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Neuro-fuzzy Network Control for a Mobile Robot

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Abstract—**A control structure that makes possible the integration of a kinematic controller and a neuro-fuzzy network (NFN) dynamic controller for mobile robots is presented. A combined kinematic/dynamic control law is developed using backstepping and stability is guaranteed by Lyapunov theory. The NFN controller proposed in this work can deal with unmodeled bounded disturbances and/or unstructured unmodeled dynamic in the mobile robot. On-line NFN parameter tuning algorithms do no require off-line learning yet guarantee small tracking errors and bounded control signals are utilized.**

Index Terms- **Mobile robot, Neuro-fuzzy networks, Lyapunov stability, Feedback control.**

I. Introduction

A mobile robot is an uncertain nonlinear dynamic system, which suffers from structured or unstructured uncertainties. The mobile robots have been used extensively in various industrial and service applications. The application ranges from security, transportation, inspection, and planetary exploration, etc. Mobile robots constitute a class of mechanical systems called nonholonomic mechanical systems characterized by kinematic constraints that are not integrable and cannot, therefore, be eliminated from the model equations. Using Lagrange formalism and differential geometry, a general dynamical model can be derived for mobile robots with nonholonomic constraints.

In the trajectory tracking problem, the mobile robot is to follow a prespecified trajectory. Using the kinematic model of the mobile robots, the tracking problem was solved in [1]. Dynamic feedback linearization has been used for trajectory tracking and posture stabilization of mobile robot systems in chained form [2]. Adaptive robust motion force control of holonomic constrained nonholonomic mobile manipulators is appeared in [3]. Also, artificial intelligence control using neural networks and fuzzy logic can be considered as an effective tool for nonlinear controller design [4].

In this paper, actuator nonlinearity is included in system dynamics. The contribution of this paper is the utilization of an NFN for estimating the nonlinear robot functions involving actuator friction nonlinearity. A rigorous design procedure with proofs is given, resulting in a kinematic tracking loop with an NFN in the feed forward loop. This paper is organized as follows. Section 2 presents the mobile robot model. Section 3 summarizes the NFN. Tracking problem definition, controller design details, and stability analysis are described in Section 4. Experimental results of the proposed controller with a mobile robot system are given in Section 5. Finally, conclusions are included in Section 6.

II. Mobile Robot

The mobile robot shown in Fig. 1 is a typical example of a nonholonomic mechanical system. It consists of a vehicle with two driving wheels mounted on the same axis, and a front free wheel [5]. The motion and orientation are achieved by independent actuators, e.g., dc motors providing the necessary torques to the rear wheels. One mobile robot with n generalized coordinates and m constraints is described by

$$M(q)\ddot{q}+V(q,\dot{q})\dot{q}+F(\dot{q})+G(q)+\tau_d = B(q)\tau - A^T(q)\lambda \quad (1)$$

where $M(q)\in R^{n\times n}$ is a symmetric, positive definite inertia matrix, $V(q,\dot{q})\in R^{n\times n}$ is the centripetal and Coriolis matrix, $F(\dot{q})\in R^{n\times 1}$ denotes the surface friction, $G(q)\in R^{n\times 1}$ is the gravitational vector, τ_d denotes the bounded unknown disturbances including unstructured unmodeled dynamics, $B(q)\in R^{n\times r}$ is the input transformation matrix, $\tau\in R^{n\times 1}$ is the input vector, $A(q)\in R^{m\times n}$ is the matrix associated with the constraints, and $\lambda\in R^{m\times 1}$ is the vector of constraint forces.

We consider that all kinematic equality constraints are independent of time, and can be expressed as follows:

$$A(q)\dot{q}=0\,. \quad (2)$$

Let $S(q)$ be a full rank matrix (n-m) formed by a set

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of smooth and linearly independent vector fields in the null space of $A(q)$, i.e.,

$$S^T(q)A^T(q) = 0\,. \tag{3}$$

According to (2) and (3), it is possible to find an auxiliary vector time function $v(t) \in R^{n-m}$ such that, for all t

$$\dot{q} = S(q)v(t)\,. \tag{4}$$

The nonholonomic constraint states that the robot can only move in the direction normal to the axis of the driving wheels, i.e., the mobile base satisfies the conditions of pure rolling and nonslipping

$$\dot{y}_c \cos\theta - \dot{x}_c \sin\theta - d\dot{\theta} = 0\,. \tag{5}$$

It is easy to verify that $S(q)$ is given by

$$S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}. \tag{6}$$

The kinematic equations of motion (4) of C, an inertial cartesian frame, in terms of its linear velocity and angular velocities are

$$v = \begin{bmatrix} v \\ \omega \end{bmatrix},\ \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{7}$$

where $|v| \le V_{\max}$ and $|\omega| \le \Omega_{\max}$, $V_{\max}$ and $\Omega_{\max}$ are the maximum linear and angular velocities of the mobile robot.

