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P·SPR·D and P·SPR·D·I Control for Linear Multi-Variable Systems
—Stabilization Based on High Gain Output Feedback—

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***Abstract*— This paper is concerned with a set-point servo problem for MIMO systems with P·SPR·D control. The P·SPR·D control is constructed by introducing a SPR (strict positive real) element instead of the I element in PID control. The purpose is to design a P·SPR·D controller for aysmptotic stabilization and to adjust P, SPR, D parameter matrices for improving convergence speed of responses under guaranteeing the stability. In our method, we consider a certain hypothetical system derived from the closed-loop system with P·SPR·D control in order to apply high gain output feedback. Then the P, SPR, D parameter matrices are adjusted by making zero dynamics of the hypothetical system asymptotically stable and performing the high gain output feedback. The proposed method is fundamentally based on the high gain output feedback theorem. The P·SPR·D control is extended to P·SPR·D·I control for improving steady state performance. The effectiveness of the method is confirmed by simulation results for unstable MIMO systems.**

I. Introduction

PID control [1], [7], [8] has been widely used as a classical dynamic controller. But it is mostly used for SISO systems, and it is often difficult to apply for MIMO systems.

As a tuning method of PID control for the MIMO system there exist several researches [2], [6] based on classical control theory. Recently, several researches [4], [5], [12], [10] adopted an approach from modern control theory which is effective for analysis of MIMO system. Refs. [4], [5] try to determine PID parameter matrices by solving LMI after one formulates PID control as static output feedback for the extended system. Ref. [12] proposed a method based on the eigenvalue assignment by the static output feedback. Ref. [10] proposes the expanded PID control of velocity type and its adjustment method by applying the high gain output feedback.

In this paper, we study a set-point servo problem of MIMO system. We propose P·SPR·D control, introducing a SPR (strict positive real) element newly, in order to apply high gain feedback for stabilization. The P·SPR·D control possesses 4 parameter matrices, K_P, K_S, K_D plus a new matrix D included in the SPR element, and manual reset quantity $\boldsymbol{m}_0$. These parameters can be decided systematically by adjusting high gain $\mathcal{L}$, based on the high gain feedback theorem [9]. Futthermore, P·SPR·D·I control is proposed to compensate the manual reset quantity $\boldsymbol{m}_0$ in order to get rid of steady state errors.

We apply the high gain output feedback to design a P·SPR·D controller aysmptotically stabilizing the closed loop system. More concretely, we consider a certain hypothetical system related to the closed loop system with P·SPR·D control and transform it into the normal form and caluculate its zero-dynamics. Then we determine the P, D parameter matrices K_P, K_D and intermediate parameter matrix H_S which stabilize the zero-dynamics. For that purpose we apply an eigenvalue assignment method by the static output feedback [12]. Then, the SPR parameter matrix K_S can be determined by multiplying the intermediate parameter matrix H_S by the high gain coefficient $\mathcal{L}$. It is noted that convergence speed of responses can be also improved by adjusting the high gain $\mathcal{L}$ under the guarantee of stabilily of the closed-loop system.

Based on the above mentioned idea, Ref. [11], [14] proposed P·SPR·D control for the regulation problem. This paper extends it to a set-point servo problem.

The effectiveness of the proposed method was confirmed with the simulation results of various plants.

II. P·SPR·D Control for a Set-point Servo Problem

Consider the following MIMO system:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$
$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{2}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^r$, $\boldsymbol{y}(t) \in \mathbb{R}^m$ are the state vector, the control vector and the output vector, respectively. The system {A,B,C} is assumed controllable and observable.

PID control is usually given as

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + K_I \int_0^t \boldsymbol{e}(\tau)d\tau + K_D\dot{\boldsymbol{e}}(t) + \boldsymbol{m}_0, \tag{3}$$

where $\boldsymbol{e}(t) = \boldsymbol{r}(t) - \boldsymbol{y}(t)$ denotes the error of output from the desired value $\boldsymbol{r}(t)$, and $K_P, K_I, K_D \in \mathbb{R}^{r\times m}$ are PID parameter matrices called Proportional, Integral, Derivative, respectively, and $\boldsymbol{m}_0$ denotes the manual reset quantity.

Let us consider a set-point servo problem with the desired output $\boldsymbol{y}(t) = \boldsymbol{y}^*$. An equilibrium state $\boldsymbol{x}_e$ holding the output at $\boldsymbol{y}^*$ must satisfy the following relation:

$$\boldsymbol{0} = A\boldsymbol{x}_e + B\overline{\boldsymbol{u}}, \quad \boldsymbol{y}^* = C\boldsymbol{x}_e$$

Since this relation consists of $(n+m)$ equations and $(n+r)$ variables, when $r \geq m$, $(r-m)$ state variables $\boldsymbol{x}_{eN}$ can be set arbitrary value $\boldsymbol{x}^*_{eN}$, but the remained state variables $\boldsymbol{x}_{eB}$ and $\overline{\boldsymbol{u}}$ are determined dependently. Putting such an equilibrium as $\boldsymbol{x}^* = \begin{bmatrix} \boldsymbol{x}^*_{eN} \\ \boldsymbol{x}_{eB}(\boldsymbol{x}^*_{eN}, \boldsymbol{y}^*) \end{bmatrix}$ and $\boldsymbol{u}^* = \overline{\boldsymbol{u}}(\boldsymbol{x}^*_{eN}, \boldsymbol{y}^*)$, we

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have

$$\mathbf{0} = A\boldsymbol{x}^* + B\boldsymbol{u}^* \tag{4}$$

$$\boldsymbol{y}^* = C\boldsymbol{x}^* \tag{5}$$

Next, letting the state error from the equilibrium $\boldsymbol{x}^*$ be

$$\boldsymbol{e}_x(t) = \boldsymbol{x}(t) - \boldsymbol{x}^* \tag{6}$$

we obtain the output error from (2) and (5) as follows.

