2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

Robust adaptive fuzzy tracking control for a class of MIMO systems: a minimal-learning-parameters algorithm

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Abstract— **A robust adaptive fuzzy tracking control problem is discussed for a class of uncertain MIMO nonlinear systems with strongly coupled interconnections. T-S fuzzy systems are used to approximate the unknown system uncertainties. Combining "dynamic surface control(DSC)"approach with "minimal learning parameters(MLP)" algorithm, a systematic procedure for controller design is developed. The key features of the proposed scheme are that, firstly, the problem of "explosion of complexity" inherent in the conventional backstepping method is circumvented, secondly, the number of parameters updated on line for each subsystem is reduced dramatically to 2, one for T-S fuzzy system and the other for the bound of disturbances, and, thirdly, the possible controller singularity problem in some of the existing adaptive control schemes with feedback linearization techniques is removed. These features result in a much simpler algorithm, which is easy to be implemented in application. It is shown that all the closed-loop signals are semi-globally uniformly ultimately bounded(SGUUB) based on Lyapunov theory. Finally, simulation results via a numerical example validate the effectiveness and performance of the proposed scheme.**

I. INTRODUCTION

In the past decades, there has been a rapid growth of research efforts aiming at the development of systematic design methods for adaptive control of MIMO nonlinear systems with unstructured uncertainty. Many remarkable results have been obtained with the help of backstepping technique combined with fuzzy systemsor neuralnetworks(NN) as approximators, refer to [1]-[5] and the references therein. Nevertheless, there exists a well-known "dimension curse" drawback in the aforementioned works, which imposes that there are many parameters need to be tuned in the fuzzy or NN approximator-based adaptive control schemes, so that the learning times tend to become unacceptably large. Fortunately, this problem was first solved recently by introducing a "minimal learning parameters(MLP)" algorithm for a class of strict feedback SISO nonlinear systems by Yang *et al* in their pioneering works [6][7], and [8]. But these results are only for nonlinear uncertain SISO systems. More recently, the idea of MLP algorithms was extended to adaptive fuzzy control scheme via backstepping technique for MIMO systems in [9][10] and [11].

However, there is another substantial drawback of "explosion of complexity" within the conventional backstepping technique in all aforementioned works, which is caused by the repeated differentiations of certain nonlinear functions as the order of the system increases. Recently, dynamic surface control (DSC) technique has been proposed to avoid this problem by introducing a first order low-pass filter at each step of the conventional backstepping design procedure[12][13]. More recently, the DSC approach was extended to an NN-based adaptive tracking control for a class of strict-feedback SISO systems with arbitrary uncertain nonlinearities in [14]. Then, the adaptive NN DSC method in [14] was extended to a class of MIMO nonlinear systems with both unknown system functions and virtual control gain functions in [15], but the proposed controller suffers from the problem of "dimension curse".

In this paper, motivated by aforementioned works in literature, a novel robust adaptive fuzzy tracking control scheme is developed for a class of nonlinear MIMO systems. By incorporating the "MLP" algorithm in [8] into the DSC technique, a systematic procedure is developed for the synthesis of stable robust adaptive fuzzy tracking controller. In our algorithm, the T-S fuzzy systems are only used to approximate those unstructured system functions, whereas the unknown virtual control gain functions do not require to be approximated. Consequently, the possible controller singularity problem can be removed. The outstanding feature is that both problems of "dimension curse" and "explosion of complexity" are avoided simultaneously. Hence, our algorithm drastically reduces the burdensome computation and is easy to be implemented in applications.

II. PRELIMINARIES

A. PROBLEM FORMULATION

Consider a class of uncertain MIMO nonlinear systems in the following form

$$\begin{cases} \dot{x}_{j,i_j} = f_{j,i_j}\left(\bar{x}_{j,i_j}\right) + g_{j,i_j}\left(\bar{x}_{j,i_j}\right) x_{j,i_j+1} + \Delta_{j,i_j}(t,x) \ , \\ \vdots \\ \dot{x}_{j,\rho_j} = f_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right) + g_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right) u_j + \Delta_{j,\rho_j}(t,x), \\ y_j = x_{j,1}, i_j = 1,\ldots,\rho_j - 1, j = 1,\ldots,m \end{cases} \tag{1}$$

where x_{j,i_j} is states of jth sub-system, $x = [x_{1,\rho_1}^T, \cdots, x_{m,\rho_m}^T]^T \in R^N$ indicates the states vector of the

This work is supported in part by the Hong Kong Research Grant Council (Grant no: CityU 112806) and the National Natural Science Foundation of China (Grant Nos.50640460116, 60874056, 60674056 and 50779033), China Postdoctoral Science Foundation (Grant No. 20070420101), Shanghai Postdoctoral Scientific Program(Grant No.07R214128), the National High Technology Research&Development (863 Plan) Program of China(Grant No. 2007AA11Z250).

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whole system, $N=\rho_1+\cdots+\rho_m$, $\bar{x}_{j,i_j}=[x_{j,1},\cdots,x_{j,i_j}]^T\in R^{i_j}$. $u_j\in R, y_j\in R$ represents the input and output of jth subsystem, respectively. f_{j,i_j} and g_{j,i_j} represent unstructured nonlinear smooth functions, respectively. j, i_j, ρ_j, m are all positive integers. The uncertain disturbances $\Delta_{j,i_j}\le d_{j,i_j}$ with d_{j,i_j} being an unknown constant.

Assumption 1: The uncertain virtual control gain functions g_{j,i_j} are confined within a certain range such that

$$0<b_{min}\le |g_{j,i_j}|\le b_{max} \tag{2}$$

where b_{min} and b_{max} are the lower and upper bound parameters, respectively. It implies g_{j,i_j} is strictly either positive or negative. Without loss of generality, we assume $0<b_{min}\le g_{j,i_j}$.

Assumption 2: The reference signal $y_{jd}(t)$ is a sufficiently smooth function of t and y_{jd}, $\dot{y_{jd}}$, $\ddot{y_{jd}}$ are bounded, that is, there exists a positive constant B_{j0}, such that $\Pi_{j0}:=\{(y_{jd},\dot{y_{jd}},\ddot{y_{jd}}):{y_{jd}}^2+\dot{y_{jd}}^2+\ddot{y_{jd}}^2\le B_{j0}\}$.

