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Incorporating communication delays in the guidance of a moving actuator/sensor for performance enhancement of controlled distributed parameter systems

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Abstract— **This paper examines the effects of position-dependent delays of mobile actuator/sensor pairs when employed for the control of spatially distributed systems. It is assumed that a collocated pair moves freely within the spatial domain in order to minimize the effects of a moving source. A time delay that depends on the distance of the actuator/sensor pair from the base station is incorporated into the guidance scheme and the supervisor has to manage conflicting objectives: that of stability robustness by keeping the moving agent close to the base station in order to minimize position-dependent delays, and of performance enhancement by commanding the mobile agent to the spatial region with the largest deviation from equilibrium, and inadvertently increasing the time delays. An algorithm that contains a time-delay management along with performance enhancement is proposed and extensive numerical studies that examine the delay effects on controller performance and agent trajectory are included.**

I. INTRODUCTION

A way to minimize the effects of moving source (disturbances) in processes governed by partial differential equations, is to consider actuating and sensing devices that can move inside the spatial domain (i.e. "chase" the moving disturbance and "undo" its effects on the process state by dispensing the appropriate control signal). The use of spatially moving and/or scheduled actuating and sensing devices for improved control and state estimation in systems governed by partial differential equations has recently been considered in the literature (see [1], [2] and references therein), directly following the recent explosion of published works on mobile sensor networks for coverage, foraging and state estimation in finite dimensional systems. However, the inclusion of the sensing and actuating devices into the dynamics of the process at which these devices are called for to perform tasks such as estimation, detection and control, have not been considered, other than few works [3], [4], [5], [6], [7], [8].

In earlier work [6], a moving actuator/sensor was utilized for the suppression of the effects of a moving source on the state of a 1D PDE. However, the effects of time delays have not been considered. Transmission delays from the mobile agent (sensor and/or actuator) due to the distance of such an agent from the base station were not considered. In a related work [9], the authors used the Fokker-Planck PDE for target tracking, but in that case the process state represented the probability density of the target movement. Here, the effects of delays, which are modeled as proportional to the distance of the agent from the base station squared, on the ability to address the effects of a moving source are considered.

The result from [6] is extended to include position-dependent delays in a class of systems with locally distributed state measurements and model such delays as proportional to the square of the distance from the base station. The guidance policy that takes into consideration the delays is presented in § III and extensive simulation studies examining the effects of the position-dependent time delays on the controller performance and the actuator/sensor trajectory are presented in § IV. Conclusions follow in § V.

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II. PROBLEM FORMULATION

We consider the employment of a mobile collocated sensor/actuator system for the improved control of the 1D PDE

$$\frac{\partial x(t,\xi)}{\partial t} = a_1\frac{\partial^2 x(t,\xi)}{\partial \xi^2} + a_2\frac{\partial x(t,\xi)}{\partial \xi} + a_3 x(t,\xi) + b_1(t,\xi),$$

with Dirichlet boundary conditions $x(t,0) = x(t,\ell) = 0$ and initial condition $x(0,\xi) = x_0(\xi)$. The function $b_1(t,\xi)$ denotes the moving disturbance in the sense of a disturbance that is *both* temporally and spatially varying. It is assumed that the disturbance spatial distribution is the same at each location but its centroid changes with time. Additionally, we allow the disturbance temporal component to be time varying. Thus

$$b_1(t,\xi) = b_{1s}(\xi;\xi_d(t))\, d(t),$$

where $\xi_d(t)$ denotes the time-varying centroid of the spatial distribution of the disturbance and $d(t)$ denotes its temporal component; for example, when the centroid of the moving disturbance is at the location $\overline{\xi}_d$, then the disturbance has a spatial component $b_{1s}(\xi;\overline{\xi}_d)$ (or acts at the support of the spatial function $b_{1s}(\xi;\overline{\xi}_d)$) and has a time intensity equal to $d(t)$. Figure 1 depicts the case where the spatial distribution is the box function and $d(t) = 5\times 10^{-2}e^{-t}\cos(2t)$ for three different values of the centroid: at $\xi_d(t_0) = 0.5$, at $\xi_d(t_1) = 0.578$ and at $\xi_d(t_2) = 0.397$. To address the effects of the moving disturbance, a moving actuating device is assumed to freely move throughout the spatial domain $\Omega = [0,\ell]$ and dispense the control signal with a chosen intensity $u(t)$. Thus, the controlled diffusion-advection PDE is rewritten

$$\begin{aligned}\frac{\partial x(t,\xi)}{\partial t} &= a_1\frac{\partial^2 x(t,\xi)}{\partial \xi^2} + a_2\frac{\partial x(t,\xi)}{\partial \xi} + a_3 x(t,\xi) \\ &\quad + b_1(t,\xi) + b_2(\xi;\xi_a(t))u(t).\end{aligned} \tag{1}$$