$$\boldsymbol{e}(t) = \boldsymbol{y}^* - \boldsymbol{y}(t) = C(\boldsymbol{x}^* - \boldsymbol{x}(t)) = -C\boldsymbol{e}_x(t) \tag{7}$$

Differentiate (6) with the use of (1), (4) to obtain the state error system

$$\begin{aligned}\dot{\boldsymbol{e}}_x(t) &= A\boldsymbol{x}(t) + B\boldsymbol{u}(t) - (A\boldsymbol{x}^* + B\boldsymbol{u}^*)\\ &= A\boldsymbol{e}_x(t) + B(\boldsymbol{u}(t) - \boldsymbol{u}^*)\end{aligned} \tag{8}$$

Accordingly, if we can asymptotically stabilize the error system (8) and can make $\boldsymbol{e}_x(t) \to \mathbf{0}$ as $t \to \infty$, we have $\boldsymbol{e}(t) \to \mathbf{0}$, that is, $\boldsymbol{y}(t) \to \boldsymbol{y}^*$. So the set-point servo problem can be solved.

In this paper, by substituting a SPR (strict positive real) element for the I element of PID control, we propose the following P·SPR·D control.

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + K_S\boldsymbol{z}(t) + K_D\dot{\boldsymbol{e}}(t) + \boldsymbol{m}_0 \tag{9}$$

$$\dot{\boldsymbol{z}}(t) = D\boldsymbol{z}(t) + \boldsymbol{e}(t), \quad \boldsymbol{z}(0) = \mathbf{0} \tag{10}$$

where (10) represents the SPR element with negative definite $D \in \mathbb{R}^{m\times m}$, and $K_S \in \mathbb{R}^{r\times m}$ denotes the SPR parameter matrix. It is noted that SPR operation (10) differs from the pure I operation. This modification results from a device for applying Proposition 1 (High Gain Output Feedback) in the next Chapter 3 without a strong assumption on the plant. In fact, when the relative degree of the plant is larger than 2, usual PID control with $D = 0$ does not satisfy the assumption in Proposition 1 and hence the high gain output feedback cannot be applied to design a PID controller (see Ref. [13] in detail).

Now we define the SPR parameter matrix K_S as

$$K_S \stackrel{\triangle}{=} H_S\mathcal{L}, \tag{11}$$

where $H_S \in \mathbb{R}^{r\times m}$ and $\mathcal{L} \in \mathbb{R}^{m\times m}$ $(\det\mathcal{L} \neq 0)$ are called the intermediate parameter matrix and the adjustable parameter matrix, respectively. So the P·SPR·D control (9) can be represented as

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + H_S\boldsymbol{z}'(t) + K_D\dot{\boldsymbol{e}}(t) + \boldsymbol{m}_0 \tag{12}$$

$$\text{where} \quad \boldsymbol{z}'(t) \stackrel{\triangle}{=} \mathcal{L}\boldsymbol{z}(t) \tag{13}$$

Since it holds from (7) and (8) that

$$\dot{\boldsymbol{e}}(t) = -C(A\boldsymbol{e}_x(t) + B(\boldsymbol{u}(t) - \boldsymbol{u}^*)), \tag{14}$$

we have

$$\begin{aligned}\boldsymbol{u}(t) = &-K_PC\boldsymbol{e}_x(t) + H_S\boldsymbol{z}'(t)\\ &-K_DC(A\boldsymbol{e}_x(t) + B(\boldsymbol{u}(t) - \boldsymbol{u}^*)) + \boldsymbol{m}_0\end{aligned}$$

by substituting (7) and (14) into (12).

Furthermore, arranging this equation, we obtain

$$\begin{aligned}\boldsymbol{u}(t) &= -(I_r + K_DCB)^{-1}K_PC\boldsymbol{e}_x(t)\\ &\quad +(I_r + K_DCB)^{-1}H_S\boldsymbol{z}'(t)\\ &\quad -(I_r + K_DCB)^{-1}K_DC(A\boldsymbol{e}_x(t) - B\boldsymbol{u}^*)\\ &\quad +(I_r + K_DCB)^{-1}\boldsymbol{m}_0\\ &= -(I_r + K_DCB)^{-1}(K_PC + K_DCA)\boldsymbol{e}_x(t)\\ &\quad +(I_r + K_DCB)^{-1}H_S\boldsymbol{z}'(t)\\ &\quad +(I_r + K_DCB)^{-1}K_DCB\boldsymbol{u}^*\\ &\quad +(I_r + K_DCB)^{-1}\boldsymbol{m}_0\\ &= -K_E\boldsymbol{e}_x(t) + K_Z\boldsymbol{z}'(t) + K_U(K_DCB\boldsymbol{u}^* + \boldsymbol{m}_0)\end{aligned} \tag{15}$$

$$\begin{aligned}\text{where } \mathrm{K_E} &\stackrel{\triangle}{=} (\mathrm{I_r} + \mathrm{K_DCB})^{-1}(\mathrm{K_PC} + \mathrm{K_DCA})\\ K_Z &\stackrel{\triangle}{=} (I_r + K_DCB)^{-1}H_S\\ K_U &\stackrel{\triangle}{=} (I_r + K_DCB)^{-1}\end{aligned}$$

