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Self-tuning control of dual-rate systems with input nonlinearities

Yongsong Xiao, Huibo Chen, Jiyang Dai, Feng Ding

***Abstract*— A polynomial transformation technique is used to obtain a model for a dual-rate nonlinear system in which the output sampling interval is an integer multiple of the control interval. Based on this model, a self-tuning control algorithm is presented by minimizing output tracking error criteria from directly the dual-rate measurement data. The self-tuning algorithm proposed can achieve virtually asymptotically optimal control and ensure the closed-loop system to be stable and globally convergent. The proposed algorithm is illustrated by examples.**

I. Problem Description

HAMMERSTEIN nonlinear systems are a class of input nonlinear ones which are characterized by static nonlinearities $f(\cdot)$ followed by linear dynamic blocks $G(z)$ [1]–[3], as depicted in Figure 1, where $u(k)$ and $y(k)$ denote the system input and output, respectively, $\hat{y}(iq+j)$ the estimated intersample output, $y_r(k)$ a deterministic reference input or desired output signal, $y_f(k)$ the feedback signal and $\hat{\boldsymbol{\theta}}$ the estimate of the parameters of $G(z)$. In general, the static

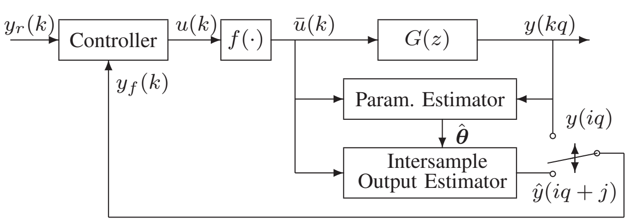

Fig. 1. The self-tuning control scheme ($j = 1, 2, \cdots, q-1$)

nonlinear part in the Hammerstein model is assumed to be a polynomial of a known order m in the input $u(k)$ as follow, e.g., [4],

$$\bar{u}(k) = f(u(k)) = c_1 u(k) + c_2 u^2(k) + \cdots + c_m u^m(k), \quad (1)$$

the linear block has the following dynamic transfer function in a unit backward shift operator z^{-1} $[z^{-1}u(k) = u(k-1)]$ with the known order n, $G(z) := B(z)/A(z)$ with

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n},$$
$$B(z) = b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}.$$

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Y.S. Xiao, H.B. Chen and F. Ding are with the Control Science and Engineering Research Center, Jiangnan University, Wuxi, P. R. China 214122. ys_xiao82@163.com (Y.S. Xiao); fding@jiangnan.edu.cn (F. Ding).

J.Y. Dai is with the Key Laboratory of Nondestructive Test (Ministry of Education), Nanchang, Nanchang Hangkong University, P.R. China 330063. djiyang@163.com

The systems with two or more different operation frequencies are called dual-rate or multirate sampled-data systems. Multirate/dual-rate systems arise in many industry applications, if the output is sampled at a slower rate than the control updating rate, then we get a dual-rate system. For such a dual-rate sampled-data system, the input data $\{u(k),\ k = 0, 1, 2, \cdots\}$ are available at each time k, but only scarce output data $\{y(kq) :\ k = 0, 1, 2, \cdots\}$ are available (q being a positive integer). We refer to the unavailable intersample outputs, $y(kq+j)$, $j = 1, 2, \cdots, q-1$, as the *missing* output samples, and to $\{u(k), y(kq)\}$ as the *dual-rate* measurement data. It is obvious that the conventional adaptive control method is not suitable for this dual-rate case directly.

In the literature, Albertos *et al*. discussed various adaptive control schemes for dual-rate systems [5]; Ishitobi *et al.* presented a least squares based adaptive control algorithm [6]; Ding and Chen presented a self-tuning method for dual-rate sampled-data systems by using the polynomial transform technique [7], [8]; and Patete *et al* considered the self-tuning control problem of minimum or non-minimum phase auto-regressive models with constant but unknown parameters [9].

This paper focuses on the self-tuning control problem of *nonlinear* dual-rate systems. The control scheme we propose is shown in Figure 1 and consists of two estimators and a controller: an estimator generating the estimate $\hat{\boldsymbol{\theta}}$ of the unknown system parameters online based on the dual-rate data $\{u(k), y(kq)\}$, an estimator computing the intersample (missing) outputs $\hat{y}(iq+j)$ using the obtained $\hat{\boldsymbol{\theta}}$ and system input $u(k)$ in order to provide a feedback signal $y_f(k)$ to the controller. That is, $y_f(k)$ connects to $y(iq)$ at times $k = iq$, and connects to $\hat{y}(iq+j)$ at $k = iq+j, j = 1, 2, \cdots, (q-1)$. This operation can be expressed in the following equation:

$$y_f(k) = \begin{cases} y(iq), & k = iq, \\ \hat{y}(iq+j), & k = iq+j,\ j = 1, \cdots, (q-1). \end{cases} \quad (2)$$

In a word, the dual-rate self-tuning control scheme here also produce a fast-rate feedback signal for controller when the intersample output is unavailable. It is easy to implement in digital computers, and practical for industry.