Equation (7) is also known as Posture kinematic model. This model could also be obtained in polar coordinates, where the posture vector is composed by the triple (e, ϕ, α) (as shown in Fig. 1), which is related to the cartesian coordinates by

$$\begin{cases} e = \sqrt{x_c^2 + y_c^2} \\ \phi = \tan^{-1}(\dfrac{y_c}{x_c}) \\ \alpha = \theta - \phi \end{cases}. \tag{8}$$

The cartesian coordinates can be calculated from polar coordinates by using

$$\begin{cases} x_c = e\cos(\phi) \\ y_c = e\sin(\phi) \end{cases} \tag{9}$$

from which, under time differentiation and solving for $\dot{e}$ and $\dot{\phi}$

$$\dot{e} = \dot{x}_c \cos(\phi) + \dot{y}_c \sin(\phi) \tag{10}$$

and

$$\dot{\phi} = \frac{1}{e}(-\dot{x}_c \sin(\phi) + \dot{y}_c \cos(\phi))\,. \tag{11}$$

By replacing (7) into (10), (11) and differentiation of the third equation of (8) with respect to time, the kinematic model in polar coordinates can be written as

$$\begin{bmatrix} \dot{e} \\ \dot{\phi} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -d\sin(\alpha) \\ \dfrac{\sin(\alpha)}{e} & d\dfrac{\cos(\alpha)}{e} \\ \dfrac{\sin(\alpha)}{e} & 1 - d\dfrac{\cos(\alpha)}{e} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{12}$$

When the reference posture is not equal to triple $(0,0,0°)$, the triple (e, ϕ, α) can not be calculated as in (8). This is the case for tracking because the reference points change with time. More precisely, in this case we have to rely on Fig. 2, where

$$e = \sqrt{e_x^2 + e_y^2} \tag{13}$$

with $[e_x\ e_y\ e_\theta]^T = [x_r\ \ y_r\ \ \theta_r]^T - [x_c\ \ y_c\ \ \theta]^T$. The sides of the triangle COR are calculated from the projections of e_x and e_y, with CO given by $e_x \cos(\theta_r) + e_y \sin(\theta_r)$ and OR by $-e_x \sin(\theta_r) + e_y \cos(\theta_r)$. Hence

$$\phi = \tan^{-1}(\frac{-e_x \sin(\theta_r) + e_y \cos(\theta_r)}{e_x \cos(\theta_r) + e_y \sin(\theta_r)}) \tag{14}$$

and then

$$\alpha = e_\theta - \phi\,. \tag{15}$$

The controller structure is described only for point stabilization. Equations (13)-(15) are needed in order to describe the robot model in polor coordinates for the tracking case, which is a more general problem than point stabilization.

The dynamical equations of the mobile base in Fig. 1 can be expressed in matrix form (1) where

$$M(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & 1 \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & md\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix}$$

$$G(q) = 0$$

$$B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R & -R \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix},\quad A^T(q) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ -d \end{bmatrix}$$

$$\lambda = -m(\dot{x}_c \cos\theta + \dot{y}_c \sin\theta)\dot{\theta}\,. \tag{16}$$

The system (1) is now transformed into a more appropriate representation for controls purposes. Differentiating (4), substituting this result in (1), and then multiplying by S^T, we can eliminate the constraint matrix $A^T(q)\lambda$. The complete equations of motion of the nonholonomic platform are given by

$$\dot{q} = Sv \tag{17}$$

$$S^T MS\dot{v} + S^T(M\dot{S} + VS)v + \overline{F}(v) + \overline{\tau}_d = S^T B\tau \tag{18}$$

where $v(t) \in R^{n-m}$ is a velocity vector. By appropriate definitions we can rewrite (18) as follows:

$$\overline{M}(q)\dot{v} + \overline{V}(q,\dot{q})v + \overline{F}(v) + \overline{\tau}_d = \overline{B}\tau$$
$$\overline{\tau} \equiv \overline{B}\tau \tag{19}$$

where $\overline{M}(q) \in R^{r\times r}$ is a symmetric positive definite inertia matrix, $\overline{V}(q,\dot{q}) \in R^{r\times r}$ is the centripetal and coriolis matrix, $\overline{F}(v) \in R^{r\times 1}$ is the surface friction, $\overline{\tau}_d$ denotes bounded unknown disturbances including unstructured unmodeled dynamics, and $\overline{\tau} \in R^{r\times 1}$ is the input vector.