The control objective is to find an adaptive fuzzy tracking controller for (1) such that all the solutions of the resulting closed-loop system are SGUUB, and the tracking error $z_{j,1}=y_j(t)-y_{jd}(t)$ can be rendered small.

B. Takagi-Sugeno type fuzzy systems

Generally, the Takagi-Sugeno(T-S) type fuzzy system[16] can be constructed by the following $K(K>1)$ fuzzy rules

R_i : If x_1 is $\Psi^i_{h_1}$AND x_2 is $\Psi^i_{h_2}$AND ... AND x_n is$\Psi^i_{h_n}$
THEN y_i is $a_{j,1}x_1+a_{j,2}x_2+\ldots+a_{in}x_n, i=1,2,...,K$

where a_{ij}, $i=1,2,\ldots,K$, $j=1,2,\ldots,n$ are unknown constants. The product fuzzy inference is employed to evaluate the ANDs in the fuzzy rules. After being defuzzified by a typical center average defuzzifier, the output of T-S fuzzy system is in the vector form

$$\hat{f}(x,A_x)=\xi(x)A_x x \tag{3}$$

where $\xi(x)=[\xi_1(x),\xi_2(x),...,\xi_K(x)]$ and $\xi_i(x)=\Pi_{j=1}^n\mu^i_{h_j}(x_j)\Big/\sum_{i=1}^K\left[\Pi_{j=1}^n\mu^i_{h_j}(x_j)\right]$, $i=1,2,\ldots,K$, called as fuzzy basis function.

$$A_x=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ a_{K1} & a_{K2} & \cdots & a_{Kn}\end{bmatrix}.$$

T-S fuzzy models are shown by Lemma 1 to be universal function approximators in the sense that they are able to approximate any smooth nonlinear functions to any degree of accuracy in any convex compact region[17].

Lemma 1: For any given real continuous function $f(x)$ on the compact set $U\in R^n$ and $\forall\varepsilon>0$, there exists a fuzzy system $\hat{f}(x,A_x)$ in the form of (3) such that

$$\sup_{x\in U}\left\|f(x)-\hat{f}(x,A_x)\right\|\le\varepsilon \tag{4}$$

where ε is called the approximating error.

C. A useful lemma

Lemma 2: For any scalar variables a and b, the following inequality holds

$$ab\le\frac{a^2}{4\gamma^2}+\gamma^2b^2 \tag{5}$$

where γ is a positive coefficient.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

A. Controller Design

Now we will incorporate the DSC technique and the MLP algorithm into a robust adaptive tracking design scheme for (1). Similar to the traditional backstepping method, the recursive design procedure contains ρ_j steps. At i_jth step, $i_j=1,\ldots,\rho_j-1$), the virtual controller α_{j,i_j+1}, $i_j=1,\ldots,\rho_j-1$ shall be developed. The control law u_j is constructed at step ρ_j. We give the procedure of the controller design as follows.

Step 1: Define 1st error variable $z_{j,1}=x_{j,1}-y_{jd}$, then

$$\dot{z}_{j,1}=g_{j,1}(\bar{x}_{j,1})x_{j,2}+f_{j,1}\left(x_{j,1}\right)+\Delta_{j,1}-\dot{y}_{jd} \tag{6}$$

According to Lemma 1, the T-S fuzzy system $\hat{f}_{j,1}\left(x_{j,1},A_{j,1}\right)$ with $A_{j,1}$ a matrix containing unknown constants and input vector $x_{j,1}\in U_{x_{j,1}}$, where $U_{x_{j,1}}$ is some compact set, is proposed here to approximate uncertain function $f_{j,1}\left(x_{j,1}\right)$. Then $f_{j,1}\left(x_{j,1}\right)$ can be expressed as

$$\begin{aligned} f_{j,1}\left(x_{j,1}\right)&=\xi_{j,1}\left(x_{j,1}\right)A_{j,1}x_{j,1}+\varepsilon_{j,1}\\ &=\xi_{j,1}\left(x_{j,1}\right)A_{j,1}z_{j,1}+\xi_{j,1}\left(x_{j,1}\right)A_{j,1}y_{jd}+\varepsilon_{j,1}\\ &=c_{\theta1}\xi_{j,1}\left(x_{j,1}\right)\omega_{j,1}+\xi_{j,1}\left(x_{j,1}\right)A_{j,1}y_{jd}+\varepsilon_{j,1}\end{aligned} \tag{7}$$

where $\varepsilon_{j,1}$ denotes approximating error. Let $c_{\theta1}=\left\|A_{j,1}\right\|$ being an unknown constant, such that $A_{j,1}=c_{\theta1}A_1^m$ and $\|A_1^m\|\le1$, so $\omega_{j,1}=A^m_{j,1}z_{j,1}$.

Substituting (7) into (6) yields

$$\dot{z}_{j,1}=g_{j,1}(\bar{x}_{j,1})x_{j,2}+c_{\theta1}\xi_{j,1}\left(x_{j,1}\right)\omega_{j,1}+v_{j,1}-\dot{y}_{jd} \tag{8}$$

where $v_{j,1}=\xi_{j,1}\left(x_{j,1}\right)A_{j,1}y_{jd}+\varepsilon_{j,1}+\Delta_{j,1}$. Note that $v_{j,1}$ can be expressed as follows

$$\left\|v_{j,1}\right\|\le\left\|\xi_{j,1}\left(x_{j,1}\right)A_{j,1}y_{jd}+\varepsilon_{j,1}+d_{j,1}\right\|\le b_{min}\theta_{j,1}\psi_{j,1} \tag{9}$$

where $\theta_{j,1}=b_{min}^{-1}\max\left(\left\|A_{j,1}y_{jd}\right\|,\left\|\varepsilon_{j,1}+d_{j,1}\right\|\right)$ and $\psi_{j,1}\left(x_{j,1}\right)=1+\left\|\xi_{j,1}\right\|$.

