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Decentralized Stochastic Guaranteed Cost Control for Uncertain Nonlinear Large-scale Interconnected Systems under Gain Perturbations

Hiroaki Mukaidani

Abstract— **This paper is concerned with the decentralized stochastic guaranteed cost control (GCC) for a class of uncertain nonlinear large-scale interconnected systems with control gain perturbations. First, the definition of the GCC problem with the deterministic and stochastic uncertainties is given. Second, a more practical model of actuator failures than outage is adopted. Based on the linear matrix inequality (LMI) approach, a method for designing reliable decentralized state feedback controllers is presented. Using the resulting control systems, asymptotically mean square stable (EMSS) is guaranteed and an adequate performance bound against high-order nonlinearity, plant uncertainty and actuator failures is attained. Finally, in order to show the effectiveness of the proposed design method, the simulation result is demonstrated.**

I. INTRODUCTION

The analysis of stochastic systems with respect to mean square stability of their equilibria has attracted many researchers. Such systems arise in a various field in control mechanical engineering. Often, these systems generally are governed by Itô stochastic differential equations. In the past few decades, many stability and control problems have been discussed for the stochastic systems [2], [3], [11].

A number of essential works on asymptotic stability of stochastic nonlinear systems have been considered. Particularly, the adaptive backstepping stabilization scheme for such systems has been studied (see, for example, [4] and the references therein for more details). These results are very elegant in theory, while the LQ control for a class of nonlinear large-scale interconnected stochastic systems with norm-bounded time-varying parameter uncertainties has not been dealt with thus far.

The problem of the decentralized robust control of large-scale interconnected systems with parameter uncertainties has been widely studied, and some solution approaches have been developed (see, for example, [5] and the references therein). However, in case where the existing approaches are applied, the deterministic systems have only been considered. Thus far, the decentralized robust control of nonlinear large-scale interconnected stochastic systems with parameter uncertainties has never been studied.

Recent advance in theory of linear matrix inequality (LMI) has allowed a revisiting of the guaranteed cost control (GCC) [6] for the large-scale interconnected systems [8], [9], [15]. Although these results are feasible for the deterministic uncertainties, the problem of guaranteed cost stabilization for the uncertain nonlinear large-scale interconnected stochastic systems has not been tackled.

In order to attain robustness against the plant uncertainty, it is generally known that feedback systems need very accurate controllers. However, due to A/D conversion, D/A conversion, finite word length and round-off errors of the numerical computations, these properties may not be guaranteed. Therefore, the implemented controller should be allowed some uncertainty of the actuator. Since controller fragility results in the performance degradation of a feedback control system, the non-fragile control problem has been an important issue. In the area of reliable control system design, in order to tolerate the failures of controllers, several design methods have been developed. Particularly, in [14], [15], a more general failure model have been adopted for actuator failures, which covers the several operations.

In this paper, the decentralized stochastic GCC problem for a class of uncertain nonlinear large-scale systems under gain perturbations is investigated. This is an extension of the work of [5], [8], [9], [15] in the sense that the nonlinear large-scale interconnected systems are included in the standard Wiener process as the stochastic systems. Furthermore, it should be noted that although the existing results [5], [8], [9], [15] are assumed to have first-order polynomial, the considered interconnections are bounded by the general polynomial-type nonlinearity. The contributions of this paper are as follows. First, the model of the actuator failures and the high-order nonlinearity are adopted to the uncertain nonlinear large-scale stochastic systems. Second, after defining the GCC problem for the nonlinear large-scale interconnected stochastic systems, a sufficient condition for the existence of the decentralized robust feedback controllers is derived by making use of the Lyapunov stability criterion such that the uncertain nonlinear large-scale interconnected stochastic systems are exponentially mean square stable (EMSS). Finally, in order to demonstrate the efficiency of the design algorithm, the numerical example is included.

Notation: The notations used in this paper are fairly standard. $\mathbf{block\ diag}$ denotes the block diagonal matrix. I_n denotes the $n \times n$ identity matrix. $\|\cdot\|$ denotes its Euclidean norm for a matrix. E denotes the expectation. $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and the maximum eigenvalues for a matrix, respectively.

H. Mukaidani is with Graduate School of Education, Hiroshima University, 1-1-1 Kagamiyama, Higashi-Hiroshima, 739-8524 Japan. mukaida@hiroshima-u.ac.jp