III. Neuro-fuzzy Networks

The NFN in Fig. 3 has a network output of the input x by the formula

$$y = \sum_{k=1}^{N_2}[w_{ko} \cdot \prod_{j=1}^{N_1}\{v_{jk} \cdot \exp(-\frac{(x_i - m_{ij})^2}{\sigma_{ij}^2})\}] \tag{20}$$

with the Gaussian function with the mean m and the standard deviation σ, $v_{jk}(=1)$, the interconnection weights from membership to rule layer, and w_{ko}, the interconnection weights from rule to output layer, N_1, the number of neurons in the membership layer, and N_2, the number of neurons in the rule layer. The NFN equation may be conveniently expressed in a vector format by defining, $\hat{W}^T = [w_{1,o}, w_{2,o}, \cdots, w_{N_2,o}] \in R^{1\times N_2}$, $\Gamma = [\Gamma_1, \Gamma_2, \cdots, \Gamma_{N_1}]^T$, $\Gamma_j(x,m,\sigma) = \exp\{-(x_i - m_{ij})^2 / \sigma_{ij}^2\}$ and a matrix format by defining $\hat{V}^T = [v_{jk}] \in R^{N_2\times N_1}$. Then, $y = \hat{W}^T\Omega(\Gamma) = \hat{W}^T\hat{V}^T\Gamma(X_i)$, $X_i \in R^{N_3\times 1}$ is the input state of the NFN. Since $v_{jk} = 1$, for notational convenience,

$$y = \hat{W}^T\Gamma(x,m,\sigma). \tag{21}$$

A general function, f, can be modeled by an NFN as:

$$f = W^{*T}\,\Gamma(x,m^*,\sigma^*) + \varepsilon \tag{22}$$

where W^* is the constant ideal weight of the current weight $\hat{W}$, m^* is the ideal mean of the current mean m, σ^* is the ideal standard deviation of the current standard deviation σ, and $\Gamma(x,m^*,\sigma^*)$ is the ideal Gaussian function of the current function Γ so that ε is bouned by a known constant ε_N, and ε is the reconstruction error due to the NFN structure.

IV. Mobile Robot Controller

In this section, we derive the kinematic controller and dynamic NFN controller of the mobile robot. The structure for the tracking control system to be derived in this section is presented in Fig. 4.

A. Kinematic controller

Consider (12) with $d = 0$. This is not required by the stability analysis of the overall control system carried out in stability proof, but rather imposed by the particular kinematic controller used here.

Let the candidate Lyapunov function be

$$V_k(e,\phi,\alpha) = \frac{1}{2}(e^2 + \alpha^2 + h\phi^2) \tag{23}$$

where h is a positive constant. Then

$$\dot{V}_k(e,\phi,\alpha) = e\cos(\alpha)v + \alpha(\omega - \frac{\sin(\alpha)}{e\alpha}(\alpha - h\phi)v). \tag{24}$$

If the linear velocity v and the angular velocity ω are made to follow the command signals v_c and ω_c, given by the feedback law

$$v_c = \begin{cases} v_c = -\gamma_1 e\cos(\alpha) \\ \omega_c = -\gamma_2\alpha - \gamma_1\cos(\alpha)\dfrac{\sin(\alpha)}{\alpha}(\alpha - h\phi) \end{cases} \tag{25}$$

where γ_1 and γ_2 are nonnegative constants, it follows that $\dot{V}_k(e,\phi,\alpha) \le 0$, which means e and α are bounded. The second time derivative is

$$\begin{aligned}\ddot{V}_k(e,\phi,\alpha) &= 2\gamma_1^2 e^2[\cos^4(\alpha) + \cos^2(\alpha)\sin^2(\alpha)] \\ &\quad + 2\gamma_2^2\alpha^2 - 2\gamma_1\gamma_2 h\phi\cos(\alpha)\sin(\alpha) \\ &\le 4\gamma_1 e^2 + 2\gamma_2^2\alpha^2\end{aligned} \tag{26}$$

thus, from Barbalat's Lemma, e and α converges to zero, which implies, form (12) and (25) that $\dot{e}$, $\dot{\phi} \to 0$. Then ϕ converges to a finite value $\overline{\phi}$. Additionally, $\dot{\alpha} = \gamma_1 h\overline{\phi}$, so $\ddot{\alpha}$ exists and is bounded, and then $\dot{\alpha} \to 0$, which implies that ϕ must converge to zero.