The objective of this paper is to design a self-tuning control algorithm so as the output $y(k)$ tracks the desired output $y_r(k)$ by minimizing the tracking error criterion function given by

$$J[u(k)] = \mathrm{E}\{[y_f(k+1) - y_r(k+1)]^2 | \mathcal{F}_{k-1}\} \quad (3)$$

and study the properties of the closed-loop system. Here, $\{\mathcal{F}_k\}$ is the σ algebra sequence generated by the observations up to and including time k.

Briefly, the paper is organized as follows. Section II present self-tuning control algorithms based on the polynomial transform technique. Sections III and IV analyze the output tracking performance and global stability of the closed-loop systems of the self-tuning control proposed. Section V gives two illustrative examples. Finally, Section VI offers some concluding remarks.

II. Control algorithm derivations

From Figure 1, we have

$$y(k)=\frac{B(z)}{A(z)}\,\bar{u}(k),\ \bar{u}(k)=f(u(k)). \tag{4}$$

The model in (4) is not suitable for dual-rate self-tuning control because it would involve the unavailable outputs. To obtain a model that we can use directly on the dual-rate data, by a polynomial transform technique, $G(z)$ can be converted into a form so that the denominator is a polynomial in z^{-q} instead of z^{-1}.

For a general discussion, let the roots of $A(z)$ be z_i to get

$$A(z)=\prod_{i=1}^{n}(1-z_iz^{-1}).$$

Define a polynomial

$$\phi_q(z):=\prod_{i=1}^{n}(1+z_iz^{-1}+z_i^2z^{-2}+\cdots+z_i^{q-1}z^{-q+1}).$$

Multiplying the numerator and denominator of $G(z)$ by $\phi_q(z)$, we get a new model:

$$P(z)=\frac{B(z)\phi_q(z)}{A(z)\phi_q(z)}=:\frac{\beta(z)}{\alpha(z)}, \tag{5}$$

with

$$\alpha(z)=1+\alpha_1z^{-q}+\alpha_2z^{-2q}+\cdots+\alpha_nz^{-qn},$$
$$\beta(z)=\beta_1z^{-1}+\beta_2z^{-2}+\cdots+\beta_{qn}z^{-qn}.$$

So we have

$$y(k)=-\sum_{i=1}^{n}\alpha_iy(k-iq)+\sum_{i=1}^{qn}\beta_if(u(k-i)). \tag{6}$$

Define the parameter vector $\boldsymbol{\theta}$ and information vector $\boldsymbol{\varphi}(k)$

$$\begin{aligned}\boldsymbol{\theta}&=[\alpha_1,\cdots,\alpha_n,\beta_1,\cdots,\beta_{qn}]^{\mathrm{T}}\in\mathbb{R}^{n_0},\\ \boldsymbol{\varphi}(k-1)&=[-y(k-q),-y(k-2q),\cdots,-y(k-nq),\\ &\qquad f(u(k-1)),\cdots,f(u(k-nq))]^{\mathrm{T}}\in\mathbb{R}^{n_0},\\ n_0&:=(q+1)n.\end{aligned}$$

Then we have

$$y(k)=\boldsymbol{\varphi}^{\mathrm{T}}(k-1)\boldsymbol{\theta}. \tag{7}$$

Let $y_r(k)$ be a desired output signal; define the output tracking error$\xi(k+1)=y(k+1)-y_r(k+1)$. In the deterministic case, if the control signal $u(k)$ is chosen according to the equation $y_r(k+1)=\boldsymbol{\varphi}^{\mathrm{T}}(k)\boldsymbol{\theta}$ obtained by minimizing the criterion function in (3), then the tracking error $\xi(k+1)$ approaches zero asymptotically.