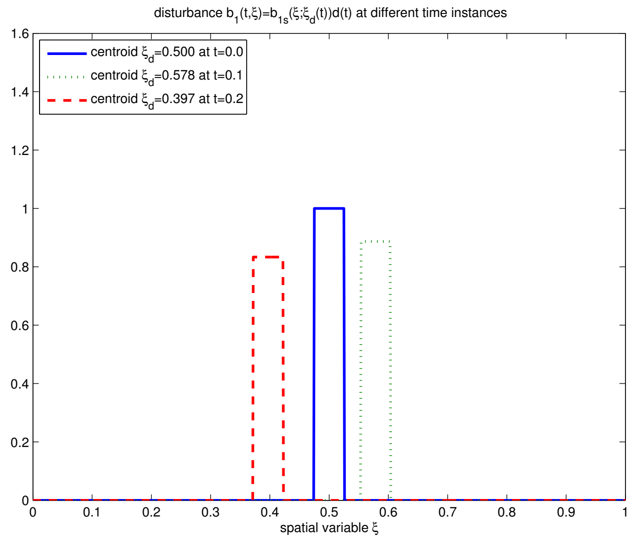

Fig. 1. Spatial distributions of moving disturbance.

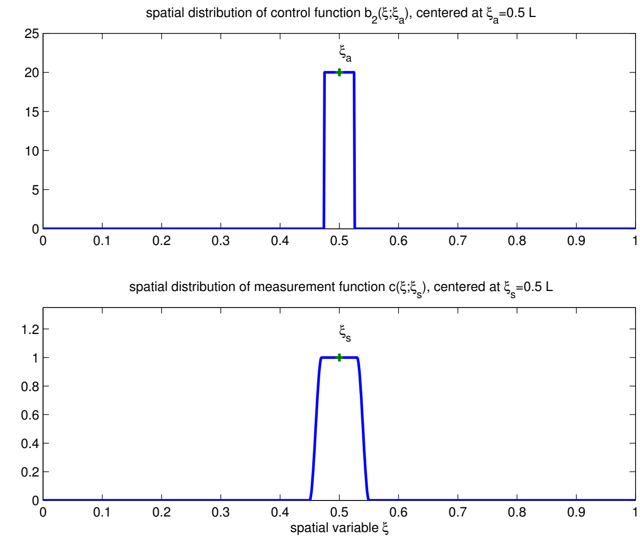

Fig. 2. Spatial distributions of input and measurement functions.

The function $b_2(\xi;\xi_a(t))$ denotes the spatial distribution of the moving actuating device that is located at the spatial position $\xi_a(t)$, which is termed the *actuator centroid*.

To minimize the complexity of the control design and the subsequent moving actuator guidance policy, a moving sensor that is collocated with the actuator can be used to provide locally distributed measurements of the state $x(t,\xi)$

$$y(t,\xi;\xi_s(t)) = c(\xi;\xi_s(t))x(t,\xi). \quad (2)$$

This mobile sensing device provides local-in-space distributed measurements of the state over the sensor range (support). In the above equation, the function $c(\xi;\xi_s(t))$ denotes the spatial distribution of the sensing device. Since it is assumed to be a moving sensor, the variation of the location of the sensor inside the spatial domain Ω is described by the centroid $\xi_s(t)$ of its spatial distribution.

In a similar fashion as in the delay-free case in [6], an effective and minimum-complexity way to minimize the effects of the moving source $b_1(t,\xi)$ is to employ a mobile collocated actuator/sensor pair. Going further in the simplification of the control and supervision structure, a static output feedback controller will be considered. For a given position of the collocated actuator/sensor $\xi_a(t) = \xi_s(t)$, the controller architecture would use a constant multiple of the measured distributed signal $u(t,\xi) = -ky(t.\xi;\xi_s(t))$ and thus the supervisor would only provide the guidance policy of the collocated mobile pair. To further minimize the complexity of the control design, it is assumed that the spatial distribution of the actuating device, denoted here by $b_2(\xi;\xi_a)$, has the same support as the spatial distribution of the sensing device, denoted in (2) by $c(\xi;\xi_s)$. Analytically, the distributed measurements from the sensor are assumed to be available over the spatial interval $[\xi_s-\Delta\xi,\xi_s+\Delta\xi]$, where the sensor support has length equal to twice the one-half spatial support of the actuating device i.e. equal to $2\Delta\xi$. An example of such a collocated pair with different spatial distributions that satisfy the assumption of *compatible* support is given in Figure 2. In this figure, the spatial distribution of the sensing device was taken to be a smoothened distribution of a polynomial function and given by

