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Signal Threshold Estimation in a Sensor Field for Undersea Target Tracking★

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Abstract— **This paper addresses the problem of online surveillance of undersea targets moving over a deployed sensor field. A real-time algorithm has been formulated to estimate the detection threshold based on the ensemble of sensor time series data collected from the track of a moving target. The probabilistic-state-machine-based algorithm is optimal in the sense of weighted linear least squares. The algorithm has been tested with sensor data from several tracks on a simulation test bed.**

1. Introduction

Distributed fields of passive sensors are often deployed to cover large areas for information gathering at moderate cost [1][2][3]. Such systems are becoming prevalent for surveillance, especially for detection of undersea targets, which allow many inexpensive sensors to be deployed in situations that would otherwise require a very large and expensive platform. Algorithms for sensor placement are scalable to the application needs and are largely fault-tolerant [4]; such applications include, but are not limited to, military surveillance, environmental and atmospheric monitoring, biological species tracking, and condition-based diagnostics and prognostics. This paper focuses on target tracking and surveillance in undersea military applications.

The task of tracking covert underwater targets includes deployment of a sensor field in the surveillance region of interest. In this paper, it is assumed that statistical distributions of expected target trajectories and the environment are known *a priori*. Given the information on placement of sensors, the objective here is to estimate track-dependent detection thresholds in real time by making trade-offs between maximization of the detection probability and minimization of the false-alarm probability.

Continuous-domain techniques are available to effectively solve the above multi-objective optimization problem offline from the corresponding Pareto set [5]. The operating point on the Pareto set is chosen from physical requirements or to reflect the user's intents, for example, in a Command & Control (C^2) environment. Generation of the Pareto set is usually computationally expensive and any perturbation of the statistical distribution of the target trajectory may require a complete re-evaluation of the optimization parameters in the above scheme [6]. Therefore, a need exists for a computationally efficient algorithm that will yield estimated results with comparable accuracy and precision. Such algorithms must be validated and calibrated by other established optimization tools [7].

In the present context of establishing the feasibility of probabilistic-state-machine-based tools for estimation of detection thresholds, the data sets have been generated from a simulation test bed at the laboratories of Naval Underseas Warfare Center (NUWC) as outputs from a given sensor field that is optimized to track a moving target. The *detection threshold* in the data set is optimal for only a given track. As the track is perturbed from the nominal condition, the performance degrades, i.e., the value of the cost functional deviates from the optimal point.

This paper investigates the feasibility of deriving a probabilistic-state-machine-based algorithm to estimate detection thresholds for tracks by using the simulation data. It is organized in four sections including the present section. Section 2 formulates a probabilistic-state-machine-based algorithm for real-time estimation of detection thresholds. Sec-

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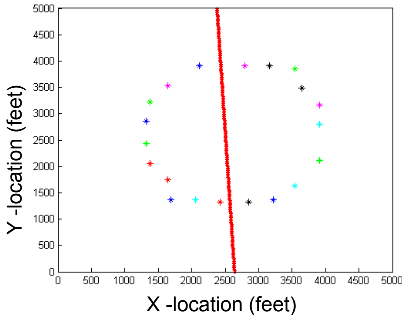

Fig. 1. Sensor placement and a target path

tion 3 presents the simulation results to establish feasibility of the proposed method for estimation of detection threshold as needed for tracking of undersea targets. The paper is concluded in Section 4 along with recommendations for future research.

2. Detection Threshold Estimation

This section formulates the problem of developing a probabilistic-state-machine-based procedure for estimation of detection thresholds for off-nominal tracks as an alternative to the conventional optimization methods.

A. The Track-before-detect Strategy

It is assumed that passive sonar sensors are deployed over a marine region, where the location of each sensor is known. As target travels across the region, each sensor picks up a noise-contaminated signal. A sensor that is closer to the target receives a signal of larger magnitude as compared to a sensor that is located farther away from the target. In this scenario, the transmission channel is assumed to be have an inverse square law relationship (with respect to distance) and the signal is contaminated with multiplicative Gaussian noise.

Fig. 1 illustrates a conceptual operation in the sensor field and depicts the track followed by the target.