Based on the model in (7), introducing a zero-mean white noise disturbance term $v(k)$, we have

$$y(k)=\boldsymbol{\varphi}^{\mathrm{T}}(k-1)\boldsymbol{\theta}+v(k). \tag{8}$$

Let $\hat{\boldsymbol{\theta}}$ be the estimate of unknown parameter vector $\boldsymbol{\theta}$, then $\hat{y}(k+1)=\boldsymbol{\varphi}^{\mathrm{T}}(k)\hat{\boldsymbol{\theta}}$ is the output prediction, which is computed by the intersample output estimator in Figure 1. According to the certainty equivalence principle [12] or minimizing the criterion function in (3), the control law takes the following form:

$$y_r(k+1)=\boldsymbol{\varphi}^{\mathrm{T}}(k)\hat{\boldsymbol{\theta}}. \tag{9}$$

Replacing k in (8) by kq gives

$$y(kq)=\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{\theta}+v(kq). \tag{10}$$

Then the recursive least squares algorithm may be used to produce the estimate $\hat{\boldsymbol{\theta}}(kq)$ of $\boldsymbol{\theta}$ at current time kq, and the algorithm is as follows:

$$\begin{aligned}\hat{\boldsymbol{\theta}}(kq)&=\hat{\boldsymbol{\theta}}(kq-q)+\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)\\ &\qquad[y(kq)-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\hat{\boldsymbol{\theta}}(kq-q)], &(11)\\ \boldsymbol{P}^{-1}(kq)&=\boldsymbol{P}^{-1}(kq-q)+\boldsymbol{\varphi}(kq-1)\boldsymbol{\varphi}^{\mathrm{T}}(kq-1), &(12)\\ \hat{\boldsymbol{\theta}}(i)&=\hat{\boldsymbol{\theta}}(kq),\ i=kq,kq+1,\cdots,kq+q-1, &(13)\\ \hat{\boldsymbol{\theta}}(kq)&=[\hat{\alpha}_1(kq),\cdots,\hat{\alpha}_n(kq),\hat{\beta}_1(kq),\cdots,\hat{\beta}_{nq}(kq)]^{\mathrm{T}} &(14)\end{aligned}$$

Based on (9), the control law is given by

$$\boldsymbol{\varphi}^{\mathrm{T}}(kq+j)\hat{\boldsymbol{\theta}}(kq)=y_r(kq+1+j),\ j=0,1,\cdots,q-1. \tag{15}$$

To initialize the control algorithm in (11)-(15), we take $\boldsymbol{P}(0)=p_0\boldsymbol{I}$ with p_0 normally a large positive number, e.g., $p_0=10^6$ and $\hat{\theta}(0)=\hat{\theta}_0$, some small real vector, e.g., $\hat{\theta}(0)=\mathbf{1}_{n_0}/p_0$.

The control signal $u(k)$ is computed by the past inputs $u(k-j)$ for $j=1,2,\cdots$, the current output $y(k)$ $(\hat{y}(k))$, past outputs $y(k-j)$ $(\hat{y}(k-j))$ for $j=1,2,\cdots$, and desired output $y_r(k+1)$ as k increases, and the input $u(k)$ is made to drive the system output at time $k+1$ to the target value $y_r(k+1)$.

Then, in our self-tuning control algorithm in (11)-(15), based on the parameter estimation, we can get the control signal $u(kq+j)$ in (15) from the following equation:

$$\begin{aligned}f(u(kq+j))&=\sum_{i=1}^{n}\hat{\alpha}_i(kq)y(kq+j+1-iq)+\\ &\quad y_r(kq+j+1)-\sum_{i=2}^{nq}\hat{\beta}_i(kq)f(u(kq+j+1-i)).\end{aligned} \tag{16}$$

Here, a difficulty arises because over the interval $[kq,\ kq+q)$, except for $j=q-1$, the above expression contains the future and past missing outputs $y(kq+j+1-iq)$. So it looks impossible to compute the control law by (16) and to realize the algorithm in (11)-(15). Our solution is based on the self-tuning control scheme stated in Section II – these unknown outputs $y(kq+j)$ in (16) are replaced by their estimates $\hat{y}(kq+j)$. So we have

$$f(u(kq+j)) = \sum_{i=1}^{n} \hat{\alpha}_i(kq)\hat{y}(kq+j+1-iq) + y_r(kq+j+1) - \sum_{i=2}^{nq} \hat{\beta}_i(kq) f(u(kq+j+1-i)). \quad (17)$$

In fact, only when $j = q-1$, the control term $u(kq+j)$ does not involve the missing outputs, and can be generated by

$$f(u(kq+q-1)) = \frac{1}{\hat{\beta}_1(kq)} \left[\sum_{i=1}^{n} \hat{\alpha}_i(kq)\hat{y}(kq+q-iq) + y_r(kq+q) - \sum_{i=2}^{nq} \hat{\beta}_i(kq) f(u(kq+q-i)) \right]. \quad (18)$$

From this equation, we can compute the control signal $u(k)$ by the inverse function $f^{-1}(\cdot)$.