Now we choose a virtual controller $\alpha_{j,2}$ for $x_{j,2}$ in the subsystem (8) as

$$\begin{aligned}\alpha_{j,2}=&-k_{j,1}z_{j,1}+\dot{y}_{jd}-\frac{\hat{\lambda}_{j,1}}{4\gamma_{j,1}^2}\xi_{j,1}\left(x_{j,1}\right)\xi_{j,1}^T\left(x_{j,1}\right)z_{j,1}\\ &-\hat{\theta}_{j,1}\psi_{j,1}\left(x_{j,1}\right)\tanh\left(\frac{\hat{\theta}_{j,1}\psi_{j,1}\left(x_{j,1}\right)z_{j,1}}{\delta_{j,1}}\right)\end{aligned} \tag{10}$$

where $k_{j,1},\gamma_{j,1}$ and $\delta_{j,1}$ are positive design constants. $\hat{\lambda}_{j,1},\hat{\theta}_{j,1}$ are the estimates of $\lambda_{j,1}=b_{min}^{-1}c_{\theta1}^2$ and $\theta_{j,1}$, respectively. The update laws of $\hat{\lambda}_{j,1}$ and $\hat{\theta}_{j,1}$ will be designed later.

Introduce a new variable $s_{j,2}$ and let $\alpha_{j,2}$ pass through a first-order filter with time constant $\eta_{j,2}$ as follows:

$$\eta_{j,2}\dot{s}_{j,2}+s_{j,2}=\alpha_{j,2},\ s_{j,2}(0)=\alpha_{j,2}(0). \tag{11}$$

Step i_j $(2\le i_j\le \rho_j-1)$: A similar procedure is employed recursively for each step i_j . Define the i_jth error variable $z_{j,i_j}=x_{j,i_j}-s_{j,i_j}$, and we have

$$\dot{z}_{j,i_j}=g_{j,i_j}(\bar{x}_{j,i_j})x_{i+1}+f_{j,i_j}\left(\bar{x}_{j,i_j}\right)+\Delta_{j,i_j}-\dot{s}_{j,i_j} \tag{12}$$

We also use a T-S fuzzy system to approximate the unknown function $f_{j,i_j}\left(\bar{x}_{j,i_j}\right)$ and obtain

$$\begin{aligned} f_{j,i_j}\left(\bar{x}_{j,i_j}\right)&=\xi_{j,i_j}\left(\bar{x}_{j,i_j}\right)A_{j,i_j}\bar{x}_{j,i_j}^T+\varepsilon_{j,i_j}\\ &=\xi_{j,i_j}A_{j,i_j}\begin{bmatrix} z_{j,1}+y_{jd}\\ z_{j,2}+s_{j,2}\\ \vdots\\ z_{j,i_j}+s_{j,i_j}\end{bmatrix}^T+\varepsilon_{j,i_j}\\ &=c_{\theta i}\xi_{j,i_j}\omega_{j,i_j}+d'_{j,i_j} \end{aligned} \tag{13}$$

where $\omega_{j,i_j}=A^m_{j,i_j}\bar{z}_{j,i_j}$, $c_{\theta i}=\left\|A^1_{j,i_j}\right\|=\lambda^{1/2}_{\max}\left(A_i^{1T}A_i^1\right)$, such that $A^1_{j,i_j}=c_{\theta i}A^m_{j,i_j}$, and $\left\|A^m_{j,i_j}\right\|\le 1$. $d'_{j,i_j}=\xi_{j,i_j}A^1_{j,i_j}y_{jd}+\xi_{j,i_j}\sum\limits_{j=2}^{i}A^j_{j,i_j}s_j+\varepsilon_{j,i_j}$.

Then (12) can be converted as follows

$$\dot{z}_{j,i_j}=g_{j,i_j}(\bar{x}_{j,i_j},w)x_{i+1}+c_{\theta i}\xi_{j,i_j}\omega_{j,i_j}+v_{j,i_j}-\dot{s}_{j,i_j} \tag{14}$$

where $v_{j,i_j}=\Delta_{j,i_j}+d'_{j,i_j}$, and

$$\begin{aligned}\left\|v_{j,i_j}\right\|&\le\left\|\xi_{j,i_j}A^1_{j,i_j}y_{jd}+\xi_{j,i_j}\sum_{j=2}^{i}A^j_{j,i_j}s_j+\varepsilon_{j,i_j}+d_{j,i_j}\right\|\\ &\le b_{min}\theta_{j,i_j}\psi_{j,i_j}\end{aligned} \tag{15}$$

where $\psi_{j,i_j}=1+\left\|\xi_{j,i_j}\right\|$, $\theta_{j,i_j}=b^{-1}_{min}\max\left(\left\|A^1_{j,i_j}y_{jd}\right\|,\left\|\sum\limits_{j=2}^{i}A^j_{j,i_j}s_j\right\|,\left\|\varepsilon_{j,i_j}+d_{j,i_j}\right\|\right)$.

Similarly, choose a virtual controller α_{j,i_j+1} as follows

$$\begin{aligned}\alpha_{j,i_j+1}=&-k_{j,i_j}z_{j,i_j}+\dot{s}_{j,i_j}-\frac{\hat{\lambda}_{j,i_j}}{4\gamma^2_{j,i_j}}\xi_{j,i_j}\left(\bar{x}_{j,i_j}\right)\xi^T_{j,i_j}\left(\bar{x}_{j,i_j}\right)z_{j,i_j}\\ &-\hat{\theta}_{j,i_j}\psi_{j,i_j}\left(\bar{x}_{j,i_j}\right)\tanh\left(\frac{\hat{\theta}_{j,i_j}\psi_{j,i_j}\left(\bar{x}_{j,i_j}\right)z_{j,i_j}}{\delta_{j,i_j}}\right)\end{aligned} \tag{16}$$

where k_{j,i_j},γ_{j,i_j} and δ_{j,i_j} are positive design constants. $\hat{\lambda}_{j,i_j},\hat{\theta}_{j,i_j}$ are the estimates of $\lambda_{j,i_j}=b^{-1}_{min}c^2_{\theta i}$ and θ_{j,i_j}, respectively. The update laws of $\hat{\lambda}_{j,i_j}$ and $\hat{\theta}_{j,i_j}$ will be designed later.

