2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Adaptive Tracking Control of Nonholonomic Mobile Robots Considering Actuator Dynamics: Dynamic Surface Design Approach

Bong Seok Park, Sung Jin Yoo, Jin Bae Park, and Yoon Ho Choi

Abstract— **In this paper, we propose an adaptive tracking control of nonholonomic mobile robots considering actuator dynamics. All parameters of robot kinematics, robot dynamics, and actuator dynamics are assumed to be uncertain. For the simple controller design, the dynamic surface control methodology is applied and extended to multi-input multi-output systems (i.e., mobile robots) that the number of inputs and outputs is different. From the Lyapunov stability theory, we derive adaptation laws and prove that all signals of a closed-loop system are semi-globally uniformly ultimately bounded. Finally, we perform compute simulations to demonstrate the performance of the proposed controller.**

I. INTRODUCTION

Over the past twenty years, the control of mobile robots has been regarded as the attractive problem due to the nature of nonholonomic constraints. Many efforts have been devoted to the tracking control of nonholonomic mobile robots [1]–[5]. However, most of the schemes have ignored the dynamics coming from electric motors. Even if some results were reported for mobile robots incorporating the actuator dynamics [6], [7], all parametric uncertainties for mobile robots were not considered at the actuator level. This is because the controller design problem would become extremely difficult as the complexity of the system dynamics increases.

The backstepping technique has been widely used as one of representative methods for controlling nonholonomic mobile robots considering kinematics and dynamics [8]–[10]. However, the backstepping design procedure has the problem of "explosion of complexity" caused by the repeated differentiations of virtual controllers. That is, the complexity of controller grows drastically as the order n of the system increases. Swaroop *et al*. [11] proposed a dynamic surface control (DSC) technique to solve this problem by introducing a first-order filtering of the synthesized virtual control law at each step of the backstepping design procedure. The DSC idea was extended to uncertain single-input single-output (SISO) [12] and multi-input multi-output (MIMO) systems [13]. Despite these efforts using the DSC technique, the DSC method is still not applied to MIMO systems (i.e., mobile robots) that have more degrees of freedom (DOFs) than the number of inputs under nonholonomic constraints.

This work has been supported by Yonsei University Institute of TMS Information Technology, by a Brain Korea 21 Program.

B. Park, S. J. Yoo and J. B. Park are with Department of Electrical and Electronic Engineering, Yonsei University, Shinchon-Dong, Seodaemum-Gu, Seoul, Korea bongseok@yonsei.ac.kr, niceguy1201@yonsei.ac.kr, jbpark@yonsei.ac.kr

Y. H. Choi is with School of Electronic Engineering, Kyonggi University, Suwon-Si, Kyonggi-Do, Korea yhchoi@kyonggi.ac.kr

Accordingly, we propose an adaptive tracking controller for path tracking of uncertain nonholonomic mobile robots considering actuator dynamics. It is assumed that all parameters of robot kinematics, robot dynamics, and actuator dynamics are uncertain. For the simple control system design, we apply the DSC technique to nonholonomic electrically driven mobile robots. The adaptive technique is used to overcome all parametric uncertainties. Based on Lyapunov stability theorem, we also prove that all of the signals in the closed-loop system are semi-globally uniformly ultimately bounded and the steady-state error can be made arbitrarily small by adjusting the design parameters.

This paper is organized as follows. Section II introduces simply the model of nonholonomic mobile robots incorporating actuator dynamics. In Section III, we propose a simple adaptive controller for nonholonomic electrically driven mobile robots with parametric uncertainties, and analyze the stability of the proposed control systems. Simulation results are discussed in Section IV. Finally, Section V gives some conclusions.

II. PROBLEM STATEMENT

We consider a mobile robot with two degrees of freedom. The kinematics and dynamics of nonholonomic mobile robots are described by the following differential equations [14]:

$$\dot{q} = J(q)z = 0.5r \begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R^{-1} & -R^{-1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1)$$

$$M\dot{z} + C(\dot{q})z + Dz = \tau \quad (2)$$

where $q = [x \quad y \quad \theta]^T \in \mathbb{R}^3$; x, y are the coordinates of the center of mass of the vehicle, and θ is the angle between the heading direction and the x axis, $z = [v_1 \quad v_2]^T \in \mathbb{R}^2$; v_1 and v_2 represent the angular velocities of right and left wheels. R is the half of the width of the mobile robot and r is the radius of the wheel,

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{11} \end{bmatrix}, \; C(\dot{q}) = 0.5R^{-1}r^2 m_c d \begin{bmatrix} 0 & \dot{\theta} \\ -\dot{\theta} & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix},$$

$$\begin{aligned} m_{11} &= 0.25R^{-2}r^2(mR^2 + I) + I_w, \\ m_{12} &= 0.25R^{-2}r^2(mR^2 - I) m = m_c + 2m_w, \\ I &= m_c d^2 + 2m_w R^2 + I_c + 2I_m, \quad \tau = [\tau_1 \quad \tau_2]^T. \end{aligned}$$

In these expressions, d is the distance from the center of mass P_c of the mobile robot to the middle point P_0 between the right and left driving wheels. m_c and m_w are the mass of the body and wheel with a motor, respectively. I_c, I_w, and I_m are the moment of inertia of the body about the vertical axis through P_c, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. The positive terms $d_{ii}, i = 1, 2$, are the damping coefficients. $\tau \in \mathbb{R}^2$ is the control torque applied to the wheels of the robot.

Property 1: [15] The inertia matrix M is symmetric and positive definite.

