2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Robust Switching-type H_∞ Filtering for Time-Varying Uncertain Time-Delay Systems

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***Abstract*— This paper is concerned with the problem of H_∞ filter design for a class of linear uncertain systems with time-varying delay. The uncertainty parameters are supposed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system model. Our proposed robust H_∞ filter is a switching-type filter, in which the filter parameters are tuned in a switching manner via a switching logic. Asymptotical stability and a prescribed H_∞ performance of the filtering error systems are guaranteed. The resultant filter design conditions are less conservative than those of filter with fixed gains. A numerical example is given to illustrate the validity of the proposed design.**

I. INTRODUCTION

Over the past few years, state estimation of dynamic systems has been extensively investigated [1], [6] and [15]. Compared with traditional Kalman filtering, the H_∞ filtering approach possesses many advantages, such as no need for priori information on the external noises and insensitiveness to uncertainty in dynamic model [16]. Hence, recently there has been substantial interest in the study of H_∞ filtering problem [2], [9], [11], [13], [18], which is designed to make the H_∞ norm of the system minimized.

On the other hand, time-delay phenomena often arises from biology, mechanics and economics intrinsically, and also appears in actuation and measurement. As is well known [8], the existence of time-delay degrades the control performance and may make the closed-loop stabilization very difficult. Recently, H_∞ filtering results have been extended to linear systems with time-delays. Both delay-independent and delay-dependent results have been proposed [4], [5], [12], [14], [15], [17] and [19]. Some of these results deal with the so-called norm-bounded uncertainty, which is somewhat conservative in many application [7] and [10].

Recently, robust H_∞ filtering linear time-delay systems with polytopic type uncertainties have been treated, based on parameter-independent Lyapunov function [4], [14], [17] or parameter-dependent Lyapunov function [5] using LMI methodologies. In fact, parameter-dependent Lyapunov method can reduce conservativeness compared with parameter-independent one when the uncertain parameters are time-invariant. Also parameter-dependent Lyapunov method can include the traditional quadratic stability approach as a special case if the time-varying parameters and their rate of variation are assumed to belong to a given convex-bounded polyhedral domain. However, while the uncertain parameters is time-varying and the bound of its derivative is unknown, only the parameter-independent Lyapunov function method can be applied.

In this paper, we investigate the robust H_∞ filtering problem for linear time-delay systems with time-varying uncertainties. The uncertainty parameters are supposed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system model. Apart from traditional filter with fixed gains, the proposed filter is a switching-type filter, which consists of a number of fixed gain filters. One of the fixed gain filters will be active at a time according to some switching laws derived from the Lyapunov stability theory. The derived filter design conditions of the switching-type filers are given in terms of LMIs, which can reduce conservativeness compared with the corresponding conditions of traditional filter with fixed gains. A numerical example is given to show the effectiveness of the proposed switching-type filter.

This paper is organized as follows. Section 2 introduces the problem and some preliminaries. It is followed by the robust switching-type H_∞ filtering design method in Section 3. An illustrative example is given in Section 4 to demonstrate the proposed method. Finally, Section 5 gives the conclusions.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. Problem Statement

Consider a linear uncertain system with time-varying delay described by

$$\begin{aligned}\dot{x}(t) &= A(\delta(t))x(t) + A_d(\delta(t))x(t-d(t)) + B_\omega\omega(t)\\ y(t) &= C_2x(t) + D_{21}\omega(t) + C_3x(t-d(t))\\ z(t) &= C_1x(t)\\ x(t) &= \phi(t), \quad t\in[-\bar{d},0)\end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state, $y(t)\in R^p$ is the measured output and $z(t)\in R^q$ is the output signal vector to be estimated, respectively. $\omega(t)\in R^v$ is the exogenous disturbance in $L_2[0,\infty)$. $\phi(t)$ is the given initial vector function that is continuous on the interval $[-\bar{d},0)$, $d(t)$ is time-varying bounded delays satisfying

$$d(t)\le\bar{d}<\infty, \quad 0\le\dot{d}(t)\le h<1$$

And

$$A(\delta(t)) = A_0+\sum_{i=1}^{N_0}\delta_i(t)A_i, \quad A_d(\delta(t)) = A_{d0}+\sum_{i=1}^{N_0}\delta_i(t)A_{di}$$

$A_0, A_i, A_d, A_{di}B_\omega, C_1, C_2, C_3$ and D_{21} are known constant matrices of appropriate dimensions. $\delta_i(t)(i=1\cdots N_0)$ are unknown time-varying uncertainty, which satisfy $\underline{\delta}_i\le\delta_i(t)\le\bar{\delta}_i$. Here $\underline{\delta}_i$ and $\bar{\delta}_i$ are known lower and upper bounds of $\delta_i(t)$, respectively. Since $C_2\in R^{p\times n}$ and $\text{rank}(C_2)=p_1\le p$, then there exists a matrix $T_c\in R^{p_1\times p}$ such that $\text{rank}(T_cC_2)=p_1$. Furthermore, there exists a matrix C_{cn} such that rank $\begin{bmatrix}T_cC_2\\ C_{cn}\end{bmatrix}=n$. Denote T_{cn} = $\begin{bmatrix}T_cC_2\\ C_{cn}\end{bmatrix}^{-1}$.