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II. PROBLEM FORMULATION

Consider a class of nonlinear large-scale interconnected stochastic systems composed of N subsystems described by the following Itô stochastic differential equations:

$$dx_i(t) = \Bigg[(A_i + \Delta A_i(t))x_i(t) + (B_i + \Delta B_i(t))u_i^F(t) + \sum_{j=1,\ j\neq i}^{N}(G_{ij} + \Delta G_{ij}(t))g_{ij}(x_i,\ x_j)\Bigg]dt + A_{i1}x_i(t)dw_i(t),\ x_i(0) = x_i^0, \tag{1a}$$

$$u_i^F(t) = \Sigma_i u_i(t) + h_i(u_i), \tag{1b}$$

$$u_i(t) = K_i x_i(t),\ i = 1, \cdots,\ N, \tag{1c}$$

where $x_i(t) \in \Re^{n_i}$, $u_i(t) \in \Re^{m_i}$ and $u_i^F(t) \in \Re^{m_i}$ are the state, the practical control and control input with failure of the ith subsystems, respectively. $w(t) = [w_1(t)\ \cdots\ w_N(t)]^T$ is a standard N-dimensional Wiener process i.e. it satisfies $E[(w(t) - w(s))(w(t) - w(s))^T] = I_N|t-s|$. A_i, B_i and A_{i1} are constant matrices of appropriate dimensions and G_{ij} are interconnection matrices between the ith subsystems and other subsystems. K_i is the fixed control gain matrix. $\Sigma_i := \mathbf{diag}\begin{pmatrix} \sigma_{i1} & \cdots & \sigma_{im_i}\end{pmatrix}$ is a diagonal positive definite matrix. The unknown vector functions $g_{ij}(x_i,\ x_j) \in \Re^{l_{ij}}$ (to simplify the notation, it will be convenient to write $g_{ij}(x_i,\ x_j) = g_{ij}$) represent the high-order interconnections among the subsystems. On the other hand, $h_i(u_i)$ denotes the control failure. It is assumed that the unknown vector functions g_{ij} and h_i are continuous and sufficiently smooth and piecewise continuous in t [5], [8], [9]. The parameter uncertainties considered here are assumed to be of the following form:

$$\begin{bmatrix} \Delta A_i(t) & \Delta B_i(t)\end{bmatrix} = D_i F_i(t)\begin{bmatrix} E_i^a & E_i^b\end{bmatrix}, \tag{2a}$$

$$\Delta G_{ij}(t) = D_{ij}F_{ij}(t)E_{ij}, \tag{2b}$$

where D_i, E_i^a, E_i^b, D_{ij} and E_{ij} are known constant real matrices of appropriate dimensions. $F_i(t) \in \Re^{p_i\times q_i}$ and $F_{ij}(t) \in \Re^{p_{ij}\times q_{ij}}$ are unknown matrix functions with Lebesgue measurable elements and satisfying

$$F_i^T(t)F_i(t) \le I_{q_i},\ F_{ij}^T(t)F_{ij}(t) \le I_{q_{ij}}. \tag{3}$$

The following conditions concerning the unknown vector functions $g_{ij}(x_i,\ x_j)$ and $h_i(u_i)$ are supposed.

Assumption 1: The control failure $h_i(u_i)$ satisfies, for each i, $h_i(u_i) := \bar{F}_i\Omega_i u_i$, where $\Omega_i := \mathbf{diag}\begin{pmatrix}\omega_{i1} & \cdots & \omega_{im_i}\end{pmatrix}$ with $\omega_{ij} > 0$ and $\bar{F}_i := \mathbf{diag}\begin{pmatrix}\bar{f}_{i1} & \cdots & \bar{f}_{im_i}\end{pmatrix}$ with $\bar{F}_i^T\bar{F}_i \le I_{m_i}$.

Assumption 2: There exist known constant matrices V_i and W_{ij} such that for all $i,\ j,\ t \ge 0$, $x_i \in \Re^{n_i}$ and $x_j \in \Re^{n_j}$, $\|g_{ij}(x_i,\ x_j)\| \le \sum_{k=1}^{p}[\alpha_k\|V_i x_i\|\|x_i\|^{k-1} + \beta_k\|W_{ij}x_j\|\|x_j\|^{k-1}]$, where α_k and β_k are positive scalar constants.

If Assumption 1 holds, there exists known constant matrix Ω_i such that for all i, $t \ge 0$, $x_i \in \Re^{n_i}$, $\|h_i(u_i)\| \le \|\Omega_i u_i\|$ [14], [15]. The value of σ_{ij}, for $j = 1,\ \dots,\ m_{m_i}$, represents the percentage of failure in the actuator j controlling the subsystem. Using this notation, each subsystem actuator failure can be represented independently. If $\sigma_{ij} = 1$ and $\omega_{ij} = 0$, it corresponds to the normal case for the jth actuator of the ith subsystem ($u_{ij}^F = u_{ij}$, where $u_i := \begin{bmatrix}u_{i1} & \cdots & u_{im_i}\end{bmatrix}$, $u_{ij} \in \Re$). When this is true for all j, $\Sigma_i = I_{m_i}$ holds, and it corresponds to the normal case in the ith channel ($u_i^F = u_i$). When $\sigma_{ij} = \omega_{ij}$, the outage case ($u_{ij}^F = 0$) would be covered. It should be noted that other cases correspond to partial failures or partial degradations of the actuators [14], [15].