By substituting (15) into (8), we obtain the closed-loop error system

$$\begin{aligned}\dot{\boldsymbol{e}}_x(t) &= A\boldsymbol{e}_x(t) + B(\boldsymbol{u}(t) - \boldsymbol{u}^*)\\ &= A\boldsymbol{e}_x(t) + B(-K_E\boldsymbol{e}_x(t) + K_Z\boldsymbol{z}'(t)\\ &\qquad +K_U(K_DCB\boldsymbol{u}^* + \boldsymbol{m}_0) - \boldsymbol{u}^*)\\ &= (A - BK_E)\boldsymbol{e}_x(t) + BK_Z\boldsymbol{z}'(t)\\ &\qquad -B(I_r + K_DCB)^{-1}\boldsymbol{u}^* + BK_U\boldsymbol{m}_0\\ &= (A - BK_E)\boldsymbol{e}_x(t) + BK_Z\boldsymbol{z}'(t) + BK_U(\boldsymbol{m}_0 - \boldsymbol{u}^*)\end{aligned} \tag{16}$$

Meanwhile, from (13) and (10) the time derivative of $\boldsymbol{z}'(t)$ becomes

$$\begin{aligned}\dot{\boldsymbol{z}}'(t) &= \mathcal{L}\Big(D\boldsymbol{z}(t) + \boldsymbol{e}(t)\Big) = \mathcal{L}\Big(D\mathcal{L}^{-1}\boldsymbol{z}'(t) - C\boldsymbol{e}_x(t)\Big)\\ &= \mathcal{L}D'\boldsymbol{z}'(t) - \mathcal{L}C\boldsymbol{e}_x(t)\end{aligned} \tag{17}$$

$$\text{where} \quad \mathrm{D'} \quad \stackrel{\triangle}{=} \quad D\mathcal{L}^{-1} \tag{18}$$

Accordingly, by combining (16) and (17), the closed-loop error system with the P·SPR·D control becomes

$$\begin{bmatrix} \dot{\boldsymbol{e}_x}(t) \\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} = \begin{bmatrix} A - BK_E & BK_Z \\ -\mathcal{L}C & \mathcal{L}D' \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}'(t) \end{bmatrix} + \begin{bmatrix} BK_U(\boldsymbol{m}_0 - \boldsymbol{u}^*) \\ \mathbf{0} \end{bmatrix} \tag{19}$$

Now let us set the manual reset quantity as $\boldsymbol{m}_0 = \boldsymbol{u}^*$. Then the closed-loop error system becomes

$$\begin{bmatrix} \dot{\boldsymbol{e}_x}(t) \\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} = \begin{bmatrix} A - BK_E & BK_Z \\ -\mathcal{L}C & \mathcal{L}D' \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}'(t) \end{bmatrix} \tag{20}$$

Next let us consider the following hypothetical system based on the closed-loop error system (20) with P·SPR·D control:

$$\begin{bmatrix} \dot{\boldsymbol{e}}_x(t) \\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} = \begin{bmatrix} A - BK_E & BK_Z \\ O & O \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}'(t) \end{bmatrix} + \begin{bmatrix} O \\ I_m \end{bmatrix} \boldsymbol{v}(t) := \widetilde{A}\widetilde{\boldsymbol{x}}(t) + \widetilde{B}\boldsymbol{v}(t) \tag{21}$$

$$\widetilde{\boldsymbol{y}}(t) = \begin{bmatrix} C & -D' \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}'(t) \end{bmatrix} := \widetilde{C}\widetilde{\boldsymbol{x}}(t) \tag{22}$$

where $\boldsymbol{v}(t) \in \mathbb{R}^m$ and $\widetilde{\boldsymbol{y}} \in \mathbb{R}^m$ are the input and the output of the hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$. Here, the input $\boldsymbol{v}(t)$ is given by the output feedback

$$\boldsymbol{v}(t) = -\mathcal{L}\widetilde{\boldsymbol{y}}(t) = -\mathcal{L}\begin{bmatrix} C & -D' \end{bmatrix}\begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}'(t) \end{bmatrix} \tag{23}$$

where $\mathcal{L}$ is the output feedback gain.

At this time, the closed-loop error system of the hypothetical system becomes

$$\begin{bmatrix} \dot{\boldsymbol{e}}_x(t) \\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} = \begin{bmatrix} A - BK_E & BK_Z \\ -\mathcal{L}C & \mathcal{L}D' \end{bmatrix}\begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}'(t) \end{bmatrix} \tag{24}$$

It is clear that (24) equals to the closed-loop error system (20) with P·SPR·D control by the output feedback gain $\mathcal{L}$ being equal to the adjustable parameter matrix of (11). Therefore, by setting

$$K_S = H_S\mathcal{L}, \quad D = D'\mathcal{L}, \tag{25}$$

(24) becomes the closed-loop error system with the P·SPR·D control

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + K_S\boldsymbol{z}(t) + K_D\dot{\boldsymbol{e}}(t) + \boldsymbol{u}^* \tag{26}$$

$$\dot{\boldsymbol{z}}(t) = D\boldsymbol{z}(t) + \boldsymbol{e}(t), \ \boldsymbol{z}(0) = \boldsymbol{0}, \quad D < 0 \tag{27}$$

III. Design of P·SPR·D Controller by High Gain Output Feedback

In our method, we use the high gain output feedback theorem [9] in order to design the P·SPR·D controller. So at the beginning we prepare some terminology.

Consider the following general MIMO system

$$\dot{\widetilde{\boldsymbol{x}}}(t) = \widetilde{A}\widetilde{\boldsymbol{x}}(t) + \widetilde{B}\boldsymbol{v}(t) \tag{28}$$

$$\widetilde{\boldsymbol{y}}(t) = \widetilde{C}\widetilde{\boldsymbol{x}}(t) \tag{29}$$

where $\widetilde{\boldsymbol{x}}(t) \in \mathbb{R}^N, \boldsymbol{v}(t) \in \mathbb{R}^m, \widetilde{\boldsymbol{y}}(t) \in \mathbb{R}^m$.

