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Adaptive Nonlinear Control Allocation of Non-minimum Phase Uncertain Systems

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Abstract— **This paper presents an adaptive nonlinear control allocation method for a general class of non-minimum phase uncertain systems. Indirect adaptive approach and Lyapunov design approach are applied to the design of adaptive control allocation. The derived adaptive control allocation law, together with a stable model reference control, guarantees that the closed-loop nonlinear system is input-to-state stable.**

Index Terms: nonlinear uncertain systems, control allocation, adaptive control, non-minimum phase systems

1. INTRODUCTION

For some nonlinear control design methods, such as model following, dynamic inversion, back stepping and sliding mode control, control allocation is an important step that maps virtual control inputs into physical actuator deflections subject to control constraints. In past decades, control allocation algorithms [1], [2], [3], [4], [5] have been extensively studied. Most of them treat the control allocation as a static optimization problem that optimizes the control allocation problem at each time instant. Different from the above static optimization algorithms, a Lyapunov design approach is used to develop a control allocation algorithm in [6], and consequently an adaptive control allocation algorithm in [7]. However, these approaches do not consider unstable internal dynamics. A control allocation approach considering both internal dynamics stabilization and actuator saturation is proposed in [8], [9] recently.

In this paper, we extend the result in [8], [9] and propose an adaptive control allocation approach for nonlinear systems with unstable internal dynamics and unknown parameters. The unknown parameters can be used to represent some fault parameters, such as actuator loss of effectiveness. Certainty equivalence indirect adaptive control approach [10] is adopted to estimate the unknown parameters. The proposed adaptive control allocation is based on a Lyapunov design approach, and the estimated parameters converge under the condition of persistent excitation. The control allocation, together with a stable model reference control law, guarantees that the closed-loop system be input-to-state stable.

2. PROBLEM FORMULATION OF CONTROL ALLOCATION

Consider the following nonlinear system:

$$\begin{cases} \dot{\mathbf{x}} = f(\mathbf{x},\mathbf{z}) + g(\mathbf{x},\mathbf{z},\mathbf{u})\vartheta \\ \dot{\mathbf{z}} = h(\mathbf{x},\mathbf{z}) + k(\mathbf{x},\mathbf{z},\mathbf{u})\vartheta \end{cases} \tag{1}$$

where $\mathbf{u} \in \mathbb{R}^m$ is the control input vector, $\mathbf{x} \in \mathbb{R}^{n_\mathbf{x}}$ is the commanded state vector with $n_\mathbf{x} \leq m$, $\mathbf{z} \in \mathbb{R}^{n_\mathbf{z}}$ is the internal state vector. The $(\mathbf{x},\mathbf{z})$ decomposition is due to some process, e.g. dynamic inversion [11]. The constant parameter vector $\vartheta \in \mathbb{R}^r$ contains unknown parameters of the nonlinear model, which may be used to represent some fault parameters, such as actuator loss of effectiveness.

Assumption 1: The functions $f(\mathbf{x},\mathbf{z})$, $g(\mathbf{x},\mathbf{z},\mathbf{u})$, $h(\mathbf{x},\mathbf{z})$,

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$k(\mathbf{x},\mathbf{z},\mathbf{u})$ as in (1) are twice continuously differentiable and are bounded for every bounded argument.

Assumption 2: The zero dynamics $\dot{\mathbf{z}} = h(\mathbf{0},\mathbf{z}) + k(\mathbf{0},\mathbf{z},\mathbf{0})\vartheta$ is unstable in Lyapunov sense.