III. Output tracking performance

Let us first introduce some definitions and assumptions. The sequence $\{v(k), \mathcal{F}_k\}$ is assumed to be a martingale difference sequence defined on a probability space $\{\Omega, \mathcal{F}, P\}$, where $\{\mathcal{F}_k\}$ is the σ algebra sequence generated by the observations up to and including time k [12]. The noise sequence $\{v(k)\}$ satisfies the following conditions:

$$(A1) \qquad \mathrm{E}[v(k)|\mathcal{F}_{k-1}] = 0, \text{ a.s.;}$$
$$(A2) \qquad \mathrm{E}[v^2(k)|\mathcal{F}_{k-1}] = \sigma^2(k) \leqslant \bar{\sigma}^2 < \infty, \text{ a.s.;}$$
$$(A3) \qquad \limsup_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} v^2(i) \leqslant \bar{\sigma}^2 < \infty, \text{ a.s.}$$

That is, $\{v(k)\}$ is an independent random noise sequence with zero mean and bounded time-varying variance.

Define

$$r(kq) := \mathrm{tr}[P^{-1}(kq)], \; r(0) := n_0/p_0.$$

It follows easily that

$$r(kq) \leqslant n_0\lambda_{\max}[P^{-1}(kq)], \quad \ln|P^{-1}(kq)| = O(\ln r(kq)),$$
$$|P^{-1}(kq)| \leqslant \lambda_{\max}^{n_0}[P^{-1}(kq)] \leqslant r^{n_0}(kq). \quad (19)$$

In order to study the output tracking performance of the self-tuning control algorithm proposed earlier, the following lemma is required.

Lemma 1: For the algorithm in (11)-(15), the following inequality holds:

$$\sum_{i=1}^{\infty} \frac{\boldsymbol{\varphi}^{\mathrm{T}}(iq-1)\boldsymbol{P}(iq)\boldsymbol{\varphi}(iq-1)}{\{\ln r(iq)\}^c} < \infty, \text{ a.s., for any } c > 1.$$

Proof can be done in a similar way in [11] and is omitted here.

We shall prove the main results of this paper by formulating a martingale process and by using stochastic process theory and the martingale convergence theorem (Lemma D.5.3 in [12]).

Theorem 1: For the system in (10), assume that (A1)-(A3) hold, $B(z)$ is stable, and the reference input $y_r(k)$ is bounded in the sense

$$(A4) \qquad |y_r(k)| < \infty.$$

Then the self-tuning control algorithm in (11)-(16) guarantees that the output tracking error at the output sampling instants has the property of minimum variance, i.e.,

1) $$\lim_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} [y_r(iq) - y(iq) + v(iq)]^2 = 0, \text{ a.s.;}$$

2) $$\limsup_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} \mathrm{E}\{[y_f(iq) - y_r(iq)]^2|\mathcal{F}_{iq-1}\} \leqslant \bar{\sigma}^2 < \infty, \text{ a.s.}$$

Proof Define the parameter estimation error vector as $\tilde{\boldsymbol{\theta}}(kq) = \hat{\boldsymbol{\theta}}(kq) - \boldsymbol{\theta}$. Using (10) and (11), we have

$$\begin{aligned}\tilde{\boldsymbol{\theta}}(kq) &= \tilde{\boldsymbol{\theta}}(kq-q) + \boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)[\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{\theta} \\ &\quad + v(kq) - \boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\hat{\boldsymbol{\theta}}(kq-q)] \\ &:= \tilde{\boldsymbol{\theta}}(kq-q) + \boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)[-\tilde{y}(kq) + v(kq)], \end{aligned} \quad (20)$$

where

$$\begin{aligned}\tilde{y}(kq) &:= \boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\tilde{\boldsymbol{\theta}}(kq-q) \\ &= \boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\hat{\boldsymbol{\theta}}(kq-q) - \boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{\theta}. \end{aligned} \quad (21)$$

By using (10) and (15), it follows that

$$\tilde{y}(kq) = y_r(kq) - y(kq) + v(kq).$$

Define a non-negative definite function

$$V(kq) = \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(kq)\boldsymbol{P}^{-1}(kq)\tilde{\boldsymbol{\theta}}(kq).$$