[Definition 1] (Relative Degree) System (28), (29) is said to have relative degree $\{q_1, q_2, \cdots, q_m\}$, when the following relations concerning $\widetilde{y}_i^{(k)}$ (k-th derivative of $\widetilde{y}_i$) hold.

1) In the neighborhood of $\widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{x}}_e$, for all $k < q_i$

$$\frac{\partial \widetilde{y}_i^{(k)}}{\partial v_j} = 0, \quad \text{for all } 1 \le j \le m$$

2) In the neighborhood of $\widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{x}}_e$, $(m \times m)$ matrix

$$\left[\frac{\partial \widetilde{y}_i^{(q_i)}}{\partial v_j}\right]_{1 \le i,j \le m}$$

is nonsingular.

If system (28), (29) has the relative degree $\{1, 1, \cdots, 1\}$ such that $\widetilde{C}\widetilde{B}$ is nonsingular, then the system can be transformed into the normal form [3]. That is, by coordinate transformation

$$\begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \widetilde{C} \\ \widetilde{T} \end{bmatrix}\widetilde{\boldsymbol{x}}, \quad \boldsymbol{\xi} \in \mathbb{R}^m, \ \boldsymbol{\eta} \in \mathbb{R}^{(N-m)} \tag{30a}$$

$$\widetilde{T}\widetilde{B} = O \tag{30b}$$

we can transform system (28), (29) into the normal form

$$\dot{\boldsymbol{\xi}}(t) = Q_{11}\boldsymbol{\xi}(t) + Q_{12}\boldsymbol{\eta}(t) + \widetilde{C}\widetilde{B}\boldsymbol{v}(t) \tag{31a}$$

$$\dot{\boldsymbol{\eta}}(t) = Q_{21}\boldsymbol{\xi}(t) + Q_{22}\boldsymbol{\eta}(t) \tag{31b}$$

$$\widetilde{\boldsymbol{y}}(t) = \boldsymbol{\xi}(t) \tag{32}$$

where $Q_{11} \in \mathbb{R}^{m \times m}$, $Q_{12} \in \mathbb{R}^{m \times (N-m)}, Q_{21} \in \mathbb{R}^{(N-m) \times m}$, $Q_{22} \in \mathbb{R}^{(N-m) \times (N-m)}$ are coefficient matrices after the coordinate transformation.

In (31b),

$$\dot{\boldsymbol{\eta}}(t) = Q_{22}\boldsymbol{\eta}(t) \tag{33}$$

is called the **zero-dynamics**. If the zero dynamics (33) is asymptotically stable, the system (28), (29) is said to be **minimum phase**.

Using the properties defined above, we have [9]:

[Propositon 1] (High Gain Output Feedback)

Suppose that system (28), (29) has relative degree $\{1, 1, \cdots, 1\}$ at an equilibrium $\widetilde{\boldsymbol{x}}_e = \boldsymbol{0}$ (i.e. $\widetilde{C}\widetilde{B}$ is nonsingular) and suppose that the system is minimum phase (i.e. the zero dynamics is asymptotically stable). Consider an output feedback control

$$\boldsymbol{v}(t) = -\mathcal{L}\widetilde{\boldsymbol{y}}(t) \tag{34}$$

with a gain matrix $\mathcal{L} \in R^{m \times m}$. Then there exist constants γ_{i0} such that the closed-loop system (28), (29), (34) is asymptotically stable, provided that $\mathcal{L}$ is chosen as $\mathcal{L} = (\widetilde{C}\widetilde{B})^{-1}(Q_{11}+\Gamma)$ with $\Gamma = \text{diag}(\gamma_1, \gamma_2, \cdots, \gamma_r)$, $\gamma_i \ge \gamma_{i0} > 0$, where Q_{11} is the matrix of (31a).

(**Proof**) Given in Ref. [9]. Q.E.D

Now let us consider to apply Proposition 1 to the hypothetical system (21), (22) in order to design the P·SPR·D controller.

First check the relative degree of system (21),(22). Differentiation of (22) becomes

$$\begin{aligned} \dot{\widetilde{\boldsymbol{y}}}(t) &= \begin{bmatrix} C & -D' \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{e}}_x(t) \\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} \\ &= \begin{bmatrix} C & -D' \end{bmatrix}\left(\begin{bmatrix} A - BK_E & BK_Z \\ O & O \end{bmatrix}\begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}'(t) \end{bmatrix}\right. \\ &\qquad \left. + \begin{bmatrix} O \\ I_m \end{bmatrix}\boldsymbol{v}(t)\right) \end{aligned} \tag{35}$$

Hence we have

$$\frac{\partial \dot{\widetilde{\boldsymbol{y}}}(t)}{\partial \boldsymbol{v}(t)} = \widetilde{C}\widetilde{B} = \begin{bmatrix} C & -D' \end{bmatrix}\begin{bmatrix} O \\ I_m \end{bmatrix} = -D'$$

To satisfy that $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ has relative degree $\{1, 1, \cdots, 1\}$, the above matrix has to be nonsingular . Therefore, let us set D' be a nonsingular matrix.