A common practice is to detect targets by setting a detection threshold D. In this setting, a sensor is said to have detected a target if it receives a signal of magnitude greater than D. Naturally, a lower value of D would result in a higher rate of false alarms due to the ambient/background noise, while a higher value of D would increase the probability of missed detection, i.e., diminish the probability of successful search. It is well known that a large number of distributed low-capability sensors would increase occurrence of false alarms. The track-before-detect strategy tends to reduce the probability of false alarms by only considering those false alarms that occur in sequences that are spatially and kinematically consistent with the motion of an expected target.

Wettergren [4] has presented an application of the track-before-detect strategy in undersea sensor networks, where the performance has been analyzed in terms of probability of successful search and probability of false alarms. In order to search the area in a cost-effective manner, each sensor in the sensor field is equipped with an autonomous detection capability, where sensor attempts to independently detect the target. The track-before-detect strategy involves fusion of multi-sensor information such that the information from multiple sensors is combined to make up for the reduced coverage provided by the individual sensors. This process requires agreement of multiple sensor decisions for reliable target detection. the rationale is that a reliable decision on target decision cannot be made until a consistent estimate of the target track is obtained from different sensors that represent different directions. In addition to reduction of the probability of false alarms, having consistent decisions from multiple sensors provides a natural mechanism of assimilating tracking information concurrently with the detection reports.

B. Formulation of the Optimization Problem

A sensor detects a target if the measured signal crosses a threshold D. In accordance with the track-before-detect paradigm, a target is said to be detected if a selected number of sensors detect the target. In addition, all of these detections must be carried out within the time interval of the target being in the search region.

The target is assumed to emit sonar energy at a level of 100dB and the ambient/background noise is assumed to be at a level of 70dB. The ocean water acts as an attenuator that is assumed to follow the inverse square law with respect to the distance from the source and the (multiplicative) noise gain is assumed to have 2db variance.

Identification of an optimal value of D involves solution of a bi-objective optimization problem. The two (conflicting) objectives are: (i) to maximize the probability of successful search (P_{SS}), and (ii) to minimize the probability of false search (P_{FS}). For a given target track, P_{SS} as a function of D is evaluated by a suitable Monte Carlo simulation. In contrast, P_{FS} is analytically evaluated [4]. Both $P_{SS}(D)$ and $P_{FS}(D)$ are monotonically decreasing functions of D.

It has been observed that the non-dominated points in the plot of $log(P_{SS})$ versus $log(P_{FS})$ form a convex curve. Thus, a single objective function J is constructed by a weighted linear combination of the objective functions P_{SS} and P_{FS}.

$$J(\alpha, D) = \alpha \log(P_{SS}(D)) - (1-\alpha)\log(P_{FS}(D)) \quad (1)$$

where α is a scalar weight ($0 < \alpha < 1$).

For a given weight α, the optimal detection threshold D^{opt} is obtained as:

$$D^{opt} = \arg\max_{D} \; J(\alpha, D) \quad (2)$$

It follows from the above expression that, for a given α, D^{opt} is dependent on the target track. However, in target detection applications, the target track is not known and the problem at hand is to identify D^{opt} in real time from the ensemble of data generated from the sensor field for the track that the target is following. A naive solution to this problem is to select an a priori value $\bar{D}$ of the detection threshold that could be set as the average of detection thresholds for all possible individual target tracks. This paper presents a method for identification of track-dependent detection threshold, which makes use of the language measure concept [8] in the setting of a probabilistic finite state automata ($PFSA$).

C. Description of the Operation Scenario

The time series data collected from k sensors are partitioned for conversion into a symbol sequence [9][10], where the number of symbols in the alphabet is a positive integer n. Then, based on the Markov assumption, a probabilistic finite state automaton ($PFSA$) is constructed from the symbol sequence [11]. The $PFSA$ is characterized by an $(n \times n)$ state transition matrix $\mathbf{\Pi}$ that is an irreducible stochastic matrix [12] by construction. The algorithm used for construction of the $PFSA$ could be non-unique because it relies on the symbol sequence that is obtained by course graining of the time series data, where symbolization may not be achieved through a generating partition [11].