In addition, the dynamic model of dc motors can be represented as follows [6]:

$$\begin{cases} \tau_m = K_T i_a, \\ u = R_a i_a + L_a \dot{i}_a + K_E \dot{\theta}_m \end{cases} \tag{3}$$

where $\tau_m = [r_{m_1} \quad r_{m_2}]^T$ is the torque generated by dc motor, $K_T = \text{diag}(k_{t_1}, k_{t_2})$ is the motor torque constant, $i_a \in \mathbb{R}^2$ is the current, $u \in \mathbb{R}^2$ is the input voltage, $R_a = \text{diag}(r_{a_1}, r_{a_2})$ is the resistance, $L_a = \text{diag}(l_{a_1}, l_{a_2})$ is the inductance, $K_E = \text{diag}(k_{e_1}, k_{e_2})$ is the back electromotive force coefficient, and $\dot{\theta}_m = [\dot{\theta}_{m_1} \quad \dot{\theta}_{m_2}]^T$ is the angular velocity of the dc motor. Here, diag$(\cdot)$ denotes the diagonal matrix.

The relationship between the dc motor and the mobile robot wheel can be written as

$$n_j = \frac{\dot{\theta}_{m_j}}{v_j} = \frac{\tau_j}{\tau_{m_j}} \tag{4}$$

where $n_j, j = 1, 2$, is the gear ratio. Using (4), the dynamic model of dc motors (3) can be rewritten as

$$\begin{cases} \tau = N K_T i_a, \\ u = R_a i_a + L_a \dot{i}_a + N K_E z. \end{cases} \tag{5}$$

where $N = \text{diag}(n_1, n_2)$.

Assumption 1: All parameters of robot kinematics (1), robot dynamics (2), and actuator dynamics (5) are constants but unknown, and lie in a compact set.

Let us define the state variables as $x_1 = q$, $x_2 = z$, and $x_3 = i_a$. Then, (1), (2), and (5) can be expressed in the following state-space form:

$$\dot{x}_1 = J(x_1) x_2 \tag{6}$$
$$\dot{x}_2 = M^{-1}(-C(\dot{x}_1)x_2 - D x_2 + N K_T x_3) \tag{7}$$
$$\dot{x}_3 = L_a^{-1}(u - R_a x_3 - N K_E x_2) \tag{8}$$

where $x_1 = [x_{11} \quad x_{12} \quad x_{13}]^T$, $x_2 = [x_{21} \quad x_{22}]^T$, and $x_3 = [x_{31} \quad x_{32}]^T$.

The control objective is to design a simple adaptive control law u for nonholonomic electrically driven mobile robots (6)-(8) to track the desired trajectory generated by the following reference robot:

$$\begin{cases} \dot{x}_r = v_r \cos\theta_r, \\ \dot{y}_r = v_r \sin\theta_r, \\ \dot{\theta}_r = \omega_r \end{cases} \tag{9}$$

where x_r, y_r, and θ_r are the position and orientation of the reference robot. v_r and ω_r are the linear and angular velocities of the reference robot, respectively.

Assumption 2: The reference signal $z_r = [v_r \quad \omega_r]^T$ is bounded, and $v_r > 0$.

Remark 1: In Assumption 2, $v_r > 0$ means that this paper is only focused on a simple controller design for *the trajectory tracking problem* of mobile robots incorporating actuator dynamics. That is, the case of $v_r = 0$ is not considered.

III. Main Results

A. Adaptive Controller Design

In this section, we develop a simple control system for nonholonomic electrically driven mobile robots. To design the adaptive control system using the DSC technique, we proceed step by step.

Step 1: Consider the robot kinematics (6). The first error surface is defined as follows:

$$\begin{bmatrix} S_{11} \\ S_{12} \\ \bar{S}_{13} \end{bmatrix} = \begin{bmatrix} \cos x_{13} & \sin x_{13} & 0 \\ -\sin x_{13} & \cos x_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_{11} \\ y_r - x_{12} \\ \theta_r - x_{13} \end{bmatrix}. \tag{10}$$

Differentiating (10) yields

$$\begin{cases} \dot{S}_{11} = \frac{r}{2R}(x_{21} - x_{22})S_{12} - \frac{r}{2}(x_{21} + x_{22}) + v_r \cos\bar{S}_{13} \\ \dot{S}_{12} = -\frac{r}{2R}(x_{21} - x_{22})S_{11} + v_r \sin\bar{S}_{13} \\ \dot{\bar{S}}_{13} = \omega_r - \frac{r}{2R}(x_{21} - x_{22}). \end{cases} \tag{11}$$

In the tracking error model (11), S_{12} can not be directly controlled. To overcome this problem, we introduce an error variable based on [16] as follows:

$$S_{13} = \bar{S}_{13} + \arctan(k_1 S_{12} v_r) \tag{12}$$

where k_1 is a positive constant. Using (12), $\dot{\bar{S}}_{13}$ in (11) is transformed into

$$\dot{S}_{13} = \omega_r - \frac{r}{2R}(x_{21} - x_{22})\left(1 + \frac{k_1 v_r S_{11}}{1 + (k_1 S_{12} v_r)^2}\right) + \alpha_1(t) \tag{13}$$

where $\alpha_1(t) = (k_1 v_r^2 \sin(S_{13} - \arctan(k_1 S_{12} v_r)) + k_1 S_{12} \dot{v}_r)/(1 + (k_1 S_{12} v_r)^2)$.

Choose a virtual control law $\bar{x}_2$ as follows:

$$\bar{x}_2 = [\bar{x}_{21} \quad \bar{x}_{22}]^T = [h_1 + h_2 \quad h_1 - h_2]^T \tag{14}$$

where

$$\begin{aligned} h_1 &= \hat{a}_1 v_r \cos(S_{13} - \arctan(k_1 S_{12} v_r)) + k_2 S_{11} \\ &\quad + \frac{1}{2}(k_1 v_r)^4 S_{11}^3 \\ h_2 &= \left(1 + \frac{k_1 v_r S_{11}}{1 + (k_1 S_{12} v_r)^2}\right)^{-1} \\ &\quad \times \left(\hat{a}_1 \alpha_2(t) v_r S_{12} + \hat{a}_2(\omega_r + \alpha_1(t)) + k_3 S_{13} + \frac{1}{2} S_{13}^3\right) \\ \alpha_2(t) &= \int_0^1 \cos\Big(-\arctan(k_1 S_{12} v_r) + \eta S_{13}\Big) d\eta \end{aligned}$$

