2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009



# A Cooperative Multi-Agent Approach for Stabilizing the Psychological Dynamics of a Two-Dimensional Crowd

K. Spieser and D.E. Davison

*Abstract*— **In this paper we extend our earlier work on the stabilization of crowds that are governed by psychological dynamics derived from Le Bon's late nineteenth-century suggestibility theory. Earlier work was restricted to the case where the crowd is one-dimensional, but now a general two-dimensional crowd is considered. The control scheme involves placing within the crowd various control agents who act as sensors, actuators, and processors. We analyze whether stabilization is possible for a given configuration of control agents, and we present an algorithm that positions control agents in a way that guarantees stabilizability. We include analysis of the control agent sensing load and requirements on inter-control-agent communication.**

## I. INTRODUCTION

Over the last century, psychologists have studied various mental processes with non-trivial and even intriguing dynamics (e.g., see [1]). Engineers of various types have used these psychological theories to help model socially-relevant phenomena for the purposes of prediction and control (e.g., see [2],[3]). The work in the present paper follows this trend. The particular dynamics studied here are based on a series of observations reported in [4] by Gustave Le Bon as he lived in the midst of the uprising of crowds in Europe. Le Bon witnessed multiple examples of "mob effects" and postulated from his observations the nature of crowds. His model reflects the idea that people in a crowd can be highly *suggestible* to influence from one another, leading to an unstable situation in which the crowd's attitude easily becomes extreme. In [5] we propose a mathematical model that is consistent with Le Bon's suggestibility theory, and we assume for the purposes of this paper that the model, summarized in Section II, is an accurate description of the psychological dynamics of people as they interact in a crowd.

This paper builds upon the work in [6] and [7], where one-dimensional (1D) crowds (i.e., queues) are studied. The main result in [6] and [7] is that it is possible to strategically place certain people in the queue (people that we call *control agents*) who can stabilize the psychological dynamics of the entire queue. Because of their importance for the present work, we summarize key results from [6] and [7] in Section III. The main contribution of the present paper is to show how a 2D crowd, assumed to be subject to Le Bon's psychological dynamics, can also be stabilized by appropriate positioning of control agents. Unlike the 1D case, however, it is generally necessary to use *multiple* control agents to stabilize a 2D crowd, and those control agents must coordinate their actions.

This research is supported by Natural Sciences and Engineering Research Council of Canada (NSERC).

The authors are with the Department of Electrical and Computer Engineering, University of Waterloo, Ontario, N2L 3G1, Canada. The corresponding author is D.E. Davison, ddavison@uwaterloo.ca.

## II. A MODEL OF PSYCHOLOGICAL DYNAMICS BASED ON LE BON'S SUGGESTIBILITY THEORY

The following model reflects how people interact, psychologically speaking, when in a crowd and subject to what Le Bon calls *mental unity*. The model, originally derived in a slightly different form in [5], is identical to the one appearing in [6] and [7]. We start by assuming the crowd consists of $n$ people who are in the psychological mental unity state. Introduce the notation $O_i$ to refer to the person in position $i$, where $1 \le i \le n$. As in [5], [6], [7], we model the state of $O_i$ as having four components:

- *Prestige of $O_i$*: $p_i[k] > 0$ is a measure of the ability of $O_i$ to influence other people.
- *Attitude (or action) of $O_i$*: $a_i[k]$ is a quantification of the behavior of $O_i$ as it relates to acceptance of an idea ($a_i[k] > 0$) or its antithesis ($a_i[k] < 0$). Attitude values for which $|a_i[k]| \approx 0$ are indicative of mild acceptance of an idea or its opposite notion, and are associated with a calm and orderly person.
- *Delayed attitude of $O_i$*: $b_i[k]$ equals $a_i[k-1]$.
- *Suggestibility of $O_i$*: $s_i[k] > 0$ is a measure of the affinity of $O_i$ to incorporate the behavior of neighboring people into his own behavior, by mimicking the actions and conduct of others.

The following nonlinear discrete-time equations describe how the people in the crowd influence one another [6], [7]:

$$
\begin{aligned}
p_i[k+1] &= c_p p_i[k] + \mu_{pa,i}|a_i[k]| && (1)\\
a_i[k+1] &= c_a a_i[k] + \mu_{apa,i} s_i^2[k] \sum_{O_j \in \mathcal{N}(O_i)} d_{i,j} p_j[k] a_j[k] && (2)\\
b_i[k+1] &= a_i[k] && (3)\\
s_i[k+1] &= \mu_{s,i} S \alpha^{c_s \beta_i[k]} && (4)\\
\beta_i[k] &:= \mu_{sa,i}(a_i[k] - b_i[k])^2 + \\
&\quad \mu_{sap,i} \sum_{O_j \in \mathcal{N}(O_i)} d_{i,j} p_j[k] |a_j[k]| + \\
&\quad \mu_{ssp,i} \sum_{O_j \in \mathcal{N}(O_i)} d_{i,j} p_j[k] (s_j[k] - s_i[k]) . && (5)
\end{aligned}
$$