Next check the minimum phase property. Since the relative degree of the hypothetical system (21), (22) is $\{1, 1, \cdots, 1\}$ from the mentioned above, we can transform this system into the normal form to obtain its zero dynamics. Hence let us consider the following transfomation for (24):

$$\begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \widetilde{C} \\ \widetilde{T} \end{bmatrix} \widetilde{\boldsymbol{x}} = \begin{bmatrix} C & -D' \\ I_n & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z}' \end{bmatrix} \tag{36}$$

where $\widetilde{T} = \begin{bmatrix} I_n & O \end{bmatrix}, \ \widetilde{T}\widetilde{B} = O,$

and $\boldsymbol{\xi} \in \mathbb{R}^m$ from $\boldsymbol{\xi} = \widetilde{\boldsymbol{y}}$ and so $\boldsymbol{\eta} \in \mathbb{R}^n$. Note that the inverse matrix of (36) becomes

$$\begin{bmatrix} C & -D' \\ I_n & O \end{bmatrix}^{-1} = \begin{bmatrix} O & I_n \\ -D'^{-1} & D'^{-1}C \end{bmatrix}$$

Therefore, from the following calculation

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{\xi}} \\ \dot{\boldsymbol{\eta}} \end{bmatrix} &= \begin{bmatrix} C & -D' \\ I_n & O \end{bmatrix} \begin{bmatrix} A - BK_E & BK_Z \\ O & O \end{bmatrix} \\ &\quad \times \begin{bmatrix} O & I_n \\ -D'^{-1} & D'^{-1}C \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} \\ &\quad + \begin{bmatrix} C & -D' \\ I_n & O \end{bmatrix} \begin{bmatrix} O \\ I_m \end{bmatrix} \boldsymbol{v}, \end{aligned} \tag{37}$$

we can obtain the normal form of the hypothetical system (21), (22):

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= -CBK_Z D'^{-1}\boldsymbol{\xi} \\ &\quad + C\Big((A - BK_E) + BK_Z D'^{-1}C\Big)\boldsymbol{\eta} - D'\boldsymbol{v} && (38a) \\ \dot{\boldsymbol{\eta}} &= -BK_Z D'^{-1}\boldsymbol{\xi} \\ &\quad + \Big((A - BK_E) + BK_Z D'^{-1}C\Big)\boldsymbol{\eta} && (38b) \\ \widetilde{\boldsymbol{y}} &= \boldsymbol{\xi} && (39) \end{aligned}$$

Accordingly, the zero dynamics is expressed as

$$\dot{\boldsymbol{\eta}} = \Big(A - B(K_E - K_Z D'^{-1}C)\Big)\boldsymbol{\eta} \tag{40}$$

To satisfy the minimum phase property, the zero dynamics (40) has to be asymptotically stable. Threfore, let us assume the following:

[Assumption 1] There exist parameter matrices H_S, K_P, K_D such that the zero dynamics

$$\dot{\boldsymbol{\eta}} = \big(A - B(K_E - K_Z D'^{-1}C)\big)\boldsymbol{\eta}$$

is asymptotically stable.

Consequently, we can apply Proposition 1 and obtain $\mathcal{L}$ of the output feedback (23) which asymptotically stabilizes (21). Thus, by setting the matrices from (25) as

$$K_S = H_S\mathcal{L}, \quad D = D'\mathcal{L},$$

P·SPR·D control

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + K_S\boldsymbol{z}(t) + K_D\dot{\boldsymbol{e}}(t) + \boldsymbol{u}^*$$

$$\dot{\boldsymbol{z}}(t) = D\boldsymbol{z}(t) + \boldsymbol{e}(t), \ \boldsymbol{z}(0) = \boldsymbol{0}, \quad D < 0$$

is obtained and an equilibrium of system (8), (7), (26), (27) is asymptotically stable. Also, from the property of high gain output feedback, we can improve the convergence speed of responses by adjusting the high gain $\mathcal{L}$ to some extent.

Meanwhile, if $\boldsymbol{m}_0 \neq \boldsymbol{u}^*$, or when one sets $\boldsymbol{m}_0 = \boldsymbol{0}$, an off-set, $\boldsymbol{e}(\infty) = \boldsymbol{y}^* - \boldsymbol{y}(\infty)$, occurs caused by the second term in the right-hand side of (19), even when a transition matrix of (24) is asymptotically stable. An amount of this off-set depends on $K_P, \ K_S, \ K_D$ and D.

The closed-loop system by the P·SPR·D control becomes finally as follows.

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{e}}_x(t) \\ \dot{\boldsymbol{z}}(t) \end{bmatrix} &= \begin{bmatrix} A - BK_E & B\widetilde{K}_Z \\ -C & D \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_x(t) \\ \boldsymbol{z}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} BK_U(\boldsymbol{m}_0 - \boldsymbol{u}^*) \\ \boldsymbol{0} \end{bmatrix} \end{aligned} \tag{41}$$

where

$$\widetilde{K}_Z \triangleq (I_r + K_D CB)^{-1} H_S \mathcal{L} = (I_r + K_D CB)^{-1} K_S$$

Consequently, an off-set becomes

$$\begin{aligned} \begin{bmatrix} \boldsymbol{e}_x(\infty) \\ \boldsymbol{z}(\infty) \end{bmatrix} &= -\begin{bmatrix} A - BK_E & B\widetilde{K}_Z \\ -C & D \end{bmatrix}^{-1} \\ &\quad \times \begin{bmatrix} BK_U(\boldsymbol{m}_0 - \boldsymbol{u}^*) \\ \boldsymbol{0} \end{bmatrix} \end{aligned} \tag{42}$$

For the regulation problem (i.e. $\boldsymbol{x}^* = \boldsymbol{0}$), however, the state $\boldsymbol{x}(t)$ can be converged to the origin by the P·SPR·D control with $\boldsymbol{m}_0 = \boldsymbol{0}$ (no off-set occurs).

Finally let us consider a counterplan in case that the manual reset quantity $\boldsymbol{m}_0 = \boldsymbol{u}^*$ cannot be calculated or is not available for the P·SPR·D control. Since the stability of transient state is guaranteed sufficiently by the high gain feedback, we devise only a countermove for a steady state error (i.e. off-set). We suggest the following two methods in order to compensate $\boldsymbol{m}_0 = \boldsymbol{u}^*$.

The first one is to use I mode such that the following equation is adopted instead of (26).

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + K_S\boldsymbol{z}(t) + K_D\dot{\boldsymbol{e}}(t) + K_I\int_0^t \boldsymbol{e}(\tau)d\tau, \tag{43}$$

which is called P·SPR·D·I control.