in which $a_1 = 1/r$, $a_2 = R/r$, k_2 and k_3 are positive constants. $\hat{a}_i$ is the estimate of a_i, $i = 1, 2$. $\hat{a}_1$ and $\hat{a}_2$ are updated as follows:

$$\dot{\hat{a}}_1 = \gamma_1 v_r \left(S_{11} \cos(S_{13} - \arctan(k_1 v_r S_{12})) + \alpha_2(t) S_{12} S_{13} \right) - \sigma_1 \gamma_1 \hat{a}_1 \quad (15)$$

$$\dot{\hat{a}}_2 = \gamma_2(\omega_r + \alpha_1(t)) S_{13} - \sigma_2 \gamma_2 \hat{a}_2 \quad (16)$$

with the initial estimates $\hat{a}_1(0) = \hat{a}_2(0) = 0$, the tuning gains γ_1, $\gamma_2 > 0$, and small gains σ_1, $\sigma_2 > 0$ for the σ-modification [17]. Then, to obtain a filtered virtual control $x_{2f} = [x_{2f_1} \quad x_{2f_2}]^T$, we pass $\bar{x}_2$ through a first-order filter

$$\tau_2 \dot{x}_{2f} + x_{2f} = \bar{x}_2, \qquad x_{2f}(0) = \bar{x}_2(0) \quad (17)$$

with a time constant $\tau_2 > 0$.

Step 2: Consider the robot dynamics (7). Define the second error surface S_2 as

$$S_2 = x_2 - x_{2f}. \quad (18)$$

Then its derivative is

$$\begin{aligned} \dot{S}_2 &= \dot{x}_2 - \dot{x}_{2f} \\ &= M^{-1}(-C(\dot{x}_1)x_2 - Dx_2 + NK_T x_3) - \dot{x}_{2f}. \end{aligned} \quad (19)$$

Choose a virtual control law $\bar{x}_3 = [\bar{x}_{31} \quad \bar{x}_{32}]^T$ to drive $S_2 \to 0$ as follows:

$$\bar{x}_3 = -k_4 S_2 - \Phi_1 \widehat{W}_1 \quad (20)$$

where k_4 is a positive constant, $\Phi_1 W_1 = -(NK_T)^{-1} C(\dot{x}_1) x_2 - (NK_T)^{-1} D x_2 - (NK_T)^{-1} M \dot{x}_{2f}$, $\widehat{W}_1$ is the estimate vector of the unknown parameter vector W_1 and it is updated by

$$\dot{\widehat{W}}_1 = \Gamma_1 \Phi_1^T S_2 - \sigma_3 \Gamma_1 \widehat{W}_1 \quad (21)$$

with the initial estimates $\widehat{W}_1(0) = 0$, a tuning gain matrix $\Gamma_1 > 0$, and a small gain $\sigma_3 > 0$. Here, Φ_1 and W_1 are defined as in (22), shown at the top of the next page.

Then, $\bar{x}_3$ is passed through a first order filter with time constant $\tau_3 > 0$ to obtain $x_{3f} = [x_{3f_1} \quad x_{3f_2}]^T$,

$$\tau_3 \dot{x}_{3f} + x_{3f} = \bar{x}_3, \qquad x_{3f}(0) = \bar{x}_3(0). \quad (23)$$

Step 3: Consider the actuator dynamics (8). To design an actual control input law u, we define the third error surface S_3 as

$$S_3 = x_3 - x_{3f}. \quad (24)$$

The time derivative of S_3 is given by

$$\dot{S}_3 = \dot{x}_3 - \dot{x}_{3f} = L_a^{-1}(u - R_a x_3 - NK_E x_2) - \dot{x}_{3f}. \quad (25)$$

We choose an actual control law u to derive $S_3 \to 0$ as follows:

$$u = -k_5 S_3 - \Phi_2 \widehat{W}_2 \quad (26)$$

where k_5 is a positive constant, $\Phi_2 W_2 = -R_a x_3 - NK_E x_2 - L_a \dot{x}_{3f}$. $\widehat{W}_2$ is the estimate vector of the unknown parameter vector W_2, and is updated by

$$\dot{\widehat{W}}_2 = \Gamma_2 \Phi_2^T S_3 - \sigma_4 \Gamma_2 \widehat{W}_2 \quad (27)$$

with the initial estimates $\widehat{W}_2(0) = 0$, a tuning gain matrix $\Gamma_2 > 0$, and a small gain $\sigma_4 > 0$. Here, Φ_2 and W_2 are defined as

$$\Phi_2 = \begin{bmatrix} -x_{31} & 0 & -x_{21} & 0 & -\dot{x}_{3f_1} & 0 \\ 0 & -x_{32} & 0 & -x_{22} & 0 & -\dot{x}_{3f_2} \end{bmatrix}$$

$$W_2 = \begin{bmatrix} r_{a_1} & r_{a_2} & n_1 k_{e_1} & n_2 k_{e_2} & l_{a_1} & l_{a_2} \end{bmatrix}^T$$

where $\dot{x}_{3f_1} = (\bar{x}_{31} - x_{3f_1})/\tau_3$ and $\dot{x}_{3f_2} = (\bar{x}_{32} - x_{3f_2})/\tau_3$.

Remark 2: Compared with [8]–[10] and [14] based on the backstepping technique, the proposed controller for nonholonomic mobile robots can overcome the "explosion of complexity" problem by using the first-order filters. Thus, the proposed controller based on the adaptive DSC technique can be simpler than the adaptive backstepping controller.

B. Stability Analysis

In this section, we show that the all signals of the proposed control system are semi-globally uniformly ultimately bounded.