In (1)–(5) the $\mu$ parameters are person-dependent positive gains used to scale the contributions of the various social effects. In (4), $\alpha > 1$ is a growth constant and $S > 0$ is a nominal suggestibility value. The constants $c_p$, $c_a$, and $c_s$ reside in the interval $(0,1)$ and capture the tendency of $p_i[k]$

978-1-4244-4524-0/09/$25.00 ©2009 AACC 

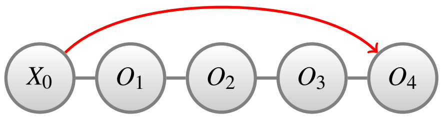


Fig. 1. A graphical display of the queue $X_0O_1O_2O_3O_4$. Neighbors are connected using line segments, and the red arrow indicates that the control agent $X_0$ is targeting $O_4$.

and $a_i[k]$ to decay towards zero and $s_i[k]$ to approach $\mu_{s,i}S$ in the absence of adequate social excitation. The quantity $\mathcal{N}(O_i)$ is the *neighbor set* of $O_i$ and consists of the set of people with which $O_i$ has direct interaction through conversation, gesturing, or other social exchanges; we assume here $\mathcal{N}(O_i)$ is time invariant. The $d_{i,j}$ terms are constants used to weight the strength of interaction between $O_i$ and $O_j$. Finally, we introduce the state of the crowd,

$$\begin{aligned} x[k] \quad = \quad & [p_1[k],\ldots,p_n[k],a_1[k],\ldots,a_n[k], \\ & b_1[k],\ldots,b_n[k],s_1[k],\ldots,s_n[k]]^T, \end{aligned} \tag{6}$$

and comment that the crowd dynamics are evidently unstable, given the positive feedback loop between the attitude and suggestibility dynamics in (1)–(5).

## III. Stabilization of 1D Crowds

In [6] we deal with the relatively simple situation in which the $n$ people of the psychological crowd are lined up in a queue, with each person able to interact with the people immediately to his left and right; the people at the ends of the queues interact with only one neighbor. For example, a queue of four people is shown in Figure 1. To control the crowd, we introduce a control agent, denoted $X_0$, and position him or her at the left end of the queue, again as shown in Figure 1 when $n=4$. We denote this controlled queue by $X_0O_1\ldots O_n$. The control agent, who should be thought of as a well-trained and disciplined individual such as a police officer, acts as a sensor, processor, and actuator. The control agent affects directly person $O_1$, and thus the neighbor set of $O_1$ needs to be augmented to include $X_0$, i.e., $\mathcal{N}(O_1)=\{X_0,O_2\}$. In anticipation of the results to follow, assume from now on that $\mathcal{N}(O_i)$ includes both people in the crowd and any control agents who directly interact with $O_i$.

Mathematically, the control agent interacts with the crowd through the following equations (where, for present purposes, take $i=0$ since $X_0$ is the only control agent):

$$p_i[k] = \hat{p}_i \tag{7}$$
$$a_i[k] = u_i[k] = u_i(x[k]) \tag{8}$$
$$s_i[k] = 0. \tag{9}$$

These three equations define the functionality of the control agent $X_0$ (or, for when we move on to multiple control agents, control agent $X_i$). Setting the prestige equal to the constant $\hat{p}_i>0$ in (7) attests to the control agent maintaining a constant level of influence. In (8), it is assumed the control agent can set its attitude state, $u_i[k]$, to any desired value in order to affect the behavior of the queue. The functional dependence of $u_i[k]$ on $x[k]$ indicates the control agent can sense the entire crowd state, i.e., a full-state-feedback scheme is assumed. Finally, setting $s_i[k]$ to zero in (9) signifies the control agent is impervious to suggestion and acts as an individual rather than a member of the psychological crowd.

Since the crowd dynamics (1)–(5) are unstable, the control objective in both [6] and the present paper is stabilization. The notion of stability, stated for the general case where $m$ control agents are present, is as follows:

**Definition [6]:** A crowd composed of $n$ people and $m$ control agents is said to be $\mathcal{C}(\lambda)$-*stabilizable* (for integer $\lambda>0$, called the *stabilization time*) if there exist $m$ causal control laws $u_1,\ldots,u_m$ capable of driving, from any initial state $x[0]$, the attitude of all crowd members to zero in no more than $\lambda$ time instants and subsequently holding all attitude states at zero. If such a collection of control laws is implemented, the crowd is said to be $\mathcal{C}(\lambda)$-*stabilized* and the control laws, $\mathcal{C}(\lambda)$-*stabilizing*. Lastly, a particular crowd member $O_i$ is said to be *zeroed* for $k\geq\tilde{k}$ if $a_i[k]=0$ for $k\geq\tilde{k}$. □

This concept of stability, essentially requiring a deadbeat response, focuses on the attitude of the crowd members, However, using the dynamics (1)–(5), it can be shown that $\mathcal{C}(\lambda)$-stability implies all state components of the crowd, and not just the attitude state components, are bounded [6].