Similarly, introduce a variable s_{j,i_j+1} and let α_{j,i_j+1} pass through the filter with time constant η_{j,i_j+1} as follows

$$\eta_{j,i_j+1}\dot{s}_{j,i_j+1}+s_{j,i_j+1}=\alpha_{j,i_j+1},\ s_{j,i_j+1}(0)=\alpha_{j,i_j+1}(0). \tag{17}$$

Step ρ_j: Define error variable $z_{j,\rho_j}=x_{j,\rho_j}-s_{j,\rho_j}$, then

$$\dot{z}_{j,\rho_j}=g_{j,\rho_j}(x)u_j+f_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)+\Delta_{j,\rho_j}-\dot{s}_{j,\rho_j} \tag{18}$$

Similarly, the unknown function $f_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)$ can be expressed as

$$\begin{aligned}f_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)&=\xi_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)A_{j,\rho_j}\bar{x}^T_{j,\rho_j}+\varepsilon_{j,\rho_j}\\ &=c_{\theta n}\xi_{j,\rho_j}\omega_{j,\rho_j}+d'_{j,\rho_j}\end{aligned} \tag{19}$$

where $\omega_{j,\rho_j}=A^m_{j,\rho_j}\bar{z}_{j,\rho_j}$, $c_{\theta n}=\left\|A^1_{j,\rho_j}\right\|=\lambda^{1/2}_{\max}\left(A_n^{1T}A_n^1\right)$, such that $A^1_{j,\rho_j}=c_{\theta n}A^m_{j,\rho_j}$, and $\left\|A^m_{j,\rho_j}\right\|\le 1$. $d'_{j,\rho_j}=\xi_{j,\rho_j}A^1_{j,\rho_j}y_{jd}+\xi_{j,\rho_j}\sum\limits_{j=2}^{n}A^j_{j,\rho_j}s_j+\varepsilon_{j,\rho_j}$.

$$\dot{z}_{j,\rho_j}=g_{j,\rho_j}(x,w)u_j+c_{\theta n}\xi_{j,\rho_j}\omega_{j,\rho_j}+v_{j,\rho_j}-\dot{s}_{j,\rho_j} \tag{20}$$

where $v_{j,\rho_j}=\Delta_{j,\rho_j}+d'_{j,\rho_j}$, and

$$\begin{aligned}\left\|v_{j,\rho_j}\right\|&\le\left\|\xi_{j,\rho_j}A^1_{j,\rho_j}y_{jd}+\xi_{j,\rho_j}\sum_{j=2}^{n}A^j_{j,\rho_j}s_j+\varepsilon_{j,\rho_j}+d_{j,\rho_j}\right\|\\ &\le b_{min}\theta_{j,\rho_j}\psi_{j,\rho_j}\end{aligned} \tag{21}$$

where $\psi_{j,\rho_j}=1+\left\|\xi_{j,\rho_j}\right\|$, and $\theta_{j,\rho_j}=b^{-1}_{min}\max\left(\left\|A^1_{j,\rho_j}y_{jd}\right\|,\left\|\sum\limits_{j=2}^{n}A^j_{j,\rho_j}s_j\right\|,\left\|\varepsilon_{j,\rho_j}+d_{j,\rho_j}\right\|\right)$.

Now, we choose the control input u_j as follows

$$\begin{aligned}u_j=&-k_{j,\rho_j}z_{j,\rho_j}+\dot{s}_{j,\rho_j}-\frac{\hat{\lambda}_{j,\rho_j}}{4\gamma^2_{j,\rho_j}}\xi_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)\xi^T_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)z_{j,\rho_j}\\ &-\hat{\theta}_{j,\rho_j}\psi_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)\tanh\left(\frac{\hat{\theta}_{j,\rho_j}\psi_{j,\rho_j}\left(\bar{x}_{j,\rho_j}\right)z_{j,\rho_j}}{\delta_{j,\rho_j}}\right)\end{aligned} \tag{22}$$

where $k_{j,\rho_j},\gamma_{j,\rho_j}$ and δ_{j,ρ_j} are positive design constants. $\hat{\lambda}_{j,\rho_j},\hat{\theta}_{j,\rho_j}$ are the estimates of $\lambda_{j,\rho_j}=b^{-1}_{min}c^2_{\theta n}$ and θ_{j,ρ_j}, respectively. The update laws of $\hat{\lambda}_{j,\rho_j}$ and $\hat{\theta}_{j,\rho_j}$ will be designed later.

Remark 1: The algorithms proposed in this paper can solve both the problem of "dimension curse" and the problem of "explosion of complexity" simultaneously, which result in a minimal learning parameterizations algorithm with a much simpler structure. Consequently, the burdensome computation of the algorithm can be reduced dramatically and it is easy to be implemented in applications.

B. Stability Analysis

Define new error variables

$$y_{j,i_j+1}=s_{j,i_j+1}-\alpha_{j,i_j+1},i_j=1,2,\ldots,\rho_j-1 \tag{23}$$

Note that $\dot{s}_{j,i_j} = -\left(s_{j,i_j} + \alpha_{j,i_j}\right)/\eta_{j,i_j} = -y_{j,i_j}/\eta_{j,i_j}$, then

$$\begin{aligned}\dot{y}_{j,2} &= \dot{s}_{j,2} - \dot{\alpha}_{j,2} \\ &= -\frac{y_{j,2}}{\eta_{j,2}} \\ &\quad + \left(\ddot{y}_{jd} - \frac{\partial \alpha_{j,2}}{\partial z_{j,1}}\dot{z}_{j,1} - \frac{\partial \alpha_{j,2}}{\partial x_{j,1}}\dot{x}_{j,1} - \frac{\partial \alpha_{j,2}}{\partial \hat{\theta}_{j,1}}\dot{\hat{\theta}}_{j,1} - \frac{\partial \alpha_{j,2}}{\partial \hat{\lambda}_{j,1}}\dot{\hat{\lambda}}_{j,1}\right) \\ &= -\frac{y_{j,2}}{\eta_{j,2}} + B_{j,2}\left(z_{j,1}, z_{j,2}, y_{j,2}, \hat{\theta}_{j,1}, \hat{\lambda}_{j,1}, y_{jd}, \dot{y}_{jd}, \ddot{y}_{jd}\right)\end{aligned} \tag{24}$$

Obviously, $B_{j,2}(\cdot)$ is a continuous function with respect to variables $\left(z_{j,1}, z_{j,2}, y_{j,2}, \hat{\theta}_{j,1}, \hat{\lambda}_{j,1}, y_{jd}, \dot{y}_{jd}, \ddot{y}_{jd}\right)$.