The second one is to use feedforward mode as follows.

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + K_S\boldsymbol{z}(t) + K_D\dot{\boldsymbol{e}}(t) + K_F(t)\boldsymbol{y}^* \tag{44}$$

$$\dot{K}_F(t) = S^{-1}\boldsymbol{e}(t)\boldsymbol{y}^{*T}, \quad S > 0, \tag{45}$$

which is called P·SPR·D+Feedforward control. Here $K_F(t)$ denotes the time-variant feedforward gain matrix and S is a weighting coefficient. Note that this idea originates in the direct adaptive control algorithm.

IV. Determination of Controller Parameter Matrices

The most important task in our method is to satisfy Assumption 1, that is, to determine P, D parameter matrices $K_P, \ K_D$ and the intermediate parameter matrix H_S such that the zero dynamics of (40),

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \Big(A - B(K_E - K_Z D'^{-1}C)\Big)\boldsymbol{\eta} \\ &= \Big(A - B(I_r + K_D CB)^{-1} \\ &\quad \times (K_P C + K_D CA - H_S D'^{-1}C)\Big)\boldsymbol{\eta}, \end{aligned} \tag{46}$$

is asymptotically stable.

In this section, we propose a method determing $K_P,\ K_D$ and H_S so as to stabilize the matrix of zero dynamics

$$A - B(I_r + K_D CB)^{-1}(K_P C + K_D CA - H_S D^{'-1} C) \tag{47}$$

We first transform the partial matrix $(I_r + K_D CB)^{-1}$ $(K_P C + K_D CA - H_S D^{'-1} C)$ of the zero dynamics (47) into

$$\begin{aligned}&(I_r + K_D CB)^{-1}(K_P C + K_D CA - H_S D^{'-1} C)\\ &= (I_r + K_D CB)^{-1}\\ &\quad\times \begin{bmatrix} K_P - H_S D^{'-1} & K_D \end{bmatrix} \begin{bmatrix} C \\ CA \end{bmatrix}\end{aligned} \tag{48}$$

By defining the following matrix

$$F_{\eta 1} = (I_r + K_D CB)^{-1}(K_P - H_S D^{'-1}) \tag{49}$$

$$F_{\eta 2} = (I_r + K_D CB)^{-1} K_D \tag{50}$$

$$F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix},\ C_\eta = \begin{bmatrix} C \\ CA \end{bmatrix}, \tag{51}$$

(48) can be represented as $F_\eta C_\eta$. Then the matrix of zero dynamics (47) can be expressed as

$$A - BF_\eta C_\eta \tag{52}$$

This can be regarded as the closed-loop system with the output feedback $-F_\eta C_\eta \boldsymbol{\eta}$ for subsystem $\{A, B, C_\eta\}$. Accordingly, in order to get matrices K_P, K_D, H_S asymptotically stabilizing the zero dynamics, we apply static output feedback $-F_\eta C_\eta \boldsymbol{\eta}$ for $\{A, B, C_\eta\}$. After determining the output feedback gain F_η stabilizing (52) , we can obtain K_P, K_D, H_S stabilizing (47) from the relations (49)∼(51).

Now, to determine such output feedback gain $F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix}$, we apply the eigenvalue assignment method with the static output feedback, which we proposed in [12], to the system $\{A, B, C_\eta\}$. That is, we can obtain the output feedback gain F_η assigning the desired eigenvalues Λ_n such that (52) is assymtotically stable, provided that $\{A, B, C_\eta\}$ be controllable and observable and the order condition $2m + r > n$ is satisfied. And when such $F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix}$ is obtained, we can determine K_D from the relation (50) as follows.

$$K_D = F_{\eta 2}(I_m - CBF_{\eta 2})^{-1} \tag{53}$$

Further, since

$$K_P = (I_r + K_D CB)F_{\eta 1} + H_S D^{'-1} \tag{54}$$

from (49), K_P can be calculated by substituting K_D of (53) and an adequate H_S into the above equation.

[Remark 1] When we apply the eigenvalue assignment method to obtain F_η stabilizing the zero dynamics, it is important how to choose the desired eigenvalues practically. So, as the adequate eigenvalues, we can use the optimal eigenvalues $\Lambda_n = \sigma(A - BK_\eta)$ which can be calculated from the optimal closed-loop matrix $A - BK_\eta$, where the $K_\eta = R^{-1}B^T P$ is obtained by solving the Riccati equation

$$PA + A^T P + Q - PBR^{-1}B^T P = O, \quad Q > 0, R > 0.$$

[Design Procedure A]

Step 1: Set the desired eigenvalues Λ_n for (52) (e.g. using the method in Remark 1).

Step 2: Apply the eigenvalue assignment method [12] to $\{A, B, C_\eta\}$, and determine the output feedback gain $F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix}$ assigning the desired eigenvalues Λ_n given in Step 1 .

Step 3: Give the nonsingular matrix $D^{'} \in \mathbb{R}^{m \times m}$ and the intermediate parameter matrix $H_S \in \mathbb{R}^{r \times m}$, determine K_D and K_P from (53), (54).

Step 4: Choose $\mathcal{L}$ in (23), applying Proposition 1, that is, $\mathcal{L} = -D'^{-1}(-CB(I_r + K_D CB)^{-1} H_S D'^{-1} + \Gamma)$, $\Gamma = \text{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_m\}$, $\gamma_i \geq \gamma_{i0} > 0$, and determine $K_S = H_S \mathcal{L}$ and $D = D'\mathcal{L}$.

Step 5: P·SPR·D controller is given by (26),(27).