It is hypothesized [13] that there exists a state weight vector $\boldsymbol{\chi} \in \mathbb{R}^n$ such that the renormalized language measure [8] of the $PFSA$ is given as

$$\boldsymbol{\nu} = \lim_{\theta \to 0^+} \theta \left(\mathbf{I} - \big(1-\theta\big)\mathbf{\Pi}\right)^{-1} \boldsymbol{\chi} \quad (3)$$

Given that the state transition probability matrix $\mathbf{\Pi}$ is irreducible, an alternative form [8] of Eq. (3) is as follows.

$$\boldsymbol{\nu} = \left(\boldsymbol{\chi}^T \mathbf{p}\right) \mathbf{1}_{(n\times 1)} \quad (4)$$

where $\mathbf{1}_{(n\times 1)} \triangleq [1\ 1\ \cdots\ 1]^T$ and the state probability vector $\mathbf{p} \in \mathbb{R}^n$ is the (unity-sum-normalized) right eigenvector of the transposed state transition matrix $\mathbf{\Pi}^T$ corresponding to its unique unity eigenvalue. Thus, the renormalized measure vector $\boldsymbol{\nu}$ can be expressed in terms of a scalar ν which is the inner product of the state probability vector $\mathbf{p}$ and the state weight vector $\boldsymbol{\chi}$. In other words, ν is the (scalar) expected value of the state weight obtained as:

$$\nu = \boldsymbol{\chi}^T \mathbf{p} \quad (5)$$

Now it is proposed that the state weight $\boldsymbol{\chi}$ be assigned such that the residue $(D_i - \bar{D})$ of the detection threshold for the i^{th} track, where $\bar{D}$ is the expected/average value of the detection threshold for all tracks, is identically equal to the parameter ν_i of the renormalized measure of the corresponding track. Then, an estimate $\widehat{D}_i$ of the detection threshold D_i for the i^{th} track is obtained from Eq. (5) as:

$$D_i - \bar{D} = \nu_i \;\Rightarrow\; \widehat{D}_i = \widehat{\boldsymbol{\chi}}^T \mathbf{p}^i + \bar{D} \quad (6)$$

The next task is to generate an estimate $\widehat{\boldsymbol{\chi}}$ of the (track-invariant) state weight vector $\boldsymbol{\chi}$ from an ensemble of track data sets. The sets of simulated track data are divided into two mutually disjoint subsets - a training set and a test set. The detection threshold for the tracks in the training set are known *a priori*.

An $(\ell \times n)$ matrix $\mathbb{H}$ is constructed from the training set as

$$\mathbb{H} = \left[\mathbf{p}^1 \ \mathbf{p}^2 \ \cdots \ \mathbf{p}^\ell\right]^T \quad (7)$$

where the $(n \times 1)$ state probability vectors $\mathbf{p}^i \in \mathbb{R}^n$, $i \in \{1, 2, 3, \cdots, \ell\}$ are respectively obtained from the ℓ tracks in the training set, where $\ell > n$. Since the matrix $\mathbb{H}$ has the full column rank n, the $(n \times n)$ matrix $\left(\mathbb{H}^T\mathbb{H}\right)$ is invertible.

Similarly, a threshold residue vector $\mathbf{D}$ consisting of the known values of the detection threshold for each of the tracks in the training set is obtained as:

$$\mathbf{D} = \left[D_1 - \bar{D} \ D_2 - \bar{D} \ \cdots \ D_\ell - \bar{D}\right]^T \quad (8)$$

where the positive scalars D_i, $i \in \{1, 2, 3, ..., \ell\}$, are the known threshold residues for the respective tracks in the training set and $\bar{D}$ is the expected/average value of detection threshold. Accordingly, a measurement model of the $(\ell \times 1)$ threshold residue vector $\mathbf{D}$ is formulated as:

$$\mathbf{D} = \mathbb{H}\boldsymbol{\chi} + \boldsymbol{\varepsilon} \quad (9)$$

where the measurement error vector ε is assumed to be additive zero-mean and the (positive definite) error covariance matrix is $\mathbf{R}$.

