol{R} = \boldsymbol{R}^T \geq 0$. Both the interconnection and the damping matrices can be state and time dependent. In the current case $\boldsymbol{R} = \boldsymbol{0}$ for the open-loop system since no damping forces are modelled.

III. CONTROL DESIGN

A controller based on the passivity property of the port-controlled Hamiltonian system will be designed in this section. Afterwards, the closed loop system will be investigated, using linear Floquet analysis, to find a controller gain k, which provides optimal stability properties.

A. Passivity based control design

A general stabilization of the system is a difficult task due to its time-varying and underactuated nature. In the port controlled Hamiltonian framework, this would require a feedback law, which reshapes the Hamiltonian of the closed loop system. An easier task would be to stabilize the open-loop equilibrium $\boldsymbol{x}^*$. The latter will be considered here. Since the Hamiltonian (which acts as storage function for the system) is positive definite, this task can be simply achieved by the feedback law $u = -ky$, where $k > 0$ (see [9, Corollary 3.3.1 p. 44]). An important condition for the control design is then that the system need be zero-state detectable, i.e. if $u = y = 0$ for $t > t_0$ the states should converge towards the equilibrium. The zero-state detectability of the system is closely related to the zeros of the input function.

The output can be written as,

$$
\begin{aligned}
y &= \boldsymbol{g}^T(\boldsymbol{x}, \nu)\frac{\partial H}{\partial \boldsymbol{x}} = \boldsymbol{b}^T(\boldsymbol{q}, \nu)\frac{\partial H}{\partial \boldsymbol{p}} \\
&= b_\theta(\boldsymbol{q}, \nu)\dot{\theta} + b_\varphi(\boldsymbol{q}, \nu)\dot{\varphi},
\end{aligned}
\tag{18}
$$

where it has been used that $\dot{\boldsymbol{q}} = \frac{\partial H}{\partial \boldsymbol{p}}$. The zeros of the input function originating from time-periodicity have no influence on the zero-state detectability since they will be countable. The generalized coordinates can induce zeros in the input function as mentioned earlier, but in the case, where the generalized velocities are different from zero, they will only occur for countable instances of time. The case where the velocity is also zero, the state will have reached the equilibrium of interest, since this is the only open-loop equilibrium. The last variable to cause zeros in the input function is the orbit inclination i. In the case of a equatorial orbit ($i = 0°$) the second term of the output will be zero. If the in-plane dynamics at the same time has reached its equilibrium, the out-of-plane dynamics will by unobservable from the output and the system is therefore not zero-state detectable in this case. For a polar orbit ($i = 90°$) the situation is similar. The in-plane dynamics will be unobservable from the output in the case where the out-of-plane dynamics have reached its equilibrium position, hence the system is not zero-state detectable in the case of a polar orbit neither.

The stability of the closed loop system can by investigated using H as Lyapunov function candidate. The time derivative of H is,

$$
\begin{aligned}
\dot{H} &= \left(\frac{\partial H}{\partial \boldsymbol{x}}\right)^T \boldsymbol{J}\frac{\partial H}{\partial \boldsymbol{x}} + \left(\frac{\partial H}{\partial \boldsymbol{x}}\right)^T \boldsymbol{g}(\boldsymbol{x}, \nu)u \\
&= -k\left(\frac{\partial H}{\partial \boldsymbol{p}}\right)^T \boldsymbol{b}(\boldsymbol{q}, \nu)\boldsymbol{b}^T(\boldsymbol{q}, \nu)\frac{\partial H}{\partial \boldsymbol{p}}.
\end{aligned}
\tag{19}
$$

Due to the zero-state detectability, the derivative of this Lyapunov candidate is negative semi-definite, however LaSalle's theorem ensures asymptotically stability of $\boldsymbol{x}^*$.

The closed loop system can be written as a port-controlled Hamiltonian system as,

$$
\dot{\boldsymbol{x}} = (\boldsymbol{J} - \boldsymbol{R})\frac{\partial H}{\partial \boldsymbol{x}}, \tag{20}
$$

where

$$
\boldsymbol{R} = \boldsymbol{g}\boldsymbol{g}^T = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_2 \end{bmatrix}. \tag{21}
$$

It is seen that the controller has added the damping matrix $\boldsymbol{R}_2 = \boldsymbol{b}\boldsymbol{b}^T$, hence the controller strategy is called damping injection. $\boldsymbol{R}_2$ will have one eigenvalue equal to zero, while the other will be positive except when $b_\theta = b_\varphi = 0$ in which case it will be zero. The lack of full rank of $\boldsymbol{R}_2$ is a consequence of the fact that the system is underactuated.