The key control concept developed in [6] is that the entire queue $X_0O_1\ldots O_n$ can be controlled by having the control agent focus all of his or her control effort on the person at the opposite end of the queue. Specifically, $X_0$ tries to zero $O_n$; i.e., $X_0$ tries to drive $a_n[k]$ to zero in finite time. We say that $X_0$ *targets* $O_n$. We also say that $X_0$ and $O_n$ are a *target pair* and use the notation $[X_0\to O_n]$. In diagrams such as Figure 1, we indicate targeting using red arrows. It is shown in [6] that it is always possible for $X_0$ to zero $O_n$ in $n$ time steps; moreover, by zeroing $O_n$, necessarily $O_1,\ldots,O_{n-1}$ are also zeroed, resulting in $\mathcal{C}(\lambda)$-stabilization with $\lambda=n$. Note that this is the shortest possible stabilization time, given the control signal takes $n$ samples to propagate the full length of the queue.

In more mathematical terms, by propagating $u_0[k]$ through the crowd dynamics, it is shown in [6] that there exist coefficients $a_n^{\overline{u}_0}[k+n]$ and $a_n^{u_0}[k+n]$, both dependent only on $x[k]$ and the latter necessarily nonzero, such that

$$a_n[k+n]=a_n^{\overline{u}_0}[k+n]+a_n^{u_0}[k+n]u_0[k]. \tag{10}$$

Hence, the causal control law

$$u_0[k]=-\frac{a_n^{\overline{u}_0}[k+n]}{a_n^{u_0}[k+n]},\ k\geq 0 \tag{11}$$

results in $a_n[k]=0$ for $k\geq n$. Using (2) recursively, it is then argued in [6] that the condition $a_n[k]=0$ for $k\geq n$ also implies $a_{n-1}[k]=0$ for $k\geq n$, which in turn implies that $a_{n-2}[k]=0$ for $k\geq n$, and so on; hence, $a_i[k]=0$ for $k\geq n$, $i=1,\ldots,n-1$, i.e., $C(n)$-stability is achieved. Simulation results for $n=3$, taken from [6], are shown in Figure 2.

The above ideas are expanded upon in [7], where various configurations of 1D crowds with *multiple* control agents are studied. One conclusion of [7] is that, for any arrangement of control agents in a 1D crowd (subject to a control agent being positioned at one end of the queue), $\mathcal{C}(\lambda)$-stabilization

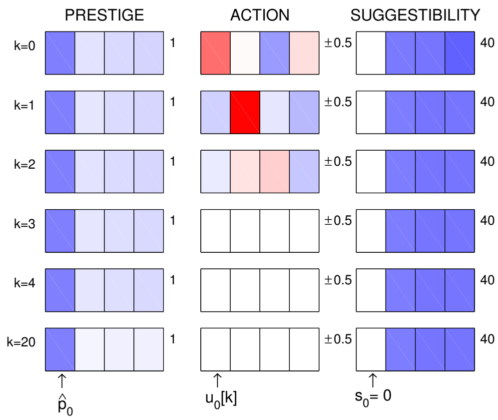


Fig. 2. Progression of the prestige, attitude (action), and suggestibility states among members of the queue $X_0O_1O_2O_3$ subject to the $\mathcal{C}(3)$-stabilizing control law (11). Blue indicates positive numbers and red indicates negative numbers. The number at the top right of each plot indicates the color saturation value, e.g., the attitude plot at $k=0$ has maximum intensity for any person with $|a_i[0]| \geq 0.5$. (This figure appears in [6].)

is possible. In particular, the idea that each control agent should target a specific crowd member is exploited, and it is shown that, for each target pair, equations of the form (10) and (11), with appropriate subscript and superscript changes, hold. Of special interest for the present paper is the queue $X_0O_1 \dots O_nX_{n+1}$, one of the 1D crowds examined in [7]. It is shown in [7] that the two control agents $X_0$ and $X_{n+1}$ can work together to stabilize the crowd by introducing the targeting schemes $[X_0 \to O_{n^*}]$ and $[X_{n+1} \to O_{n^*+1}]$, with $n^* := \lceil n/2 \rceil$. We summarize the following three results from [7] dealing with the queue $X_0O_1 \dots O_nX_{n+1}$ since we pursue the same line of reasoning for 2D crowds:

$R_1$: First, it is shown that the equivalent of (10) holds for each of the two target pairs, i.e.,

$$a_{n^*}[k+n^*] = a_{n^*}^{\overline{u^1}}[k+n^*] + a_{n^*}^{u^1}[k+n^*]u_0[k] \quad (12)$$

$$a_{n^*+1}[k+(n-n^*)] = a_{n^*+1}^{\overline{u^2}}[k+(n-n^*)] + a_{n^*+1}^{u^2}[k+(n-n^*)]u_{n+1}[k]. \quad (13)$$

It follows that there exist control laws $u_0$ and $u_{n+1}$ to zero both of the targets $O_{n^*}$ and $O_{n^*+1}$. Note that the coefficients in (12) and (13) depend on the states of all of $O_1, \dots, O_n$; thus, both control agents must be able to sense the entire crowd state. In addition, if $n$ is odd then the coefficients in (12) also depend on information about $X_{n+1}$, namely $u_{n+1}[k]$ and $\hat{p}_{n+1}$. Thus, in addition to synchronization of time steps and agreement on how to assign targets, the two control agents require special communication if $n$ is odd. Specifically, if $n$ is odd then $X_{n+1}$ must compute its control signal first and communicate that value, as well as the value $\hat{p}_{n+1}$, to $X_0$ so that $X_0$ can then compute its control signal.