Assumption 1: System (1) is asymptotically stable.

This work is supported in part by Program for New Century Excellent Talents in University (NCET-04-0283), the Funds for Creative Research Groups of China (No. 60521003), Program for Changjiang Scholars and Innovative Research Team in University (No. IRT0421), the State Key Program of National Natural Science of China (Grant No. 60534010), the Funds of National Science of China (Grant No. 60674021, 60804024) and the Funds of PhD program of MOE, China (Grant No. 20060145019), and the 111 Project(B08015).

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For traditional robust filtering, the following form filter is usually used.

$$\begin{aligned} \dot{\xi}_1(t) &= A_{Ff}\xi_1(t) + B_{Ff}y(t) \\ z_{F1}(t) &= C_{Ff}\xi_1(t) \end{aligned} \tag{2}$$

where $A_{Ff} \in R^{n\times n}$, $B_{Ff} \in R^{n\times p}$ and $C_{Ff} \in R^{q\times n}$ are the filter parameter matrices to be determined. Here, we assume that the filter is of the same order as the system model.
Denote $x_{ef}(t) = [x^T(t)\ \xi_1^T(t)]^T$ and $z_{ef}(t) = z(t) - z_{F1}(t)$. Then combining (2) with (1), the filtering error dynamic can be obtained as

$$\begin{aligned} \dot{x}_{ef}(t) &= A_{ef}x_{ef}(t) + A_{efd}x_{ef}(t-d(t)) + B_{ef}\omega(t) \\ z_{ef}(t) &= C_{ef}x_{ef}(t) \end{aligned} \tag{3}$$

where

$$A_{ef} = \begin{bmatrix} A(\delta) & 0 \\ B_{Ff}C_2 & A_{Ff} \end{bmatrix}, \quad A_{efd} = \begin{bmatrix} A_d(\delta) & 0 \\ B_{Ff}C_3 & 0 \end{bmatrix},$$

$$B_e = \begin{bmatrix} B_\omega \\ B_{Ff}D_{21} \end{bmatrix}, \quad C_{ef} = [C_1 \quad -C_{Ff}].$$

In this paper, the following robust filter with switching-type gains is considered.

$$\begin{aligned} \dot{\xi}(t) &= A_F(\hat{\delta}(t))\xi(t) + B_F(\hat{\delta}(t))y(t) \\ z_F(t) &= C_F(\hat{\delta}(t))\xi(t) \end{aligned} \tag{4}$$

where $\hat{\delta}_i(t) (i = 1 \cdots N_0)$ are the estimations of $\delta_i(t)$, which will be obtained according to the designed switching laws. $A_F(\hat{\delta}) \in R^{n\times n}$, $B_F(\hat{\delta}) \in R^{n\times p}$ and $C_F(\hat{\delta}) \in R^{m\times n}$ have the following forms, that is

$$A_F(\hat{\delta}) = A_{F0} + \sum_{i=1}^{N_0} \hat{\delta}_i A_{Fi}, \quad B_F(\hat{\delta}) = B_{F0} + \sum_{i=1}^{N_0} \hat{\delta}_i B_{Fi},$$

$$C_F(\hat{\delta}) = C_{F0} + \sum_{i=1}^{N_0} \hat{\delta}_i C_{Fi}$$

where A_{F0}, A_{Fi}, B_{F0}, B_{Fi}, C_{F0}, C_{Fi} are fixed parameter matrices to be designed. Here, the designed filter is of the same order as the system model.
Denote $x_e(t) = [x^T(t)\ \xi^T(t)]^T$ and $z_e(t) = z(t) - z_F(t)$. Applying the robust filter (4) to the system (1), it follows

$$\begin{aligned} \dot{x}_e(t) &= A_e x_e(t) + A_{ed}x_e(t-d(t))B_e\omega(t) \\ z_e(t) &= C_e x_e(t) \end{aligned} \tag{5}$$

where

$$A_e = \begin{bmatrix} A(\delta) & 0 \\ B_F(\hat{\delta})C_2 & A_F(\hat{\delta}) \end{bmatrix}, \quad A_{ed} = \begin{bmatrix} A_d(\delta) & 0 \\ B_F(\hat{\delta})C_3 & 0) \end{bmatrix},$$

$$B_e = \begin{bmatrix} B_\omega \\ B_F(\hat{\delta})D_{21} \end{bmatrix}, \quad C_e = [C_1 \quad -C_F(\hat{\delta})]$$

The purpose of this paper is to develop delay-dependent conditions for the existence of the robust switching-type H_∞ filter (4) for linear time-delay system (1). Specially, for given $\gamma > 0$, find a filter of the form (4) such the corresponding error dynamics (5) is asymptotically stable and satisfies $\|T_{z_e\omega}\|_\infty < \gamma$ under zero-initial conditions for any nonzero $\omega(t) \in L_2[0,\infty]$ and all admissible uncertainties.