B. Dynamic controller

Given the desired velocity $v_c(t)$, define now the auxiliary velocity tracking error as

$$e_c = v_c - v \tag{27}$$

Differentiating (27) and using (19), the mobile robot dynamics may be written in terms of the velocity tracking error as

$$\overline{M}(q)\dot{e}_c = -\overline{V}(q,\dot{q})e_c - \overline{\tau} + f(\zeta) + \overline{\tau}_d \tag{28}$$

where the important nonlinear mobile robot function is

$$f(\zeta) = \overline{M}(q)\dot{v}_c + \overline{V}(q,\dot{q})v_c + \overline{F}(v). \tag{29}$$

The vector ζ required to compute $f(\zeta)$ can be defined as

$$\zeta \equiv [v^T \quad v_c^T \quad \dot{v}_c^T]^T \tag{30}$$

which can be measured. Function $f(\zeta)$ contains all the mobile robot parameters such as masses, moments of inertia, friction coefficients, and so on. These quantities are often imperfectly known and difficult to determine. It is assumed $\bar{\tau}_d < \tau_M$, with τ_M being a known positive constant.

In applications the nonlinear robot function $f(\zeta)$ is at least partially unknown. Therefore, a suitable control input for velocity following is given by the computed torque like control

$$\bar{\tau} = \hat{f} + K_4 e_c - \gamma \tag{31}$$

with K_4 a diagonal positive definite gain matrix, and $\hat{f}(\zeta)$ an estimate of the robot function $f(\zeta)$ that is provided by the NFN. The robustifying signal $\gamma(t)$ is required to compensate the unmodeled unstructured disturbances. Using this control in (28), the closed loop system becomes

$$\bar{M}(q)\dot{e}_c = -(K_4 + \bar{V})e_c + \tilde{f} + \bar{\tau}_d + \gamma \tag{32}$$

where the velocity tracking error is driven by the functional estimation error

$$\tilde{f} = f - \hat{f}\,. \tag{33}$$

Some definitions are required in order to proceed.

Definition 1. We denote by $\|\cdot\|$ any suitable vector norm. when it is required to be specific we denote the p-norm by $\|\cdot\|_p$

Definition 2: given $A = [a_{ij}]$, $B \in R^{m\times n}$ the Frobenius norm is defined by

$$\| A \|_F^2 = tr\{A^T A\} = \sum_{i,j} a_{ij}^2 \tag{34}$$

with $tr\{\cdot\}$ the trace. The associated inner product is $< A, B >_F = tr\{A^T B\}$. The Frobenius norm cannot be defined as the induced matrix norm for any vector norm, but is compatable with the 2-norm so that $\| Ax\|_2 \le \| A\|_F \| x\|_2$, with $A \in R^{m\times n}$ and $x \in R^n$.

Definition 3 : For notational convenience, we define the matrix of all the NFN parameters as $\hat{Z} = diag\{\hat{W}, \hat{m}, \hat{\sigma}\}$.

Definition 4: Define the parameter estimation error as $\tilde{W} = W^* - \hat{W}$, $\tilde{m} = m^* - \hat{m}$, $\tilde{\sigma} = \sigma^* - \hat{\sigma}$ and $\tilde{Z} = Z - \hat{Z}$.

Definition 5: Define the membership layer output error for a given x as

$$\tilde{\Gamma} = \Gamma - \hat{\Gamma} = \Gamma(x, m^*, \sigma^*) - \Gamma(x, \hat{m}, \hat{\sigma})\,. \tag{35}$$

The Taylor series expansion of $\Gamma(x)$ for a given x may be written as

$$\Gamma(x, m^*, \sigma^*) = \Gamma(x, \hat{m}, \hat{\sigma}) - \Gamma_m \tilde{m} + \Gamma_\sigma \tilde{\sigma} + O(x, \tilde{m}, \tilde{\sigma}) \tag{36a}$$

with

$$\Gamma_m(z) = \left.\frac{\partial \Gamma(z)}{\partial z}\right|_{m=\hat{m}},\ \Gamma_\sigma(z) = \left.\frac{\partial \Gamma(z)}{\partial z}\right|_{\sigma=\hat{\sigma}} \tag{36b}$$

and $O(x, \tilde{m}, \tilde{\sigma})$ denoting the higher-order terms in the Taylor series. We have

$$\tilde{\Gamma} = \Gamma_m \tilde{m} + \Gamma_\sigma \tilde{\sigma} + O(x, \tilde{m}, \tilde{\sigma})\,. \tag{36c}$$

Definition 6: The operators $diag(\cdot)$ and $tr\{\cdot\}$ have the following property:

$$\begin{aligned} tr(\tilde{Z}^T (Z - \tilde{Z})) &= < \tilde{Z}, Z > - \| Z \|^2 \le \| \tilde{Z} \| \cdot \| Z \| - \| \tilde{Z} \|^2 \\ &\le \| \tilde{Z} \| Z_M - \| \tilde{Z} \|^2 \end{aligned}\,. \tag{37}$$

The following mild assumption always hold in practical applications.