On the other hand, it is known that $x_i(t)$ will be bounded whenever the trajectory $x_i(t)$ is confined to a compact set [13]. Hence, the above Assumption 2 satisfies the inequality $\|g_{ij}(x_i,\ x_j)\| \le \sum_{k=1}^{p}[\alpha_k\gamma_i^{k-1}\|V_i x_i\| + \beta_k\gamma_j^{k-1}\|W_{ij}x_j\|] = \|\tilde{V}_i x_i\| + \|\tilde{W}_{ij}x_j\|$, where $\tilde{V}_i := \sum_{k=1}^{p}\alpha_k\gamma_i^{k-1}V_i$ and $\tilde{W}_{ij} := \sum_{k=1}^{p}\beta_k\gamma_j^{k-1}W_{ij}$ for all $\|x_i(t)\| \le \gamma_i$. It seems that this constraint assumption $\|x_i(t)\| \le \gamma_i$ is suitable because all the trajectories have to be stable. It may be noted that if $p = 1$, $\alpha_1 = \beta_1 = 1$, Assumption 2 is the same as the existing one [5], [8], [9].

Assumption 3: For all $i,\ j$,

$$U_i := 2\sum_{j=1,\ j\neq i}^{N}(\tilde{V}_i^T\tilde{V}_i + \tilde{W}_{ji}^T\tilde{W}_{ji}) > 0,\ S_i := \Omega_i^T\Omega_i > 0.$$

Associated with system (1) is the cost function

$$J = \sum_{i=1}^{N}E\left[\int_0^{\infty}[x_i^T(t)Q_i x_i(t) + u_i^T(t)R_i u_i(t)]dt\right], \tag{4}$$

where Q_i and R_i are given as the positive definite symmetric matrices.

The following concept is standard in the stability theory of stochastic systems (see e.g., [1], [11], [12] and the references therein for more details).

Definition 1: The stochastic system is said to be EMSS if it satisfies the following equation.

$$\exists\rho,\ \psi > 0,\ E\|x(t)\|^2 \le \rho e^{-\psi(t-t_0)}E\|x(t_0)\|^2.$$

Lemma 1: [11], [12] The trivial solution of a stochastic differential equation as follows:

$$dx(t) = f(t,\ x)dt + g(t,\ x)dw(t), \tag{5}$$

where $x(t) = \begin{bmatrix}x_1^T(t) & \cdots & x_N^T(t)\end{bmatrix}^T$ with $f(t,\ x)$ and $g(t,\ x)$ sufficiently differentiable maps, is EMSS if there exists a function $V(x(t))$ which satisfies the following inequalities

$$a_1\|x(t)\|^2 \le V(x(t)) \le a_2\|x(t)\|^2,\ a_1,\ a_2 > 0, \tag{6a}$$

$$\mathcal{D}V(x(t)) := \frac{\partial V(x(t))}{\partial x}f(t,\ x) + \frac{1}{2}\mathbf{Tr}\left[g^T(t,\ x)\frac{\partial^2 V(x(t))}{\partial x^2}g(t,\ x)\right] \le -c\|x(t)\|^2,\ c > 0 \tag{6b}$$

for $x(t) \neq 0$.

Based on reference [6], the definition of the GCC for the nonlinear large-scale interconnected stochastic systems with the deterministic uncertainties is given below.

Definition 2: A decentralized fragile controller $u_i^F(t) = \Sigma_i u_i(t) + h_i(u_i)$ is said to be the GCC for the uncertain nonlinear large-scale interconnected stochastic systems (1) and the cost function (4) if the closed-loop systems are EMSS and the closed-loop value of the cost function (4) satisfies the bound $J \le \mathcal{J}$ for all admissible uncertainties.

The objective of this paper is to design a decentralized reliable linear guaranteed cost controller $u_i^F(t) = \Sigma_i K_i x_i(t) + h_i(u_i)$, $i = 1, \ldots, N$ for the nonlinear large-scale interconnected stochastic systems (1) with uncertainties (2), (3) and the actuator failure.

III. PRELIMINARY RESULT

Now, a sufficient condition for existence of the state feedback guaranteed cost controller for the uncertain nonlinear large-scale interconnected stochastic systems (1) is established.

Theorem 1: Consider the nonlinear large-scale interconnected stochastic systems (1) with the uncertainties (2) and (3) under Assumptions 1 and 2. If there exists symmetric positive definite matrix $P_i \in \Re^{n_i \times n_i}$ such that the matrix inequality (7) holds, the fragile controllers $u_i^F(t) = \Sigma_i u_i(t) + h_i(u_i)$, $i = 1, \cdots, N$ are the guaranteed cost controller,

$$\Lambda_i = \begin{bmatrix} \Xi_i & P_i\tilde{G}_{i1} & \cdots & P_i\tilde{G}_{iN} & C_i \\ \tilde{G}_{i1}^T P_i & -I_{l_{i1}} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{G}_{iN}^T P_i & 0 & \cdots & -I_{l_{iN}} & 0 \\ C_i^T & 0 & \cdots & 0 & R_i - I_{m_i} \end{bmatrix} < 0, \quad (7)$$