An estimate $\widehat{\chi}$ of the track-invariant $(n \times 1)$ vector χ is obtained by the (weighted) linear least square method based on the information obtained from the training set. This task is performed by orthogonal projection of the $(\ell \times 1)$ threshold residue vector $\mathbf{D}$ onto the column space of $\mathbb{H}$ such that

$$\widehat{\chi} = \left[\mathbb{H}^T \mathbf{R}^{-1} \mathbb{H}\right]^{-1} \mathbb{H}^T \mathbf{R}^{-1} \mathbf{D} \tag{10}$$

Note that $\widehat{\chi}$ is an unbiased estimate of χ and if the measurement noise ε in Eq. (9) is jointly Gaussian, then $\widehat{\chi}$ is also the minimum-variance estimate of χ.

If the information on the measurement noise covariance matrix is not available, it is logical to assume (e.g., for identical sensors) that $\mathbf{R} \sim \mathbf{I}_{\ell \times \ell}$. In that case, Eq. (10) reduces to

$$\widehat{\chi} = \left[\mathbb{H}^T \mathbb{H}\right]^{-1} \mathbb{H}^T \mathbf{D} \tag{11}$$

Following Eq. (11), a probabilistic-state-machine-based estimate of the threshold residue vector $\mathbf{D}$ is obtained in terms of an estimate of the track-invariant state weight vector χ as

$$\widehat{\mathbf{D}} = \mathbb{H}\, \widehat{\chi} \;\Leftrightarrow\; \widehat{D}_i = \widehat{\chi}^T \mathbf{p}^i + \bar{D},\; i \in \{1, 2, \cdots, \ell\} \tag{12}$$

Remark 2.1: The estimated state weight vector $\widehat{\chi}$ is a linear functional in the space $\mathbb{R}^n$ because $\widehat{\chi} : \mathbb{R}^n \to \mathbb{R}$ is a linear map, as seen in Eq. (6). If the scalar D is replaced by a vector parameter of dimension $m \leq n$, then $\widehat{\chi}$ will become a linear transformation from $\mathbb{R}^n$ onto $\mathbb{R}^m$. In a more general case, a nonlinear transformation should be sought to address this identification problem. That is, it might be necessary to find a homeomorphism between the range space of $\widehat{\chi}$ and the space of the parameter vector that replaces the scalar D.

Remark 2.2: The algorithm in Eq. (12) for estimation of detection threshold is sufficiently simple for real-time execution on (limited memory) sensor nodes. In the present form, the algorithm is formulated based on the principle of linear least squares and is data driven in the absence of additional pertinent information such as a model of the underlying physical process and statistics of the environmental noise. Should this information be available, it is envisioned that combined model-based and data-driven algorithms for (possibly nonlinear) estimation of the state weight vector χ could be constructed for execution in real time on (limited memory) sensor nodes in a sensor network.

3. Results and Discussion

This section presents the results generated from the simulated data from a typical undersea sensor field. The ensemble of time series data is obtained from a set of 20 sensors in the given sensor field for each of 21 different tracks. The optimal detection threshold for each track corresponds to the value of the cost weight $\alpha = 0.95$ (see Eq. (1)). The objective here is to demonstrate efficacy of probabilistic-state-machine-based estimation of the detection threshold for individual tracks. Data from 10 tracks out of the 21 tracks have been used to for the purpose of learning and the parameter vector χ is estimated corresponding to the cost weight $\alpha = 0.95$. The performance is then tested on the remaining 11 tracks.