Similarly, we have

$$\begin{aligned}\dot{y}_{j,i_j+1} = \dot{s}_{j,i_j+1} - \dot{\alpha}_{(j,i_j+1)} = -\frac{y_{j,i_j+1}}{\eta_{j,i_j+1}} + B_{j,i_j+1} \\ \left(\bar{z}_{j,i_j+1}, y_{j,2}, \ldots, y_{j,i_j}, \bar{\hat{\theta}}_{j,i_j}, \bar{\hat{\lambda}}_{j,i_j}, y_{jd}, \dot{y}_{jd}, \ddot{y}_{jd}\right)\end{aligned} \tag{25}$$

where $i_j = 2, \ldots, \rho_j - 1$.

Consider $x_{j,i_j+1} = z_{j,i_j+1} + s_{j,i_j+1}$ and $s_{j,i_j+1} = y_{j,i_j+1} + \alpha_{j,i_j+1}$, the overall error systems can be expressed as

$$\begin{aligned}\dot{z}_{j,1} &= g_{j,1}z_{j,2} + g_{j,1}y_{j,2} + g_{j,1}\alpha_{j,2} \\ &\quad + c_{\theta 1}\xi_{j,1}\left(x_{j,1}\right)\omega_{j,1} + v_{j,1} - \dot{y}_{jd} \\ \dot{z}_{j,i_j} &= g_{j,i_j}z_{j,i_j+1} + g_{j,i_j}y_{j,i_j+1} + g_{j,i_j}\alpha_{j,i_j+1} \\ &\quad + c_{\theta i}\xi_{j,i_j}\left(\bar{x}_{j,i_j}\right)\omega_{j,i_j} + v_{j,i_j} - \dot{s}_{j,i_j}, i_j = 2, \ldots, \rho_j - 1, \\ &\vdots \\ \dot{z}_{j,\rho_j} &= g_{j,\rho_j}u_j + c_{\theta n}\xi_{j,\rho_j}(x)\omega_{j,\rho_j} + v_{j,\rho_j} - \dot{s}_{j,\rho_j}\end{aligned} \tag{26}$$

Now we propose our main result as follows.

Theorem 1: Consider the closed-loop system composed of (24)$\sim$(26), the virtual controllers (10),(16), the controller (22), and the updated laws for λ_{j,i_j} and $\hat{\theta}_{j,i_j}$ in the following equation (27), given any positive number p_j, for all initial conditions satisfying $\Pi_j := \left\{\sum_{j=1}^{\rho_j}\left(z_j^2 + \tilde{\theta}_j^T b_{min}\Gamma_{j,1}^{-1}\tilde{\theta}_j + \tilde{\lambda}_j^T b_{min}\Gamma_{j,2}^{-1}\tilde{\lambda}_j\right) + \sum_{j=2}^{\rho_j} y_j^2 < 2p_j\right\}$, there exist k_{j,i_j}, γ_{j,i_j}, δ_{j,i_j}, η_{j,i_j}, σ_{j,i_j} and Γ_{j,i_j}, such that the solution of the closed-loop is uniformly ultimately bounded. Furthermore, given any $\mu_{j,1} > 0$, we can tune our controller parameters such that the output error $z_{j,1} = y_{j,1}(t) - y_{jd}(t)$ satisfies $\lim_{t\to\infty}\left|z_{j,1}(t)\right| \le \mu_{j,1}$.

$$\left\{\begin{aligned}\dot{\hat{\lambda}}_{j,i_j} &= \Gamma_{j,1}\left[\Phi_{j,i_j}\left(\bar{x}_{j,i_j}\right)z_{j,i_j}^2 - \sigma_{j,1}\left(\hat{\lambda}_{j,i_j} - \lambda_{j,i_j}^0\right)\right] \\ \dot{\hat{\theta}}_{j,i_j} &= \Gamma_{j,2}\left[\psi_{j,i_j}\left(\bar{x}_{j,i_j}\right)\left\|z_{j,i_j}\right\| - \sigma_{j,2}\left(\hat{\theta}_{j,i_j} - \theta_{j,i_j}^0\right)\right]\end{aligned}\right. \tag{27}$$

where $\Phi_{j,i_j} = \frac{1}{4\gamma_{j,i_j}^2}\xi_{j,i_j}\left(\bar{x}_{j,i_j}\right)\xi_{j,i_j}^T\left(\bar{x}_{j,i_j}\right)$, λ_{j,i_j}^0, θ_{j,i_j}^0, $\Gamma_{j,1}$, $\Gamma_{j,2}$, $\sigma_{j,1}$ and $\sigma_{j,2}$ are design parameters.

Proof: Choosing the Lyapunov candidate function as

$$\begin{aligned}V_j =& \frac{1}{2}\sum_{i_j=1}^{\rho_j}\left(z_{j,i_j}^2 + \tilde{\theta}_{j,i_j}^T b_{min}\Gamma_{j,2}^{-1}\tilde{\theta}_{j,i_j} + \tilde{\lambda}_{j,i_j}^T b_{min}\Gamma_{j,1}^{-1}\tilde{\lambda}_{j,i_j}\right) \\ &+ \frac{1}{2}\sum_{i_j=1}^{\rho_j-1} y_{j,i_j+1}^2\end{aligned} \tag{28}$$

where $\tilde{\theta}_{j,i_j} = \theta_{j,i_j} - \hat{\theta}_{j,i_j}$, $\tilde{\lambda}_{j,i_j} = \lambda_{j,i_j} - \hat{\lambda}_{j,i_j}$. The time derivative of V_j along the system trajectories is

$$\begin{aligned}\dot{V}_j =& \sum_{i_j=1}^{\rho_j}\left(z_{j,i_j}\dot{z}_{j,i_j} - \tilde{\theta}_{j,i_j}^T b_{\min}\Gamma_{j,2}^{-1}\dot{\hat{\theta}}_{j,i_j} - \tilde{\lambda}_{j,i_j}^T b_{\min}\Gamma_{j,1}^{-1}\dot{\hat{\lambda}}_{j,i_j}\right) \\ &+ \sum_{i=1}^{n-1} y_{j,i_j+1}\dot{y}_{j,i_j+1}\end{aligned} \tag{29}$$