Since ϑ is an unknown parameter vector, an adaptive law based on a certainty equivalence indirect adaptive control approach is used to obtain its estimate $\hat{\vartheta}$, which is discussed in Section 3. In this section, the control allocation is based on the nonlinear system with the estimate $\hat{\vartheta}$, namely,

$$\begin{cases} \dot{\mathbf{x}} = f(\mathbf{x},\mathbf{z}) + g(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} \\ \dot{\mathbf{z}} = h(\mathbf{x},\mathbf{z}) + k(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} \end{cases} \tag{2}$$

The objective of control allocation is for the $\mathbf{x}$-subsystem to track a reference model which represents the desired dynamics of the closed-loop system and $\mathbf{z}$-subsystem to follow some stable model. To implement the objective, we assume that the asymptotically stable reference model for the system (2) is described as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_d\mathbf{x} + \mathbf{B}_d\mathbf{r} \\ \dot{\mathbf{z}} = \mathbf{A}_s\mathbf{z} \end{cases} \tag{3}$$

with system matrices $\mathbf{A}_d \in \mathbb{R}^{n_\mathbf{x}\times n_\mathbf{x}}$ and $\mathbf{A}_s \in \mathbb{R}^{n_\mathbf{z}\times n_\mathbf{z}}$, input matrix $\mathbf{B}_d \in \mathbb{R}^{n_\mathbf{x}\times n_\mathbf{r}}$ and reference vector $\mathbf{r} \in \mathbb{R}^{n_\mathbf{r}}$.

Assumption 3: The matrices $\mathbf{A}_d$ and $\mathbf{A}_s$ are Hurwitz and $\mathbf{r}$ is continuously differentiable.

The system (2) matches the reference model (3) if

$$\begin{cases} g(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} = \tau(\mathbf{x},\mathbf{z},\mathbf{r}) \\ h(\mathbf{x},\mathbf{z}) + k(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} = \mathbf{A}_s\mathbf{z} \end{cases} \tag{4}$$

where $\tau \in \mathbb{R}^{n_\mathbf{x}}$ is a given virtual control vector in terms of a state feedback law

$$\tau(\mathbf{x},\mathbf{z},\mathbf{r}) = \mathbf{A}_d\mathbf{x} + \mathbf{B}_d\mathbf{r} - f(\mathbf{x},\mathbf{z}) \tag{5}$$

Define

$$\begin{aligned} \sigma_{g\tau}(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u}) &= g(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} - \tau(\mathbf{x},\mathbf{z},\mathbf{r}) \\ \sigma_{kh}(\mathbf{x},\mathbf{z},\hat{\vartheta},\mathbf{u}) &= h(\mathbf{x},\mathbf{z}) + k(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} - \mathbf{A}_s\mathbf{z} \end{aligned} \tag{6}$$

The main objective of control allocation problem is then to minimize the cost function:

$$\begin{aligned} J_1(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u}) =& \frac{1}{2}\sigma_{g\tau}^T(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u})\mathbf{H}_1\sigma_{g\tau}(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u}) \\ &+ \frac{1}{2}\sigma_{kh}^T(\mathbf{x},\mathbf{z},\hat{\vartheta},\mathbf{u})\mathbf{H}_2\sigma_{kh}(\mathbf{x},\mathbf{z},\hat{\vartheta},\mathbf{u}) \end{aligned} \tag{7}$$

where $0 < \mathbf{H}_1 \in \mathbb{R}^{n_\mathbf{x}\times n_\mathbf{x}}$ and $0 < \mathbf{H}_2 \in \mathbb{R}^{n_\mathbf{z}\times n_\mathbf{z}}$ are known weighting matrices.

The secondary objective is to minimize power consumption

$$J_2(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T\mathbf{H}_3\mathbf{u} \tag{8}$$

where $0 < \mathbf{H}_3 \in \mathbb{R}^{m\times m}$ is a known weighting matrix and $\|\mathbf{H}_3\| \ll \|\mathbf{H}_1\|$ and $\|\mathbf{H}_3\| \ll \|\mathbf{H}_2\|$.

Now the control allocation problem is formulated in terms of solving the following nonlinear minimization problem:

$$\min_{\mathbf{u}} J(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u}) \quad \text{subject to} \quad \mathbf{u} \in \Omega \tag{9}$$

where $J(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u}) = J_1(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u}) + J_2(\mathbf{u})$ and

$$\Omega \triangleq \left\{ \mathbf{u} = [u_1 \cdots u_m]^T \,\middle|\, \underline{u}_i \le u_i \le \bar{u}_i, i = 1,2,\cdots,m \right\} \tag{10}$$

with $\underline{u}_i$ and $\bar{u}_i$ $(i = 1,2,\cdots,m)$ being the lower and upper control limits, respectively.