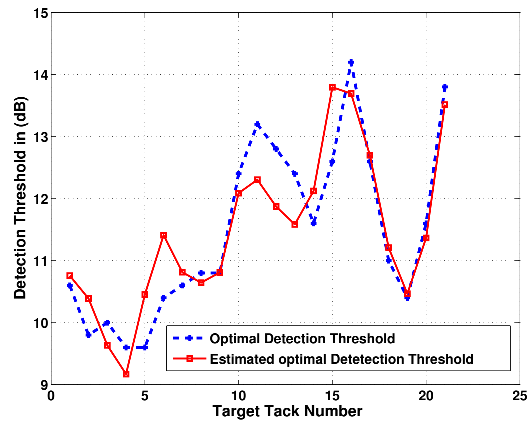

Fig. 2. Estimated and Computed Detection Threshold

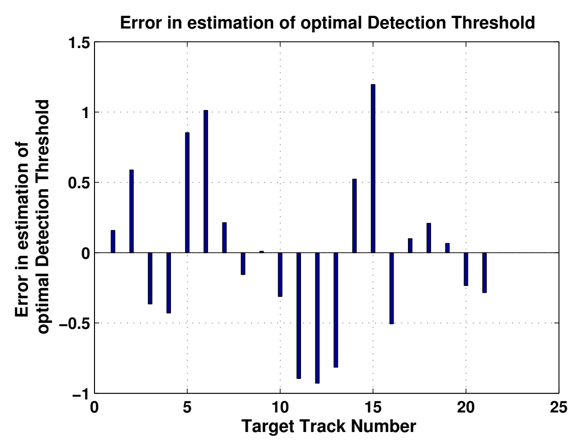

Fig. 3. Error in Estimated Detection Threshold

Figure 2 shows both the computed values of D and the respective estimated values $\widehat{D}$. These results have been generated by partitioning the signal space by a symbol

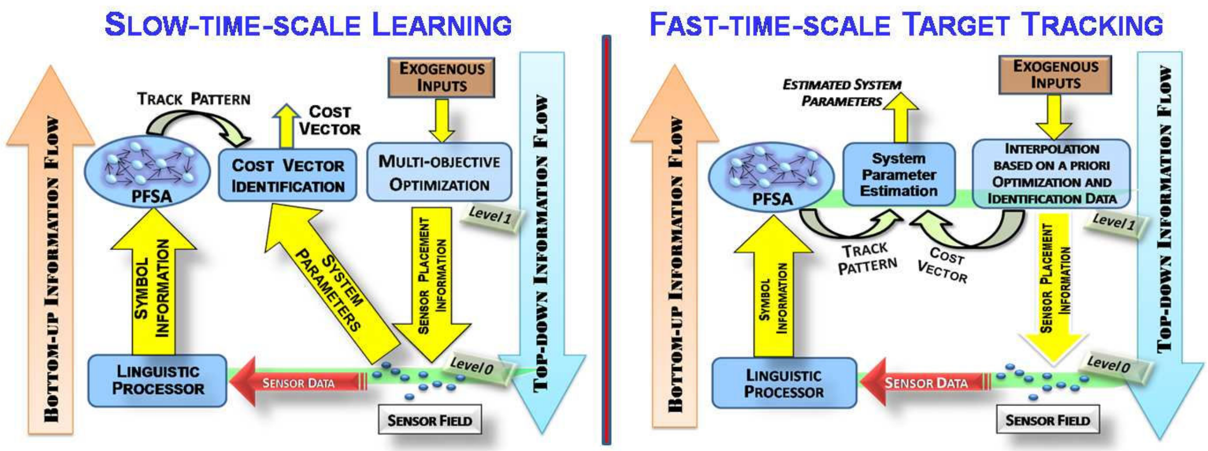

Fig. 4. Depiction of information flow

alphabet Σ with cardinality $|\Sigma| = 4$ and then constructing a $PFSA$ with ($n = 4$) states [11]. Hence, the $\boldsymbol{\chi}$ vector lies in $\mathbb{R}^4$. Figure 3 depicts the estimation error $\delta_i \triangleq \widehat{D}_i - D_i$ in the i^{th} track for $i = \{1, 2, \cdots . 21\}$. The mean of the estimation error was evaluated to be $m_\delta \sim 10^{-6}$ dB, which shows that the estimate is unbaised. The standard deviation of the estimation error was found to be $\sigma_\delta = 0.59$ dB.

