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# Uncertainty reduction on temperature differential measurements using the calibration by comparison method

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***Abstract*—In several multi-sensor temperature control applications the objective is to regulate the difference between the measurements while leaving wider tolerance to their absolute value. In this short note, the analysis of the uncertainty of the difference between two measurements due to sensor calibration error is analyzed. It is shown how a calibration by comparison method, which can be performed with out requesting expert calibration assistantship, can result in an important uncertainty reduction.**

## I. Introduction

TEMPERATURE effects in industrial processes becomes more relevant as manufacturing scales becomes smaller. As a result, industry is continuously increasing the demand for precise temperature control. It is commonly found that temperature must be accurately homogenized while leaving broader tolerance to its absolute value. In this cases a common solution is to use multiple sensors and actuators to actively homogenize the temperature. Homogenized temperature means that the *difference* between the temperature at different points is small. In this paper the uncertainty on the difference between two temperature measurements is called differential uncertainty for short. Experiment designer could spend his efforts reducing random measurement errors below the required precision, however, calibration error of sensors will left a residual systematic error which can only be tackled by recalibrating the sensors. In this paper the analysis is limited to the effect of calibration error on differential uncertainty, all other uncertainty sources should be combined by following guidelines found in [1]. In general, the differential uncertainty is the addition of the uncertainty of each individual measurement if different sensors are used (uncorrelated calibration error). However, if only one sensor is used to perform both measurements (correlated calibration error) differential uncertainty is reduced. The extreme case is found if in both measurement the output of the sensor is exactly the same, then the differential uncertainty vanishes, this is, the two points are known to be exactly at the same temperature. If the outputs of the sensor are *slightly* different, one should not add again the uncertainties as is done in [2], but should expect the uncertainty to remain small. A method to find a number for this uncertainty, based on the sensor response curve (model) and the Gauss-Markov estimation method is explained here together with an analysis of the effects that a calibration by comparison method would produce. The general solution of the problem for any kind of sensors and with additional details and examples can be found in [3].

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## II. Differential uncertainty

### A. One sensor for both measurements

A sensor model must be assumed: for thermistors this model is usually the Steinhart and Hart equation. Sensor model parameters are to be identified based on a calibrated sensor response in which $N$ pairs of the nominal temperature $\theta'_i$ and the output resistance $R_i$ of the sensor are listed together with the uncertainty $u_{\theta i}$ on $\theta'_i$ due to uncertainty during calibration or construction.

As it is well known, a Gauss-Markov method estimates the $n$ parameters $\mathbf{p} = [p_1 \ \cdots \ p_n]$ of the function $\theta = f(R, \mathbf{p})$ assuming linear dependence on $\mathbf{p}$, so each pair of values on the table is considered to be obtained as

$$\theta'_i = \mathbf{w}(R_i)\mathbf{p} + \varepsilon_i , \tag{1}$$

where $\mathbf{p}$ represent the 'true' parameters of the sensor, to be estimated, and $\varepsilon$ is a normally distributed random variable with zero mean and variance $\sigma_\varepsilon^2$ . This analysis assumes $R_i$ is known without uncertainty (or as precisely as necessary). The data included in the table can be rearranged in matrix form as $\mathbf{y}' = W(\mathbf{R})\mathbf{p} + \boldsymbol{\varepsilon}$, where $\boldsymbol{\theta}' = [\theta'_1 ... \theta'_N]^T$, $\mathbf{R} = [R_1 ... R_N]^T$, $\boldsymbol{\varepsilon} = [\varepsilon_1 ... \varepsilon_N]^T$, and $E[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T] = Q^2$ is the covariance matrix of $\boldsymbol{\varepsilon}$. The estimation of $\mathbf{p}$ minimizing the sum of the squares of the residuals $v_i$ defined as $v_i = \theta'_i - f(R, \hat{\mathbf{p}})$ is

$$\hat{\mathbf{p}} = \left(W^T Q^{-2} W\right)^{-1} W^T Q^{-2} \boldsymbol{\theta}' , \tag{2}$$

where the dependence of $W$ on $\mathbf{R}$ is omitted for visual clarity. The Gauss-Markov estimation, under the assumptions previously mentioned, satisfies

$$E[\hat{\mathbf{p}}] = \mathbf{p}$$
$$E\left[(\hat{\mathbf{p}}-\mathbf{p})(\hat{\mathbf{p}}-\mathbf{p})^T\right] = \left(W^T Q^{-2} W\right)^{-1} = S^2 . \quad (3)$$

The variance of a predicted temperature obtained from a resistance value $R_1$ using the model $\hat{\theta}_1 = \mathbf{w}(R_1)\hat{\mathbf{p}}$ is

$$E\left[\left(\hat{\theta}_1 - \theta_1\right)^2\right] = \left(\mathbf{w}_1\hat{\mathbf{p}}_1 - \mathbf{w}_1\mathbf{p}_1\right)^2 = \mathbf{w}_1 S^2 \mathbf{w}_1^T , \quad (4)$$

where $\mathbf{w}(R_1) = \mathbf{w}_1$ for short. A second absolute measurement using the sensor but with output resistance $R_2$, will have a variance obtained by a similar expression as (4) just changing subscript 1 by 2. The variance of the difference between both absolute measurements is

$$\sigma_d^2 = \left(\mathbf{w}_2 - \mathbf{w}_1\right) S^2 \left(\mathbf{w}_2 - \mathbf{w}_1\right)^T . \quad (5)$$

Notice that if $R_1 = R_2$, then $\mathbf{w}_1 = \mathbf{w}_2$, and $\sigma_d^2$ is zero as expected.