Define

$$\Delta(\mathbf{u}) = [S(u_1) \;\; S(u_2) \;\; \cdots \;\; S(u_m)] \tag{11}$$

with

$$S(u_i) = \min((u_i - \underline{u}_i)^3, (\bar{u}_i - u_i)^3, 0), i = 1,2,\cdots,m \tag{12}$$

Then the constraint condition $\mathbf{u} \in \Omega$ is equivalent to

$$\Delta(\mathbf{u}) = 0 \tag{13}$$

By introducing the Lagrangian

$$L(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u},\lambda) = J(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u}) + \Delta(\mathbf{u})\lambda \tag{14}$$

where $\lambda \in \mathbb{R}^m$ is a Lagrange multiplier, the optimization problem (9) is reformulated as

$$\min_{\mathbf{u},\lambda} L(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u},\lambda) \tag{15}$$

The following additional assumption is made:

Assumption 4: There exists a constant $\gamma_1 > 0$ such that $\frac{\partial^2 L}{\partial \mathbf{u}^2} \ge \gamma_1 \mathbf{I}_m$, $\forall \mathbf{u} \in \Omega$.

With Assumptions 1 and 4, the following lemma is immediate ([12], p. 42).

Lemma 1: If Assumptions 1 and 4 hold, the problem (15) achieves local minima if and only if $\frac{\partial L}{\partial \lambda} = 0$ and $\frac{\partial L}{\partial \mathbf{u}} = 0$.

Remark 1: It should be noted that Assumption 4 is not a very strong condition, as it is satisfied by all control-affine nonlinear systems. Furthermore, Lemma 1 holds for global minima for such systems.

Define

$$V_m(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u},\lambda)=\frac{1}{2}\left[\left(\frac{\partial L}{\partial \mathbf{u}}\right)^T\frac{\partial L}{\partial \mathbf{u}}+\left(\frac{\partial L}{\partial \lambda}\right)^T\frac{\partial L}{\partial \lambda}\right] \quad (16)$$

The Lyapunov-like function (16) is designed to attract $(\mathbf{u},\lambda)$ to minimize L for all reference $\mathbf{r}$ and estimate $\hat{\vartheta}$.

3. Adaptive Parameter Estimation

Section 2 presents the idea of the control allocation based on the estimate of the unknown parameter ϑ in the nonlinear uncertain system (1). In this section, we will discuss how to estimate the unknown parameter ϑ.

Assume that the states $\mathbf{x}$ and $\mathbf{z}$, and control input $\mathbf{u}$ of the system (1) are available for measurement. We define the serial-parallel model [10] as

$$\begin{cases} \dot{\hat{\mathbf{x}}} = f(\mathbf{x},\mathbf{z}) + g(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} + \mathbf{A}_{\mathbf{x}}(\mathbf{x}-\hat{\mathbf{x}}) \\ \dot{\hat{\mathbf{z}}} = h(\mathbf{x},\mathbf{z}) + k(\mathbf{x},\mathbf{z},\mathbf{u})\hat{\vartheta} + \mathbf{A}_{\mathbf{z}}(\mathbf{z}-\hat{\mathbf{z}}) \end{cases} \quad (17)$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{z}}$ and $\hat{\vartheta}$ are the estimates of $\mathbf{x}$, $\mathbf{z}$ and ϑ as in (1), and $\mathbf{A}_{\mathbf{x}} > 0$ and $\mathbf{A}_{\mathbf{z}} > 0$ are positive definite diagonal matrices. Define the estimation error vectors $\tilde{\vartheta}$ and ϵ as

$$\tilde{\vartheta} \triangleq \vartheta - \hat{\vartheta}, \quad \epsilon \triangleq \begin{bmatrix} \epsilon_{\mathbf{x}} \\ \epsilon_{\mathbf{z}} \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{x}-\hat{\mathbf{x}} \\ \mathbf{z}-\hat{\mathbf{z}} \end{bmatrix} \quad (18)$$