The properties of the closed-loop system are influenced by the zero dynamics. Notice from the similar transformation (36) that the variable $\boldsymbol{\eta}$ of the zero dynamics in the extended system coincides with the state variable $\boldsymbol{x}$. Then, we know that the actual response of the state $\boldsymbol{x}(t)$ approaches asymptotically to the response of $\boldsymbol{\eta}(t)$ of the zero dynamics (40), when the diagonal matrix Γ is made larger and larger to achieve high gain $\mathcal{L}$. Accordingly, we may adopt a design policy of determining P, SPR, D parameter matrices such that the zero dynamics matrix $(A - B(K_E - K_Z D'^{-1} C)$ comes close to the ideal closed-loop matrix of the desired response.

As a practical design policy, it may be effective to decide $\mathcal{L}$ subjectively, observing the output responses concretely, since D, K_P, K_S, K_D can be computed immediately given $\mathcal{L}$ in our method.

V. Numerical Example

Consider an example of 5 dimensional 2-input 2-output unstable system

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \boldsymbol{u}(t) \tag{55}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \boldsymbol{x}(t) \tag{56}$$

Its eigenvalues are $\{\ 0,\ 0,\ -1,\ 1 \pm i\ \}$ which implies that the plant is unstable.

Set the desired eigenvalues Λ_n from Step 1 of Design Procedure A. By using the method in Remark 1 with $P = I_5, R = I_2$, the desired eigenvalues are obtained:

$$\Lambda_n = \{-1.104 \pm 1.264i,\ -1.670 \pm 0.5475i,\ -0.8819\}$$

From Step 2, by applying the eigenvalue assignment method [12], F_η which assigns Λ_n is obtained as

$$F_\eta = \begin{bmatrix} 7.245 & -5.815 & 4.587 & -0.6357 \\ -8.815 & -0.7322 & -2.839 & -3.480 \end{bmatrix}$$

By Step 3, setting

$$D' = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ H_S = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix},$$

we can calculate P, D parameter matrices as

$$K_P = \begin{bmatrix} 6.245 & -6.315 \\ -9.314 & -1.732 \end{bmatrix},\ K_D = \begin{bmatrix} 4.587 & -0.6357 \\ -2.839 & -3.480 \end{bmatrix}$$

By step 4, choosing $\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2\}$ as (a) $\Gamma = \mathrm{diag}\{0.1, 0.1\}$, (b) $\Gamma = \mathrm{diag}\{1, 1\}$, (c) $\Gamma = \mathrm{diag}\{5, 5\}$, (d) $\Gamma = \mathrm{diag}\{10, 10\}$ and setting $K_S = H_S\mathcal{L},\ D = D'\mathcal{L}$, we obtained the simulation results as shown in Fig.1. Here the desired output is given as $\boldsymbol{y}^* = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$.

By making γ_i larger, we can see from Fig.1 that transient responses get improved from the unstable response to convergent ones.

Fig.2 shows the simulation results by the P·SPR·D·I control with $K_I = \begin{bmatrix} 0.05 & 0.05 \\ 0.05 & 0.05 \end{bmatrix}$, when $\boldsymbol{u}^*$ is not available. It is observed that no off-set yields and convergence speed is very quick.

Fig.3 shows the simulation results by the P·SPR·D+Feedforward control with $S = \begin{bmatrix} 1000 & 100 \\ 100 & 1000 \end{bmatrix}$. Although the output converged to the desired value $\boldsymbol{y}^*$ (no off-set) by this method also, it was observed that the P·SPR·D·I control achived better performance always than the P·SPR·D+Feedforward control as far as the set-point servo problem is concerned.

VI. Concluding Remarks

We proposed P·SPR·D control and P·SPR·D·I one for the set-point servo problem of general MIMO systems. The P·SPR·D control was derived based on the concept that one makes a system minimum phase and uses high gain output feedback. The zero dynamics could be asymptotically stabilized by using the SPR element.

Note that, however, $\boldsymbol{z}(t) \to \boldsymbol{0}$ as $t \to \infty$, when $\boldsymbol{e}(t) = \boldsymbol{y}^* - \boldsymbol{y}(t) \to \boldsymbol{0}$ in (10). So the SPR mode does not contribute to get rid of steady state errors. Therefore, the set-point servo controller must be P·SPR·D+$\boldsymbol{u}^*$ as (26) or P·SPR·D·I as (43).

We can determine the controller parameter matrices systematically by Design Procedure A. Note that the P·SPR·D or P·SPR·D·I control are useful not only for asymptotic stabilization but also for improving the convergence speed by adjusting the high gain $\mathcal{L}$. An advantage of using the SPR element is that one can establish systematic adjustment of controller parameters, based on the high gain output feedback.

It is also remarked that the use of SPR element contributes powerfully to stabilizing the closed-loop system. Namely, by adding the SPR to PI or PID, we can improve stabilization ability to a great extent.

Implementation of a controller with the SPR element is not difficult by a digital processor.

The robustness of P·SPR·D control, where the zero dynamics may be stabilized by H^∞ output feedback, is considered an interesting future topic.