B. Closed loop analysis

This section investigates the stability properties of the closed loop system, for different values of k, using Floquet analysis. A linearised version of the closed loop system can be written as,

$$
\dot{\boldsymbol{x}} = \boldsymbol{A}(\nu)\boldsymbol{x}, \tag{22}
$$

where $\boldsymbol{A}(\nu)$ is the $T = 2\pi$-periodic system matrix,

$$
\begin{aligned}
\boldsymbol{A}(\nu) = &\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \end{bmatrix} \\
&- \frac{1}{2}k \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2\cos^2 i & -\sin(2i)\cos\nu \\ 0 & 0 & -\sin(2i)\cos\nu & 2\sin^2 i\cos^2\nu \end{bmatrix}.
\end{aligned}
\tag{23}
$$

The independent solutions of (22) can be written in a fundamental matrix $\boldsymbol{\Phi}(\nu)$ for which,

$$
\dot{\boldsymbol{\Phi}}(\nu) = \boldsymbol{A}(\nu)\boldsymbol{\Phi}(\nu). \tag{24}
$$

It is seen that $\boldsymbol{\Phi}(\nu + T)$ is also a fundamental matrix, hence the connection between $\boldsymbol{\Phi}(\nu)$ and $\boldsymbol{\Phi}(\nu + T)$ can be written,

$$
\boldsymbol{\Phi}(\nu + T) = \boldsymbol{\Phi}(\nu)\boldsymbol{M}, \tag{25}
$$

where $\boldsymbol{M}$ is the nonsingular monodromy matrix and the characteristic multipliers ρ_i can be found as the eigenvalues of $\boldsymbol{M}$. The stability of the system is determined from ρ_i (see [10]) and can be summarized as,

- If one characteristic multiplier is numerically lager than one $|\rho_i| > 1$, the system is unstable.
- If all characteristic multipliers are numerically less than one $|\rho_i| < 1$, the system is asymptotically stable.
- If the multipliers of unit length ($|\rho_i| = 1$) have equal algebraic and geometrical multiplicity and the remaining multipliers have $|\rho_i| < 1$ a periodic solution exist.

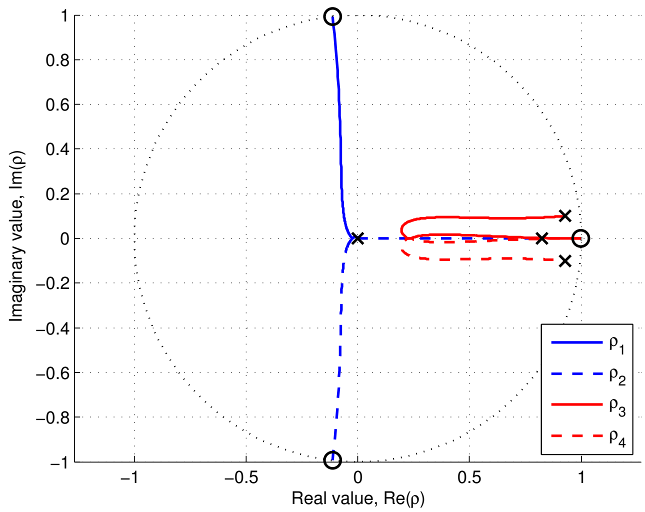

Fig. 3. The characteristic multipliers shown in the complex plane. $k = 0$ is marked with $\circ$ and $k = 100$ is marked with $\times$.

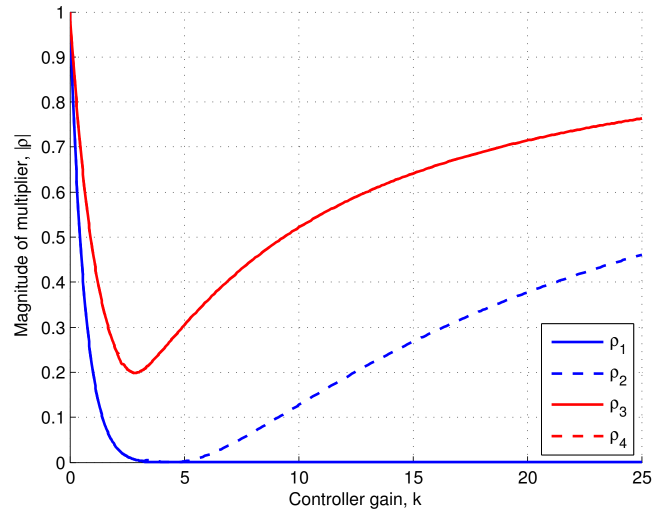

Fig. 4. The absolute value of the characteristic multipliers as function of the controller gain k.