and denote

$$\mathbf{A}_\epsilon \triangleq \begin{bmatrix} \mathbf{A}_{\mathbf{x}} & 0 \\ 0 & \mathbf{A}_{\mathbf{z}} \end{bmatrix}, \; \mathcal{G}(\mathbf{x},\mathbf{z},\mathbf{u}) \triangleq \begin{bmatrix} g(\mathbf{x},\mathbf{z},\mathbf{u}) \\ k(\mathbf{x},\mathbf{z},\mathbf{u}) \end{bmatrix} \quad (19)$$

From (1) and (17), we obtain the error system as follows

$$\dot{\epsilon} = \mathcal{G}(\mathbf{x},\mathbf{z},\mathbf{u})\tilde{\vartheta} - \mathbf{A}_\epsilon \epsilon \quad (20)$$

It can be seen that the eigenvalues of $-\mathbf{A}_\epsilon$ determine the rate of convergence of ϵ.

The essential idea behind the on-line parameter estimation is to adjust the parameter vector $\hat{\vartheta}$ continually so that ϵ approaches zero as time increases.

By collecting the $\mathbf{x}$ and $\mathbf{z}$ dynamics in (1), the ϵ dynamics in (20), $\dot{t}=1$, and $\dot{\vartheta}=0$, as well as $\dot{\mathbf{u}}$, $\dot{\lambda}$ and $\dot{\hat{\vartheta}}$ (which are given by (28) and (29) later) together, we have the following augmented time-invariant system

$$\dot{\zeta} = F(\zeta), \;\; \zeta(0) = \zeta_0 \quad (21)$$

with $\zeta = [t,\mathbf{x},\mathbf{z},\epsilon,\vartheta,\hat{\vartheta},\mathbf{u},\lambda]^T \in \mathbb{R}^q$. The inclusion of time t as a state in ζ is to allow $\mathbf{r}$ in (3) to be any continuously differentiable time function and convert the original time-varying system into the time-invariant system (21).

Now we introduce the adaptive Lyapunov-like function

$$V(\zeta) = V_m(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u},\lambda) + \frac{1}{2}\tilde{\vartheta}^T\mathbf{Q}\tilde{\vartheta} + \frac{1}{2}\epsilon^T\mathbf{Q}_\epsilon\epsilon \quad (22)$$

where the symmetric $\mathbf{Q} > 0$ and diagonal $\mathbf{Q}_\epsilon > 0$ are known weighting matrices. The first term $V_m(\mathbf{x},\mathbf{z},\mathbf{r},\hat{\vartheta},\mathbf{u},\lambda)$ as defined in (16) corresponds to the necessary and sufficient condition to achieve local minima of (15). The remaining terms form a standard Lyapunov-like function for adaptive on-line parameter estimation where $\tilde{\vartheta}$ and ϵ are required to converge to zero.

4. Main Results

Before the main result, we present several definitions related to set-stability as follows.

Definition 1: [13]: For the continuous system (21), introduce a closed positively invariant set

$$\mathcal{A} := \{\zeta \in \mathbb{R}^q | V(\zeta) = 0\} \quad (23)$$

with $V(\zeta)$ as in (22). The distance from a point $\zeta \in \mathbb{R}^q$ to the set $\mathcal{A}$ is defined by

$$|\zeta|_{\mathcal{A}} := \inf_{y\in\mathcal{A}} \|\zeta - y\| \quad (24)$$

Definition 2: [14]: The closed positively invariant set $\mathcal{A}$ of the time-invariant system (21) is stable if, for each $\eta > 0$, there is $\delta > 0$ such that

$$|\zeta_0|_{\mathcal{A}} < \delta \Longrightarrow |\zeta|_{\mathcal{A}} < \eta, \;\; \forall t \geq 0 \quad (25)$$