where $\Lambda_i \in \Re^{\bar{N} \times \bar{N}}$, $C_i := P_i\tilde{B}_i + K_i^T \Sigma_i R_i$, $\bar{N} = n_i + m_i + \sum_{j=1,\ j\neq i} l_{ij}$, $\Xi_i := \tilde{A}_i^T P_i + P_i\tilde{A}_i + A_{i1}^T P_i A_{i1} + U_i + \bar{R}_i + K_i^T S_i K_i$, $\tilde{A}_i := \bar{A}_i + D_i F_i(t)\bar{E}_i$, $\tilde{B}_i := B_i + D_i F_i(t) E_i^b$, $\tilde{G}_{ij} := G_{ij} + D_{ij} F_{ij}(t) E_{ij}$, $\bar{A}_i := A_i + B_i \Sigma_i K_i$, $\bar{E}_i := E_i^a + E_i^b \Sigma_i K_i$ and $\bar{R}_i := Q_i + K_i^T \Sigma_i R_i \Sigma_i K_i$.

In other words, the closed-loop systems are EMSS for $\|x_i(0)\| < \delta_i$ and the corresponding value of the cost function (4) satisfies the following inequality (8) for all admissible uncertainties $F_i(t)$ and $F_{ij}(t)$.

$$J < \sum_{i=1}^{N} E[x_i^T(0) P_i x_i(0)]. \quad (8)$$

Remark 1: Note that there exists no matrix $P_i\tilde{G}_{ii}$, $i = 1, \cdots, N$ in the matrix Λ_i.

Proof: Combining the guaranteed cost controller $u_i^F(t) = \Sigma_i K_i x_i(t) + h_i(u_i)$, $i = 1, \cdots, N$ with (1) gives a closed-loop uncertain stochastic system of the form

$$dx_i(t) = \left[\tilde{A}_i x_i(t) + \tilde{B}_i h_i + \sum_{j=1,\ j\neq i}^{N} \tilde{G}_{ij} g_{ij}\right] dt + A_{i1} x_i(t) dw_i(t). \quad (9)$$

Suppose now there exists the symmetric positive definite matrix $P_i > 0$, $i = 1, \cdots, N$ such that the matrix inequality (7) holds for all admissible uncertainties. In order to prove EMSS of the closed-loop uncertain stochastic system (9), let us define the following Lyapunov function candidate

$$V(x(t)) = x^T(t)\mathbf{P}x(t) = \sum_{i=1}^{N} x_i^T(t) P_i x_i(t) > 0, \quad (10)$$

where $\mathbf{P} := \textbf{block diag}\begin{pmatrix} P_1 & \cdots & P_N \end{pmatrix}$

First, using the fact $\lambda_{\min}(\mathbf{P})\|x(t)\|^2 \le V(x(t)) \le \lambda_{\max}(\mathbf{P})\|x(t)\|^2$, the condition (6a) holds. Second, in order to prove the formula (6b), the stochastic differential is given by

$$\begin{aligned} \mathcal{D}V(x(t)) = \sum_{i=1}^{N} \Bigg\{ & x_i^T(t)(\tilde{A}_i^T P_i + P_i\tilde{A}_i) x_i(t) \\ & + h_i^T \tilde{B}_i^T P_i x_i(t) + x_i^T(t) P_i \tilde{B}_i h_i \\ & + \sum_{j=1,\ j\neq i}^{N} \left[g_{ij}^T \tilde{G}_{ij}^T P_i x_i(t) + x_i^T(t) P_i \tilde{G}_{ij} g_{ij} \right] \\ & + x_i^T(t) A_{i1}^T P_i A_{i1} x_i(t) \Bigg\}. \end{aligned}$$

Since $\sum_{i=1}^{N}\sum_{j=1,\ j\neq i}^{N}(2x_i^T \tilde{V}_i^T \tilde{V}_i x_i + 2x_i^T \tilde{W}_{ji}^T \tilde{W}_{ji} x_i - g_{ij}^T g_{ij}) = \sum_{i=1}^{N}\sum_{j=1,\ j\neq i}^{N}(2x_i^T \tilde{V}_i^T \tilde{V}_i x_i + 2x_j^T \tilde{W}_{ij}^T \tilde{W}_{ij} x_j - g_{ij}^T g_{ij})$, it follows that

$$\begin{aligned} & \mathcal{D}V(x(t)) \\ & = \sum_{i=1}^{N} \Bigg\{ z_i^T(t)\Lambda_i z_i(t) - x_i^T(t) Q_i x_i(t) \\ & \quad -(\Sigma_i K_i x_i + h_i)^T R_i (\Sigma_i K_i x_i + h_i) \\ & \quad -(x_i^T K_i^T S_i K_i x_i - h_i^T h_i) \\ & \quad - \sum_{j=1,\ j\neq i}^{N} (2x_i^T \tilde{V}_i^T \tilde{V}_i x_i + 2x_j^T \tilde{W}_{ij}^T \tilde{W}_{ij} x_j - g_{ij}^T g_{ij}) \Bigg\}, \end{aligned}$$

where $z_i(t) = \begin{bmatrix} x_i^T(t) & g_{i1}^T & \cdots & g_{iN}^T & h_i^T \end{bmatrix}^T \in \Re^{\bar{N}}$ and Ξ_i and Λ_i are given in (7).