The general complex solution of (22) can, in the case where an eigenvalue of multiplicity m has m independent eigenvectors, be written as,

$$\boldsymbol{x}(\nu) = \sum_{i=1}^{2n} c_i \rho_i^{\frac{\nu}{T}} \boldsymbol{p}_i(\nu), \tag{26}$$

where $\boldsymbol{p}_i$ is a T-periodic function and c_i is a constant. From (26) it is clear the numerically largest multiplier will dominate the response as well as decide the stability (in agreement of the above scheme). This multiplier will determine the convergence of the solution. We will denote the numerically largest characteristic multiplier *the stability deciding multiplier*. In this work the characteristic multipliers are found numerically by solving (24) with the initial condition $\boldsymbol{\Phi}(0) = \boldsymbol{I}$. The monodromy matrix can then by found from (25) as $\boldsymbol{M} = \boldsymbol{\Phi}(T)$.

Fig. 3 shows the evolution of the characteristic multipliers in the complex plane for increasing controller gain. The absolute values are shown in Fig. 4. The inclination is $i = 45^\circ$ in this case. For $k = 0$ the figures show that two periodic solutions exist with different frequencies corresponding to the natural frequencies of the in- and out-of-plane motions[1]. The in-plane motion has a natural frequency of $\sqrt{3}\omega_o$ corresponding to ρ_1 and ρ_2 while the out-of-plane natural frequency equals $2\omega_o$ corresponding to ρ_3 and ρ_4. $\rho_3 = \rho_4 = 1$ since the out-of-plane natural frequency is a multiple of the orbit rate, which corresponds to the frequency of the time variation of $\boldsymbol{A}(\nu)$. For increasing k the multipliers are moving towards the origin, reaching a minimum of the absolute value at $k \approx 3$. Afterwards three multipliers converge to one, while the last one converges to zero, resulting in an increasing stability deciding multiplier. It is seen that the system is asymptotically stable for all $k \neq 0$, which is in agreement with the stability proof of the previous section. Fig. 5 shows three simulations of the closed loop system for different controller gains. The gains are chosen to illustrate the influence of the deciding multiplier on the system response.

Due to the zeros of the input function the controller will only be stable in a certain range of orbit inclinations. Fig. 6 shows the stability deciding characteristic multiplier as function of the controller gain for different inclinations. In case of either an equatorial or a polar orbit, the characteristic multiplier is one for all k, and periodic solutions will occur. This is in agreement with the analysis of the zero-state detectability from the previous section. As already mentioned, the out-of-plane motion is unactuated in the equatorial case

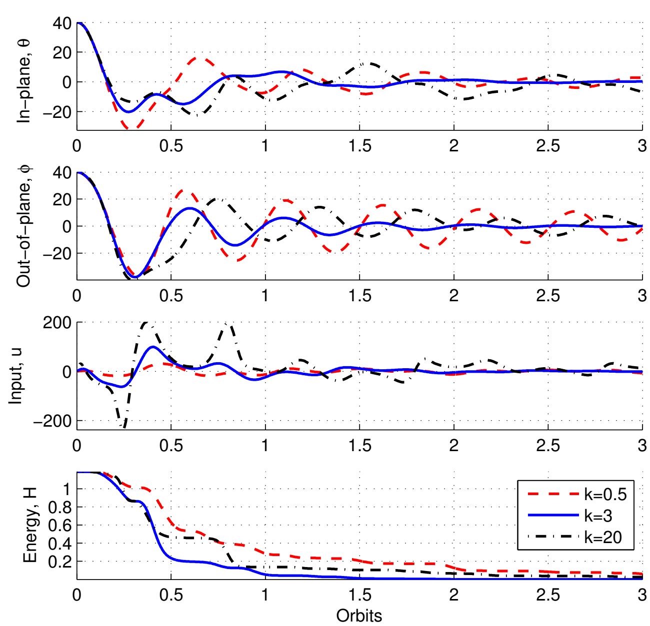

Fig. 5. Simulation of nonlinear closed loop system for different gains k.

[1]In the case $k = 0$ the eigenvalues can be found analytically and it can be checked the that geometric multiplicity of ρ_3 and ρ_4 equals two.