Assumption 1: On any compact subset of R^n, the ideal NFN parameters are bounded by known positive values so that $\| m^* \|_F \le m_M$, $\| \sigma^* \|_F \le \sigma_M$, $\| W^* \|_F \le W_M$, or $\| Z \|_F \le Z_M$ with Z_M known.

Assumption 2: The desired reference trajectory is bounded so that $\| q_r \| \le q_M$ with q_M a known scalar bound.

We will use an NFN to approximate $f(\zeta)$ for computing the control in (31). By placing into (31) the NFN approximation equation given by (21), the control input then becomes

$$\bar{\tau} = \hat{W}^T \Gamma(x, \hat{m}, \hat{\sigma}) + K_4 e_c - \gamma \tag{38}$$

with $\gamma(t)$ a function to be detailed subsequently that provides robustness in the face of robot kinematics and higher order terms in the Taylor series.

Using this controller, the closed loop velocity error dynamics become

$$\begin{aligned} \bar{M}(q)\dot{e}_c &= -(K_4 + \bar{V})e_c + W^{*T}\, \Gamma(x, m^*, \sigma^*) \\ &\quad - \hat{W}^T \Gamma(x, \hat{m}, \hat{\sigma}) + (\varepsilon + \bar{\tau}_d + \gamma) \end{aligned}\,. \tag{39}$$

Denoting $\hat{\Gamma} = \Gamma(x, \hat{m}, \hat{\sigma})$, and adding and subtracting $W^{*T}\,\hat{\Gamma}$ yields

$$\begin{aligned} \bar{M}(q)\dot{e}_c &= -(K_4 + \bar{V})e_c + \tilde{W}^T \hat{\Gamma} + W^{*T}\, \tilde{\Gamma} \\ &\quad + (\varepsilon + \bar{\tau}_d + \gamma) \end{aligned}\,. \tag{40}$$

Adding and subtracting $\hat{W}^T \tilde{\Gamma}$ yields

$$\begin{aligned} \bar{M}(q)\dot{e}_c &= -(K_4 + \bar{V})e_c + \tilde{W}^T \hat{\Gamma} + \hat{W}^T \tilde{\Gamma} + \tilde{W}^T \tilde{\Gamma} \\ &\quad + (\varepsilon + \bar{\tau}_d + \gamma) \end{aligned}\,. \tag{41}$$

Using the Taylor series approximation for $\tilde{\Gamma}$, the closed loop error system becomes

$$\begin{aligned} \bar{M}(q)\dot{e}_c &= -(K_4 + \bar{V})e_c + \tilde{W}^T \hat{\Gamma} + \hat{W}^T (\Gamma_m \tilde{m} + \Gamma_\sigma \tilde{\sigma}) \\ &\quad + (\delta + \gamma + \varepsilon + \bar{\tau}_d) \end{aligned} \tag{42}$$

where the disturbance term are

$$\delta(t) = \tilde{W}^T (\Gamma_m \tilde{m} + \Gamma_\sigma \tilde{\sigma}) + W^T O(x, \tilde{m}, \tilde{\sigma})\,. \tag{43}$$

Assumption 3: The disturbance term in (43), δ is bounded by a constant δ_N, i.e., $\delta < \delta_N$.

It remains now to show how to select the tuning algorithms for the NFN parameters $\hat{Z}$, and the robustifying term $\gamma(t)$ so that robust stability and tracking performance are guaranteed.