$R_2$: Second, it is argued in [7] that, if both $O_{n^*}$ and $O_{n^*+1}$ are zeroed, then necessarily all other crowd members are zeroed. Hence, $\mathcal{C}(\lambda)$-stabilization is achieved.

$R_3$: Third, the stabilization time is simply the longer of the lengths of the two "subqueues" $X_0O_1 \dots O_{n^*}$ and $O_{n^*+1} \dots O_nX_{n+1}$, i.e., $\lambda = \max\{n^*, n-n^*\}$.

We next extend this thinking to general 2D crowds. More details of this extension are available in [8].

## IV. Stabilization of 2D Crowds for Specified Control Agent Positioning and Targeting

Consider a 2D crowd composed of $n$ people and a set of $m$ control agents that have the task of cooperatively stabilizing the crowd. Assume throughout this section that the neighbors of each of the $n+m$ people are prespecified. Consistent with the notation developed in [7], we now use superscripts to refer to the crowd members and the control agents, i.e., $O^1,\dots,O^n$ and $X^1,\dots,X^m$. (We reserve subscripts to refer to absolute position numbers, and we use superscripts to refer to a particular crowd member or control agent relative to other crowd members or control agents, respectively. For 2D crowds, there is no natural absolute position numbering scheme, so we use superscripts exclusively and show crowd member interactions in a graph.)

The main purpose of this section is to build upon the three results given at the end of Section III to determine if the $m$ control agents can $\mathcal{C}(\lambda)$-stabilize the 2D crowd. As earlier, we exploit the idea of targeting. Hence, assume each control agent, say $X^j$, has a prespecified target in the crowd, say $T^j$ (where $T^j = O^i$ for some value of $i$). For conciseness, denote this targeting relationship by $[X^j \to T^j]$.

To proceed, we need some definitions and notation:

- A *path* $\ell_{j,k}$ is a sequence of distinct people consisting of a single control agent and one or more crowd members such that the sequence begins with $X^j$, the sequence terminates with $O^k$, and adjacent members of the sequence are neighbors of one another.
- The *length of path* $\ell_{j,k}$ is denoted by $|\ell_{j,k}|$ and equals the number of people in $\ell_{j,k}$ excluding the control agent.
- In general, there may be multiple paths, still finite in number, linking $X^j$ with $O^k$. We denote the set of all such paths, sorted by path length, in the sequence $L_{j,k} := (\ell_{j,k}^1, \ell_{j,k}^2, \ell_{j,k}^3, \dots)$, where $\ell_{j,k}^1, \ell_{j,k}^2, \ell_{j,k}^3, \dots$ are paths from $X^j$ to $O^k$ and $|\ell_{j,k}^1| \leq |\ell_{j,k}^2| \leq |\ell_{j,k}^3| \leq \dots$. Assuming $L_{j,k}$ has at least one element, we call $\ell_{j,k}^1$ the *shortest path* from $X^j$ to $O^k$ and we call $|\ell_{j,k}^1|$, also denoted by $d(X^j, O^k)$, the *distance* between $X^j$ and $O^k$. If $L_{j,k}$ has no elements, we define $d(X^j, O^k) = \infty$.
- Analogous to the above, we define a *path* from $O^i$ to $O^j$ to be a sequence of distinct crowd members such that the sequence begins with $O^i$, the sequence terminates with $O^j$, and adjacent members of the sequence are neighbors of one another. The *length* of the path from $O^i$ to $O^j$ equals the number of crowd members in the sequence, excluding $O^i$. The *distance* between two control crowd members $O^i$ and $O^j$, denoted $d(O^i, O^j)$, is the length of the shortest path from $O^i$ to $O^j$ or, if there is no path from $O^i$ to $O^j$, define $d(O^i, O^j) = \infty$.

In addition, the following six assumptions are made:

$A_1$: Each control agent has exactly one neighbor, and the neighbors of control agents are distinct.

$A_2$: Each control agent targets a specific member of the crowd, but there are no duplicate targets. Hence, the collection of targets (called the *target set*) contains exactly $m$ crowd members.

$A_3$: There exists at least one path from $X^i$ to $T^i$, $i = 1, \ldots, m$.

$A_4$: Control agents can communicate among themselves, with no time delays.

$A_5$: Each control agent can sense the full state of the crowd.

$A_6$: Each control agent is at least as close to its target as is any other control agent. That is, let $p_{ij} := d(X^i, T^i) - d(X^j, T^i)$ for $1 \leq i \leq m$, $1 \leq j \leq m$. Then assume $p_{ij} \leq 0$.

Assumptions $A_1$ and $A_2$ allow us to readily extend the 1D results to 2D crowds; Assumption $A_3$ is necessary for $X^i$ being able to zero its target, $T^i$; Assumptions $A_4$–$A_6$ are made to keep the analysis relatively straightforward.