Definition 3: **(Persistence of Excitation (PE))** The continuous signal matrix $\mathcal{G}(\mathbf{x}(t),\mathbf{z}(t),\mathbf{u}(t)) \in \mathbb{R}^{(n_{\mathbf{x}}+n_{\mathbf{z}})\times r}$ as in (19) is PE with a level of excitation $\gamma_0 > 0$ over the time interval $[t_1,t_2]$ with $t_2 > t_1 \geq 0$, if

$$\mathcal{G}^T\mathcal{G} \geq \gamma_0 \mathbf{I}_r, \quad \forall t \in [t_1,t_2] \quad (26)$$

Denote

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=\begin{bmatrix}\dfrac{\partial^2 L}{\partial \mathbf{u}^2} & \dfrac{\partial^2 L}{\partial\lambda\partial \mathbf{u}}\\ \dfrac{\partial^2 L}{\partial \mathbf{u}\partial\lambda} & \mathbf{0}_{m\times m}\end{bmatrix}\begin{bmatrix}\dfrac{\partial L}{\partial \mathbf{u}}\\ \dfrac{\partial L}{\partial \lambda}\end{bmatrix},\quad \Xi=\begin{bmatrix}\left(\dfrac{\partial^2 L}{\partial \mathbf{x}\partial \mathbf{u}}\right)^T\\ \left(\dfrac{\partial^2 L}{\partial \mathbf{z}\partial \mathbf{u}}\right)^T\end{bmatrix} \quad (27)$$

Lemma 2: Under Assumptions 1 and 4, $\alpha=\beta=0$ if and only if $\dfrac{\partial L}{\partial \mathbf{u}}=\dfrac{\partial L}{\partial \lambda}=0$.

Proof: It is omitted due to limited space.

Now we are ready to state the main result of this paper.

Theorem 1: Consider the system (1), (3) and (20) and Assumptions 1-4. For given positive definite matrices $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, Γ_1, Γ_2 and $\mathbf{Q}$, positive definite diagonal matrices $\mathbf{Q}_\epsilon$ and $\mathbf{A}_\epsilon$, and positive constant ω, the set $\mathcal{A}$ as in (23) is closed positively invariant and stable, the closed-loop nonlinear system is input-to-state stable, and $\left(\dfrac{\partial L}{\partial \mathbf{u}},\dfrac{\partial L}{\partial \lambda},\epsilon\right)\to 0$ as $t\to\infty$ if the dynamic update law

$$\begin{cases}\dot{\mathbf{u}}=-\Gamma_1\alpha+\xi_1\\ \dot{\lambda}=-\Gamma_2\beta+\xi_2\end{cases} \quad (28)$$

and adaptive law

$$\dot{\hat{\vartheta}}=\mathbf{Q}^{-1}\mathcal{G}^T\left(\Xi\frac{\partial L}{\partial \mathbf{u}}+\mathbf{Q}_\epsilon\epsilon\right) \quad (29)$$

are adopted. Here $L\in\mathbb{R}$, $\Xi\in\mathbb{R}^{(n_\mathbf{x}+n_\mathbf{z})\times m}$, $\epsilon\in\mathbb{R}^{n_\mathbf{x}+n_\mathbf{z}}$ and $\mathcal{G}\in\mathbb{R}^{(n_\mathbf{x}+n_\mathbf{z})\times r}$ are as in (14), (27), (18) and (19), respectively, $\alpha\in\mathbb{R}^m$ and $\beta\in\mathbb{R}^m$ are as in (27), and ξ_1, $\xi_2\in\mathbb{R}^m$ satisfy

$$\alpha^T\xi_1+\beta^T\xi_2+\delta+\omega V_m=0 \quad (30)$$

with V_m as in (16) and

$$\begin{aligned}\delta=&\left(\frac{\partial L}{\partial \mathbf{u}}\right)^T\frac{\partial^2 L}{\partial \mathbf{x}\partial \mathbf{u}}\Big[f+g\hat{\vartheta}\Big]+\left(\frac{\partial L}{\partial \mathbf{u}}\right)^T\frac{\partial^2 L}{\partial \mathbf{r}\partial \mathbf{u}}\dot{\mathbf{r}}\\ &+\left(\frac{\partial L}{\partial \mathbf{u}}\right)^T\frac{\partial^2 L}{\partial \mathbf{z}\partial \mathbf{u}}\Big[h+k\hat{\vartheta}\Big]+\left(\frac{\partial L}{\partial \mathbf{u}}\right)^T\frac{\partial^2 L}{\partial\hat{\vartheta}\partial \mathbf{u}}\dot{\hat{\vartheta}}\end{aligned} \quad (31)$$