It is easy to verify that the inequalities $2x_i^T \tilde{V}_i^T \tilde{V}_i x_i + 2x_j^T \tilde{W}_{ij}^T \tilde{W}_{ij} x_j \ge g_{ij}^T g_{ij}$ and $u_i^T S_i u_i = x_i^T K_i^T S_i K_i x_i \ge h_i^T h_i$ hold under Assumptions 1 and 2. By using the above inequality, it immediately follows that

$$\begin{aligned} & \mathcal{D}V(x(t)) \\ & < -\sum_{i=1}^{N} [x_i^T(t) Q_i x_i(t) + (\Sigma_i K_i x_i + h_i)^T R_i (\Sigma_i K_i x_i + h_i)] \\ & < -\lambda_{\min}(\mathbf{R})\|x(t)\|^2 < 0, \end{aligned} \quad (11)$$

where $\mathbf{R} := \textbf{block diag}\begin{pmatrix} \tilde{R}_1 & \cdots & \tilde{R}_N \end{pmatrix}$, $\tilde{R}_i := Q_i + \tilde{K}_i^T R_i \tilde{K}_i$ and $\tilde{K}_i := (\Sigma_i + \bar{F}_i \Omega_i) K_i$.

Hence, $V(x(t))$ is a Lyapunov function for the closed-loop uncertain stochastic system (9). Therefore, since the following inequality holds:

$$E\|x(t)\|^2 \le \frac{\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{P})} E\|x(0)\|^2 \exp\left[-\frac{\lambda_{\min}(\mathbf{R})}{\lambda_{\max}(\mathbf{P})} t\right],$$

the closed-loop uncertain stochastic systems (9) are EMSS. Moreover, applying Itô's formula results in

$$\begin{aligned} & dV(x(t)) \\ &= \mathcal{D}V(x(t))dt + 2\sum_{i=1}^{N} x_i^T(t)A_{i1}^T P_i x_i(t) dw_i(t) \\ &< -\sum_{i=1}^{N} x_i^T(t)\tilde{R}_i x_i(t)dt + 2\sum_{i=1}^{N} x_i^T(t)A_{i1}^T P_i x_i(t) dw_i(t). \end{aligned} \tag{12}$$

Furthermore, by integrating both sides of the inequality (12) from 0 to T and using the initial conditions, the following inequality holds

$$\begin{aligned} & E[V(x(T))] - E[V(x(0))] \\ < \quad & -\sum_{i=1}^{N} E\left[\int_0^T x_i^T(t)\tilde{R}_i x_i(t)dt\right]. \end{aligned} \tag{13}$$

Since the closed-loop uncertain stochastic systems (9) are EMSS, that is, $\lim_{T\to\infty} E\|x(T)\|^2 \to 0$, $V(x(T)) \to 0$ holds. Thus the following inequality holds.

$$\begin{aligned} J &= \sum_{i=1}^{N} E\left[\int_0^\infty x_i^T(t)\tilde{R}_i x_i(t)dt\right] < E[V(x(0))] \\ &= \sum_{i=1}^{N} E[x_i^T(0)P_i x_i(0)] = \mathcal{J}. \end{aligned}$$

The proof of Theorem 1 is completed. ■

IV. MAIN RESULT

Theorem 2: Under Assumptions 1 and 2, suppose there exist the constant positive parameters $\mu_i > 0$, $\varepsilon_i > 0$ and $\phi_i > 0$ such that for all $i = 1, \ldots, N$ the LMI (14) have the symmetric positive definite matrices $X_i > 0 \in \Re^{n_i \times n_i}$ and a matrix $Y_i \in \Re^{m_i \times n_i}$.

If such conditions are met, the decentralized linear state feedback controllers

$$u_i^F(t) = \Sigma_i u_i(t) + h_i(u_i), \ i = 1, \cdots, \ N, \tag{15}$$

with $u_i(t) = K_i x_i(t) = Y_i X_i^{-1} x_i(t)$ are the guaranteed cost controllers and

$$J < \sum_{i=1}^{N} E[x_i^T(0) X_i^{-1} x_i(0)] \tag{16}$$

is the guaranteed cost.