Define the boundary layer errors as

$$y_2 = x_{2f} - \bar{x}_2, \quad (28)$$

$$y_3 = x_{3f} - \bar{x}_3. \quad (29)$$

The estimate errors are defined as $\tilde{a}_i = a_i - \hat{a}_i, \widetilde{W}_i = W_i - \widehat{W}_i, i = 1, 2$. Then, the derivative of y_2 and y_3 are

$$\dot{y}_2 = \dot{x}_{2f} - \dot{\bar{x}}_2 = -\frac{y_2}{\tau_2} + \Xi_1(S_1, S_2, y_2, z_r, \hat{a}_1, \hat{a}_2) \quad (30)$$

$$\dot{y}_3 = \dot{x}_{3f} - \dot{\bar{x}}_3 = -\frac{y_3}{\tau_3} + \Xi_2(S_1, S_2, S_3, y_2, y_3, z_r, \widehat{W}_1) \quad (31)$$

where $S_1 = [S_{11} \quad S_{12} \quad S_{13}]^T$. Ξ_1 and Ξ_2 are defined as in (32), shown at the top of the next page.

Consider the Lyapunov function candidate as follows:

$$V = V_1 + V_2 \quad (33)$$

where

$$V_1 = \frac{1}{2r} S_{11}^2 + \frac{1}{2r} S_{12}^2 + \frac{R}{2r} S_{13}^2 + \frac{1}{2\gamma_1} \tilde{a}_1^2 + \frac{1}{2\gamma_2} \tilde{a}_2^2 \quad (34)$$

$$V_2 = \frac{1}{2} \left(S_2^T (M^{-1} NK_T)^{-1} S_2 + S_3^T L_a S_3 + y_2^T y_2 + y_3^T y_3 + \widetilde{W}_1^T \Gamma_1^{-1} \widetilde{W}_1 + \widetilde{W}_2^T \Gamma_2^{-1} \widetilde{W}_2 \right). \quad (35)$$

Theorem 1: Consider the nonholonomic electrically driven mobile robot (6)-(8) with parametric uncertainties controlled by the adaptive control law (26). If the proposed control system satisfies Assumptions 1-2 and the unknown parameters a_1, a_2, W_1, and W_2 are trained by the adaptation laws (15), (16), (21), and (27), respectively, then for any initial conditions satisfying $V(0) \leq \mu$ where μ is any

$$\begin{cases} \Phi_1 &= \begin{bmatrix} -\dot{x}_{13}x_{22} & 0 & -x_{21} & 0 & -\dot{x}_{2f_1} & -\dot{x}_{2f_2} & 0 & 0 \\ 0 & \dot{x}_{13}x_{21} & 0 & -x_{22} & 0 & 0 & -\dot{x}_{2f_2} & -\dot{x}_{2f_1} \end{bmatrix}, \\ W_1 &= \begin{bmatrix} \frac{r^2 m_c d}{2Rn_1 k_{t_1}} & \frac{r^2 m_c d}{2Rn_2 k_{t_2}} & \frac{d_{11}}{n_1 k_{t_1}} & \frac{d_{22}}{n_2 k_{t_2}} & \frac{m_{11}}{n_1 k_{t_1}} & \frac{m_{12}}{n_1 k_{t_1}} & \frac{m_{11}}{n_2 k_{t_2}} & \frac{m_{12}}{n_2 k_{t_2}} \end{bmatrix}^T, \\ \dot{x}_{2f_1} &= (\bar{x}_{21} - x_{2f_1})/\tau_2, \quad \dot{x}_{2f_2} = (\bar{x}_{22} - x_{2f_2})/\tau_2. \end{cases} \tag{22}$$

$$\begin{cases} \Xi_1(S_1, S_2, y_2, z_r, \hat{a}_1, \hat{a}_2) = -\begin{bmatrix} \frac{\partial v}{\partial v_r}\dot{v}_r + \frac{\partial v}{\partial S_1}\dot{S}_1 + \frac{\partial v}{\partial \hat{a}_1}\dot{\hat{a}}_1 + \frac{\partial w}{\partial z_r}\dot{z}_r + \frac{\partial w}{\partial S_1}\dot{S}_1 + \frac{\partial w}{\partial \hat{a}_1}\dot{\hat{a}}_1 + \frac{\partial w}{\partial \hat{a}_2}\dot{\hat{a}}_2 \\ \frac{\partial v}{\partial v_r}\dot{v}_r + \frac{\partial v}{\partial S_1}\dot{S}_1 + \frac{\partial v}{\partial \hat{a}_1}\dot{\hat{a}}_1 - \frac{\partial w}{\partial z_r}\dot{z}_r - \frac{\partial w}{\partial S_1}\dot{S}_1 - \frac{\partial w}{\partial \hat{a}_1}\dot{\hat{a}}_1 - \frac{\partial w}{\partial \hat{a}_2}\dot{\hat{a}}_2 \end{bmatrix}, \\ \Xi_2(S_1, S_2, S_3, y_2, y_3, z_r, \widehat{W}_1) = k_4\dot{S}_2 + \dot{\Phi}_1\widehat{W}_1 + \Phi_1\dot{\widehat{W}}_1. \end{cases} \tag{32}$$

positive constant, there exists a set of gains $k_1, \ldots, k_5$, τ_{i+1}, γ_i, Γ_i, and σ_j, where $i = 1, 2$ and $j = 1, 2, 3, 4$, such that the error states are semi-globally uniformly ultimately bounded and can be made arbitrarily small.

Proof: We first consider the Lyapunov function candidate V_1. Noting that $\sin(S_{13} - \arctan(k_1 v_r S_{12})) = -\sin(\arctan(k_1 S_{12} v_r)) + S_{13}\alpha_2(t)$, the time derivative of V_1 along (11)–(16), (18), and (28) yields