$$c(\xi;\xi_s) = \begin{cases} 1 & \text{if } \xi \in [\xi_s - 0.6\Delta\xi, \xi_s + 0.6\Delta\xi] \\ 1 - 3\xi_l^2 - 2\xi_l^3 & \text{if } \xi \in [\xi_s - \Delta\xi, \xi_s - 0.6\Delta\xi] \\ 1 - 3\xi_r^2 + 2\xi_r^3 & \text{if } \xi \in [\xi_s + 0.6\Delta\xi, \xi_s + \Delta\xi] \\ 0 & \text{otherwise.} \end{cases}$$

where $\xi_r = \frac{\xi-\xi_s-0.6\Delta\xi}{0.4\Delta\xi}$ and $\xi_l = \frac{\xi-\xi_s+0.6\Delta\xi}{0.4\Delta\xi}$. Such a distribution was used in the simulation studies presented in Section IV. In the same figure, the spatial distribution of the collocated actuating device was taken to be the boxcar function, which approximates the spatial delta function

$$b_2(\xi;\xi_s) = \begin{cases} \frac{1}{2\Delta\xi} & \text{if } \xi \in [\xi_s - \Delta\xi, \xi_s + \Delta\xi] \\ 0 & \text{otherwise} \end{cases}$$

We can formally make the assumption of *compatible support* of the spatial distributions of the actuating and sensing devices that additionally requires that the support of the actuator *be contained in* the support of the sensor. Such an assumption significantly reduces the controller complexity in the sense that the control term as appears in weak form does not require additional modifications:

$$\begin{aligned} \langle B_2(\xi_a)u(t),\phi\rangle &= \int_0^\ell b_2(\xi;\xi_a(t))u(t)\phi(\xi)\,d\xi \\ &= \int_{\xi_s-\Delta\xi}^{\xi_s+\Delta\xi} \frac{1}{2\Delta\xi}u(t)\phi(\xi)\,d\xi, \text{ since } \mathcal{D}(b_2) \subseteq \mathcal{D}(c) \\ &= \frac{-k}{2\Delta\xi}\int_{\xi_s-\Delta\xi}^{\xi_s+\Delta\xi} y(t,\xi;\xi_s)\phi(\xi)\,d\xi \\ &= \frac{-k}{2\Delta\xi}\int_{\xi_s-\Delta\xi}^{\xi_s+\Delta\xi} c(\xi;\xi_s)x(t,\xi)\phi(\xi)\,d\xi. \end{aligned}$$

The above demonstrates that when the sensor distribution is defined over the domain of the actuator i.e. $[\xi_a-\Delta\xi,\xi_a+$

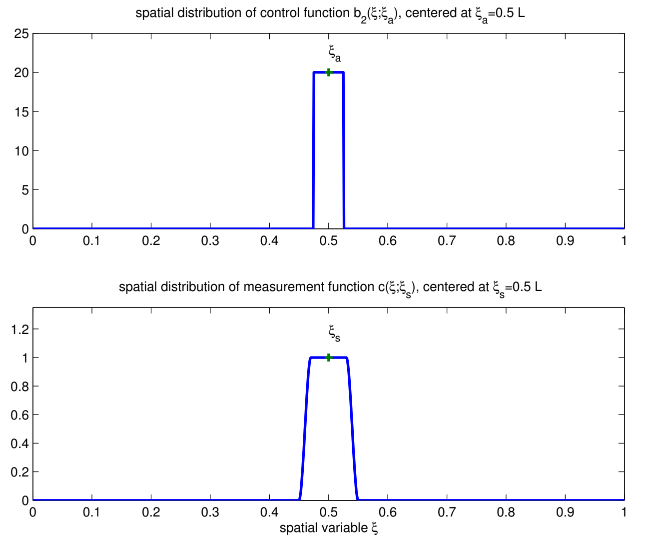

Fig. 3. Combined effects of collocated distributions for different supports: (a) support of b_2 inside support of c, (b) support of c inside support of b_2.