By using Lemma 2, and mentioning that

$$\begin{aligned}c_{\theta i}\xi_{j,i_j}\omega_{j,i_j}z_{j,i_j} \le& b_{min}\frac{\hat{\lambda}_{j,i_j}}{4\gamma_{j,i_j}^2}\xi_{j,i_j}\xi_{j,i_j}^T z_{j,i_j}^2 \\ &+ b_{min}\frac{\tilde{\lambda}_{j,i_j}}{4\gamma_{j,i_j}^2}\xi_{j,i_j}\xi_{j,i_j}^T z_{j,i_j}^2 + \gamma_{j,i_j}^2\omega_{j,i_j}^T\omega_{j,i_j}\end{aligned} \tag{30}$$

$v_{j,i_j}z_{j,i_j} \le g_{j,i_j}\hat{\theta}_{j,i_j}\psi_{j,i_j}\left(\bar{x}_{j,i_j}\right)\left\|z_{j,i_j}\right\| + b_{min}\tilde{\theta}_{j,i_j}\psi_{j,i_j}\left(\bar{x}_{j,i_j}\right)\left\|z_{j,i_j}\right\|$, $g_{j,i_j}\hat{\theta}_{j,i_j}\psi_{j,i_j}\left\|z_{j,i_j}\right\| - g_{j,i_j}\hat{\theta}_{j,i_j}\psi_{j,i_j}z_{j,i_j}\tanh\left(\frac{\hat{\theta}_{j,i_j}\psi_{j,i_j}z_{j,i_j}}{\delta_{j,i_j}}\right) \le g_{j,i_j}\delta_{j,i_j} \le b_{max}\delta_{j,i_j}$, and $\tilde{\theta}_{j,i_j}^T\left(\hat{\theta}_{j,i_j} - \theta_{j,i_j}^0\right) \ge \frac{1}{2}\left|\tilde{\theta}_{j,i_j}\right|^2 - \frac{1}{2}\left|\theta_{j,i_j}^* - \theta_{j,i_j}^0\right|^2$, one has

$$\begin{aligned}\dot{V}_j \le& \sum_{i_j=2}^{\rho_j-1}\left(-\left(b_{min}k_{j,i_j} - 2 - \frac{1+b_{max}}{\eta_{j,i_j}}\right)z_{j,i_j}^2 + \frac{b_{max}}{4}z_{j,i_j+1}^2\right) \\ &- \left(b_{\min}k_{j,1} - 2 - \frac{b_{max}+1}{4}\right)z_{j,1}^2 - \left(b_{min}k_{j,\rho_j} - \frac{1+b_{max}}{\eta_{j,i_j}}\right)z_{j,\rho_j}^2 \\ &- \sum_{i_j=1}^{\rho_j}\left(\frac{\sigma_{j,2}}{2\lambda_{\max}\left(b_{min}\Gamma_{j,2}^{-1}\right)}\tilde{\theta}_{j,i_j}^T\Gamma_{j,2}^{-1}\tilde{\theta}_{j,i_j}\right) \\ &- \sum_{i_j=1}^{\rho_j}\left(\frac{\sigma_{j,1}}{2\lambda_{\max}\left(b_{min}\Gamma_{j,1}^{-1}\right)}\tilde{\lambda}_{j,i_j}^T\Gamma_{j,1}^{-1}\tilde{\lambda}_{j,i_j}\right) \\ &+ \sum_{i_j=1}^{\rho_j-1}\left(\frac{b_{max}}{4}y_{j,i_j+1}^2 - \frac{3-b_{max}}{4\eta_{j,i_j+1}}y_{j,i_j+1}^2 + \left|y_{j,i_j+1}B_{j,i_j+1}\right|\right) \\ &+ \sum_{i_j=1}^{\rho_j}\left(\gamma_{j,i_j}^2\omega_{j,i_j}^T\omega_{j,i_j} + \delta'_{j,i_j}\right)\end{aligned} \tag{31}$$

where $\delta'_{j,i_j} = (b_{max}+1)B_0^2 + b_{max}\delta_{j,i_j} + \frac{\sigma_{j,1}}{2}\left|\lambda_{j,i_j}^* - \lambda_{j,i_j}^0\right|^2 + \frac{\sigma_{j,2}}{2}\left|\theta_{j,i_j}^* - \theta_{j,i_j}^0\right|^2$.

Since B_{j,i_j+1}, $i_j = 1, 2, \ldots, \rho_j - 1$ has a maximum M_{j,i_j+1}(see [14] for details), let $\frac{1}{\eta_{j,i_j+1}} = \left(\frac{3-b_{max}}{4}\right)^{-1}\left(\frac{b_{max}}{4} + \frac{M_{j,i_j+1}^2}{2\alpha} + \alpha_0\right)$, and note that $\left|B_{j,i_j+1}y_{j,i_j+1}\right| \le \frac{y_{j,i_j+1}^2 B_{j,i_j+1}^2}{2\alpha} + \frac{\alpha}{2}$, where α_0 and α are positive constants. Then it yields

$$\frac{b_{max}}{4} y_{j,i_j+1}^2 - \frac{(3-b_{max})}{4\eta_{j,i_j+1}} y_{j,i_j+1}^2 + \left|B_{i+1} y_{j,i_j+1}\right|$$
$$\leq -\alpha_0 y_{j,i_j+1}^2 + \frac{\alpha}{2} \quad (32)$$