$$\begin{aligned} \dot{V}_1 &= \frac{1}{r}S_{11}\dot{S}_{11} + \frac{1}{r}S_{12}\dot{S}_{12} + \frac{R}{r}S_{13}\dot{S}_{13} - \frac{1}{\gamma_1}\tilde{a}_1\dot{\hat{a}}_1 - \frac{1}{\gamma_2}\tilde{a}_2\dot{\hat{a}}_2 \\ &= S_{11}\Bigg[-\frac{\bar{x}_{21} + \bar{x}_{22}}{2} - \frac{S_{21} + S_{22} + y_{21} + y_{22}}{2} \\ &\qquad + a_1 v_r \cos(S_{13} - \arctan(k_1 S_{12} v_r))\Bigg] \\ &\quad + S_{13}\Bigg[a_2(\omega_r + \alpha_1(t)) + a_1 v_r \alpha_2(t) S_{12} \\ &\qquad - \left(1 + \frac{k_1 v_r S_{11}}{1 + (k_1 S_{12} v_r)^2}\right)\left(\frac{\bar{x}_{21} - \bar{x}_{22}}{2}\right)\Bigg] \\ &\quad - \left(1 + \frac{k_1 v_r S_{11}}{1 + (k_1 S_{12} v_r)^2}\right)\left(\frac{S_{21} - S_{22} + y_{21} - y_{22}}{2}\right)S_{13} \\ &\quad - \frac{1}{r}S_{12}v_r\sin(\arctan(k_1 S_{12} v_r)) - \frac{1}{\gamma_1}\tilde{a}_1\dot{\hat{a}}_1 - \frac{1}{\gamma_2}\tilde{a}_2\dot{\hat{a}}_2 \\ &= -k_2 S_{11}^2 - \frac{1}{r}S_{12}v_r\sin(\arctan(k_1 S_{12} v_r)) - k_3 S_{13}^2 \\ &\quad - \frac{1}{2}(k_1 v_r)^4 S_{11}^4 - \frac{1}{2}S_{13}^4 + \sigma_1\tilde{a}_1\hat{a}_1 + \sigma_2\tilde{a}_2\hat{a}_2 \\ &\quad - \left(\frac{S_{21} + S_{22} + y_{21} + y_{22}}{2}\right)S_{11} \\ &\quad - \left(1 + \frac{k_1 v_r S_{11}}{1 + (k_1 S_{12} v_r)^2}\right)\left(\frac{S_{21} - S_{22} + y_{21} - y_{22}}{2}\right)S_{13} \end{aligned} \tag{36}$$

Second, consider the Lyapunov function candidate V_2. Using (19), (20), (24), (25), (26), and (29), we can obtain

$$(M^{-1}NK_T)^{-1}\dot{S}_2 = -k_4 S_2 + y_3 + S_3 + \Phi_1\widetilde{W}_1 \tag{37}$$

$$L_a\dot{S}_3 = -k_5 S_3 + \Phi_2\widetilde{W}_2 \tag{38}$$

The time derivative of V_2 along (30), (31), (37), and (38) is given by

$$\begin{aligned} \dot{V}_2 =& S_2^T(M^{-1}NK_T)^{-1}\dot{S}_2 + S_3^T L_a\dot{S}_3 + y_2^T\dot{y}_2 + y_3^T\dot{y}_3 \\ &- \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 - \widetilde{W}_2^T\Gamma_2^{-1}\dot{\widehat{W}}_2 \\ =& S_2^T(-k_4 S_2 + S_3 + y_3 + \Phi_1\widetilde{W}_1) + S_3^T(-k_5 S_3 + \Phi_2\widetilde{W}_2) \\ &+ y_2^T\left(-\frac{y_2}{\tau_2} + \Xi_1\right) + y_3^T\left(-\frac{y_3}{\tau_3} + \Xi_2\right) \\ &- \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 - \widetilde{W}_2^T\Gamma_2^{-1}\dot{\widehat{W}}_2. \end{aligned} \tag{39}$$

Substituting (21) and (27) into (39) yields

$$\begin{aligned} \dot{V}_2 =& -k_4\|S_2\|^2 - k_5\|S_3\|^2 - \frac{1}{\tau_2}\|y_2\|^2 - \frac{1}{\tau_3}\|y_3\|^2 \\ &+ S_2^T S_3 + S_2^T y_3 + y_2^T\Xi_1 + y_3^T\Xi_2 \\ &+ \sigma_3\widetilde{W}_1^T\widehat{W}_1 + \sigma_4\widetilde{W}_2^T\widehat{W}_2 \end{aligned} \tag{40}$$

Finally, consider the Lyapunov function candidate V. Substituting (36) and (40) into the time derivative of V, we have

$$\begin{aligned} \dot{V} \le& -k_2 S_{11}^2 - \frac{1}{r}S_{12}v_r\sin(\arctan(k_1 S_{12} v_r)) - k_3 S_{13}^2 \\ &- \frac{1}{2}(k_1 v_r)^4 S_{11}^4 - \frac{1}{2}S_{13}^4 - k_4\|S_2\|^2 - k_5\|S_3\|^2 \\ &- \frac{1}{\tau_2}\|y_2\|^2 - \frac{1}{\tau_3}\|y_3\|^2 + \|S_2\|\|S_3\| + \|S_2\|\|y_3\| \\ &+ \|y_2\|\|\Xi_1\| + \|y_3\|\|\Xi_2\| + \sigma_1\tilde{a}_1\hat{a}_1 + \sigma_2\tilde{a}_2\hat{a}_2 \\ &+ \sigma_3\widetilde{W}_1^T\widehat{W}_1 + \sigma_4\widetilde{W}_2^T\widehat{W}_2 \\ &+ \frac{1}{2}(|S_{21}| + |S_{22}| + |y_{21}| + |y_{22}|)|S_{11}| \\ &+ \left|1 + \frac{k_1 v_r S_{11}}{1 + (k_1 v_r S_{12})^2}\right|\frac{|S_{21}| + |S_{22}| + |y_{21}| + |y_{22}|}{2}|S_{13}| \end{aligned}$$