B. Preliminaries

Lemma 1[20]: Let $x(t) \in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any matrices $M_1, M_2 \in R^{n\times n}$ and $X = X^T > 0$, and a scalar function $h := h(t) \leq 0$:

$$\begin{aligned} &-\int_{t-h}^{t} \dot{x}^T(s)X\dot{x}(s)ds \\ &\leq \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \\ &+ \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} X^{-1} \begin{bmatrix} M_1 & M_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \end{aligned} \tag{6}$$

Lemma 2: Consider the closed-loop system described by (2). Then the following statements are equivalent:
(i) there exist symmetric positive-definite matrices P_a, Q, S, matrices M_1, M_2 and a filter described by (3) such that for $\delta_i \in [\underline{\delta_i}, \bar{\delta}_i]$

$$\begin{bmatrix} \Delta_0 & \begin{bmatrix} P_aB_{ef} \\ 0 \end{bmatrix} & \begin{bmatrix} \bar{d}M_1^T \\ 0 \\ \bar{d}M_2^T \\ 0 \end{bmatrix} & \begin{bmatrix} \bar{d}A^TS \\ 0 \\ \bar{d}A_d^TS \\ 0 \end{bmatrix} & \begin{bmatrix} C_{ef}^T \\ 0 \end{bmatrix} \\ * & -\gamma^2 I & 0 & \bar{d}B_\omega^T & 0 \\ * & * & -S & 0 & 0 \\ * & * & * & -\bar{d}S & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{7}$$

where

$$\Delta_0 = \begin{bmatrix} \Xi_0 & P_aA_{edf} + \begin{bmatrix} -M_1^T + M_2 & 0 \\ 0 & 0 \end{bmatrix} \\ * & -(1-h)Q + \begin{bmatrix} -M_2^T + M_2 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

with $\Xi_0 = P_aA_{ef} + A_{ef}^TP_a + Q + \begin{bmatrix} M_1^T + M_1 & 0 \\ 0 & 0 \end{bmatrix}$
(ii) there exist symmetric matrices $Y, N, \bar{Q}, \bar{S}$ with $0 < N < Y$, $\bar{Q} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} \\ * & \bar{Q}_{22} \end{bmatrix}$, matrices M_1, M_2 and a filter described by (3) with $A_{Ff} = A_{Fe0}$, $B_{Ff} = B_{Fe0}$ and $C_{Ff} = C_{Fe0}$ such that for $\delta_i \in [\underline{\delta_i}, \bar{\delta}_i]$

$$V_{a1} := \begin{bmatrix} \Theta_1 & \Theta_2 & \Theta_4 & 0 & \Theta_7 & \bar{d}\bar{M}_1^T & \bar{d}A^T\bar{S} & C_1^T \\ * & \Theta_3 & \Theta_5 & 0 & \Theta_8 & 0 & 0 & -C_{Fe0}^T \\ * & * & \Theta_6 - (1-h)\bar{Q}_{11} & -(1-h)\bar{Q}_{12} & 0 & \bar{d}\bar{M}_2^T & \bar{d}A_d^T\bar{S} & 0 \\ * & * & * & -(1-h)\bar{Q}_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{S} & 0 & 0 \\ * & * & * & * & * & * & -\bar{d}\bar{S} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{8}$$

where

$$\begin{aligned}\Theta_1 &= YA(\delta) - NB_{Fe0}C_2 + \bar{M}_1 + \bar{Q}_{11} \\ &\quad + [YA(\delta) - NB_{Fe0}C_2 + \bar{M}_1]^T \\ \Theta_2 &= -NA_F(\delta) - A^T(\delta)N + C_2^T B_{Fe0}^T N + \bar{Q}_{12} \\ \Theta_3 &= NA_{Fe0} + (NA_{Fe0})^T + \bar{Q}_{22} \\ \Theta_4 &= YA_d(\delta) - NB_{Fe0}C_3 - \bar{M}_1^T + \bar{M}_2 \\ \Theta_5 &= -NA_d(\delta) + NB_{Fe0}C_3 \\ \Theta_6 &= -\bar{M}_2 - \bar{M}_2^T \\ \Theta_7 &= YB_\omega - NB_{Fe0}D_{21} \\ \Theta_8 &= -NB_\omega + NB_{Fe0}D_{21}\end{aligned}$$

Proof: Due to the space of limitation, the proof is omitted here. □

Algorithm 1: Let γ denotes the robust H_∞ performance bound of the closed-loop system (3). Let $NA_{Ff} = \bar{A}_{Ff}$ and $NB_{Ff} = \bar{B}_{Ff}$.

$$\min \eta \text{ s.t. } 0 < N < Y \quad (8)$$

where $\eta = \gamma^2$. Then the resultant gains of robust filter (2) are $A_{Ff} = \bar{A}_{Ff}N^{-1}$, $B_{Ff} = \bar{B}_{Ff}N^{-1}$ and C_{Ff}.