$\Delta\xi] \subseteq [\xi_s - \Delta\xi, \xi_s + \Delta\xi]$, then the control term in the spatial integral will be truncated but the control signal will be delivered by the actuating device throughout the support of the actuator. Only the portion of the measured state that falls inside the domain of the actuator will be utilized. The opposite, where the support of the actuator is larger than the support of the sensor would result in portions of the actuator support receiving zero control signal, if the proposed control policy is used. Both scenarios are depicted in Figure 3.

Assumption 1 (Compatible supports): To minimize controller design complexity and ensure real-time feasibility of the proposed moving collocated output feedback controller, one requires that the support of the actuating device be inside the spatial domain of definition of the sensing device; this means $\mathcal{D}(b_2) \subseteq \mathcal{D}(c)$. For simplicity, in this work, it will be assumed that the actuator spatial domain is equal to the sensor spatial domain (i.e. $\mathcal{D}(b_2) \equiv \mathcal{D}(c)$).

Remark 1: In the compatibility assumption above, it was assumed that the spatial range of influence of the actuating device was equal to that of the collocated sensing device. Following the demonstration in Figure 3, when the support of the actuator is larger than that of the sensor, it means that the control signal is not delivered in the support that falls outside the intersection of the two devices; i.e. the part of the spatial support $[\xi_a - \Delta\xi, \xi_s - \Delta\xi) \cup (\xi_s\Delta\xi, \xi_a + \Delta\xi]$ of the actuator will deliver zero control effort. On the other hand, if the spatial distribution of the actuating device is inside the support of the sensing device, it means that the portions $[\xi_s - \Delta\xi, \xi_a - \Delta\xi) \cup (\xi_a\Delta\xi, \xi_s + \Delta\xi]$ of the measured information are not utilized in the feedback control, but that every spatial point of the spatial support of the actuating device receives a control signal.

The added feature in this work, as compared to the earlier work [6], is the inclusion of transmission delays that are directly proportional to the sensor/actuator position inside Ω. Towards that end, we make the following assumption.

Assumption 2 (position-dependent delay): The measured signal from the collocated sensor/actuator that is transmitted to the base station is delayed by a *time-and-space dependent delay*. Such a delay is assumed to be proportional to the square of the distance of the collocated device from the left boundary, considered to be the base station, and given by

$$\tau(\xi_s(t)) = \alpha\xi_s^2(t), \tag{3}$$

where the proportionality constant α has units of $[T][D^{-2}]$. The *measured* output signal is given by

$$y_{meas}(t,\xi;\xi_s(t)) = c(\xi;\xi_s(t))x(t,\xi)$$

whereas the *transmitted* output signal is given by

$$y_{trans}(t,\xi;\xi_s(t)) = y_{meas}(t-\tau,\xi;\xi_s(t)).$$

With the assumptions of compatible supports and position-dependent delays, the controlled process (1), its distributed output (2) with the delay given in (3) can be written as

$$\begin{aligned} \frac{\partial x(t,\xi)}{\partial t} &= a_1\frac{\partial^2 x(t,\xi)}{\partial \xi^2} + a_2\frac{\partial x(t,\xi)}{\partial \xi} + a_3 x(t,\xi) \\ &\quad + b_1(t,\xi) + b_2(\xi;\xi_s(t))u(t),\ x(0,\xi) = x_0(\xi) \qquad (4) \\ y_{meas}(t,\xi;\xi_s(t)) &= c(\xi;\xi_s(t))\,x(t,\xi) \\ y_{trans}(t,\xi;\xi_s) &= y_{meas}(t-\tau_k,\xi;\xi_s), \quad \tau_k = \alpha.\xi_s^2(t_k) \end{aligned}$$

In view of Assumptions 1,2 and Remark 1, the control and monitoring objectives can be stated.

Problem statement: *Consider the controlled process with collocated moving actuator/sensor pair that satisfy the compatibility assumption (Assumption 1). The objective becomes that of proposing a sensor centroid guidance policy and an associated control law that would minimize the effects of the moving disturbance on the distributed state $x(t,\xi)$ despite the position-dependent delay as described in Assumption 2.*

III. Guidance policy of sensor/actuator

While it has been established that a mobile actuator, capable of moving throughout the spatial domain, can better affect a process compared to a stationary actuator, a possible implementation drawback arises. This is because of the control design complexity which now has to provide a control signal at each centroid location of the actuating device. An optimal control strategy that would incorporate the motion of the mobile actuator into the optimization procedure would provide a truly optimal control signal and optimal guidance policy of the mobile actuator, but might not be feasible due to the on-line computational burden. Instead, a suboptimal control policy is proposed, whereby the feedback architecture is simplified to that of a static output feedback of the form $u = -ky$. In other words, the supervisor has to provide the guidance policy of the actuator which would collect the measured signal from the collocated sensor and simply amplify it and feed it back to the actuating device.