Let $\sigma_{j,1} \Big/ 2\lambda_{\max}\left(b_{min}\Gamma_{j,1}^{-1}\right) = \sigma_{j,2} \Big/ 2\lambda_{\max}\left(b_{min}\Gamma_{j,2}^{-1}\right) = \alpha_0$, and $k_{j,1} = b_{min}^{-1}(3 + \frac{b_{max}+1}{4} + \alpha_0)$, $k_{j,i_j} = b_{min}^{-1}(3 + \frac{1+b_{max}}{4\eta_{j,i_j}} + \frac{b_{max}}{4} + \alpha_0)(i_j = 2, \ldots, \rho_j - 1)$, $k_{j,\rho_j} = b_{min}^{-1}(\frac{b_{max}+1}{4\eta_{j,i_j}} + \alpha_0 + 1)$, then (31) can be expressed as

$$\begin{aligned} \dot{V}_j \leq & -\alpha_0 \sum_{i_j=1}^{\rho_j} \left(\tilde{\theta}_{j,i_j}^T b_{min}\Gamma_{j,2}^{-1}\tilde{\theta}_{j,i_j} + \tilde{\lambda}_{j,i_j}^T b_{min}\Gamma_{j,1}^{-1}\tilde{\lambda}_{j,i_j}\right) \\ & -(\alpha_0+1)\sum_{i_j=1}^{\rho_j} z_{j,i_j}^2 - \alpha_0 \sum_{i_j=1}^{\rho_j-1} y_{j,i_j+1}^2 \\ & + \sum_{i_j=1}^{\rho_j}\left(\gamma_{j,i_j}^2 \omega_{j,i_j}^T \omega_{j,i_j}\right) + \rho \\ \leq & -2\alpha_0 V_j - \left\|z_j\right\|^2 + \gamma^2\|\omega\|^2 + \rho \end{aligned} \quad (33)$$

where $\rho = \sum_{i_j=1}^{\rho_j}\left(\delta'_{j,i_j}\right) + \sum_{i_j=1}^{\rho_j-1}(\alpha/2)$, $\gamma = (\gamma_{j,1}^2 + \gamma_{j,2}^2 + \cdots + \gamma_{j,\rho_j}^2)^{1/2}$, $\omega = \left[\omega_{j,1}, \omega_{j,2}, \ldots, \omega_{j,\rho_j}\right]^T$.

Note that $\omega_{j.i_j} = A_{j.i_j}^m \bar{z}_{j.i_j}^T$ and $\|A_{j.i_j}^m\| \leq 1$, it gives

$$\omega = \begin{bmatrix} \omega_{j,1} \\ \omega_{j,2} \\ \vdots \\ \omega_{j,\rho_j} \end{bmatrix} = \begin{bmatrix} A_{j,1}^m & 0 & \cdots & 0 \\ A_{j,2}^{m1} & A_{j,2}^{m2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ A_{j,\rho_j}^{m1} & A_{j,\rho_j}^{m2} & \cdots & A_{j,\rho_j}^{mn} \end{bmatrix} \begin{bmatrix} z_{j,1} \\ z_{j,2} \\ \vdots \\ z_{j,\rho_j} \end{bmatrix}$$
$$= Az_j$$

and

$$\| \omega \| \leq \| A \| \| z_j \| \leq \| z_j \| \quad (34)$$

Now, if choosing $\gamma < 1$, then (33) becomes

$$\dot{V}_j \leq -2\alpha_0 V_j + \rho = -c_{j,1} V_j + \rho \quad (35)$$

where $c_{j,1} = (2\alpha_0)$.

From (35), we obtain

$$V_j(t) \leq \frac{\rho}{c_{j,1}} + \left(V_j(t_0) - \frac{\rho}{c_{j,1}}\right) e^{-(t-t_0)}$$

It follows that the solutions of the closed-loop control system are uniformly ultimately bounded, which implies that, for any $\mu_{j,1} > (\rho/c_{j,1})^{1/2}$, there exists a constant $T > 0$ such that $\left\|z_{j,1}(t)\right\| \leq \mu_{j,1}$ for all $t \geq t_0 + T$. The last statement holds readily since $(\rho/c_{j,1})^{1/2}$ can be made arbitrarily small if the design parameters $\gamma_{j,1}$, $\delta_{j,1}$, $\eta_{j,2}$, $\sigma_{j,1}$ and $\Gamma_{j,1}$ are chosen appropriately.

■

IV. APPLICATION EXAMPLES

Consider the following MIMO nonlinear system with strong coupled interconnections:

$$\begin{cases} \dot{x}_{1,1} = g_{1,1}x_{1,2} + f_{1,1} + \Delta_{1,1} \\ \dot{x}_{1,2} = g_{1,2}u_1 + f_{1,2} + \Delta_{1,2} \\ \dot{x}_{2,1} = g_{2,1}x_{2,2} + f_{2,1} + \Delta_{2,1} \\ \dot{x}_{2,2} = g_{2,2}u_2 + f_{2,2} + \Delta_{2,2} \\ y_1 = x_{1,1}, y_2 = x_{2,1}, \end{cases} \quad (36)$$

where $f_{1,1} = x_{1,1}e^{-0.5x_{1,1}}$, $g_{1,1} = 1 + x_{1,1}^2$, $\Delta_{1,1} = 0.5x_{1,1}^2 x_{2,1}x_{2,2}\sin(t)$, $f_{1,2} = x_{1,1}x_{1,2}^2$, $g_{1,2} = 3 + \cos(x_{1,1}x_{1,2})$, $\Delta_{1,2} = 0.2\cos(x_{1,1}^2 + x_{1,2}^2)x_{2,1}^2 x_{2,2}$, $f_{2,1} = 0.5x_{2,1}x_{1,2}$, $g_{2,1} = 2 + \sin^3(x_{2,1}x_{1,2}) + x_{1,1}$, $\Delta_{2,1} = 0.6\sin(x_{2,1}x_{1,1}x_{1,2})$, $f_{2,2} = (x_{2,1}x_{2,2} + x_{1,1}x_{1,2})$, $g_{2,2} = 2 + \cos(x_{2,1}x_{1,1})$, and $\Delta_{2,2} = 0.5(x_{2,1}^2 + x_{2,2}^2)\sin(x_{1,1}x_{1,2})\sin^2(t)$.