Proof: Let us introduce the matrices $X_i := P_i^{-1}$ and $Y_i := K_i P_i^{-1}$. Pre- and post-multiplying both sides of the inequality (14) by

$$\textbf{block diag}\left[\begin{array}{cccccccc} P_i & I_{n_i} & I_{q_i} & I_{l_{i1}} & I_{s_{i1}} & \cdots & I_{l_{iN}} & I_{s_{iN}} \quad I_{n_i} \\ & & I_{m_i} & I_{n_i} & I_{m_i} & I_{q_i} & I_{n_i} & \end{array}\right]$$

and using Schur complement [10], the LMI (14) holds if and only if the following inequality (17) holds.

$$\mathcal{F}_i := \begin{bmatrix} \Gamma_i & A_{i1}^T & P_i G_{i1} & \cdots & P_i G_{iN} & P_i B_i \\ A_{i1} & -P_i^{-1} & 0 & \cdots & 0 & 0 \\ G_{i1}^T P_i & 0 & \Theta_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ G_{iN}^T P_i & 0 & 0 & \cdots & \Theta_N & 0 \\ B_i^T P_i & 0 & 0 & \cdots & 0 & \Theta_B \end{bmatrix} < 0, \tag{17}$$

where $\Gamma_i := \bar{A}_i^T P_i + P_i \bar{A}_i + U_i + \bar{R}_i + K_i^T S_i K_i + (\mu_i + \phi_i) P_i D_i D_i^T P_i + P_i H_i P_i + \mu_i^{-1} \bar{E}_i^T \bar{E}_i$, $\Theta_j := \varepsilon_i^{-1} E_{ij}^T E_{ij} - I_{l_j}$, $\Theta_B := \phi_i^{-1} E_i^{bT} E_i^b + R_i - I_{m_i}$.

Using a standard matrix inequality for all admissible uncertainties (2) and (3), the matrix inequality (18) holds.

$$\begin{aligned} & 0 > \mathcal{F}_i \\ & \geq \begin{bmatrix} \Phi_i & A_{i1}^T & P_i G_{i1} & \cdots & P_i G_{iN} & \bar{C}_i \\ A_{i1} & -P_i^{-1} & 0 & \cdots & 0 & 0 \\ G_{i1}^T P_i & 0 & -I_{l_{i1}} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ G_{iN}^T P_i & 0 & 0 & \cdots & -I_{l_{iN}} & 0 \\ \bar{C}_i^T & 0 & 0 & \cdots & 0 & -I_{m_i} \end{bmatrix} \\ & \quad + \bar{\mathbf{D}} F_i(t) (\bar{\mathbf{E}} + \tilde{\mathbf{E}})^T + (\bar{\mathbf{E}} + \tilde{\mathbf{E}}) F_i^T(t) \bar{\mathbf{D}}^T \\ & \quad + \mathbf{D}_i \mathbf{F}_i(t) \mathbf{E}_i + \mathbf{E}_i^T \mathbf{F}_i^T(t) \mathbf{D}_i^T = \mathcal{L}_i. \end{aligned} \tag{18}$$

where $\Phi_i := \bar{A}_i^T P_i + P_i \bar{A}_i + U_i + \bar{R}_i + K_i^T S_i K_i$, $\bar{C}_i := P_i B_i + K_i^T \Sigma_i R_i$,

$$\begin{aligned} \bar{\mathbf{D}} &:= \left[\begin{array}{cccccc} (P_i D_i)^T & 0 & 0 & \cdots & 0 & 0 \end{array}\right]^T, \\ \bar{\mathbf{E}} &:= \left[\begin{array}{cccccc} \bar{E}_i & 0 & 0 & \cdots & 0 & 0 \end{array}\right]^T, \\ \tilde{\mathbf{E}} &:= \left[\begin{array}{cccccc} 0 & 0 & 0 & \cdots & 0 & E_i^b \end{array}\right]^T, \\ \mathbf{D}_i &:= \left[\begin{array}{ccccccc} 0 & 0 & P_i D_{i1} & \cdots & P_i D_{iN} & 0 \\ & & & O & & \end{array}\right], \\ \mathbf{F}_i(t) &:= \textbf{block diag}\left(\begin{array}{cccccc} 0 & 0 & F_{i1}(t) & \cdots & F_{iN}(t) & 0 \end{array}\right), \\ \mathbf{E}_i &:= \textbf{block diag}\left(\begin{array}{cccccc} 0 & 0 & E_{i1} & \cdots & E_{iN} & 0 \end{array}\right). \end{aligned}$$

Finally, using Schur complement [10] for $\mathcal{L}_i < 0$ results in $\Lambda_i < 0$. Hence, the closed-loop stochastic systems are EMSS. On the other hand, since the results of the cost bound (16) can be proved by using similar arguments for the proof of Theorem 1, it is omitted. ■

Since the LMI (14) consists of a solution set of $(\mu_i, \ \varepsilon_i, \ \phi_i, \ X_i, \ Y_i)$, various efficient convex optimization algorithms can be applied. Moreover, its solutions represent the set of guaranteed cost controllers. This parameterized representation can be exploited to design the guaranteed cost controllers, which minimizes the value of the guaranteed cost for the closed-loop uncertain nonlinear large-scale interconnected stochastic systems. Consequently, solving the following optimization problem allows us to determine the