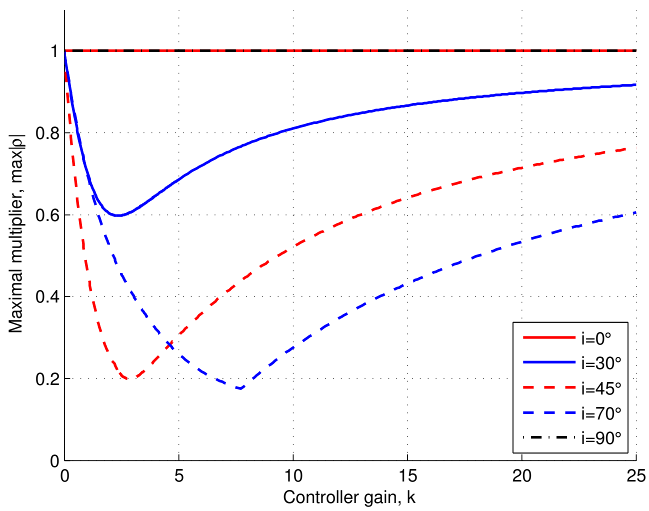

Fig. 6. The absolute value of the deciding multiplier for different i.

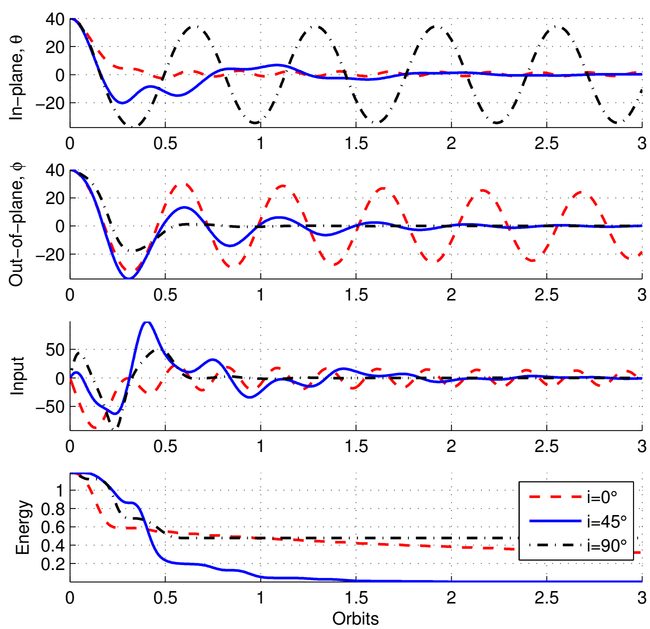

Fig. 7. Simulation of nonlinear closed loop system for different orbit inclination i. The controller gain is chosen as $k = 3$.

since $b_\varphi = 0$, hence in the linear approach, the out-of-plane motion will oscillate with its natural frequency. For polar orbits b_θ vanished in the linear approach, hence the motion is unactuated and similar to the equatorial orbit, the in-plane motion will oscillate with its natural frequency.

Fig 7 shows simulation of the closed loop system for different inclinations. The figure shows that for the equatorial orbit the out-of-plane motion is poorly damped, while the in-plane motion is poorly damped for the polar orbit.

IV. DISCUSSION

The controller designed in this paper has distinct advantages to other approaches. With this approach, the zeros of the input functions are not leading to singularities in the control law, which in turn gives to a large operational region. The approach is balancing the trade-off between performance and robustness in favour of the robustness of the control. This is a known property of a passivity based control design (see [11]). Robustness is quite important in a practical context, since the uncertainty in the magnetic field is quite large. The performance is limited by the minimum of the stability deciding multiplier, which lead to slow control action compared to other approaches.

Earlier papers have emphasized that the current which can be induced along the tether is limited (see e.g. [6]. This can prevent the choice of an optimal control gain, which will lead to a longer settling time for the controller However, it will not have any influence on the stability of the controller.

V. CONCLUSION

A controller that provides asymptotically stability for the open-loop equilibrium of a tethered satellite system was designed in this paper, using an electrodynamic tether as actuator. The design was based on a port-controlled Hamiltonian formulation of the system and stability was shown using the Hamiltonian as a Lyapunov function. The performance of the closed loop system was investigated using Floquet theory and a controller gain was found that minimize the settling time. The performance was investigated, primarily as a function of orbit inclination. As a salient feature, it was shown that damping was injected for all values of inclination, except when pure equatorial or polar orbits were considered. These orbits lead to nonactuated out-of-plane and in-plane dynamics, respectively, as should be expected.

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