### B. *Two sensors, one for each measurement*

The general expression for the differential uncertainty using two different sensors (before recalibrating them by comparison) is obtained following the same procedure as in the previous section, the result is

$$\sigma_d^2 = \mathbf{v}_k^T S_B^2 \mathbf{v}_k + \mathbf{w}_j^T S_A^2 \mathbf{w}_j , \quad (6)$$

where subscripts $A$ and $B$ denotes the sensor associated to each variable and $\mathbf{w}_k$ has been replaced by $\mathbf{v}_k$ to emphasize that the two sensors may have different model structure. Specifically, $\mathbf{w}_j$ corresponds to sensor A model while $\mathbf{v}_k$ refers to that of sensor B.

## III. Calibration by Comparison

Recalibrating sensor B by comparing its output with that of sensor A is done by exposing both to the same temperature $\theta_i$ for $N_C$ different points. Each time both sensor are exposed to $\theta_i$ the following equation must hold

$$\theta_i = \mathbf{w}_i^T \mathbf{p}_A = \mathbf{v}_i^T \mathbf{p}_B, \quad i = 1, ..., N_C . \quad (7)$$

Only $N_C = n_B$ equations like (7) must be obtained for solving $\mathbf{p}_B$, where $n_B$ is the number of parameters of sensor B model and $V_C$ is a square matrix built similarly as $W$. The estimator $\hat{\mathbf{p}}_B$ is a random variable obtained as a linear transformation of another random variable $\hat{\mathbf{p}}_A$. Defining $G = V_C^{-1} W_C$, statistics of $\hat{\mathbf{p}}_B$ are found to be

$$\mathrm{E}\{\hat{\mathbf{p}}_B\} = G\mathrm{E}\{\hat{\mathbf{p}}_A\}\mathbf{p}_A = G\mathbf{p}_A = \mathbf{p}_B$$
$$\mathrm{E}\left\{(\hat{\mathbf{p}}_B - \mathbf{p}_B)(\hat{\mathbf{p}}_B - \mathbf{p}_B)^T\right\} = G S_A G^T = S_B^2 . \quad (8)$$

Finally the general solution for the variance of the error of a differential measurement $\theta_k - \theta_j$ performed by using sensor B to measure $\theta_k$ and sensor A to measure $\theta_j$, after recalibrating sensor B by comparing its output with that of sensor A, is

$$\sigma_d^2 = \mathbf{w}_j^T S_A^2 \mathbf{w}_j + \mathbf{v}_k^T G S_A G^T \mathbf{v}_k - \mathbf{w}_j^T S_A^2 \mathbf{v}_k - \mathbf{v}_k^T G S_A G^T \mathbf{w}_j . \quad (9)$$

Equation (9) must be evaluated for each particular case. However, if sensor A and sensor B are interchangeable sensors, then $[x_{A1} ... x_{ANB}] \approx [x_{B1} ... x_{BNB}]$, $V_C \approx W_C$, $G \approx I$ and $S_B^2 \approx S_A^2 = S^2$, converting (9) into (5). The latter means that differential uncertainty using interchangeable sensors, after recalibrating one of them by comparison, is approximately the same as if only one sensor were used. Not the pervious us true only if the effect of other sources of uncertainty such as resistance measurement and calibration well temperature homogeneity are negligible.

## IV. Example

Assume two interchangeable thermistors with calibration uncertainty of $\sigma_\theta = 100\ m\mathrm{K}$. Their response is simulated using the Steinhart and Hart [4] equation such that interchangeability is satisfied. Using manufacturer data and (9), differential uncertainty for different measurement gradients is plotted in figure 1, to be interpreted as follows: if the output of sensor A and sensor B indicates a gradient of 20 K, the temperature difference between point A and point B is 20 K with an uncertainty due to calibration error of less than 20 mK.

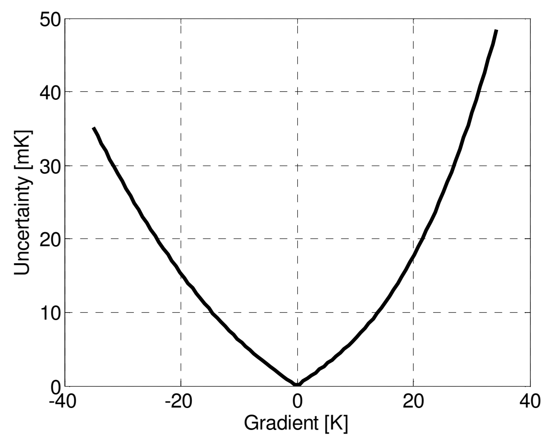


Fig. 1. Simulated differential measurement uncertainty as a function of the size of the gradient for interchangeable thermistors after calibration by comparison

## V. Conclusion

It has been shown how to find an estimation of the uncertainty for differential measurements after a calibration by comparison between sensors is performed. The results suggest that such a calibration may result useful in reducing the differential uncertainty and therefore may be considered as a useful procedure to be performed with out recalibrating sensors against more precise and expensive standards.

## References

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