To illustrate many of the above notions, consider the crowd of Figure 3. There are three paths from $X^1$ to $O^6$, namely

$$
\begin{aligned}
\ell^1_{1,6} &= (X^1, O^1, O^2, O^3, O^6), \\
\ell^2_{1,6} &= (X^1, O^1, O^2, O^3, O^5, O^7, O^6), \\
\ell^3_{1,6} &= (X^1, O^1, O^2, O^3, O^5, O^7, O^8, O^6),
\end{aligned}
$$

which have lengths of 4, 6, and 7, respectively. Therefore, $d(X^1, O^6) = 4$. Similarly, $d(X^2, O^8) = 3$, $d(X^3, O^7) = 3$, $d(X^3, O^6) = 4$, and $d(O^5, O^7) = 1$. Finally, the setup is consistent with Assumptions $A_1$, $A_2$, $A_3$, and $A_6$.

### *Generalization of Result $R_1$*

We now reconsider Result $R_1$ in the context of 2D crowds. Focus on control agent $X^i$ and its target, $T^i$. By Assumption $A_3$, we know there is a path connecting $X^i$ to $T^i$. As in the analysis of 1D crowds, we conclude therefore that an expression of the form (10) still exists (with the obvious notational changes); consequently, it is possible for $X^i$ to zero $T^i$ in exactly $d(X^i, T^i)$ time steps with a control law of the form (11) (again with obvious notational changes). The coefficient terms in the equivalent of (10), as always, depend on the state of the crowd, but only on the state of those crowd members that are sufficiently close to the target $T^i$. Indeed, by considering the rate at which information propagates through the crowd, we conclude that the coefficients depend on the state of a crowd member $O^j$ (and, consequently, $X^i$ needs to sense the state of $O^j$) if and only if

$$
d(O^j, T^i) \leq d(X^i, T^i). \tag{14}
$$

In addition, the coefficients depend on the values of other control agents that are sufficiently close to the target $T^i$. Specifically, if a control agent $X^j$ satisfies

$$
d(X^j, T^i) \leq d(X^i, T^i), \tag{15}
$$

then the coefficients depend on $\hat{p}^j$ and on $u^j[k]$. (More generally, the coefficients depend on $u^j[k], \ldots, u^j[k + p_{ij}]$, but Assumption $A_6$ implies here $p_{ij} = 0$.) If (15) is satisfied, then it is necessary for $X^j$ to compute its control signal, and pass that information (in addition to the value of $\hat{p}^j$) to $X^i$ since $X^i$ requires that information to compute $u^i[k]$.

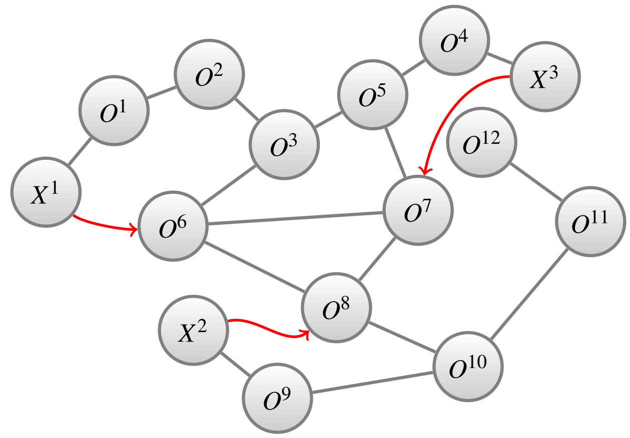


Fig. 3. A 2D crowd of 12 people with 3 control agents. As with queues, neighbors are connected using line segments, and the red arrows indicate targeting by the control agents.

As an example, consider the crowd in Figure 3 and focus on the target pair $[X^2 \to O^8]$. There is at least one path from $X^2$ to $O^8$, so $O^8$ can be zeroed in $d(X^2, O^8) = 3$ time steps. Moreover, the coefficients in the equivalent of (10) depend on the states of all the crowd members except for that of $O^1$; hence, control agent $X^2$ needs to sense the state of all crowd members except that of $O^1$. Note that $X^1$ and $X^3$ are more than 3 steps away from $O^8$, so the coefficients in the equivalent of (10) do not depend on $u^1$, $\hat{p}^1$, $u^3$, or $\hat{p}^3$; consequently, $X^1$ and $X^3$ do not have to pass any information to $X^2$ in order for $X^2$ to compute its control signal.

Algorithm 1, on the next page, provides a systematic way to test whether all targets in a target set can be zeroed using control laws of the form (11). The input of the algorithm consists of: $n$, $m$, the neighbor sets of all people, and a targeting scheme for the control agents. The output of the algorithm is a set of target pairs, denoted $\Upsilon$. Inclusion of the target pair $[X^i \to T^i]$ in $\Upsilon$ guarantees the target $T^i$ can be zeroed by $X^i$ using a control law of the form (11), and guarantees that the control law can be calculated without having to simultaneously solve multiple (nonlinear) equations of the form (10). If $\Upsilon$ contains all $m$ target pairings then the control agents $X^1, \ldots, X^m$ are able to zero the entire target set. In terms of special communication requirements among control agents, if Case (b) in Step 2 of Algorithm 1 is invoked, then it is necessary that control agent $X^j$ pass information to $X^i$, as discussed after (15). In terms of sensing load, control agent $X^i$ needs to sense the state of any crowd member, $O^j$, that satisfies (14).