III. ROBUST H_∞ SWITCHING-TYPE FILTER DESIGN

Theorem 1: Consider the filtering error system (5), and let $\gamma > 0, \bar{d} > 0$ and $0 < h < 1$ be given scalars. If there exist positive definite matrices N, Y, Q_{11}, Q_{22}, S with $0 < N < Y$ and matrices $Q_{12}, M_1, M_2, A_{F0}, A_{Fi}, B_{F0}, B_{Fi}, C_{F0}, C_{Fi}, i = 1 \cdots N_0$ such that for $\delta_i(t), \hat{\delta}_i(t) \in [\underline{\delta}_i, \bar{\delta}_i]$ the following matrix inequalities hold:

$$\begin{bmatrix} T_1 & T_2 & T_4 & 0 & T_7 & \bar{d}M_1^T & \bar{d}A^TS & C_1^T \\ * & T_3 & T_5 & 0 & T_8 & 0 & 0 & -C_F^T(\delta) \\ * & * & T_6 - (1-h)Q_{11} & -(1-h)Q_{12} & T_9 & \bar{d}M_2^T & \bar{d}A_d^TS & 0 \\ * & * & * & -(1-h)Q_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & T_{10} - \gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & * & -\bar{d}S & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (9)$$

with

$$\begin{aligned}A_F(\delta) &= A_{F0} + \sum_{i=1}^{N_0} \delta_i A_{Fi}, \quad B_F(\delta) = B_{F0} + \sum_{i=1}^{N_0} \delta_i B_{Fi} \\ T_1 &= \Psi_1 + \Psi_1^T \\ \Psi_1 &= YA(\delta) - NB_F(\delta)C_2 + M \\ &\quad + \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i)[-2N_3^T NB_{Fi}C_2 + NB_{Fi}C_2] \\ T_2 &= -NA_F(\delta) - A^T(\delta)N + C_2^T B_F^T(\delta)N \\ &\quad + \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i)[-N_3^T NA_{Fi} - C_2^T B_{Fi}^T N] + Q_{12} \\ T_3 &= NA_F(\delta) + (NA_F(\delta))^T + Q_{22} \\ T_4 &= YA_d(\delta) - NB_F(\delta)C_3 - M_1^T + M_2 + \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \\ &\quad \times [-2N_3^T NB_{Fi}C_3 + NB_{Fi}C_3 + 2C_2^T B_{Fi}^T NN_4] \\ T_5 &= -NA_d(\delta) + NB_F(\delta)C_3 \\ &\quad + \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i)[-C_3^T B_{Fi}^T N + N_4^T NA_{Fi}] \\ T_6 &= \sum_{i=1}^{N_0} 2(\hat{\delta}_i - \delta_i)[N_4^T NB_{Fi}C_3 + (N_4^T NB_{Fi}C_3)^T] \\ &\quad - M_2 - M_2^T \\ T_7 &= YB_\omega - NB_F(\hat{\delta})D_{21} + \sum_{i=1}^{N_0} 2(\hat{\delta}_i - \delta_i) \\ &\quad \times [C_2^T B_{Fi}^T NN_2 + NB_{Fi}D_{21} - N_3^T NB_{Fi}D_{21}] \\ T_8 &= -NB_\omega + NB_F(\hat{\delta})D_{21} \\ &\quad + \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i)[A_{Fi}^T NN_2 - 2NB_{Fi}D_{21}] \\ T_9 &= 2\sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i)[N_4TNB_{Fi}D_{21} + C_3^T B_{Fi}^T NN_2] \\ T_{10} &= 2\sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i)(N_2^T NB_{Fi}D_{21} + D_{21}^T B_{Fi}^T NN_2) \\ N_1 &= T_{cn}\begin{bmatrix} T_c \\ 0 \end{bmatrix}, \quad N_2 = T_{cn}\begin{bmatrix} T_cD_{21} \\ 0 \end{bmatrix} \\ N_3 &= T_{cn}\begin{bmatrix} 0 \\ C_{cn} \end{bmatrix}, N_4 = T_{cn}\begin{bmatrix} T_cC_3 \\ 0 \end{bmatrix}\end{aligned}$$

and also $\hat{\delta}_i(t)$ is determined according to the switching law

$$\hat{\delta}_i = \begin{cases} \bar{\delta}_i, & \text{if } U_i + \bar{U}_i < 0 \\ \underline{\delta}_i, & \text{if } U_i + \bar{U}_i \geq 0 \end{cases}, \quad i = 1 \cdots N_0 \quad (10)$$

$$\begin{aligned}U_i &= \xi^T NA_{Fi}\xi - y^T N_1^T NA_{Fi}\xi + \xi^T NB_{Fi}y \\ &\quad - y^T N_1^T NB_{Fi}y \\ \bar{U}_i &= \xi^T NB_{Fi}y - y^T N_1^T NB_{Fi}y\end{aligned}$$

Then, the filter error system (5) is asymptotically stable with an H_∞ disturbance attenuation level γ.

Proof: Choose the following Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t).$$

where

$$\begin{aligned}V_1(t) &= x_e^T(t)Px_e(t), \quad V_2(t) = \int_{t-d(t)}^{t} x_e^T(\alpha)Qx_e(\alpha)d\alpha, \\ V_3(t) &= \int_{-\bar{d}}^{0}\int_{t+\beta}^{t} \dot{x}_e^T(\alpha)H^TSH\dot{x}_e(\alpha)d\alpha d\beta.\end{aligned}$$

with $H = \begin{bmatrix} I & 0 \end{bmatrix}, Q = \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$, and $S > 0$.