Using (10), (20) and (21), we have

$$\begin{aligned} & V(kq) \\ &= \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(kq-q)\boldsymbol{P}^{-1}(kq)\tilde{\boldsymbol{\theta}}(kq-q) \\ &\quad +2\tilde{\boldsymbol{\theta}}^{\mathrm{T}}(kq-q)\boldsymbol{\varphi}(kq-1)[-\tilde{y}(kq)+v(kq)] \\ &\quad +\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)[-\tilde{y}(kq)+v(kq)]^2 \\ &= \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(kq-q)[\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{\varphi}(kq-1) + \\ &\quad \boldsymbol{P}^{-1}(kq-q)]\tilde{\boldsymbol{\theta}}(kq-q) + 2\tilde{y}(kq)[-\tilde{y}(kq)+v(kq)] \\ &\quad +\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)[-\tilde{y}(kq)+v(kq)]^2 \\ &= V(kq-q) - [1-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)]\tilde{y}^2(kq) \\ &\quad +\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)v^2(kq) \\ &\quad +2[1-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)]\tilde{y}(kq)v(kq). \end{aligned}$$

Noting that $\tilde{y}(kq)$, $\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)$ are uncorrelated with $v(kq)$ and are $\mathcal{F}_{kq-1}$-measurable, taking the conditional expectation on both sides of the up equation with respect to $\mathcal{F}_{kq-1}$ and using (A1)-(A2) give

$$\begin{aligned}\mathrm{E}[V(kq)|\mathcal{F}_{kq-1}] &\leqslant 2\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)\bar{\sigma}^2 \\ &\quad +V(kq-q) - [1-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)]\tilde{y}^2(kq).\end{aligned}$$

Let

$$W(kq) := \frac{V(kq)}{[|\ln r(kq)|]^c}, \; c > 1.$$

Noting that $\ln r(kq)$ is non-decreasing, we have

$$\begin{aligned}
&\mathrm{E}[W(kq)|\mathcal{F}_{kq-1}] \\
&\leqslant \frac{V(kq-q)}{[\ln r(kq)]^c} - \frac{1-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)}{[\ln r(kq)]^c}\tilde{y}^2(kq) \\
&\quad + \frac{2\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)}{[\ln r(kq)]^c}\bar{\sigma}^2 \\
&\leqslant W(kq-q) - \frac{1-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)}{[\ln r(kq)]^c}\tilde{y}^2(kq) \\
&\quad + \frac{2\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)}{[\ln r(kq)]^c}\bar{\sigma}^2. \qquad (22)
\end{aligned}$$

In terms of Lemma 1, we can see that the sum of the last right-hand term of (22) for k from $k=1$ to $k=\infty$ is finite. Since

$$\begin{aligned}
&1-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1) \\
&=[1+\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq-q)\boldsymbol{\varphi}(kq-1)]^{-1} \geqslant 0,
\end{aligned}$$

applying the martingale convergence theorem (Lemma D.5.3 in [12]) to (22), we conclude that $W(kq)$ converges a.s. to a finite random variable, say, W_0; i.e.,

$$W(kq) = \frac{V(kq)}{[\ln r(kq)]^c} \to W_0 < \infty, \text{ a.s.},$$

and also

$$\sum_{k=1}^{\infty} \frac{1-\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1)}{[\ln r(kq)]^c}\tilde{y}^2(kq) < \infty, \text{ a.s.}$$

Due to $\boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{P}(kq)\boldsymbol{\varphi}(kq-1) \leqslant c$ with c being a constant less than unity [12], we have

$$\sum_{i=1}^{\infty} \frac{\tilde{y}^2(iq)}{[\ln r(iq)]^c} < \infty, \text{ a.s.} \qquad (23)$$

As $r(kq) \to \infty$, using the Kronecker lemma (Lemma D.5.5 in [12]) yields

$$\lim_{k\to\infty} \frac{1}{[\ln r(kq)]^c} \sum_{i=1}^{k} \tilde{y}^2(iq) = 0, \text{ a.s.}$$

Since $[\ln r(kq)]^c = o(r(kq))$, we have

$$\lim_{k\to\infty} \frac{k}{r(kq)} \frac{1}{k} \sum_{i=1}^{k} \tilde{y}^2(iq) = 0, \text{ a.s.} \qquad (24)$$

Since $B(z)$ is stable, applying Lemma B.3.3 in [12] to (10) and using (A3) yield

$$\frac{1}{k}\sum_{i=1}^{k} u^2(iq) \leqslant \frac{c_1}{k}\sum_{i=1}^{k} y^2(iq) + c_2, \text{ a.s.},$$

where c_i represent finite positive constants. According to the definitions of $r(kq)$ and $\varphi(kq)$, it is not difficult to get

$$\begin{aligned}
\frac{r(kq)}{k} &\leqslant \frac{c_3}{k}\sum_{i=1}^{k} y^2(iq) + c_4 \\
&= \frac{c_3}{k}\sum_{i=1}^{k} [y_r(iq) - \tilde{y}(iq) + v(iq)]^2 + c_4 \\
&\leqslant \frac{c_5}{k}\sum_{i=1}^{k} \tilde{y}^2(iq) + c_6, \text{ a.s.}
\end{aligned}$$