The reference signals $y_{1d} = 0.5(\sin(t) + \sin(0.5t))$, $y_{2d} = \sin(t)$. The initial conditions for $x_{1,1}$, $x_{1,2}$, $x_{2,1}$ and $x_{2,2}$ are [0, -0.2, 0, -0.2].

Now, the virtual controller (10), parameters adaptation laws (36) and the control law (22) are applied to system (36), where $\rho_j = 2$, $j = 1,2$ and $i_j = 1,2$.

In simulation, define five fuzzy sets for each variable with labels $A_{hi}^1(NL)$, $A_{hi}^2(NM)$, $A_{hi}^3(ZE)$, $A_{hi}^4(PM)$, $A_{hi}^5(PL)$ which are characterized by the following membership functions

$$\begin{aligned} & \mu_{A_{hi}^1} = \exp\left[-(x+1)^2\right], \mu_{A_{hi}^2} = \exp\left[-(x+0.5)^2\right] \\ & \mu_{A_{hi}^3} = \exp\left[-x^2\right], \quad \mu_{A_{hi}^4} = \exp\left[-(x-0.5)^2\right] \\ & \mu_{A_{hi}^5} = \exp\left[-(x-1)^2\right] \end{aligned} \quad (37)$$

The controller parameters are chosen as $k_{j,i_j} = [20,20,20,20]$, $\delta_{j,i_j} = 100$. $\Gamma_{1,i_j,k} = \Gamma_{2,i_j,k} = [50,2,20,4]$, $\sigma_{1,i_j,k} = \sigma_{2,i_j,k} = [0.02,0.5,0.005,0.1]$, $k = 1,2$. The time constants $\eta_{j,i_j} = 0.1$. The effectiveness and good performance of the proposed algorithms are illuminated in Figs.1∼3.

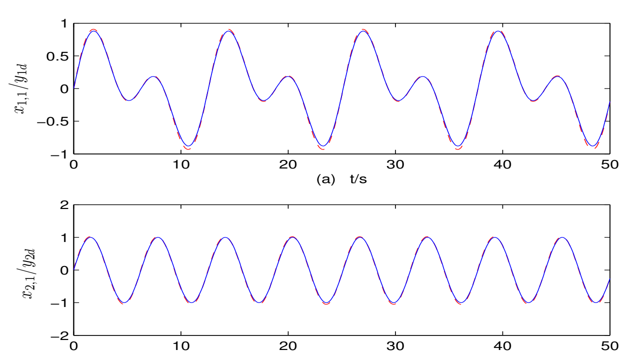

Fig. 1. Simulation results for system output and reference signal: (a)y_1 (solid line) and y_{1d} (dashed line), (b)y_2(dashed line) and y_{2d}(solid line).

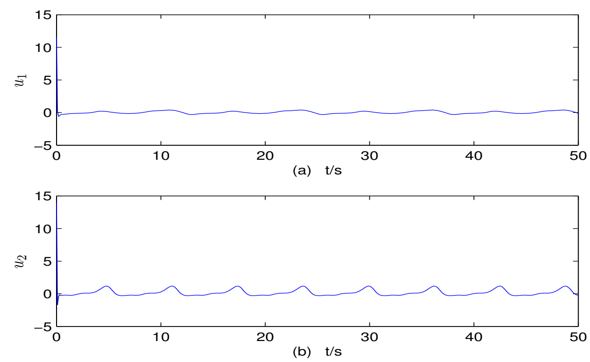

Fig. 2. Simulation results for proposed controller: (a) u_1, (b) u_2.

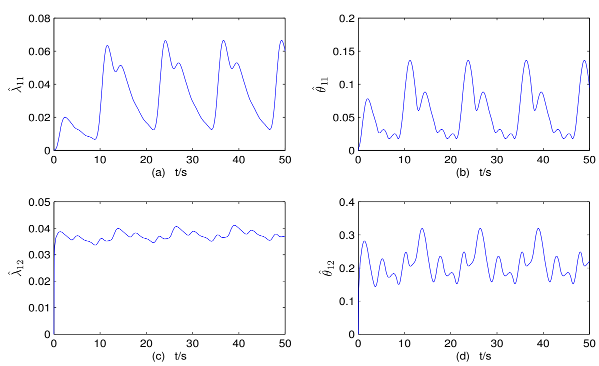

Fig. 3. Parameters adaptation: (a)$\hat{\lambda}_{1,1}$, (b)$\hat{\theta}_{1,1}$, (c)$\hat{\lambda}_{1,2}$, (d)$\hat{\theta}_{1,2}$.

V. CONCLUSION

In this paper, the tracking control problem has been considered for a class of uncertain MIMO nonlinear systems with strong coupled interconnections. Combining the DSC technique with the MLP algorithms, a robust adaptive fuzzy tracking control algorithm is developed, which guarantees the closed-loop system is SGUUB. The main features of the proposed algorithms are that the adaptive mechanism with minimal learning parameters is achieved, and the problem of "explosion of complexity" existing in the conventional backstepping method, as well as the possible controller singularity problem in some of the existing adaptive control schemes with feedback linearization techniques, are circumvented, so that the algorithm is in a much simpler form and its computational load is reduced dramatically. Thus it is much easier to implement this algorithm for applications. Simulation results of a numerical example have been presented to illustrate the effectiveness of the proposed algorithm.

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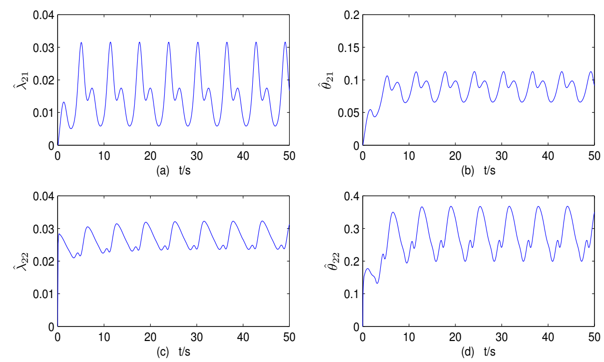

Fig. 4. Parameters adaptation: (a)$\hat{\lambda}_{2,1}$, (b)$\hat{\theta}_{2,1}$, (c)$\hat{\lambda}_{2,2}$, (d)$\hat{\theta}_{2,2}$.

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