Theorem 1: Consider a nonholomic system (17) and (18). Take the control $\bar{\tau}$ for (19) as (38) with robustifying term

$$\gamma(t) = -(\delta_N + \tau_M) e_s \tag{44}$$

where $e_s = e_c / \| e_c \|$. Let NFN parameter tuning be provided by

$$\dot{\hat{W}} = F\hat{\Gamma}e_c^T + kF \| e_c \| \hat{W}$$
$$\dot{\hat{m}} = Ge_c^T \hat{W}\Gamma_m + kG \| e_c \| \hat{m} \qquad (45)$$
$$\dot{\hat{\sigma}} = He_c^T \hat{W}\Gamma_\sigma + kH \| e_c \| \hat{\sigma}$$

where F, G, and H are positive definite design parameter matrices, $k > 0$. The parameter estimates ,$\hat{W}$, $\hat{m}$, $\hat{\sigma}$ will remain bounded and the tracking error $e_c(t)$ evoles within a practical bound

$$\| e_c \| \le \frac{\frac{k}{4} Z_M^2 + \varepsilon_N}{K_4}. \qquad (46)$$

Proof: Consider Lyapunov candidates:

$$V(e, \phi, \alpha, e_c, \tilde{W}, \tilde{m}, \tilde{\sigma}) = V_k + V_1 \qquad (47)$$

where

$$V_1 = \frac{1}{2}[e_c^T \overline{M} e_c + tr\{\tilde{W}F^{-1}\tilde{W}\} + tr\{\tilde{m}^T G^{-1}\tilde{m}\} + tr\{\tilde{\sigma}^T H^{-1}\tilde{\sigma}\}]. \qquad (48)$$

Differentiating yields

$$\dot{V} = \dot{V}_k + \dot{V}_1 = e\dot{e} + h\phi\dot{\phi} + \alpha\dot{\alpha} + \dot{V}_1 = e\cos(\alpha)v + \alpha\{w - \frac{\sin(\alpha)}{e \cdot \alpha}(\alpha - h\phi)v\} + \dot{V}_1. \qquad (49)$$

Differentiating V_1 and substituting from (42), we obtain

$$\dot{V}_1 = -e_c^T K_4 e_c + \frac{1}{2} e_c^T (\dot{\overline{M}} - 2\overline{V})e_c + tr\{\tilde{W}^T (F^{-1}\dot{\tilde{W}} + \hat{\Gamma}e_c^T)\} + tr\{\tilde{m}^T (G^{-1}\dot{\tilde{m}} + e_c^T \hat{W}^T \Gamma_m)\} + tr\{\tilde{\sigma}(H^{-1}\dot{\tilde{\sigma}} + e_c^T \hat{W}\Gamma_\sigma)\} + e_c^T (\delta + \gamma + \varepsilon + \overline{\tau}_d). \qquad (50)$$

The skew symmetry property, $\dot{\overline{M}} - 2\overline{V} = 0$, makes the second term zero, and since $\dot{\tilde{W}} = -\dot{\hat{W}}$, $\dot{\tilde{m}} = -\dot{\hat{m}}$, $\dot{\tilde{\sigma}} = -\dot{\hat{\sigma}}$, the tuning rules yield

$$\begin{aligned}\dot{V}_1 &= -e_c^T K_4 e_c + k \| e_c \| tr\{\tilde{W}^T (W^* - \tilde{W})\} + k \| e_c \| tr\{\tilde{m}^T (m^* - \tilde{m})\} + k \| e_c \| tr\{\tilde{\sigma}^T (\sigma^* - \tilde{\sigma})\} + e_c^T (\delta + \gamma + \varepsilon + \overline{\tau}_d) \\ &= -e_c^T K_4 e_c + k \| e_c \| tr\{\tilde{Z}^T (Z - \tilde{Z})\} + e_c^T (\delta + \gamma + \varepsilon + \overline{\tau}_d)\end{aligned}. \qquad (51)$$

From definition 6 and robustifying term (44), these results

$$\begin{aligned}\dot{V}_1 &= -e_c^T K_4 e_c + k \| e_c \| \cdot \| \tilde{Z} \|_F (Z_M - \| \tilde{Z} \|_F) + \| e_c \| (\delta + \gamma + \varepsilon + \overline{\tau}_d) \\ &\le -e_c^T K_4 e_c + k \| e_c \| \cdot \| \tilde{Z} \|_F (Z_M - \| \tilde{Z} \|_F) - \| e_c \| (\delta_N + \tau_M) + \| e_c \| (\delta + \overline{\tau}_d) + \| e_c \| \varepsilon_N\end{aligned}. \qquad (52)$$

Substituting (52) and (25) into (49), we obtain

$$\dot{V} \le -\gamma_1^2 e^2 \cos^2(\alpha) - \gamma_2 \alpha^2 - e_c^T K_4 e_c + k \| e_c \| \| \tilde{Z} \|_F (Z_M - \| \tilde{Z} \|_F) + \| e_c \| \varepsilon_N. \qquad (53)$$