Thus, from (24)

$$0 = \lim_{k\to\infty} \frac{\frac{1}{k}\sum_{i=1}^{k} \tilde{y}^2(iq)}{\frac{r(kq)}{k}} \geqslant \lim_{k\to\infty} \frac{\frac{1}{k}\sum_{i=1}^{k} \tilde{y}^2(iq)}{\frac{c_5}{k}\sum_{i=1}^{k} \tilde{y}^2(iq) + c_6} \geqslant 0, \text{ a.s.},$$

and hence

$$\lim_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} [y_r(iq) - y(iq) + v(iq)]^2 = 0, \text{ a.s.} \qquad (25)$$

Since

$$\begin{aligned}
&\mathrm{E}\{[y_r(kq) - y(kq) + v(kq)]^2|\mathcal{F}_{kq-1}\} \\
&= \mathrm{E}[(y_r(kq) - y(kq))^2 + 2y_r(kq)v(kq) \\
&\quad -2y(kq)v(kq) + v^2(kq)|\mathcal{F}_{kq-1}] \\
&= \mathrm{E}[(y_r(kq) - y(kq))^2|\mathcal{F}_{kq-1}] + 0 - 2\sigma^2(kq) + \sigma^2(kq) \\
&= \mathrm{E}[(y_r(kq) - y(kq))^2|\mathcal{F}_{kq-1}] - \sigma^2(kq), \text{ a.s.},
\end{aligned}$$

and $y_f(kq) = y(kq)$ at the output sampling instants, we have

$$\begin{aligned}
&\limsup_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} \mathrm{E}\{[y_f(iq) - y_r(iq)]^2|\mathcal{F}_{iq-1}\} \\
&= \limsup_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} \mathrm{E}\{[y(iq) - y_r(iq)]^2|\mathcal{F}_{iq-1}\} \\
&= \limsup_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} \sigma^2(iq) \leqslant \bar{\sigma}^2, \text{ a.s.}
\end{aligned}$$

This proves Theorem 1. □

IV. Global convergence

Next, we analyze global stability of the self-tuning closed-loop system. From (2) and (10), we have

$$y_f(kq) = y(kq) = \boldsymbol{\varphi}^{\mathrm{T}}(kq-1)\boldsymbol{\theta} + v(kq), \qquad (26)$$

$$y_f(kq+j) = \hat{y}(kq+j), \ j=1,2,\cdots,q-1. \qquad (27)$$

From Figure 1 and (10), since $v(kq)$ is a "white" noise, the best estimates of all missing output $y(kq+j)$ are

$$\hat{y}(kq+j+1) = \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(kq+j)\hat{\boldsymbol{\theta}}(kq), \ j=0,1,\cdots,q-2.$$

The missing output estimates $\hat{y}(kq+j)$ can also be computed from the recursive equation:

$$\begin{aligned}
\hat{y}(kq+j+1) &= \sum_{i=1}^{nq} \hat{\beta}_i(kq) f(u(kq+j+1-i)) - \\
&\sum_{i=1}^{n} \hat{\alpha}_i(kq)\hat{y}(kq+j+1-iq), j=0,1,\cdots,q-2. \quad (28)
\end{aligned}$$

Comparing (17) with (28), we find that the missing intersample output estimates $\hat{y}(kq+j),\ j=1,2,\cdots,\ q-1$, equal the desired outputs $y_r(kq+j)$; so we have

$$y_r(kq+j)=\hat{y}(kq+j)=\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(kq+j)\hat{\boldsymbol{\theta}}(kq). \qquad (29)$$

It is easy to understand that the unknown intersample outputs $y(kq+j)$ are replaced by the desired outputs $y_r(kq+j)$ because our goal is to make $y(k)$ track $y_r(k)$. Hence, combining (18) with (29) generates the control signal sequence $\{u(kq+j),\ j=0,1,\cdots,q-1\}$ based on the parameter estimates $\hat{\boldsymbol{\theta}}(kq)$ obtained. Thus, the following theorem is easily established.