As it was already established in the literature on mobile sensor networks, the equations of motion of the mobile devices must be incorporated into the guidance optimization scheme, as a mobile agent will have inertia and velocity constraints. To incorporate velocity constraints of the moving sensor, one may view the system in (4) as a hybrid system with the change in the sensor position occurring at discrete time instances. Furthermore, the distance that it can traverse over a prescribed time interval is bounded by velocity considerations. This significantly minimizes the complexity of the guidance scheme. Towards that, we divide the time interval $[t_0,t_f]$ into n equidistant subintervals $[t_0,t_1,t_2\ldots,t_f]$, with $t_{k+1}=t_k+\Delta t$. In a given interval $[t_k,t_k+\Delta t]$, the sensor is constrained to traverse at a maximum of a distance $\pm\Delta\xi$ from the current centroid position $\xi_s(t_k)$. Therefore, the maximum average speed is bounded by $\upsilon_{av}=\frac{\Delta\xi}{\Delta t}$. The moving sensor guidance policy would then consider a given speed υ_{av} and a prescribed time interval Δt and propose an updated sensor position from $\xi_s(t_k)$ to $\xi_s(t_{k+1})$ where

$$\xi_s(t_k)-\Delta\xi \le \xi_s(t_{k+1}) \le \xi_s(t_k)+\Delta\xi.$$

Alternatively, this can be viewed as a constraint on the next iterate of the sensor centroid position $\xi_s(t_k+\Delta t)=\xi_s(t_{k+1})\in[\xi_s(t_k)-\Delta\xi,\xi_s(t_k)+\Delta\xi]$, or even as employing the projection operator to restrict the next iterate to $[\xi_s(t_k)-\Delta\xi,\xi_s(t_k)+\Delta\xi]$.

The above constraint can be used in conjunction with a sensor centroid guidance policy. In view of the real-time implementation ability of the guidance policy resulting from possible on-line guidance optimization, the proposed guidance policy (whether constrained or not) is based on the *maximum deviation policy* summarized in [6], [10]. However, one may combine the Lyapunov-based guidance policy that was also presented in [10] along with the adaptation of the feedback gain using the adaptive scheme in [6]. In order to minimize computational burden resulting from the on-line computations, we consider the constrained maximum deviation policy with location-dependent delays.

As was already presented in [6] for the delay-free case, the maximum deviation guidance policy moves the centroid of the sensor from the current position $\xi_s(t_k)$ to the next position $\xi_s(t_{k+1})$ by finding the maximum deviation of the measured state $c(\xi;\xi_s(t_k))x(t,\xi)$ from the equilibrium over the sensor support at the current position $\xi_s(t_k)$ and moves the sensor to that maximum. However, in light of the position-dependent delays, the correct policy might be to keep the sensor centroid at the previous position, but the delayed state measurements would be predicting a new position within $[\xi_s(t_k)-\Delta\xi,\xi_s(t_k)+\Delta\xi]$. Figure 4 demonstrates the effects of position-dependent delays in the correct prediction of the centroid guidance. The current sensor centroid is at $\xi_s(t_k)=0.6$, but the delayed output (solid blue) has the centroid at $\xi_s(t_k-\tau_k)=0.5$. The maximum deviation policy for the current measured output would predict the new centroid position at $\xi_s(t_{k+1})=0.63$ whereas using the delayed output, the prediction would place the centroid at $\xi_s(t_{k+1})=0.53$. The collocated actuator can be more effective at the correct predicted position of $\xi_s(t_{k+1})=0.63$ since the state has a larger deviation from its equilibrium, whereas the delayed measurements predicts the position at $\xi_s(t_{k+1})=0.53$ and hence the actuator will not be as effective. A direct consequence of the delay effects is the performance degradation