$$
\begin{bmatrix}
\Psi_i & X_iA_{i1}^T & L_i^T & G_{i1} & 0 & \cdots & G_{iN} & 0 & X_i & Y_i^T\Sigma_i & X_i & \tilde{C}_i & 0 & Y_i^T \\
A_{i1}X_i & -X_i & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_i & 0 & -\mu_i I_{q_i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
G_{i1}^T & 0 & 0 & -I_{l_{i1}} & E_{i1}^T & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & E_{i1} & -\varepsilon_i I_{s_{i1}} & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G_{iN}^T & 0 & 0 & 0 & 0 & \cdots & -I_{l_{iN}} & E_{iN}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & E_{iN} & -\varepsilon_i I_{s_{iN}} & 0 & 0 & 0 & 0 & 0 & 0 \\
X_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -Q_i^{-1} & 0 & 0 & 0 & 0 & 0 \\
\Sigma_i Y_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -R_i^{-1} & 0 & 0 & 0 & 0 \\
X_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & -U_i^{-1} & 0 & 0 & 0 \\
\tilde{C}_i^T & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & R_i - I_{m_i} & E_i^{bT} & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & E_i^b & -\phi_i I_{q_i} & 0 \\
Y_i & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_i^{-1}
\end{bmatrix} < 0, \quad (14)
$$

where $\Psi_i := A_iX_i + B_iY_i + (A_iX_i + B_iY_i)^T + (\mu_i + \phi_i)D_iD_i^T + H_i$, $L_i := E_i^aX_i + E_i^bY_i$, $H_i := \sum_{j=1,\ j\neq i}^{N} \varepsilon_i D_{ij}D_{ij}^T$, $\tilde{C}_i := B_i + Y_i^T\Sigma_iR_i$.

optimal bound.

$$
\mathcal{D}_0 : \min_{\sum_i \mathcal{X}_i} \sum_{i=1}^{N} \gamma_i^2 E[Z_i] = \mathcal{J},
$$

$$
\mathcal{X}_i \in (\mu_i,\ \varepsilon_i,\ \phi_i,\ X_i,\ Y_i,\ Z_i) \quad (19)
$$

such that the LMI (14) and

$$
\begin{bmatrix} -Z_i & I_{n_i} \\ I_{n_i} & -X_i \end{bmatrix} < 0. \quad (20)
$$

Finally, the optimization problem that should be solved is given.

Theorem 3: If the above optimization problem has the solution μ_i, ε_i, ϕ_i, X_i, Y_i and Z_i, then the controller of the form (15) are the decentralized linear state feedback controllers, which ensure the minimization of the guaranteed cost (16) for the uncertain nonlinear large-scale interconnected stochastic systems.

Proof: By Theorem 2, the controllers (15) constructed from the feasible solutions μ_i, ε_i, ϕ_i, X_i, Y_i and Z_i are the decentralized reliable linear guaranteed cost controllers of the uncertain nonlinear large-scale interconnected stochastic systems (1). It follows that

$$
\begin{aligned}
J &< \sum_{i=1}^{N} E[x_i^T(0)X_i^{-1}x_i(0)] \le \sum_{i=1}^{N} \|x_i(0)\|^2 E[X_i^{-1}] \\
&\le \min_{\sum_i \mathcal{X}_i} \sum_{i=1}^{N} \gamma_i^2 E[Z_i] = \mathcal{J}. \quad (21)
\end{aligned}
$$

Thus, the minimization of $\sum_{i=1}^{N} \gamma_i^2 E[Z_i]$ implies the minimum value $\mathcal{J}$ of the guaranteed cost for uncertain nonlinear large-scale interconnected stochastic systems (1). The optimality of the solution of the optimization problem follows from the convexity of the objective function under the LMI constraints. This is the required result. ■

It should be noted that the original optimization problem for the guaranteed cost (21) can be decomposed to the following reduced optimization problems (22) because each optimization problem (22) is independent of other LMI. Hence, the optimization problems (22) for each independent subsystem can be solved.

$$
\mathcal{J} = \min_{\sum_i \mathcal{X}_i} \left(\sum_{i=1}^{N} \gamma_i^2 E[Z_i] \right) = \sum_{i=1}^{N} \left(\min_{\mathcal{X}_i} \gamma_i^2 E[Z_i] \right), \quad (22)
$$

$$
\mathcal{D}_i : \min_{\mathcal{X}_i} \gamma_i^2 E[Z_i],\ i = 1,\ \ldots,\ N.
$$

A design procedure for constructing the guaranteed cost controller is given below.

Step 1. Starting for any γ_i, calculate $\tilde{V}_i$ and $\tilde{W}_{ij}$.

Step 2. Find $\mathcal{X}_i$ such that the LMI's (14) and (20) is feasible. If the LMI's (14) and (20) are not feasible, decrease the design parameter γ_i and go to Step 1. If γ_i is less than some prescribed computational accuracy, then stop and declare that the GCC fails. Otherwise, proceed Step 3.

Step 3. Minimize $\gamma_i^2 E[Z_i]$ over $\mathcal{X}_i$ satisfying the LMI's (14) and (20).

Step 4. If a solution is available, obtain the gain matrix $K_i = Y_iX_i^{-1}$ for each subsystem and the cost bound.