Theorem 2: Assume that the conditions of Theorem 1 hold, $A(z)$ and $B(z)$ both are stable and $f(\cdot)$ is invertible. Then the self-tuning control algorithm in (11)-(14), (18) ensures the closed-loop system to be stable and globally convergent with probability 1; moreover,

- The input and output variables are uniformly bounded,

$$\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}[u^2(i)+y^2(i)+y_f^2(i)]<\infty,\ \text{a.s.}$$

- The average output tracking error is equal to and less than $\bar{\sigma}^2/q$,

$$\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\mathrm{E}\{[y_f(i)-y_r(i)]^2|\mathcal{F}_{i-1}\}\leqslant\frac{\bar{\sigma}^2}{q},\ \text{a.s.}$$

Proof Since $y_r(k)$ is bounded, it is easy to get that the outputs $y(kq)$ at the output sampling instants are uniformly bounded from Theorem 1 and (A3), i.e.,

$$\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}y^2(iq)\leqslant\delta_y<\infty,\ \text{a.s.}$$

Also, the intersample output estimates $\hat{y}(kq+j),\ j=1,2,\cdots,(q-1)$, satisfy

$$\hat{y}(kq+j)=y_r(kq+j),\ j=1,2,\cdots,q-1.$$

So $y_f(kq+j)$ is bounded. According to (26) and (27), $y_f(k)$ is bounded. Since $A(z)$ and $B(z)$ are stable, so are $\alpha(z)$ and $\beta(z)$; and $u(k)$ is bounded in terms of Lemma B.3.3 in [12]. Hence we have

$$\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}u^2(i)<\infty,\ \text{a.s.},$$
$$\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}y^2(i)<\infty,\ \text{a.s.},$$
$$\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}y_f^2(i)<\infty,\ \text{a.s.},$$

which mean that all the input and output variables are uniformly bounded. Also,

$$\begin{aligned}&\limsup_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\mathrm{E}\{[y_f(i)-y_r(i)]^2|\mathcal{F}_{i-1}\}\\&=\limsup_{k\to\infty}\frac{1}{kq}\sum_{i=1}^{k}\mathrm{E}\{[y(iq)-y_r(iq)]^2|\mathcal{F}_{iq-1}\}+\\&\limsup_{k\to\infty}\frac{1}{kq}\sum_{j=1}^{q-1}\sum_{i=1}^{k}\mathrm{E}\{[\hat{y}(iq+j)-y_r(iq+j)]^2|\mathcal{F}_{iq+j-1}\}.\end{aligned}$$

Since $y_f(i)=\hat{y}(i)=y_r(i)$ at the missing output sampling instants, the last term on the right-hand side is zero, and the first term is no more than $\bar{\sigma}^2/q$ from Theorem 1. This proves Theorem 2. □

Theorem 2 indicates that the proposed self-tuning control scheme in the dual-rate setting can achieve the property of minimum variance at the output sampling instants, just like the Åström-Wittenmark STR. Between the output sampling instants, we have $\hat{y}_f(kq+j)=\hat{y}(kq+j)=y_r(kq+j)$, $j=1,2,\cdots,q-1$, which implies zero tracking error for intersampling instants.

The persistent excitation condition is required for the convergence of the parameter estimation. Like in conventional discrete-time systems [12], self-tuning control algorithms do not guarantee the convergence of the parameter estimation to their true values.

In order to avoid generating $u(k)$ with too large magnitudes, for a given small positive ε, if $|\hat{\beta}_1(kq)|<\varepsilon$, we take $\hat{\beta}_1(kq)=\mathrm{sgn}[\hat{\beta}_1(kq)]\varepsilon$, where the sign function is

$$\mathrm{sgn}(x)=\begin{cases}1, x\geqslant 0,\\0, x<0.\end{cases}$$

V. Example

In this section, we illustrate the results reported with two simulation examples.

Example 1 Consider a second-order system,

$$G(z)=\frac{B(z)}{A(z)}=\frac{1.0z^{-1}+0.50z^{-2}}{1-1.50z^{-1}+0.70z^{-2}},$$

and the monotone nonlinear function,

$$\begin{aligned}\bar{u}(k)&=f(u(k))=c_1u(k)+c_2u^2(k)+c_3u^3(k)\\&=u(k)+2u^2(k)+3u^3(k).\end{aligned}$$

We take the noise sequence $\{v(k)\}$ to be a white noise sequence with zero mean and variance $\sigma^2=0.20^2$ and the desired output to be

$$y_r(500i+j)=(-1)^i,\ i=0,1,2,\cdots,\ j=1,2,\cdots,500.$$

The self-tuning control algorithm in Section II is applied to this system. The output $y(k)$ and the desired output $y_r(k)$ are shown in Figure 2 with $q=2$. Figure 3 with $q=1$ shows the simulated results of the Åström-Wittenmark self-tuning regulator (A-W STR) of conventional discrete-time systems.