Consider sets $A_1 := \{\frac{1}{r}S_{11}^2 + \frac{1}{r}S_{12}^2 + \frac{R}{r}S_{13}^2 + S_2^T(M^{-1}NK_T)^{-1}S_2 + y_2^T y_2 + (1/\gamma_1)\tilde{a}_1^2 + (1/\gamma_2)\tilde{a}_2^2 \le 2\mu\}$ and $A_2 := \{\frac{1}{r}S_{11}^2 + \frac{1}{r}S_{12}^2 + \frac{R}{r}S_{13}^2 + S_2^T(M^{-1}NK_T)^{-1}S_2 + S_3^T L_a S_3 + \sum_{i=1}^{2}(y_{i+1}^T y_{i+1} + (1/\gamma_i)\tilde{a}_i^2) + \widetilde{W}_1^T\Gamma_1^{-1}\widetilde{W}_1 \le 2\mu\}$. Since A_1 and A_2 are compact in $\mathbb{R}^9$ and $\mathbb{R}^{21}$, respectively, there exist positive constants p_1, p_2 such that $\|\Xi_1\| \le p_1$ on A_1 and $\|\Xi_2\| \le p_2$ on A_2. Using the fact $\left|\frac{k_1 v_r S_{11}}{1 + (k_1 v_r S_{12})^2}\right| \le |k_1 v_r S_{11}|$ and Young's inequality (i.e., $z_1 z_2 \le \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$), we obtain

$$\begin{aligned} \dot{V} \le& -(k_2 - 1)S_{11}^2 - \frac{1}{r}S_{12}v_r\sin(\arctan(k_1 S_{12} v_r)) \\ &- (k_3 - 1)S_{13}^2 - (k_4 - \frac{7}{4})\|S_2\|^2 - (k_5 - \frac{1}{2})\|S_3\|^2 \end{aligned}$$

$$
\begin{aligned}
&- (\frac{1}{\tau_2} - \frac{3}{4})\|y_2\|^2 - (\frac{1}{\tau_3} - \frac{1}{2})\|y_3\|^2 + \frac{\|y_2\|^2\|\Xi_1\|^2}{2\delta_1} \\
&+ \frac{\|y_3\|^2\|\Xi_2\|^2}{2\delta_2} + \frac{\delta_1}{2} + \frac{\delta_2}{2} + \sigma_1(|\tilde{a}_1|a_1 - |\tilde{a}_1|^2) \\
&+ \sigma_2(|\tilde{a}_2|a_2 - |\tilde{a}_2|^2) + \sigma_3(\|\widetilde{W}_1\|\|W_1\| - \|\widetilde{W}_1\|^2) \\
&+ \sigma_4(\|\widetilde{W}_2\|\|W_2\| - \|\widetilde{W}_2\|^2)
\end{aligned}
$$

where δ_1 and δ_2 denote positive constants. If we choose $k_2 = 1 + k_2^*, k_3 = 1 + k_3^*, k_4 = (7/4) + k_4^*, k_5 = (1/2) + k_5^*, (1/\tau_2) = (3/4) + (p_1^2/2\delta_1) + \tau_2^*$, and $(1/\tau_3) = (1/2) + (p_2^2/2\delta_2) + \tau_3^*$, then

$$
\begin{aligned}
\dot{V} \leq &- k_2^* S_{11}^2 - \frac{1}{r} S_{12} v_r \sin(\arctan(k_1 S_{12} v_r)) - k_3^* S_{13}^2 \\
&- k_4^* \|S_2\|^2 - k_5^* \|S_3\|^2 - \frac{1}{2}\sigma_3\|\widetilde{W}_1\|^2 - \frac{1}{2}\sigma_4\|\widetilde{W}_2\|^2 + \varepsilon \\
&- \sum_{i=1}^{2} \Bigg\{ \tau_{i+1}^* \|y_{i+1}\|^2 + \frac{1}{2}\sigma_i \tilde{a}_i^2 \\
&\qquad + \left(1 - \frac{\|\Xi_i\|^2}{p_i^2}\right) \frac{p_i^2 \|y_{i+1}\|^2}{2\delta_i} \Bigg\} \\
\leq &- 2\zeta(V - V_p) + \varepsilon
\end{aligned} \tag{41}
$$

where $k_2^*, k_3^*, k_4^*, k_5^* > 0$, $\tau_2^*, \tau_3^* > 0$, and $\varepsilon = (\delta_1 + \delta_2 + \sigma_1 a_1^2 + \sigma_2 a_2^2 + \sigma_3\|W_1\|^2 + \sigma_4\|W_2\|^2)/2$, and $V_p = (1/2r)(S_{12}^2 - S_{12} v_r \sin(\arctan(k_1 S_{12} v_r)))$. The constant ζ is $0 < \zeta < \min[r k_2^*, 1, \frac{r}{R} k_3^*, M_m k_4^*, \frac{1}{L_{a,M}} k_5^*, \tau_2^*, \tau_3^*, (\sigma_1 \gamma_1)/2, (\sigma_2\gamma_2)/2, (\sigma_3\Gamma_{1,m})/2, (\sigma_4\Gamma_{2,m})/2]$ where $\Gamma_{i,m}$, $i = 1, 2$, and M_m are the minimum eigenvalues of Γ_j and $M^{-1} N K_T$, respectively, and $L_{a,M}$ is the maximum eigenvalue of L_a. Since $S_{12} v_r \sin(\arctan(k_1 S_{12} v_r)) \geq 0$ for all S_{12} and all $t \geq 0$, (41) implies $\dot{V} \leq 0$ on $V = \mu$ when $\zeta > \varepsilon/2(\mu - V_p)$. Therefore, $V \leq \mu$ is an invariant set, i.e., if $V(0) \leq \mu$, then $V(t) \leq \mu$ for all $t \geq 0$. Therefore, we can prove all error signals in the closed-loop system are semi-globally uniformly ultimately bounded. Besides, by increasing the design parameter ζ, i.e., adjusting k_j^*, τ_2^*, τ_3^*, γ_1, γ_2, Γ_1, Γ_2, and σ_l ($j = 1, \ldots, 5$, $l = 1, \ldots, 4$), the errors in the controlled closed-loop system can be made arbitrarily small. ∎

Remark 3: In this remark, we comment that h_2 in (14) is well defined for all $t \geq 0$ [16]. For any $k_1, k_2, k_3 \geq 0$, consider a set $\Psi(k_1, k_2, k_3) = \{(S_{11}, S_{12}, S_{13}, \hat{a}_1, \hat{a}_2) \in \mathbb{R}^5 : k_1 k_2 k_3 |S_{11}| < 1\}$. Then, let Ω_1 and Ω be sets given by $\Omega_1 = \{(S_{11}, S_{12}, S_{13}, \hat{a}_1, \hat{a}_2) \in \mathbb{R}^5 : V_1(t, S_{11}, S_{12}, S_{13}, \hat{a}_1, \hat{a}_2) < \varpi, \ \forall t \geq 0\}$ and $\Omega = \{V(t) \leq \mu, \ \forall t \geq 0\}$ where $0 < \varpi \leq \mu$ is a largest constant such that $\Omega_1 \subset \Psi(||k_1||_\infty, ||k_2||_\infty, ||k_3||_\infty) \subset \Omega$. From (41), since S_{11}, S_{12}, S_{13}, $\hat{a}_1$, and $\hat{a}_2$ remain in an invariant set Ω, $h_2(t)$ is well defined for all $t \geq 0$.