Since the first two terms in (53) are negative, there results

$$\begin{aligned}\dot{V} &\le -e_c^T K_4 e_c + k \| e_c \| \cdot \| Z \|_F (Z_M - \| \tilde{Z} \|_F) + \| e_c \| \varepsilon_N \\ &\le \| e_c \| \{-K_4 \| e_c \| + k \| Z \|_F Z_M - k \| \tilde{Z} \|_F^2 + \varepsilon_n\} \\ &\le \| e_c \| \{-K_4 \| e_c \| - k(\| \tilde{Z} \|_F - \frac{1}{2} Z_M)^2 + \frac{1}{4} k Z_M^2 + \varepsilon_N\}\end{aligned} \qquad (54)$$

which is guaranteed to be negative as long as

$$\| e_c \| \ge \frac{\frac{k}{4} Z_M^2 + \varepsilon_N}{K_4} \qquad (55)$$

or

$$k(\| \tilde{Z} \|_F - \frac{1}{2} Z_M)^2 \ge \frac{k}{4} Z_M^2 + \varepsilon_N \qquad (56)$$

which is equivalent to

$$\| \tilde{Z} \|_F \ge \sqrt{\frac{\frac{k}{4} Z_M^2 + \varepsilon_N}{k}} + \frac{1}{2} Z_M. \qquad (57)$$

V. EXPERIMENTAL RESULTS

In this section, we illustrate the effectiveness of a proposed NFN controller for a mobile robot. The dynamic NFN controller is implemented on a mobile robot. Fig 5(a) shows the experimental set up for a mobile robot. The vehicle parameters are $m = 10[Kg]$ and $I = 5[Kg \cdot m^2]$. The wheels have a radius $r = 0.05[m]$ and are mounted on an axle of length $2R = 0.35[m]$. The wheels are drived by motors having rated torque $20[mN \cdot m]$ at $3000[rpm]$ and $24[V]$ rated voltage. Each motor is equipped with an incremental encoder counting $600[pulse/turn]$ and a gear. As shown in Fig. 5(b), the control algorithm is implemented at a $100[Hz]$ sampling rate via PC microcontroller. Wheel PWM duty cycle commands are sent to the robot and the encoders measure $\Delta\varphi_R$ and $\Delta\varphi_l$ for odometric computation. If $\Delta\varphi_R$ and $\Delta\varphi_l$ be the wheel angular displacements measured during sampling time T_s by the encoders, the robot linear and angular displacements are constructed as $\Delta s = (r/2)(\Delta\varphi_R + \Delta\varphi_l)$, $\Delta\theta = (r/2R)(\Delta\varphi_R - \Delta\varphi_l)$. The posture estimated at time $t_k = kT_s$ is

$$\hat{q}(k) = \begin{pmatrix} \hat{x}_k \\ \hat{y}_k \\ \hat{\theta}_k \end{pmatrix} = \hat{q}_{k-1} + \begin{pmatrix} \cos\overline{\theta}_k & 0 \\ \sin\overline{\theta}_k & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta s \\ \Delta\theta \end{pmatrix} \qquad (58)$$

where $\overline{\theta}_k = \hat{\theta}_{k-1} + \Delta\theta / 2$. The NFN input vector x can be taken as $x = [v_c \quad \dot{v}_c \quad \mathrm{sgn}(\Delta\varphi) \quad v]^T$. The number of nodes in successive layer of the NFN is 4-9-9-1. The reference trajectory is generated by the following velocities;

$$v_r = 1.1[m/\sec]$$

$$w_r = -5.7 + 28\sin(t/2)[\deg ree/\sec]. \qquad (59)$$

Fig. 6 shows the tracking response with friction nonlinearity. The performance degraded by the friction effects. However, the proposed NFN controller shows an improvement in trajectory response compared with the feedback controller.

VI. Conclusions

The NFN dynamic controller with a kinematic controller for tracking of nonholonomic mobile robots has been developed. In fact, perfect knowledge of the mobile robot parameters is unattainable, e.g., the friction nonlinearity is very difficult to model by conventional techniques. To confront this, an NFN dynamic controller with guaranteed performance has been derived. There is not need of a prior information of the parameters of the mobile robot, because the NFN learns them on the fly. The proposed controller is shown to be asymptotically stable through theoretical proof and experiment with a mobile robot.

References

[1] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noquchi, "A stable tracking control method for autonomus mobile robot," *in Proc. IEEE Int. Conf. Robot. Automat.*, Sacramento, vol. 1, Cincinati, OH, May 1990, pp. 384-389.

[2] G. Oriolo, A. De Luca, and M. Vendittelli, "WMR control via dynamic feedback linearization: design, implementation and experimental validation," *IEEE Trans. Control Systems Technology*, vol. 10, no. 6, pp. 835-852, Nov. 2002.