As an illustration, consider again the crowd in Figure 3. In Step 1 of Algorithm 1, recognize that $d(X^2, O^8) < d(X^1, O^8)$ and $d(X^2, O^8) < d(X^3, O^8)$. Hence, we set $\Upsilon = \{[X^2 \to O^8]\}$. In Step 2, we observe that $d(X^3, O^7) < d(X^1, O^7)$ and $d(X^3, O^7) < d(X^2, O^7)$, so we update $\Upsilon$ to $\Upsilon = \{[X^2 \to O^8], [X^3 \to O^7]\}$. We now repeat Step 2, focusing on the remaining target pair, $[X^1 \to O^6]$. Evidently $d(X^1, O^6)$ is not less than $d(X^2, O^6)$ or $d(X^3, O^6)$; however, the target pairs $[X^2 \to O^8]$ and $[X^3 \to O^7]$ already appear in $\Upsilon$, so we augment $\Upsilon$ to get $\Upsilon = \{[X^2 \to O^8], [X^3 \to O^7], [X^1 \to O^6]\}$. Algorithm 1 then terminates. Since $\Upsilon$ contains all three target

**Algorithm 1: Determining if All Members of the Target Set Can be Zeroed**

*Step 1:* Determine if there exists a target pair, $[X^i \to T^i]$, for which $d(X^i,T^i) < d(X^j,T^i)$ for all $j \neq i$. If no such target pair exists, the algorithm terminates with $\Upsilon = \emptyset$. Otherwise, define $\Upsilon := \{[X^i \to T^i]\}$ and proceed to Step 2.

*Step 2:* Determine if there exists a target pair, $[X^i \to T^i]$, not already in $\Upsilon$, for which:

(a) $d(X^i,T^i) < d(X^j,T^i)$ for all $j \neq i$, or

(b) for each $j$ such that $d(X^j,T^i) \leq d(X^i,T^i)$ it follows that $[X^j \to T^j]$ already belongs to $\Upsilon$.

If no such target pair exists, the algorithm terminates. Otherwise, add the element $[X^i \to T^i]$ to the set $\Upsilon$, and repeat Step 2.

**Algorithm 2: Determining if People Outside the Target Set Are Zeroed When the Target Set is Zeroed**

*Step 1:* Initialize $\alpha = 0$ and initialize $\Omega_0$ to the set of crowd members that are known to be zeroed for $k \geq \tilde{k}$.

*Step 2:* Determine if there exists a crowd member, say $O^i$, in $\Omega_\alpha$ such that (i) $O^i$ has exactly one neighbor that does not belong to $\Omega_\alpha$, and (ii) that neighbor is not a control agent.

(a) If there is no such crowd member, the algorithm terminates with $\Omega_\infty := \Omega_\alpha$.

(b) Otherwise, we conclude that the neighbor, denoted $O^h$, is zeroed for $k \geq \tilde{k}$. Define $\Omega_{\alpha+1} := \Omega_\alpha \cup \{O^h\}$, increment $\alpha$ by 1, and repeat Step 2.

**Algorithm 3: Stabilizing a 2D Crowd**

*Step 1:* Initialize $m = n$. Position each control agent so it is the neighbor of a distinct crowd member, and have each control agent target its neighbor. Algorithm 1 and the fact that every crowd member is a target imply the crowd is $\mathcal{C}(1)$-stabilizable.

*Step 2:* Subject to Assumptions $A_1$–$A_3$ and $A_6$, determine all possible arrangements of $m-1$ control agents, and, for each of these, identify all possible targeting schemes. The number of such configurations is finite, but possibly very large.

*Step 3:* For each configuration from Step 2, use Algorithm 1 to determine if all members of the target set can be zeroed and, if so, use Algorithm 2 to determine if zeroing the target set implies all members of the crowd are zeroed.

(a) If both the above conditions hold for at least one configuration, then such a configuration (with $m-1$ control agents) is $\mathcal{C}(\lambda)$-stabilizing; decrement $m$ by 1 and repeat Step 2.

(b) If, on the other hand, none of the configurations from Step 2 satisfy the above conditions, then Algorithm 3 terminates with the conclusion that the crowd may be $\mathcal{C}(\lambda)$-stabilized using one of the previously-identified $\mathcal{C}(\lambda)$-stabilizing $m$-control-agent configurations.

pairs, the three control agents in Figure 3 can in fact zero their respective targets. In terms of special communication requirements, $X^1$ needs to know the values $u_2[k]$, $\hat{p}^2$, $u_3[k]$, and $\hat{p}^3$ before it can compute $u_1[k]$. In terms of sensing load, $X^1$ needs to measure the state of all members of the crowd, $X^2$ needs to measure the state of all members of the crowd except for that of $O^1$, and $X^3$ needs to measure the state of all members of the crowd except for that of $O^1$ and $O^{12}$.