Remark 4: In the adaptation laws (15), (16), (21), and (27), a σ-modification [17] is used for preventing parameter drift to infinity. We can also apply an e-modification [18] and a projection operator method [19] in place of the σ-modification.

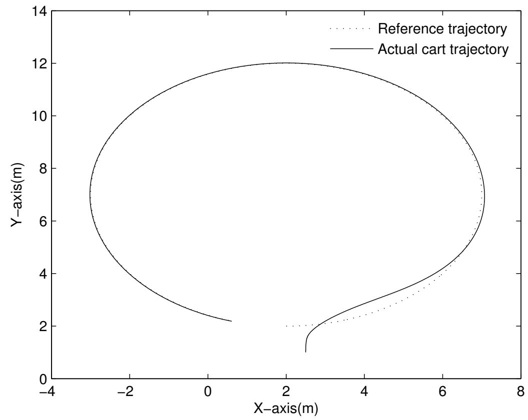

Fig. 1. Trajectory tracking result of the mobile robot.

IV. SIMULATIONS

In this section, we perform the simulation for the tracking control of the nonholonomic electrically driven mobile robot to demonstrate the validity of the proposed control method. The physical parameters for the mobile robot are chosen as $R = 0.75, d = 0.3, r = 0.15, m_c = 30, m_w = 1, I_c = 15.625, I_w = 0.005, I_m = 0.0025$, and $d_{11} = d_{22} = 5$. The parameters for the motor dynamics are chosen as $R_a = \text{diag}(1.6, 1.6), L_a = \text{diag}(0.048, 0.048), K_E = \text{diag}(0.19, 0.19), K_T = \text{diag}(0.2613, 0.2613)$, and $N = \text{diag}(62.55, 62.55)$. In this simulation, we assume that all of these parameters are unknown.

The controller parameters and adaptation gains for the proposed control systems are chosen as $k_1 = 2, k_2 = 2, k_3 = 1, k_4 = 1, k_5 = 1, \sigma_i = 0.001, i = 1, \ldots, 4, \gamma_1 = \gamma_2 = 2, \tau_2 = \tau_3 = 0.01, \Gamma_1 = \text{diag}(0.0001)$, and $\Gamma_2 = \text{diag}(0.2, 0.2, 0.2, 0.2, 0.00001, 0.00001)$. The reference linear and angular velocity is given by $v_r = 1\ m/s$ and $\omega_r = 0.2\ m/s$. The initial postures for the reference robot and the actual robot are $(x_r, y_r, \theta_r) = (2, 2, 0)$ and $(x, y, \theta) = (2.5, 1, \pi/2)$, respectively. The simulation results are shown in Figs. 1 and 2. Fig. 1 shows the tracking result. In Fig. 2(a), the control is started at $t = 0$ and all state errors converge to zero quickly in less than a few seconds. Fig. 2(b) shows the boundedness of the control input. The estimates of unknown parameters in the closed-loop system are shown in Figs. 2(c)-(d).

V. CONCLUSIONS

In this paper, a simple adaptive controller for nonholonomic electrically driven mobile robots with parametric uncertainties has been proposed. The dynamics, the kinematics, and the motor dynamics of mobile robots with parametric uncertainties have been considered. The DSC technique has been extended to design the controller for path tracking of mobile robots including actuator dynamics, and the adaptive control technique has been applied to deal with parametric uncertainties. From the Lyapunov stability theory, we have