[3] Z. Li, S. S. Ge, and A. Ming, "Adaptive robust motion /force control of holonomic constrained nonholonomic mobile manipulators," *IEEE Trans. Systems, Man, and Cybernetics B*, vol. 37, no. 3, pp. 607-616, June 2007.

[4] J. O. Jang, H. T. Chung, and G. J. Jeon, "Neuro-fuzzy controller for a XY positioning table," *Intelligent Automation and Soft Computing*, vol. 13, no. 2, pp. 153-169, April 2007.

[5] R. C. Luo and Kuo L. Su, "Multilevel multisensor based intelligent recharging system for mobile robot," *IEEE Trans. Ind. Electronics*, vol. 55, no. 1, pp. 270-279, Jan. 2008.

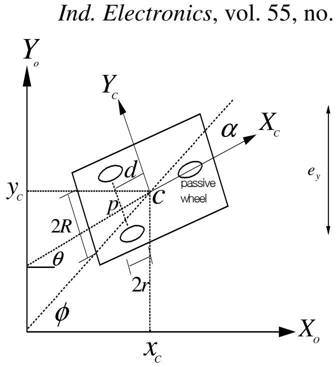


Fig. 1. Mobile robot.

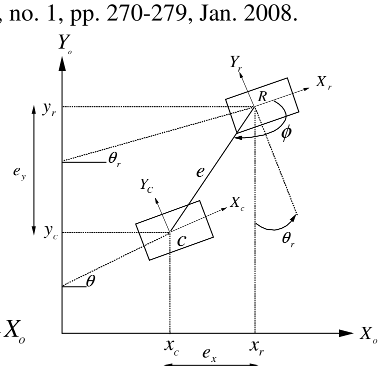


Fig. 2. Reference and coordinate system.

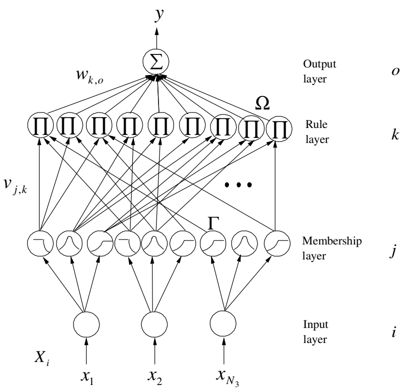


Fig. 3. Neuro-fuzzy network.

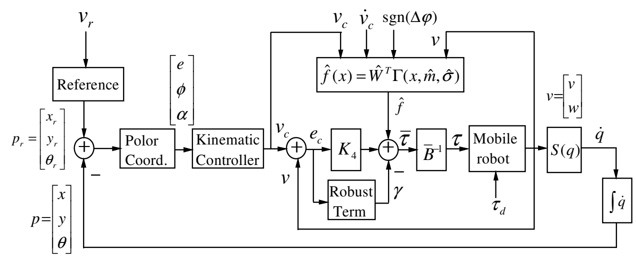
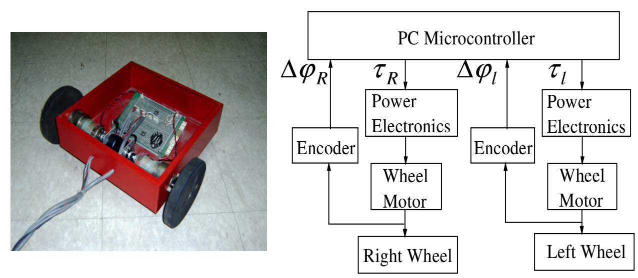


Fig. 4. Proposed NFN controller.



(a) (b)

Fig. 5. Experimental setup for a mobile robot.

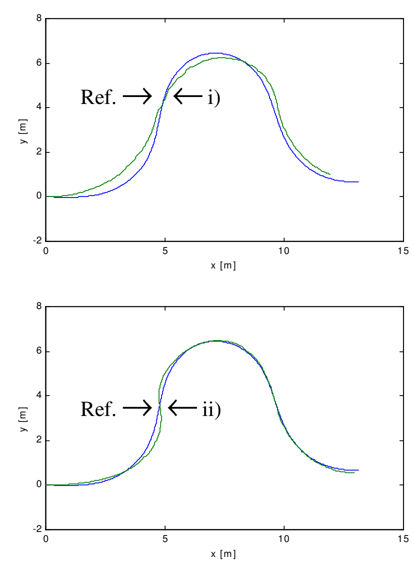


Fig. 6. Experimental tracking response of a mobile robot, i)without compensation, ii) with an NFN controller.