Then the derivative of $V(t)$ along any trajectory of the filtering error system (5) is given by

$$\dot{V}_1(t) = 2x_e^T(t)P\dot{x}_e(t) \quad (11)$$

$$\dot{V}_2(t) \leq x_e^T(t)Qx_e(t) - (1-h)x_e^T(t-d(t))Qx_e(t-d(t)) \quad (12)$$

$$\begin{aligned}\dot{V}_3(t) &\leq \bar{d}\eta_0 \begin{bmatrix} A^T \\ A_d^T \\ B_\omega^T \end{bmatrix} S \begin{bmatrix} A & A_d & B_\omega \end{bmatrix} \eta_0^T \\ &\quad - \int_{t-d(t)}^{t} \dot{x}^T(\alpha)S\dot{x}(\alpha)d\alpha \quad (13)\end{aligned}$$

with $\eta_0 = \begin{bmatrix} x^T(t) & x^T(t-d(t)) & \omega^T(t) \end{bmatrix}$.
Using Lemma 2, it follows

$$-\int_{t-d(t)}^{t} \dot{x}^T(s) S \dot{x}(s) ds \tag{14}$$

$$\leq \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}$$
$$+ \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} S^{-1} \begin{bmatrix} M_1 & M_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix} \tag{15}$$

Then A_e and A_{ed} can be written as

$$A_e = A_{ea} + A_{eb}, \quad A_{ed} = A_{e1} + A_{e2}$$

where

$$A_{ea} = \begin{bmatrix} A(\delta) & 0 \\ B_F(\delta)C_2 & A_F(\delta) \end{bmatrix},$$
$$A_{eb} = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} 0 & 0 \\ B_{Fi}C_2 & A_{Fi} \end{bmatrix}$$
$$A_{e1} = \begin{bmatrix} A_d(\delta) & 0 \\ B_F(\delta)C_3 & 0 \end{bmatrix}, \quad A_{e2} = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} 0 & 0 \\ B_{Fi}C_3 & 0 \end{bmatrix}$$

Let P be of the following form,

$$P = \begin{bmatrix} Y & -N \\ -N & N \end{bmatrix}$$

with $0 < N < Y$, which implies $P > 0$. From (1), it follows

$$T_c C_2 x = T_c [y - D_{21}\omega - C_3 x(t-d(t))] \tag{16}$$

Thus

$$x = T_{cn} \begin{bmatrix} T_c C_2 x \\ C_{cn} x \end{bmatrix} = N_1 y - N_2 \omega + N_3 x - N_4 x(t-d(t)) \tag{17}$$

with $N_1 = T_{cn} \begin{bmatrix} T_c \\ 0 \end{bmatrix}, N_2 = T_{cn} \begin{bmatrix} T_c D_{21} \\ 0 \end{bmatrix}, N_3 = T_{cn} \begin{bmatrix} 0 \\ C_{cn} \end{bmatrix}, N_4 = T_{cn} \begin{bmatrix} T_c C_3 \\ 0 \end{bmatrix}$.

Furthermore, we have

$$PA_{ea} = \begin{bmatrix} YA(\delta) - NB_F(\delta)C_2 & -NA_F(\delta) \\ -NA(\delta) + NB_F(\delta)C_2 & NA_F(\delta) \end{bmatrix}$$

and

$$PA_{eb} = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} -NB_{Fi}C_2 & -NA_{Fi} \\ NB_{Fi}C_2 & NA_{Fi} \end{bmatrix}$$

which follows

$$[x^T \ \xi^T] PA_{eb} [x^T \ \xi^T]^T$$
$$= \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \{ -x^T NB_{Fi}C_2 x - x^T NA_{Fi}\xi$$
$$+ \xi^T NB_{Fi}C_2 x + \xi^T NA_{Fi}\xi \}$$

By (16) and (17), it is easy to see

$$- x^T NB_{Fi}C_2 x$$
$$= (\omega^T N_2^T - x^T N_3^T + x^T(t-d(t))N_4^T) NB_{Fi}$$
$$\times (C_2 x + D_{21}\omega + C_3 x(t-d(t))) + x^T NB_{Fi}D_{21}\omega$$
$$- y^T N_1^T NB_{Fi} y + x^T NB_{Fi}C_3 x(t-d(t))$$
$$- x^T NA_{Fi}\xi = -(y^T N_1^T - \omega^T N_2^T + x^T N_3^T$$
$$- x^T(t-d(t))N_4^T) NA_{Fi}\xi$$
$$\xi^T NB_{Fi}C_2 x = \xi^T NB_{Fi}(y - D_{21}\omega - C_3 x(t-d(t)))$$