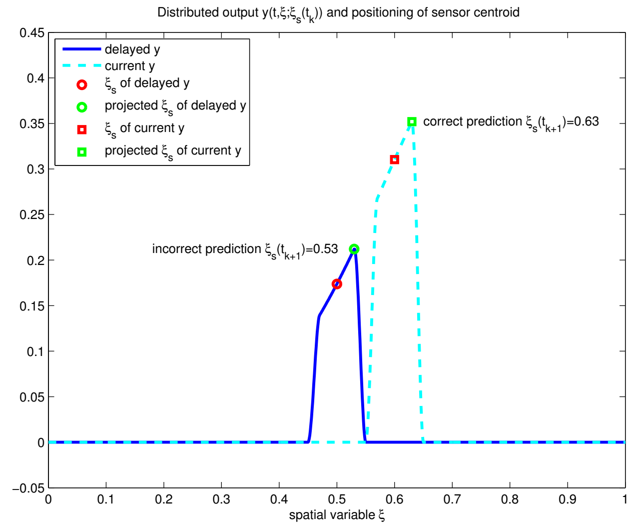

Fig. 4. Effects of delay in centroid prediction: delayed output (—), correct output (− −), centroid of delayed output at $\xi_s(t_k)=0.5$ (○), incorrect prediction of centroid at $\xi_s(t_k)=0.53$ (○), centroid of current output at $\xi_s(t_k)=0.6$ (□), correct prediction of centroid at $\xi_s(t_k)=0.63$ (□).

of the moving actuator/sensor. Added to that is the stability effects of delayed measurements on the closed loop system.

Following the above, the sensor motion using the maximum deviation guidance policy can be expressed by

$$\xi_s(t_{k+1})=\arg\max_{\xi\in[\xi_s(t_k)-\Delta\xi,\xi_s(t_k)+\Delta\xi]}\left|y_{meas}(t-\tau_k,\xi;\xi_s(t_k))\right|$$

where the position-dependent delay is given by (3). Associated with the above guidance policy, is the control policy. In view of the previous arguments on complexity reduction, a simplified static output feedback control law is proposed

$$u(t,\xi;\xi_s(t))=-k\,y(t,\xi;\xi_s(t))=-k\,c(\xi;\xi_s(t))\,x(t,\xi)$$

where the static scalar gain $k>0$ may be chosen arbitrarily. However, in view of the position-dependent delay, the actual feedback controller will involve delayed measurement

$$u(t,\xi;\xi_s(t))=-k\,y_{meas}(t-\tau_k,\xi;\xi_s(t_k)). \tag{5}$$

When the PDE in (4) is written in abstract form and includes the delayed feedback in (5), it results in

$$\dot{X}(t)=\mathcal{A}X(t)-k\mathcal{B}_2(t_k)\mathcal{C}(t_k)X(t-\tau_k)+\mathcal{B}_1(t)d(t).$$

Even in the case of a fixed-in-space actuator/sensor pair, the system will be expressed by

$$\dot{X}(t)=\mathcal{A}X(t)-k\mathcal{B}_2\mathcal{C}X(t-\tau_k)+\mathcal{B}_1(t)d(t). \tag{6}$$

On one hand, the supervisor would want to move the actuator/sensor pair close to the base station in order to minimize the delay. This would eliminate the destabilizing effects of time delays in the closed loop system (6). However, the performance-based guidance policy would move the actuator/sensor to the location with the largest deviation from the equilibrium. If such a deviation is far away from the base station, then the corresponding delay would significantly increase and possibly destabilize the system. To ameliorate this conflicting requirements of stability robustness versus performance, a modification to the algorithm in [6] is proposed here and which attempts to minimize time delays by keeping the mobile sensor/actuator close to the base station whenever a certain level of performance is attained, and to move the agent in the spatial region that is needed the most while at the same time ensuring that the resulting predicted time delays will not destabilize the system.

Algorithm 1: sensor/actuator guidance based on maximum state deviation using static feedback and delay management