### *Generalization of Result $R_2$*

We now turn to Result $R_2$. If all members of the target set can be zeroed, does it follow that other people in a 2D crowd are necessarily zeroed? The analysis in Section III readily generalizes. Indeed, consider a subset of the crowd, denoted $\Omega_\alpha$, in which all members of $\Omega_\alpha$ are zeroed for $k \geq \tilde{k}$ (for some integer $\tilde{k} \geq 1$). Assume person $O^i \in \Omega_\alpha$ has exactly one neighbor that does not belong to $\Omega_\alpha$, and that the neighbor is not a control agent. Call the neighbor $O^h$. In this case, (2) (written for $O^i$, with an abuse of our subscript/superscript notation) simplifies to $0 = 0 + \mu_{apa,i} s_i^2[k] p^h[k] a^h[k]$ for $k \geq \tilde{k}$; since the $\mu$ parameters, prestige, and suggestibility are positive quantities, it follows that $a^h[k] = 0$ for $k \geq \tilde{k}$. In other words, $O^h$ is necessarily zeroed for $k \geq \tilde{k}$. To demonstrate this idea, consider again the crowd in Figure 3. Assuming that the target set $\{O^6, O^7, O^8\}$ is zeroed, one conclusion we can reach is that $O^{10}$ is also zeroed since it is the only neighbor of $O^8$ that is not in the target set; similarly, we can conclude $O^5$ and $O^3$ are zeroed. Once these conclusions are made it is then possible to re-apply the above idea repeatedly to deduce that other crowd members are also zeroed.

We exploit the above thinking in Algorithm 2, shown at the left. The input to the algorithm is the same as that for Algorithm 1. Algorithm 2 determines, for a given set of crowd members that are zeroed for $k \geq \tilde{k}$, whether there is a larger set of crowd members, denoted $\Omega_\infty$, that are also necessarily zeroed for $k \geq \tilde{k}$. In typical use, $\Omega_0$ in Algorithm 2 is initialized to the target set, assuming that Algorithm 1 indicates each member of the target set can be zeroed. If the output of Algorithm 2, $\Omega_\infty$, includes all $n$ crowd members, then $\mathcal{C}(\lambda)$-stabilization is guaranteed. If $\Omega_\infty$ does not include all $n$ crowd members, then we can reach no conclusion about $\mathcal{C}(\lambda)$-stabilizability.

As an example, in Figure 3 we have already determined using Algorithm 1 that all members of the target set can be zeroed. Applying Algorithm 2 with $\Omega_0 = \{O^6, O^7, O^8\}$ results in $\Omega_\infty := \{O^6, O^7, O^8, O^5, O^3, O^{10}, O^4, O^2, O^1\}$. Since $O^9$, $O^{11}$, and $O^{12}$ are not included in $\Omega_\infty$, we cannot deduce whether or not $\mathcal{C}(\lambda)$-stabilization of the crowd in Figure 3 is possible.

### *Generalization of Result $R_3$*

Suppose that Algorithm 1 indicates all members of the target set can be zeroed and, moreover, that Algorithm 2 guarantees $\mathcal{C}(\lambda)$-stabilization. Then the stabilization time is determined by the longest distance between control agents and their respective targets: $\lambda = \max_{1 \leq i \leq m} d(X^i, T^i)$.

## V. Stabilization of 2D Crowds for Unspecified Control Agent Positioning and Targeting

The question naturally arises if a different placement of control agents and associated targeting scheme for the crowd in Figure 3 would be more successful at stabilization than that indicated in the figure. Here we present a brute-force approach, Algorithm 3, that results in a placement of control agents and a targeting scheme that guarantees $\mathcal{C}(\lambda)$-stabilizability with control laws of the form (11).

The input to Algorithm 3 is $n$ and the neigbor sets of all crowd members. The algorithm starts off by introducing $n$ control agents. Each of the $n$ control agents is positioned adjacent to a unique crowd member, and each control agent targets its neighbor. While resource intensive, this control strategy is guaranteed by Algorithm 1 to provide $\mathcal{C}(1)$-stabilizability. Algorithm 3 then iteratively removes control agents and considers new targeting schemes, at each stage applying Algorithms 1 and 2 to determine if the particular control agent arrangement and targeting scheme can $\mathcal{C}(\lambda)$-stabilize the crowd. For some value of $m$, it will not be possible to conclude (via Algorithms 1 and 2) $\mathcal{C}(\lambda)$-stabilizability, at which point Algorithm 3 terminates. The output of the algorithm is a $m$, a (possibly non-unique) control agent arrangement, and a targeting scheme that guarantees $\mathcal{C}(\lambda)$-stabilizability for $\lambda$ in the range $1 \leq \lambda \leq n$ and for $m$ in the range $1 \leq m \leq n$.

Algorithm 3 was applied to find an arrangement of control agents and a targeting scheme to $\mathcal{C}(\lambda)$-stabilize the 12-person crowd in Figure 3. We started with twelve control agents as shown in Figure 4. Following Algorithm 3, we reduced the number of control agents by removing control agents, choosing targeting schemes, and using Algorithms 1 and 2 to determine if the resulting crowd is still $\mathcal{C}(\lambda)$-stabilizable. As detailed in [8], the algorithm terminates after eight control agents are removed. Figure 5 shows the resulting arrangement of the four control agents and their targets; the stabilization time is $\lambda = 4$ time steps, and the communication and sensing requirements are indicated in the caption of the figure.