Thus,

$$x_e^T PA_{eb} x_e = \eta^T A_{Pe} \eta + \eta^T B_{Pe} \omega + U$$
$$+ \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \omega^T N_2^T NB_{Fi}D_{21}\omega$$

where $\eta^T = \begin{bmatrix} x_e^T & x_e^T(t-d(t)) \end{bmatrix} = \begin{bmatrix} x^T(t) & \xi^T(t) & x^T(t-d(t)) & \xi^T(t-d(t)) \end{bmatrix}$

$$A_{Pe} = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix}, B_{pe} = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

with

$$\Psi_{11} = \begin{bmatrix} -N_3^T NB_{Fi}C_2 & -N_3^T NA_{Fi} \\ 0 & 0 \end{bmatrix},$$
$$\Psi_{12} = \begin{bmatrix} -N_3^T NB_{Fi}C_3 + NB_{Fi}C_3 & 0 \\ -NB_{Fi}C_3 & 0 \end{bmatrix},$$
$$\Psi_{21} = \begin{bmatrix} N_4^T NB_{Fi}C_2 & N_4^T NA_{Fi} \\ 0 & 0 \end{bmatrix},$$
$$\Psi_{22} = \begin{bmatrix} N_4^T NB_{Fi}C_3 & 0 \\ 0 & 0 \end{bmatrix}$$
$$F_1 = \begin{bmatrix} C_2^T B_{Fi}^T N N_2 + NB_{Fi}D_{21} - N_3^T NB_{Fi}D_{21} \\ A_{Fi}^T N N_2 - NB_{Fi}D_{21} \end{bmatrix}$$
$$F_2 = \begin{bmatrix} C_3^T B_{Fi}^T N N_2 + N_4^T NB_{Fi}D_{21} \\ 0 \end{bmatrix}$$
$$U = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) U_i,$$
$$U_i = \xi^T NA_{Fi}\xi - y^T N_1^T NA_{Fi}\xi + \xi^T NB_{Fi} y$$
$$- y^T N_1^T NB_{Fi} y$$

On the other hand, we have

$$PA_{e1} = \begin{bmatrix} YA_d(\delta) - NB_F(\delta)C_3 & 0 \\ -NA_d(\delta) + NB_F(\delta)C_3 & 0 \end{bmatrix}$$

and

$$PA_{e2} = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} -NB_{Fi}C_3 & 0 \\ NB_{Fi}C_3 & 0 \end{bmatrix}$$

which follows

$$x_e^T(t) PA_{eb} x_e^T(t-d(t))$$
$$= \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \{ -x^T NB_{Fi}C_3 x(t-d(t))$$
$$+ \xi^T NB_{Fi}C_3 x(t-d(t)) \}$$

By (16) and (17), it is easy to see

$$- x^T NB_{Fi}C_3 x(t-d(t))$$
$$= (\omega^T N_2^T - x^T N_3^T + x^T(t-d(t))N_4^T) NB_{Fi}$$
$$\times (C_2 x + D_{21}\omega + C_3 x(t-d(t))) + x^T NB_{Fi}D_{21}\omega(t)$$
$$- y^T N_1^T NB_{Fi} y + x^T NB_{Fi}C_2 x(t)$$
$$\xi^T NB_{Fi}C_3 x(t-d(t)) = \xi^T NB_{Fi}(y - D_{21}\omega - C_2 x(t))$$

Thus,

$$x_e^T PA_{e2} x_e(t-d(t)) = \eta^T \bar{A}_{Pe} \eta + \eta^T \bar{B}_{Pe} \omega + \bar{U}$$
$$+ \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \omega^T N_2^T NB_{Fi}D_{21}\omega$$

where

$$\bar{A}_{Pe}=\sum_{i=1}^{N_0}(\hat{\delta}_i-\delta_i)\begin{bmatrix}\Phi_{11} & \Phi_{12}\\ \Phi_{21} & \Phi_{22}\end{bmatrix},\bar{B}_{Pe}=\sum_{i=1}^{N_0}(\hat{\delta}_i-\delta_i)\begin{bmatrix}F_3\\ F_4\end{bmatrix}$$

with

$$\begin{aligned}
\Phi_{11}&=\begin{bmatrix}-N_3^TNB_{Fi}C_2+NB_{Fi}C_2 & 0\\ -NB_{Fi}C_2 & 0\end{bmatrix},\\
\Phi_{12}&=\begin{bmatrix}-N_3^TNB_{Fi}C_3 & 0\\ 0 & 0\end{bmatrix},\\
\Phi_{21}&=\begin{bmatrix}N_4^TNB_{Fi}C_2 & 0\\ 0 & 0\end{bmatrix},\quad \Phi_{22}=\begin{bmatrix}N_4^TNB_{Fi}C_3 & 0\\ 0 & 0\end{bmatrix}\\
F_3&=\begin{bmatrix}C_2^TB_{Fi}^TNN_2+NB_{Fi}D_{21}-N_3^TNB_{Fi}D_{21}\\ -NB_{Fi}D_{21}\end{bmatrix}\\
F_4&=\begin{bmatrix}C_3^TB_{Fi}^TNN_2+N_4^TNB_{Fi}D_{21}\\ 0\end{bmatrix}\\
\bar{U}&=\sum_{i=1}^{N_0}(\hat{\delta}_i-\delta_i)\bar{U}_i,\\
\bar{U}_i&=\xi^TNB_{Fi}y-y^TN_1^TNB_{Fi}y.
\end{aligned}$$