Furthermore, if the matrix $\mathcal{G}(\mathbf{x},\mathbf{z},\mathbf{u})$ as in (19) is PE for all $t\in[t_1,\infty)$ with $t_1\geq 0$, then the estimate $\hat{\vartheta}\to\vartheta$ as $t\to\infty$.

Proof: It is omitted due to limited space.

To solve (30) for ξ_1 and ξ_2, one method is to solve a least-square problem subject to (30). This leads to the Lagrangian

$$l(\xi_1,\xi_2,\nu)=\frac{1}{2}(\xi_1^T\xi_1+\xi_2^T\xi_2)+\nu(\alpha^T\xi_1+\beta^T\xi_2+\delta+\omega V_m) \quad (32)$$

where $\nu\in\mathbb{R}$ is a Lagrange multiplier. The first order optimality conditions

$$\frac{\partial l}{\partial \xi_1}=0,\ \frac{\partial l}{\partial \xi_2}=0,\ \frac{\partial l}{\partial \nu}=0 \quad (33)$$

lead to the following system of linear equations

$$\begin{bmatrix}I_m & 0 & \alpha\\ 0 & I_m & \beta\\ \alpha^T & \beta^T & 0\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\\ \nu\end{bmatrix}=\begin{bmatrix}0\\ 0\\ -\delta-\omega V_m\end{bmatrix} \quad (34)$$

Remark 2: It is noted that Equation (34) always has a unique solution for ξ_1 and ξ_2 if any one of α and β is nonzero. If $\alpha=0$ and $\beta=0$ under Assumptions 1 and 4, $\xi_1=\xi_2=0$ defines the solution.

5. EXAMPLE

In this section, we use a prototype planar vertical takeoff and landing (PVTOL) aircraft to demonstrate the proposed approach. The aircraft equations (see [9] for more details) are given by

$$\begin{cases}\ddot{y}=-u_1\vartheta_1\sin\phi+\epsilon_c u_2\vartheta_2\cos\phi\\ \ddot{z}=u_1\vartheta_1\cos\phi+\epsilon_c u_2\vartheta_2\sin\phi-1\\ \ddot{\phi}=u_2\vartheta_2\end{cases} \quad (35)$$

where y and z represent the normalized position of the aircraft center of mass. ϕ represents the roll angle. The control inputs u_1 and u_2 represent the normalized thrust and roll moment, respectively. As the aircraft model (35) is normalized, all state and input variables are dimensionless. "-1" is the normalized gravitational acceleration and ϵ_c is the coupling coefficient between the roll moment and the lateral acceleration of the aircraft. ϑ_1 and ϑ_2 are unknown parameters which might be used to represent the efficiency of the control inputs u_1 and u_2, respectively. Here, we set $\epsilon_c=0.5$ which makes the system strongly non-minimum phase [9].

When considering the altitude z as the controlled output, system (35) can be written in the form of equation (1)

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ u_1\vartheta_1 \cos z_3 + \epsilon_c u_2 \vartheta_2 \sin z_3 \end{bmatrix} \\ \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ 0 \\ z_4 \\ 0 \end{bmatrix} + \begin{bmatrix} w_y \\ -u_1\vartheta_1 \sin z_3 + \epsilon_c u_2 \vartheta_2 \cos z_3 \\ 0 \\ u_2\vartheta_2 \end{bmatrix} \end{cases}$$

where $\mathbf{x} = [x_1, x_2]^T = [z, \dot{z}]^T$ is the commanded state vector, $\mathbf{z} = [z_1, z_2, z_3, z_4]^T = [y, \dot{y}, \phi, \dot{\phi}]^T$ is the internal state vector, and w_y is the lateral gust disturbance. We impose the control constraints:

$$|u_1| \leq 1.2, \quad |u_2| \leq 1.4 \tag{36}$$

Reference Model Parameters: In this example, the stable reference model is given as in (3) with

$$\mathbf{A}_d = \begin{bmatrix} 0 & 1 \\ -64 & -16 \end{bmatrix}, \quad \mathbf{B}_d = \begin{bmatrix} 0 \\ 64 \end{bmatrix}$$

$$\mathbf{A}_s = \begin{bmatrix} 0 & 1.0000 & 0 & 0 \\ 0.6427 & 2.1726 & -4.3131 & -2.2408 \\ 0 & 0 & 0 & 1.0000 \\ 1.2854 & 4.3452 & -6.6262 & -4.4816 \end{bmatrix}$$

Control Parameters: Based on the guidelines given in Sections 2-4, the following control parameters are chosen: $\mathbf{H}_1 = \mathbf{I}_2$, $\mathbf{H}_2 = \mathbf{I}_4$, $\mathbf{H}_3 = 10^{-6}\mathbf{I}_2$, $\mathbf{Q} = \mathbf{I}_2$, $\mathbf{Q}_\epsilon = diag\{1, 1, 1, 1, 1, 300\}$, $\mathbf{A}_\mathbf{z} = 20\mathbf{I}_4$, $\mathbf{A}_\mathbf{x} = 4\mathbf{I}_2$, $\Gamma_1 = 2\mathbf{I}_2$, $\Gamma_2 = 20\mathbf{I}_2$ and $\omega = 300$.

Persistence of Excitation: In this example, we have

$$\mathcal{G}^T\mathcal{G} = \begin{bmatrix} u_1^2 & 0 \\ 0 & \epsilon_c^2 u_2^2 \end{bmatrix}$$

According to Theorem 1, the estimate $\hat{\vartheta} \to \vartheta$ as $t \to \infty$ if the PE condition (26) is satisfied (i.e., if $u_1 \neq 0$ and $u_2 \neq 0$).

Simulation: Set the initial position of the aircraft as $y(0) = 0.05$, $z(0) = 0.05$, the initial roll angle as $\phi(0) = 0.1rad$ and the initial control efficiency parameter estimates $\hat{\vartheta}_1(0) = 1$ and $\hat{\vartheta}_2(0) = 1$. In this simulation, there is no fault in PVTOL aircraft during the first 20 seconds, i.e. $\vartheta_1 = \vartheta_2 = 1$. However, from the 20th second onwards, the u_1 actuator loss of effectiveness occurs with $\vartheta_1 = 0.9$ and from the 25th second onwards, the u_2 actuator loss of effectiveness occurs with $\vartheta_2 = 0.25$. In this simulation, the reference input $\mathbf{r}$ is given by

$$\mathbf{r} = \begin{cases} r_f\left[6(\frac{t}{t_1})^5 - 15(\frac{t}{t_1})^4 + 10(\frac{t}{t_1})^3\right], \; 0 \leq t < t_1 \\ r_f, \; t_1 \leq t < t_2 \\ -r_f\left[6(\frac{t-t_2}{t_f-t_2})^5 - 15(\frac{t-t_2}{t_f-t_2})^4 + 10(\frac{t-t_2}{t_f-t_2})^3\right] + r_f, \; t_2 \leq t < t_f \\ 0, \; t \geq t_f \end{cases}$$

with $t_1 = 10s$, $t_2 = 20s$, $t_f = 30s$ and $r_f = 1$, and the lateral gust disturbance w_y is described by

$$w_y = \begin{cases} 0, & t < 30s \\ 0.15, & 30 \leq t < 32s \\ 0, & 32 \leq t < 34s \\ -0.15, & 34 \leq t < 36s \\ 0, & t \geq 36s \end{cases} \tag{37}$$