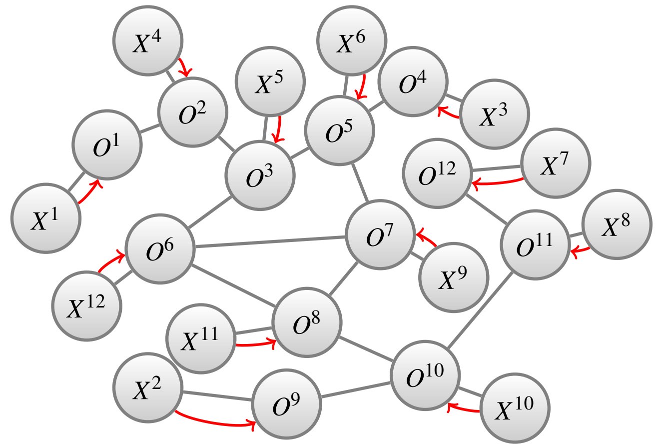


Fig. 4. A control configuration that uses twelve control agents to $\mathcal{C}(1)$-stabilize the crowd.

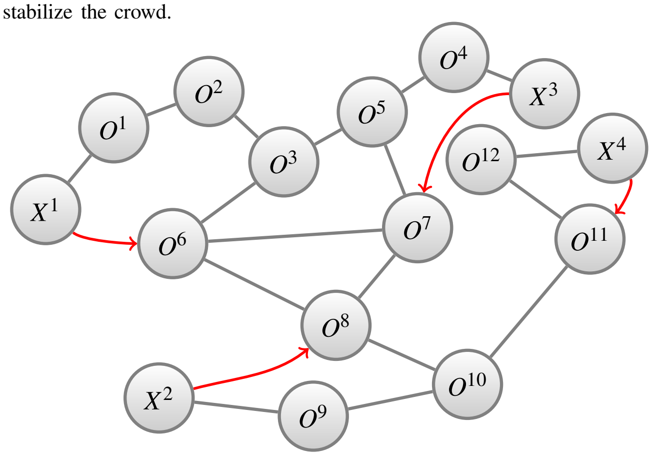


Fig. 5. A control configuration that uses just four control agents to $\mathcal{C}(4)$-stabilize the crowd. The only communication requirement is that $X^2$ and $X^3$ must compute their control signals and pass that information to $X^1$. In terms of sensing loads, $X^1$ must sense the state of all crowd members, $X^2$ must sense the state of all crowd members except that of $O^1$, $X^3$ must sense the state of all crowd members except that of $O^1$ and $O^{12}$, and $X^4$ needs to sense only the state of $O^8$, $O^9$, $O^{10}$, $O^{11}$, and $O^{12}$.

## VI. Conclusions

Having devised an algorithm to $\mathcal{C}(\lambda)$-stabilize any 2D crowd, we have identified goals for future work, including: (a) seek to better understand the class of crowds that can be stabilized for a given $m$, $1 \leq m \leq n-1$; (b) improve the efficiency of Algorithm 3, perhaps using results from graph theory; (c) determine how to deal with the removal of Assumption $A_1$, which is unrealistic in many social settings; (d) determine if there is an advantage to removing Assumptions $A_2$ and $A_6$; (e) incorporate *a priori* constraints on sensing load or communication among control agents (see Assumptions $A_4$ and $A_5$); (f) introduce time variations in the neighbor sets, allowing for movement among both crowd members and control agents; (g) study the tradeoff between $m$ and $\lambda$; (h) introduce model uncertainty; (i) investigate and include "sensor dynamics" to model errors in human perception processes; (j) include more recent social psychological theories (e.g., theories on conformity, social learning, and decision making [1]); and (k) in the long-term, collaborate with social psychologists to try to (ethically!) demonstrate the results of this work experimentally.

## References


[1] D. G. Myers and S. J. Spencer, *Social Psychology*, 3rd ed. McGraw Hill Ryerson, 2006.

[2] J. D. Sterman, *Business Dynamics: Sytems Thinking and Modeling for a Complex World*. Irwin/McGraw-Hill, 2000.

[3] J. Sime, "Crowd psychology and engineering," *Safety Science*, vol. 21, pp. 1–14, 1995.

[4] G. LeBon, *The Crowd: A Study of the Popular Mind*. Ernest Benn Limited, 1952, english Translation of original French version in 1896.

[5] K. Bergey, K. Spieser, and D. E. Davison, "The psychological dynamics of students in a classroom: Modeling and control strategies based on suggestibility theory," in *Conference on Control Applications*. Singapore: IEEE, Oct. 2007.

[6] K. Spieser and D. E. Davison, "Stabilizing the psychological dynamics of people in a queue," in *Proceedings of the American Control Conference*. Seattle, WA: AACC, Jun. 2008.

[7] ——, "Multi-agent stabilization of the psychological dynamics of one-dimensional crowds," *Automatica*, vol. 45, 2009.

[8] K. Spieser, "Stabilizing the psychological dynamics of people in a crowd," MASc thesis, University of Waterloo, 2008.