1) using velocity considerations υ_{av} and sensor support specifications $(\Delta\xi)$, find the smallest time interval Δt which takes into consideration data processing delays and dwell time [11], [12]
2) (initialization) first consider the interval $[t_0, t_0+\Delta t)$
3) place (move) the sensor at an initial location $\xi_s(t_0)$ that maximizes observability of the associated pair $(\mathcal{A}, \mathcal{C}(\xi_s(t_0))$ representing the process and output operators in the abstract formulation of (1)
4) implement the locally distributed static control law
$$u(t,\xi;\xi_s(t_0)) = -kc(\xi;\xi_s(t_0))x(t,\xi), \quad t \in [t_0, t_0+\Delta t]$$
5) find location of maximum state deviation from equilibrium over the span of the sensing device (sensor support) at current centroid location $\xi_s(t_0)$
$$\xi_s(t_1) = \arg \min_{\xi_s(t_0)-\Delta\xi \le \xi \le \xi_s(t_0)+\Delta\xi} \left| c(\xi;\xi_s(t_0))x(t,\xi) \right|$$
6) move sensor at new position $\xi_s(t_1)$ and implement
$$u(t,\xi;\xi_s(t_1)) = -k y_{meas}(t-\tau_1,\xi;\xi_s(t_1)), \quad t \in [t_1, t_1+\Delta t]$$
   a) if at the end of $[t_1, t_1+\Delta t]$, the maximum deviation of the measured state does not subside, then move towards the base station in order to reduce the time delays in the closed loop system
   b) if the measured state's maximum deviation is reduced by the end of $[t_1, t_1+\Delta t]$, then move sensor towards the new maximum deviation within the support of the sensor at the current position
7) consider next interval with $t_{k+1} = t_k + \Delta t$, and perform search in step 5 over current sensor span $[\xi_s(t_k) - \Delta\xi, \xi_s(t_k) + \Delta\xi]$ and repeat step 6 for new interval.

The well-posedness of the combined sensor/actuator motion and static output feedback (5) applied to the system (4) can be argued within the context of switched infinite dimensional systems as detailed in [2], [12].

IV. Results

The PDE in (1) was simulated using 80 linear elements [13] in $\Omega = [0,1]$ and an initial condition $x(0,\xi) = \sin(\pi\xi)e^{-7(\xi-\ell)^2}$. The coefficients in (1) were $a_1 = 0.005$, $a_2 = 0.15$, $a_3 = 0.003$. The moving source was taken as
$$b_1(t,\xi) = 10^{-5}\left(0.3\cos(\tfrac{9\pi t}{t_f}) + 0.5\right) \times \\ (H(\xi+\xi_c(t)+\Delta\xi) - H(\xi-\xi_c(t)-\Delta\xi))$$

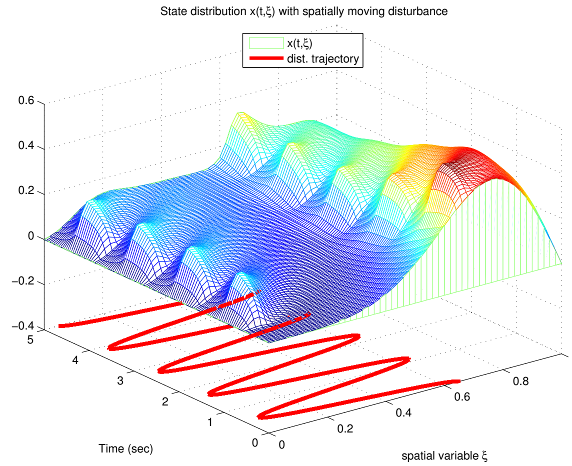

Fig. 5. Effects of spatially moving disturbance on open loop state.

where $\xi_c(t)$ denotes the centroid of the moving source and $\Delta\xi = \ell/20$ denotes the one-half of the spatial support of the spatial distribution of the moving source. The same spatial support $\Delta\xi$ was used for the moving sensing and actuating devices. The closed loop system was simulated in the time interval $[0,5]$ with a maximum velocity $\upsilon_{av} = 0.5$, thus resulting in $n = \frac{t_f \upsilon_{av}}{\Delta\xi} = 50$ subintervals and therefore $\Delta t = \frac{t_f}{n} = 0.1$, i.e. the switching times at which the sensor/actuator pair was being moved were occurring every $\Delta t = 0.1$ time units. The moving source, as described above, had its own centroid moving with a lower speed of $\upsilon_{source} = 0.1111$.

The effects of the moving disturbance on the state are seen in Figure 5, which necessitate the use of a mobile actuator. The L_2 state norm for both a stationary and moving sensor/actuator pair for two different values of a time delay are presented in Figure 6. It can be observed that large delays have a detrimental effect on controller performance. The spatial distribution of the state at the final time is presented for these two cases in Figure 7, and the effects of the time delays on the sensor trajectory are presented in Figure 8.

V. Conclusions

This paper examined the effects position-dependent delays on the performance of a moving sensor/actuator pair employed for the control of a process governed by a 1-D diffusion-advection PDE. It was assumed that an unknown source was moving within the spatial domain thereby forcing the state to deviate from its equilibrium, and therefore a moving actuator/sensor would be used to counterbalance the effects of this moving disturbance. While in the delay-free case it was shown that a moving actuator/sensor pair can have a superior performance in suppressing the effects of a moving disturbance, in the nonzero delay such a performance was shown to deteriorate for large values of the time delay. Such a delay was proportional to the distance of the mobile actuator/sensor pair from the base station.