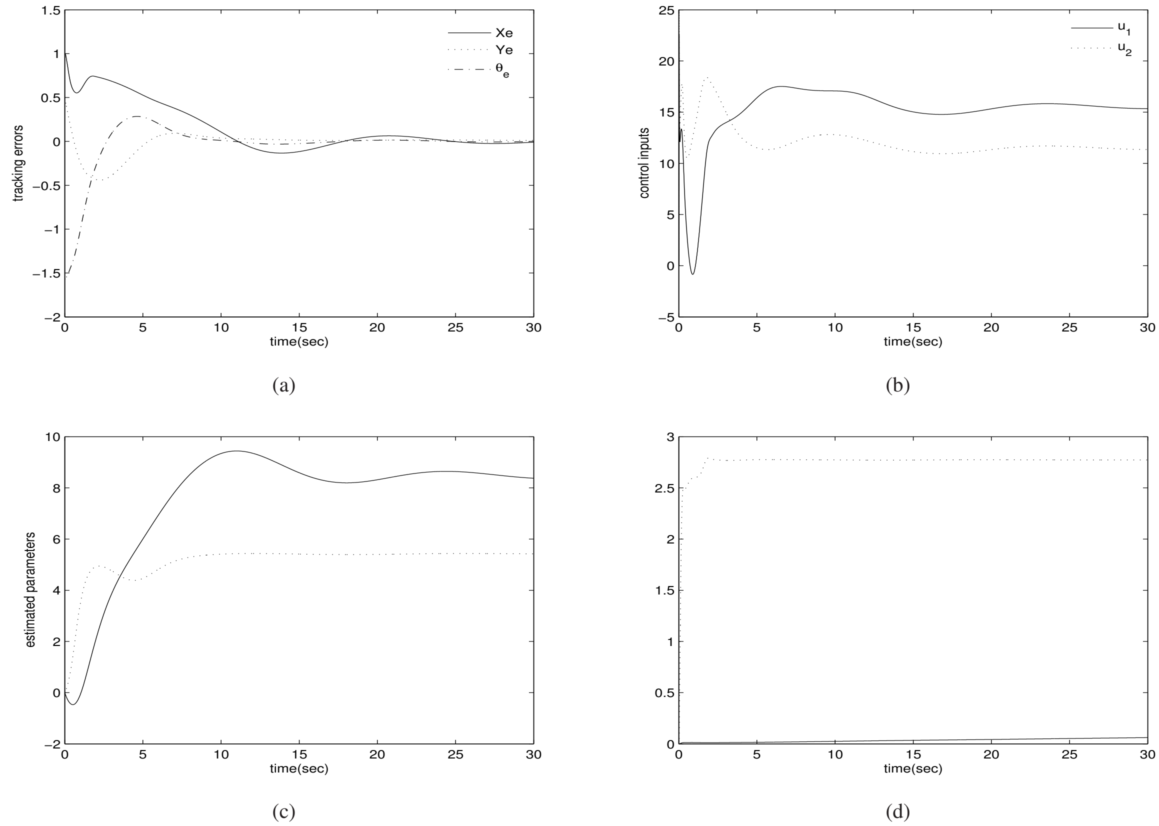

Fig. 2. Simulation results (a) Tracking errors x_e, y_e, θ_e (b) Control inputs (c) Estimated parameters (solid : $\hat{a}_1$, dotted : $\hat{a}_2$) (d) Estimated parameters (solid : $||\widehat{W}_1||$, dotted : $||\widehat{W}_2||$)

proved that all signals in the closed-loop system are semi-globally uniformly ultimately bounded. Finally, from the simulation results, it has been shown that the proposed controller has good tracking performance and the robustness against the parametric uncertainties.

REFERENCES

[1] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for a nonholonomic mobile robot", in *Proc. IEEE International Conf. on Intelligent Robotics and Automation,* 1991, pp. 1236-1241.

[2] H. S. Shim, and Y. G. Sung, "Stability and four-posture control for nonholonomic mobile robots", *IEEE Trans. Robot. Automat.*, vol. 20, no. 1, pp. 148-154, 2004.

[3] J. M. Yang, and J. H. Kim, "Sliding mode control of trajectory tracking of nonholonomic wheeled mobile robots", *IEEE Trans. Robot. Automat.*, vol. 15, no. 3, pp. 578-587, 1999.

[4] C. D. Sousa, E. M. Hemerly, and R. K. H. Galvão, "Adaptive control for mobile robot using wavelet networks", *IEEE Trans. Sys., Man, and Cybern.*, vol. 32, no. 4, pp. 493-504, 2002.

[5] W. E. Dixon, M. S. de Queiroz, D. M. Dawson, and T. J. Flynn, "Adaptive tracking and regulation of a wheeled mobile robot with controller/update law modularity", *IEEE Trans. Contr. Syst. Technol.*, vol. 12, no. 1, pp. 138-147, 2004.

[6] T. Das, and I. N. Kar "Design and implementation of an adaptive fuzzy logic-based controller for wheeled mobile robots", *IEEE Trans. Contr. Syst. Technol.,* vol. 14, no. 3, pp. 501-510, 2006.

[7] K. Kozlowski, and J. Majchrzak, "A backstepping approach to control a nonholonomic mobile robot", in *Proc. IEEE International Conf. on Robotics and Automation,* 2002, pp. 3972-3977.

[8] W. Dong, W. Huo, and S. K. Tso, and W. L. Xu, "Tracking control of uncertain dynamic nonholonomic system and its application to wheeled mobile robots", *IEEE Trans. Robot. Automat.*, vol. 16, no. 6, pp. 870-874, 2000.

[9] R. Fierro, and F. L. Lewis, "Control of a nonholonomic mobile robot: Backstepping kinematics into dynamics", *Journal of Robotic System.*, Vol. 14, No. 3, pp. 149-163, 1997.

[10] T. Fukao, H. Nakagawa, and N. Adachi, "Adaptive tracking control of a nonholonomic mobile robot", *IEEE Trans. Robot. Automat.*, Vol. 16, No. 5, pp. 609-615, 2000.

[11] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, "Dynamic surface control for a class of nonlinear systems", *IEEE Trans. Autom. Control*, Vol. 45, No. 10, pp. 1893-1899, 2000.

[12] P. P. Yip, and J. K. Hedrick, "Adaptive dynamic surface control: A simplified algorithm for adaptive backstepping control of nonlinear systems", *Int. J. Control*, Vol. 71, No. 5, pp. 959-979, 1998.

[13] S. J. Yoo, J. B. Park, and Y. H. Choi, "Adaptive dynamic surface control of flexible-joint robots using self-recurrent wavelet neural networks", *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, Vol. 36, No. 6, pp. 1342-1355, 2006.

[14] K. D. Do, Z. P. Jiang, and J. Pan, "Simultaneous tracking and stabilization of mobile robots: an adaptive approach", *IEEE Trans. Automatic Control*, vol. 49, no. 7, pp. 1147-1152, 2004.

[15] J. E. Slotine, and W. Li, "On the adaptive control of robot manipulators", *Int. J. Robot. Res.*, vol. 6, no. 3, pp. 49-59, 1987.

[16] Z. P. Jiang, and H. Nijmeijer, "Tracking control of mobile robots: A case study in backstepping", *Automatica,* vol. 33, no. 7, pp. 1393-1399, 1997.

[17] P. A. Ioannou and P. V. Kokotovic. *Adaptive Systems with Reduced Models,* NY: Springer-Verlag, 1983.

[18] K. S. Narendra and A. M. Annaswamy, "A new adaptive law for robust adaptation without persistent excitation," *IEEE Trans. Automatic Control,* vol. 32, no. 2, pp. 134-145, 1987.

[19] L. X. Wang, *A Course in Fuzzy Systems and Control*, Upper Saddle River, NJ: Prentice-Hall, 1997.