A. Multi-level Optimization Using Language Measure

In a tactical operation, such as the one described in this paper, there are two types of information inputs. One is the input given by a commanding/supervising officer, and the other is the information derived from the online track data collected from the sensor field. In this case, the input given by the commanding officer is the cost weight α (see Eq. (1)). The language-measure-theoretic approach [8] allows separation of the two inputs; the value of χ vector is dependent on α while the state probability vector $\mathbf{p}$ is a function of the target track i.

$$\widehat{D}_i(\alpha) = \chi^T(\alpha)\mathbf{p}^i \tag{13}$$

Hierarchical modeling of the decision space enables extension of problem to a multi-level optimization scheme. The Pareto decision point determines an optimal characteristic vector by maximization of the language measure of the linguistic model at each level with respect to the assigned cost vector on the corresponding states of the linguistic model (e.g., a finite-state automaton). The language-measure-theoretic syntactic modifications are optimized via a numerically efficient combinatorial scheme leading to an optimized decision hierarchy. The computational cost of language-measure-theoretic optimization is estimated to be relatively small implying that optimality will be maintained by re-optimization in real time or near-real time as the commander's cost objectives vary and the mission states evolve. Figure 4 illustrates the concept of hierarchical learning and real-time execution for typical track-before detection of undersea targets as explained in the following two paragraphs.

The left hand plate in Fig. 4 shows the learning scheme which is executed in real time on a slow scale with a set of exogenous inputs. These inputs include adjustments of weights in the multi-objective cost functional for a given sensor field). The generated information on sensor placement is passed to the sensor field in the top-down direction to update the system parameters (e.g., reference values of detection thresholds for individual tracks). At the lower level, the time series data from the sensor field for each track are used for generation of a symbol sequence [11] that is transmitted bottom-up for construction of a probabilistic finite state automaton ($PFSA$). The quasi-static state probability vectors of the $PFSA$ serve as patterns of individual tracks. At the upper level, a combination of the track patterns and system parameters yield a single cost vector for all tracks. Thus, the output of the learning scheme is a cost vector that is a function of the exogenous inputs for a given sensor field.

The right hand plate in Fig. 4 shows the target tracking scheme which is executed in real time on a fast scale based on exogenous inputs. These inputs include the cost vector that was obtained in the learning scheme for a specified weight in the multi-objective cost functional for a given sensor field. As a target follows a track, time series data from the sensor field are collected for online generation of a symbol sequence that is transmitted bottom-up to compute the state probability vector (i.e., a track pattern) from the $PFSA$ that was constructed in the learning scheme. At the upper level, A scalar function (e.g., an inner product) of the online track pattern vector and the cost vector (that was computed in the learning scheme) yields the estimated system parameters (e.g., detection threshold) for the track that is just completed by the target. Thus, the output of the target tracking scheme is estimated system parameters that are functions of the exogenous inputs (i.e., the cost vector) and the online tracking signals from the sensor field.

4. Conclusions and Recommendations for Future Research

This paper addresses online surveillance of undersea targets as a real-time track-before-detect problem. As the target moves across the sensor field, each sensor collects time series data; the sensors that are closer to the path of the target capture stronger signals. A track-before-detect algorithm has been formulated to estimate the track-dependent detection threshold based on the ensemble of time series data from the sensor field. The objective here is to obtain a suboptimal trade-off between the probabilities of false alarm and successful search based on a specific cost functional. However, the probabilistic-state-machine-based algorithm is optimal in the sense of weighted linear least squares. The algorithm has been tested with sensor data from several tracks on a simulated sensor field. The results suggest that the probabilistic-state-machine-based approach is feasible for real-time estimation of detection threshold as needed for tracking of undersea targets. However, further investigation needs to be conducted with rich experimental data to firmly validate efficacy of the proposed track-before-detect algorithm for online surveillance of undersea targets.

Future research would involve extension of the probabilistic-state-machine-based approach to the challenging problem of optimal sensor placement [14] for online tracking of undersea targets. Current optimization algorithms, such as GANBI [7], are computationally expensive in terms of both execution time and memory requirements. It is necessary to construct a mathematically rigorous and computationally inexpensive (i.e., fast execution with low memory requirements) formal-language-theoretic algorithm to support the optimization objectives similar to what the existing tools provide. Once calibrated with results generated from existing optimization algorithms, it is expected that the formal-language-theoretic algorithm would provide solutions for sensor placement in real tome for modest perturbations in the nominal target statistics. It is envisioned that this algorithm will be locally executable on individual nodes of a sensor network in the undersea environment.

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