Using the adaptive control allocation law designed by the proposed approach, we obtain the simulation results as shown in Figure 1. From Figure 1, it is observed that both control u_1 and u_2 are kept within the given control constraints $|u_1| \leq 1.2$ and $|u_2| \leq 1.4$. This indicates that the proposed approach can handle control constraints very well. It is also shown that z and $\dot{z}$ track r and $\dot{r}$ well, which indicates that the control allocation function works well. In addition, the simulation results in Figure 1 show that the internal states y, $\dot{y}$, ϕ and $\dot{\phi}$ are stabilized, which manifests that the unstable internal dynamics can be stabilized by the proposed approach. Moreover, from the response of $\hat{\vartheta}_1$ in Figure 1, we can see that $\hat{\vartheta}_1$ converges to its true value of $\vartheta_1 = 1$ and keeps this value until the failure of the actuator u_1 occurs at the 20th second. Then $\hat{\vartheta}_1$ converges to its new true value of $\vartheta_1 = 0.9$. Similar behavior can be observed from the $\hat{\vartheta}_2$ response, except for the following: First, although the failure of the actuator u_2 occurs at the 25th second with the new true value of $\vartheta_2 = 0.25$, $\hat{\vartheta}_2$ remains unchanged until the 30th second when the lateral gust wind w_y as in (37) is imposed on the PVTOL aircraft. Then, from the 30th second on, $\hat{\vartheta}_2$ converges to the true value of $\vartheta_2 = 0.25$. This is because that in this particular simulation, u_1 satisfies the PE condition ($u_1 \neq 0$) all the time while u_2 satisfies the condition ($u_2 \neq 0$) during $0 \sim 5s$ and $30 \sim 42s$ only.

Fortunately, the non-convergence of $\hat{\vartheta}_2$ has no effect on the responses of the system states as at this time $u_2 = 0$.

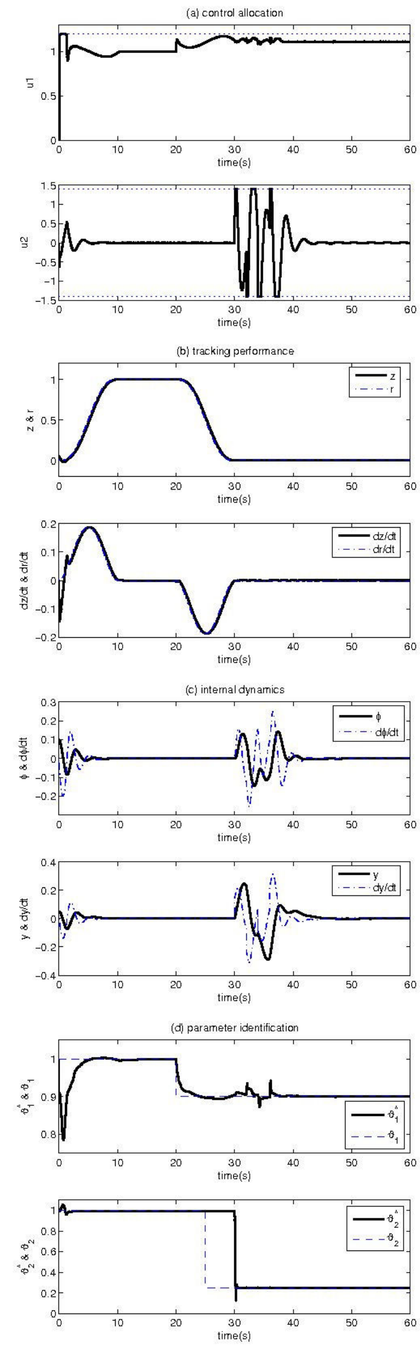

Fig. 1. Responses of PVTOL

6. CONCLUSION

In this paper, an adaptive control allocation approach is proposed for a general class of nonlinear systems with unstable internal dynamics and unknown parameters. The proposed allocation approach is based on Lyapunov design approach and indirect adaptive control approach. The derived allocation law guarantees the closed-loop stability and satisfies control constraints. The PVTOL aircraft example demonstrates the effectiveness of the proposed approach.

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