Then from the derivative of $V(t)$ along the closed-loop system (5), it follows

$$\dot{V}_1(t)\le\begin{bmatrix}\eta\\ \omega\end{bmatrix}^T\begin{bmatrix}\Omega_0 & \begin{bmatrix}PB_e+F_1+F_3\\ F_2+F_4\end{bmatrix}\\ * & \Phi_1\end{bmatrix}\begin{bmatrix}\eta\\ \omega\end{bmatrix}+2(U+\bar{U})$$

where

$$\Omega_0=\begin{bmatrix}\Gamma_1 & \Theta_1\\ * & \Theta_2\end{bmatrix}+\begin{bmatrix}C_e^T & 0\end{bmatrix}\begin{bmatrix}C_e\end{bmatrix}$$

$$\Phi_1=2\sum_{i=1}^{N_0}(\hat{\delta}_i-\delta_i)(N_2^TNB_{Fi}D_{21}+D_{21}^TB_{Fi}^TNN_2)-\gamma^2I$$

with

$$\begin{aligned}
\Gamma_1&=PA_{ea}+A_{ea}^TP\\
&\quad+\sum_{i=1}^{N_0}(\hat{\delta}_i-\delta_i)[\Phi_{11}+\Psi_{11}+(\Phi_{11}+\Psi_{11})^T]\\
\Theta_1&=PA_{e1}+\sum_{i=1}^{N_0}(\hat{\delta}_i-\delta_i)[\Phi_{12}+\Phi_{21}^T+\Psi_{12}+\Psi_{21}^T]\\
\Theta_2&=\sum_{i=1}^{N_0}(\hat{\delta}_i-\delta_i)[\Phi_{22}+\Psi_{22}+(\Phi_{22}+\Psi_{22})^T]
\end{aligned}$$

Furthermore, from (11) and (16) we can obtain that

$$\begin{aligned}
&\dot{V}(t)+z_e^T(t)z_e(t)-\gamma^2\omega^T(t)\omega(t)\\
&\le\begin{bmatrix}\eta\\ \omega\end{bmatrix}^T\Omega_1\begin{bmatrix}\eta\\ \omega\end{bmatrix}+2(U+\bar{U})
\end{aligned}\tag{18}$$

where

$$\begin{aligned}
\Omega_1=&\begin{bmatrix}\Omega_2+\Pi & \begin{bmatrix}PB_e+F_1+F_3\\ F_2+F_4\end{bmatrix}\\ * & \Phi_1\end{bmatrix}\\
&+\bar{d}\begin{bmatrix}A^T\\ 0\\ A_d^T\\ 0\\ B_\omega^T\end{bmatrix}S\begin{bmatrix}A & 0 & A_d & 0 & B_\omega\end{bmatrix}
\end{aligned}$$

with

$$\begin{aligned}
&\Omega_2=\\
&\begin{bmatrix}Q+\begin{bmatrix}M_1^T+M_1 & 0\\ 0 & 0\end{bmatrix} & \begin{bmatrix}-M_1^T+M_2 & 0\\ 0 & 0\end{bmatrix}\\ 0 & \Theta_0\end{bmatrix}\\
&+\Omega_0\\
&\Pi=\bar{d}\begin{bmatrix}M_1^T\\ 0\\ M_1^T\\ 0\end{bmatrix}S^{-1}\begin{bmatrix}M_1 & 0 & M_2 & 0\end{bmatrix},\quad Q=\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}\\
&\Theta_0=-(1-h)Q+\begin{bmatrix}-M_2^T+M_2 & 0\\ 0 & 0\end{bmatrix}
\end{aligned}$$

The design condition $\dot{V}(t)+z_e^T(t)z_e(t)-\gamma^2\omega^T(t)\omega(t)\le 0$ is reduced to

$$\Omega_1<0\tag{19}$$

and

$$U+\bar{U}\le 0.\tag{20}$$

Since y and ξ are available on line, the switching law can be chosen as (10). So (20) can be achieved.
Notice that

$$PB_e=\begin{bmatrix}YB_\omega-NB_F(\hat{\delta})D_{21}\\ -NB_\omega+NB_F(\hat{\delta})D_{21}\end{bmatrix}$$

It is easy to see $\Omega_1<0$ is equivalent to

$$\begin{aligned}
&\Omega_4=\\
&\begin{bmatrix}\Omega_2 & \begin{bmatrix}PB_e+F_1+F_3\\ F_2+F_4\end{bmatrix} & \begin{bmatrix}\bar{d}M_1^T\\ 0\\ \bar{d}M_2^T\\ 0\end{bmatrix} & \begin{bmatrix}\bar{d}A^TS\\ 0\\ \bar{d}A_d^TS\\ 0\end{bmatrix} & \begin{bmatrix}C_e^T\\ 0\end{bmatrix}\\
* & \Phi_1 & 0 & \bar{d}B_\omega^T & 0\\
* & * & -S & 0 & 0\\
* & * & * & -\bar{d}S & 0\\
* & * & * & * & -I\end{bmatrix}\\
&<0
\end{aligned}\tag{21}$$