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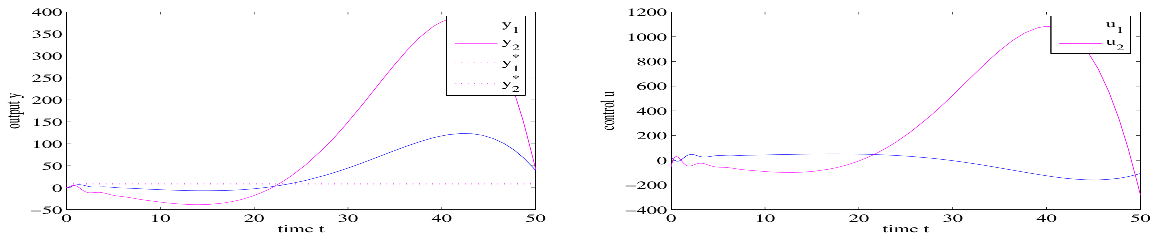
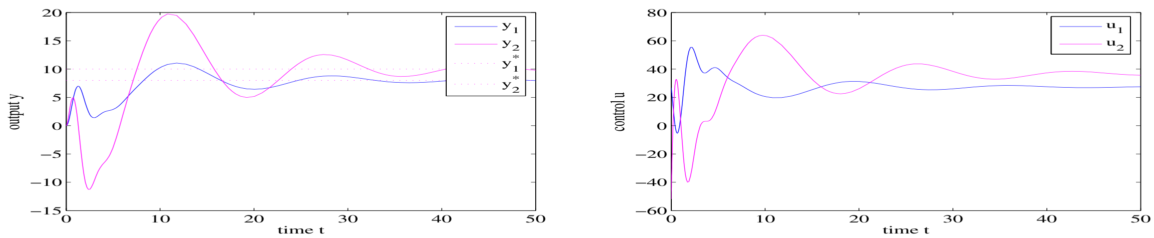
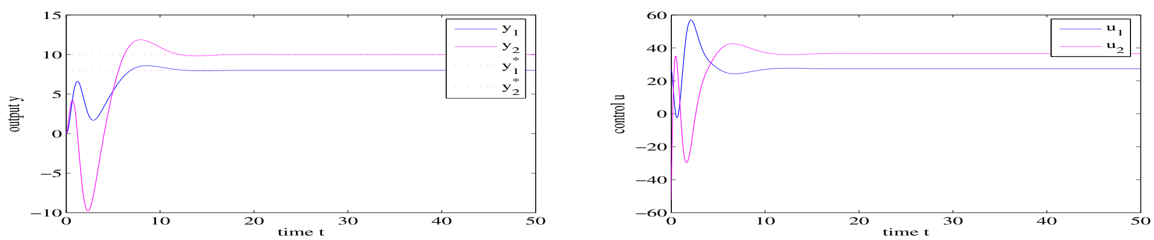


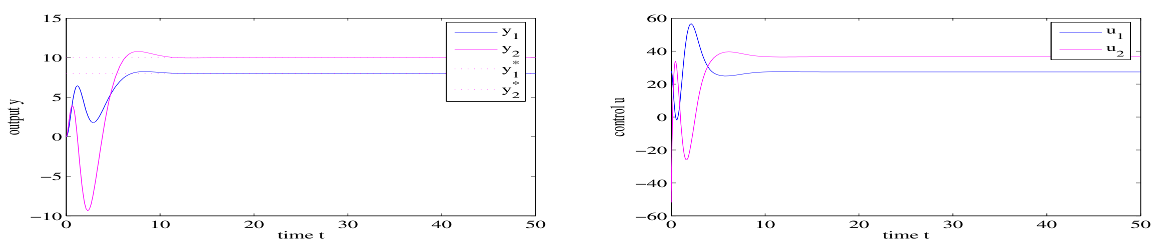
$(a)\quad \Gamma = \mathrm{diag}\{0.1, 0.1\}$



$(b)\quad \Gamma = \mathrm{diag}\{1, 1\}$

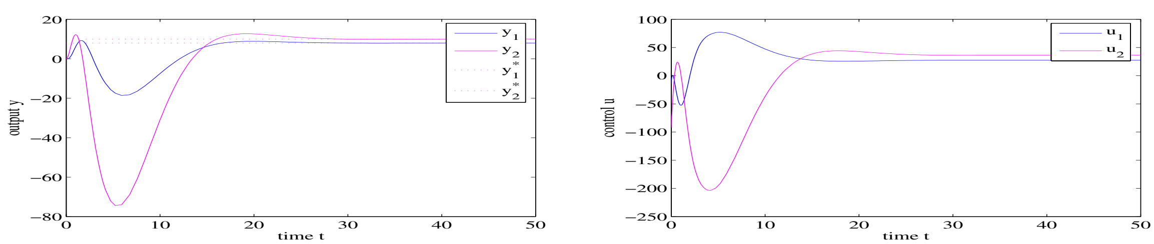


$(c)\ \Gamma = \mathrm{diag}\{5, 5\}$



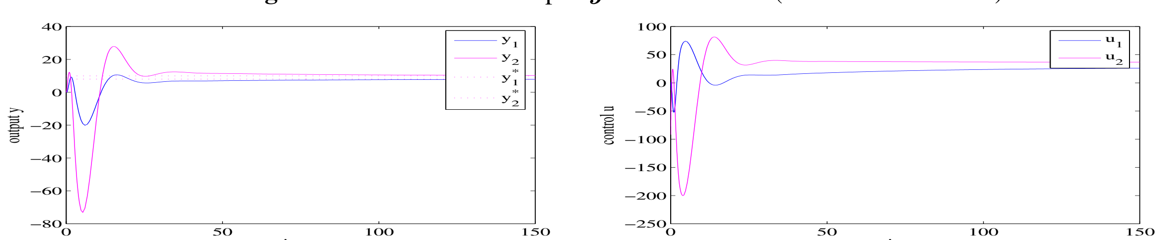
$(d)\ \Gamma = \mathrm{diag}\{10, 10\}$

Fig.1 Time histories of output $\boldsymbol{y}$ and control $\boldsymbol{u}$ by adjusting Γ (P·SPR·D Control)



$(d)\quad \Gamma = \mathrm{diag}\{10, 10\}$

Fig.2 Time histories of output $\boldsymbol{y}$ and control $\boldsymbol{u}$ (P·SPR·D·I Control)



$(d)\quad \Gamma = \mathrm{diag}\{10, 10\}$

Fig.3 Time histories of output $\boldsymbol{y}$ and control $\boldsymbol{u}$ (P·SPR·D+Feedforward Control)