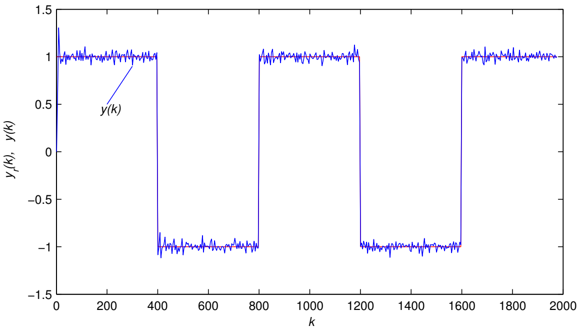

Fig. 2. $y(k)$ and $y_r(k)$ versus k $(q = 2, \sigma^2 = 0.20^2)$

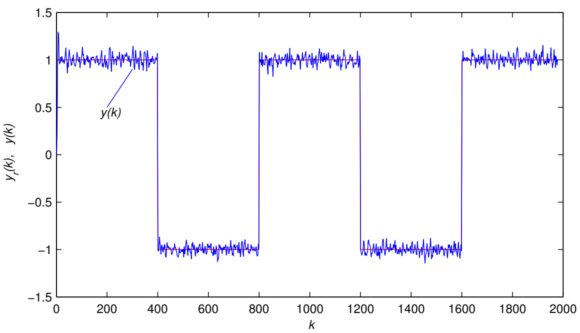

Fig. 3. $y(k)$ and $y_r(k)$ versus k $(q = 1, \sigma^2 = 0.20^2)$

Example 2 Consider a third-order system,

$$G(z) = \frac{B(z)}{A(z)} = \frac{0.50z^{-1} + 0.30z^{-2} - 0.10z^{-3}}{1 + 0.50z^{-1} - 0.40z^{-2} - 0.40z^{-3}}.$$

The non-monotone nonlinear function,

$$\bar{u}(k) = f(u(k)) = 2 - u(k) - 2u^2(k) + u^3(k).$$

Simulation conditions are same as before, but $\bar{\sigma}^2 = 0.15^2$ and $\bar{\sigma}^2 = 0.30^2$. Results are shown in Figures 4 and 5.

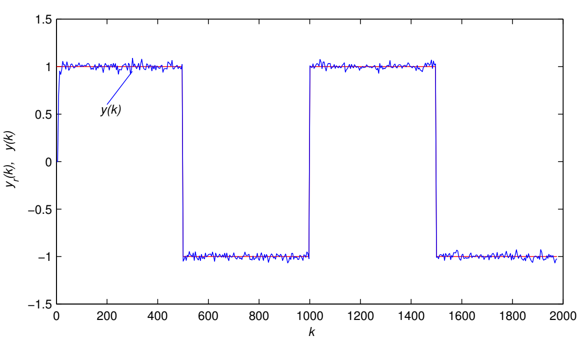

Fig. 4. $y(k)$ and $y_r(k)$ versus k $(q = 2, \sigma^2 = 0.15^2)$

From Figures 2 to 5, we can see that the control algorithm proposed in this paper can achieve less and more stationary average tracking error than the A-W STR algorithm for different nonlinear systems. Thus, the closed-loop tracking performance is satisfactory.

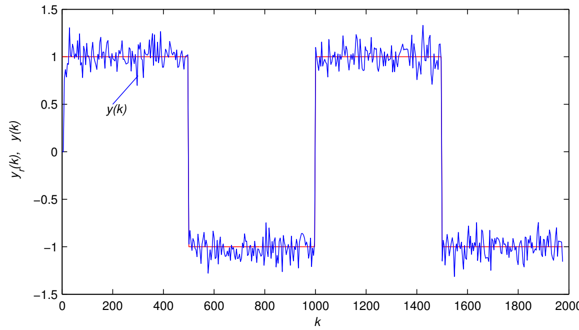

Fig. 5. $y(k)$ and $y_r(k)$ versus k $(q = 2, \sigma^2 = 0.30^2)$

VI. Conclusions

In this paper, we propose a self-tuning control algorithm based on only available *dual-rate data* for a nonlinear system, namely Hammerstein system. The algorithm generates a relatively fast-rate control signal from an online parameter identification scheme which estimates fast-rate models for Hammerstein systems involving dynamic linear blocks and static nonlinear blocks. It is shown in the the theorems that the proposed control algorithm can achieve desired tracking control objective under certain conditions.

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