If (9) holds, which implies $\Omega_4<0$. Thus it follows $\Omega_1<0$. Together with the switching laws (10) , we can get $\dot{V}(t)\le 0$. Furthermore, we have

$$\dot{V}(t)+z_e^T(t)z_e(t)-\gamma^2\omega^T(t)\omega(t)\le 0.$$

Integrate the above-mentioned inequalities from 0 to ∞ on both sides, we obtain

$$V(\infty)-V(0)+\int_0^\infty z_e(t)^Tz_e(t)dt\le\gamma^2\int_0^\infty\omega(t)^T\omega(t)dt.$$

which implies that the H_∞ disturbance attenuation of the closed-loop system (5) is no larger than γ holds. □

Theorem 2: If the condition in Lemma 1 holds for the closed-loop system (3) with traditional robust filter (2), then the condition in Theorem 1 holds for the closed-loop system (5) with robust switching-type filter (4).
Proof: The proof can be easily obtained from Theorem 1, so we omit it here. □
Algorithm 2: Let γ denotes the robust H_∞ performance bound of the closed-loop system (5). Let $NA_{F0}=\bar{A}_{F0}$, $NA_{Fi}=\bar{A}_{Fi}$, $NB_{F0}=\bar{B}_{F0}$.and $NB_{Fi}=\bar{B}_{Fi}$.

$$\min\eta\ \text{ s.t. }\ 0<N<Y\ \text{and}\ (9),$$

where $\eta=\gamma^2$. Then the resultant gains of robust switching-type filter (4) are $A_{F0}=\bar{A}_{F0}N^{-1}$, $A_{Fi}=\bar{A}_{Fi}N^{-1}$, $B_{F0}=\bar{B}_{F0}N^{-1}$, $B_{Fi}=\bar{B}_{Fi}N^{-1}$, C_{F0} and C_{Fi}, $i=1\cdots N_0$.

IV. Numerical Example

Consider the following linear time-delay system (1) with time-varying uncertainty satisfying

$$A(\delta) = \begin{bmatrix} -2 & 5 \\ -1 & -4 \end{bmatrix} + \delta_1(t) \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} + \delta_2(t) \begin{bmatrix} 0 & 0.1 \\ 1 & 0.1 \end{bmatrix}$$

$$A_d(\delta) = \begin{bmatrix} -0.1 & 0.4 \\ 0.2 & 0.3 \end{bmatrix} + \delta_1(t) \begin{bmatrix} 0.2 & 0.1 \\ 0.05 & 0 \end{bmatrix} + \delta_2(t) \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \; B_\omega = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}, \; C_2 = \begin{bmatrix} 3 & 0 \\ 1 & 0 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}, \; D_{21} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} \; x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

with $\delta_1(t) = 0.5\cos(t)$, $\delta_2(t) = \sin(t)$ and $d(t) = \frac{1}{2}\sin(t) + \frac{1}{2}$. Here we chose $T_c = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

Using Matlab LMI tool box [3], Algorithm 1 and Algorithm 2, we get the H_∞ performance index is 1.1179 with the robust switching-type filter while that of traditional robust filter with fixed gains is 1.3037. Just as the theory has proved the robust switching-type H_∞ filter design method is less conservative than traditional robust filter with fixed gains.

The simulations are carried out with the following disturbance $\omega(t) = \begin{bmatrix} \omega_1(t) & \omega_2(t) \end{bmatrix}^T$, where

$$\omega_1(t) = \omega_2(t) = \begin{cases} 1, & 1 \le t \le 2 \text{ (seconds)} \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 and Figure 2 are the responses curves of system states with the robust switching-type H_∞ filter and traditional robust H_∞ filter with fixed gains, respectively. It is easy to see that our robust switching-type H_∞ filter has more disturbance attenuation ability than that of traditional robust filter with fixed gains as theory has proved.

V. Conclusions

This paper has proposed a novel robust H_∞ filter design procedure for linear uncertain systems with time-varying delay. The uncertainty parameters are supposed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system model. A new switching-type filter is established based on LMI method and switching laws to guarantee asymptotic stability and a prescribed H_∞ performance level of the error systems for all admissible uncertainties. The derived design conditions are less conservative than those of the corresponding filter with fixed gains, which has also been demonstrated by an illustrative example.

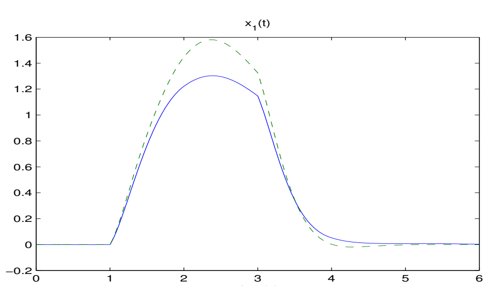

Fig. 1. Response curve of the first state with robust switching-type filter (solid) and traditional robust filter with fixed gains (dashed).

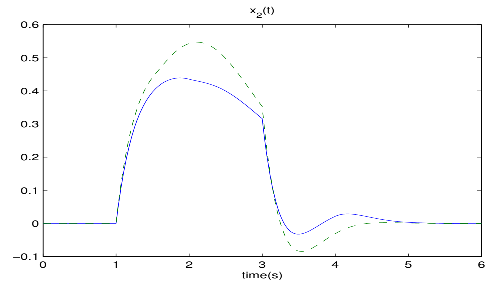

Fig. 2. Response curve of the second state with robust switching-type filter (solid) and traditional robust filter with fixed gains (dashed).

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