To improve the performance of the moving actuator/sensor pair under the influence of a position-dependent delay, a

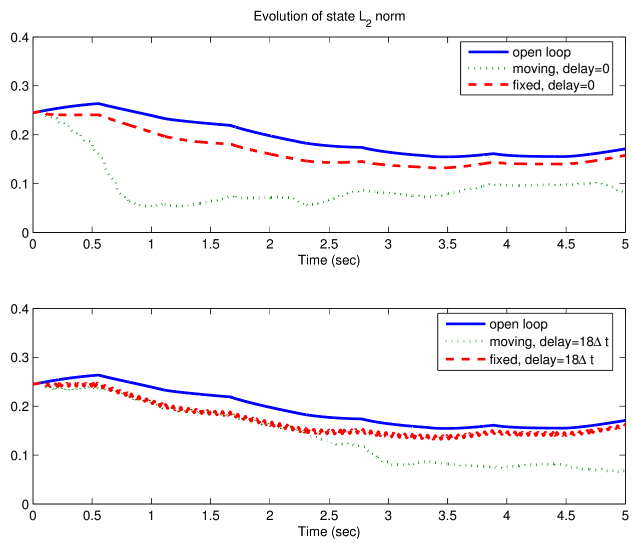

Fig. 6. Effect of delays on L_2 state norm for fixed and moving actuator/sensor.

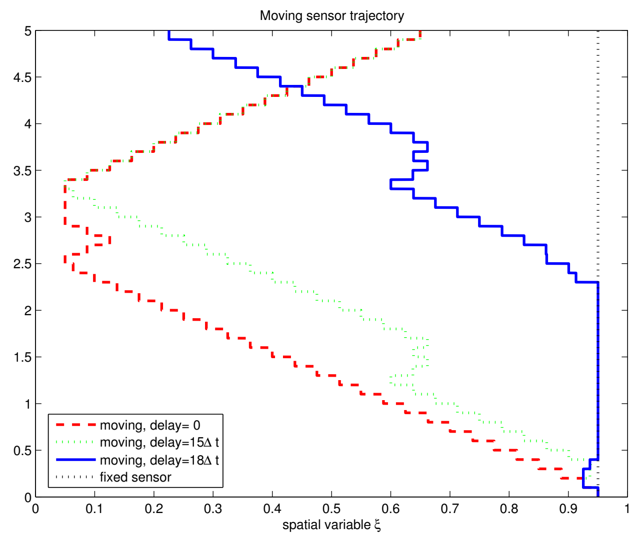

Fig. 8. Sensor/actuator trajectory for different time delays.

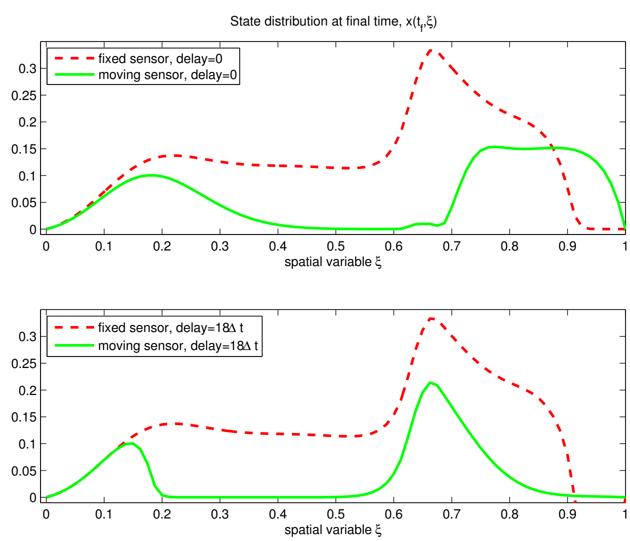

Fig. 7. Spatial distribution of state at final time $t = 5$ sec.

robust modification scheme was incorporated into the guidance scheme which allowed the moving actutor/sensor pair to move closer to the base station minimizing the time delays at the expense of performance. A balance between stability robustness and performance was introduced and allowed the pair to move to a position further from the base station if it deemed such a move as performance enhancing and to move closer to the base station if it considered that the closed loop stability was compromised by the increase in time-delays.

Extensive simulation studies revealed that a moving sensor/actuator pair capable of providing locally distributed measurements and dispensing locally distributed control action at the spatial range of the sensor, can significantly minimize the effects of a moving source on the process state.

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