V. NUMERICAL EXAMPLE

Consider the uncertain nonlinear large-scale interconnected stochastic systems. Each system has two states and one control input. The system matrices and the unknown functions with the uncertainties are given as follows.

$$
A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1.5 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},
$$

$$
A_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & -1.2 \end{bmatrix},\ A_{11} = \begin{bmatrix} -0.2 & 0 \\ 0.1 & 0.5 \end{bmatrix},
$$

$$
A_{21} = \begin{bmatrix} 0 & -0.3 \\ 0 & 0.1 \end{bmatrix},\ A_{31} = \begin{bmatrix} 0.1 & -0.1 \\ 0 & 0.2 \end{bmatrix},
$$

$$
B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix},\ B_3 = \begin{bmatrix} 1.2 \\ 0 \end{bmatrix},
$$

$$
g_{ij}(x_i,\ x_j) = \|x_i\|^2\|x_j\|^2,
$$

$$
D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ D_2 = \begin{bmatrix} 1.2 \\ 1 \end{bmatrix},\ D_3 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},
$$

$$
E_i^a = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix},\ E_i^b = \begin{bmatrix} 0.01 \end{bmatrix},
$$

$$
G_{12} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},\ G_{13} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\ G_{23} = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix},
$$

$$G_{21}=\begin{bmatrix}0.2\\0.1\end{bmatrix},\ G_{31}=\begin{bmatrix}0.3\\0\end{bmatrix},\ G_{32}=\begin{bmatrix}0.2\\0\end{bmatrix},$$
$$V_i=W_{ij}=0.1I_2,\ D_{ij}=\begin{bmatrix}0\\1\end{bmatrix},$$
$$E_{12}=E_{13}=\begin{bmatrix}0.015\end{bmatrix},\ E_{23}=E_{21}=\begin{bmatrix}0.01\end{bmatrix},$$
$$E_{31}=E_{32}=\begin{bmatrix}0.03\end{bmatrix},$$
$$\sigma_{11}=0.8,\ \sigma_{21}=0.9,\ \sigma_{31}=0.7,\ \omega_{i1}=0.1,$$
$$R_i=0.1,\ Q_i=0.1I_2,\ i,\ j=1,\ 2,\ 3,\ i\neq j.$$

These nonlinear large-scale stochastic systems cannot be treated using the technique in [5], [8], [9] because the interconnection term cannot be bounded by a linear combination of the state $x_i(t)$ and the high-order interconnections are included. Furthermore, the stochastic uncertainties exist.

First, let us consider usefulness of the proposed reliable control technique for a failure scenario described by the following model.

$$\begin{aligned}u_1^F(t)&=(0.8+0.1\bar{f}_{11})K_1x_1(t),\ |\bar{f}_{11}|\le 1,\\u_2^F(t)&=(0.9+0.1\bar{f}_{21})K_1x_1(t),\ |\bar{f}_{21}|\le 1,\\u_3^F(t)&=(0.7+0.1\bar{f}_{31})K_1x_1(t),\ |\bar{f}_{31}|\le 1,\end{aligned}$$

where $u_i^F(t)=\Sigma_iu_i(t)+h_i(u_i)=\sigma_{i1}u_i(t)+h_i(u_i)\in\Re$, $h_i(u_i)=\bar{F}_i\Omega_iu_i=\bar{f}_{i1}\omega_{i1}u_i\in\Re$, $u_i(t)=K_ix_i(t),\ i=1,\ 2,\ 3.$

This mean that the control designer allows a failure of the order of 80% in the actuator of subsystem $i=1$ with an error of the order of 10%. The tolerances assumed for the order actuators are interpreted in the same way. That is, using the above notation, the control failure can be described appropriately.

Second, taking the norm of $g_{ij}(x_i,\ x_j)$ yields $\|g_{ij}(x_i,\ x_j)\|=\|x_i\|^2\|x_j\|^2\le 0.5(\|x_i\|^4+\|x_j\|^4)$. Hence, there exists $\tilde{V}_i=\tilde{W}_{ij}=0.5\gamma_i^3I_2,\ i,\ j=1,\ 2,\ 3,\ i\neq j$. The design parameter is selected as $\gamma_i=2$ tentatively. By applying Theorem 3 and solving the corresponding optimization problem (22), the decentralized linear optimal state feedback controllers are given as

$$\begin{aligned}K_1&=\begin{bmatrix}-1.0076e+01 & -7.6097\end{bmatrix},\\K_2&=\begin{bmatrix}-6.0261 & -5.2210\end{bmatrix},\\K_3&=\begin{bmatrix}-8.9032 & -2.8391e-01\end{bmatrix}.\end{aligned}$$

Consequently, the optimal guaranteed cost of the closed-loop uncertain stochastic systems is $\mathcal{J}=4.76287775e{+}01$, where $\min_{\mathcal{X}_1}J_1=2.8208888e+01$, $\min_{\mathcal{X}_2}J_2=1.0721752e+01$ and $\min_{\mathcal{X}_3}J_3=8.6981375$.

It should be noted that although the stochastic uncertainty exists as compared with the existing results [5], [8], [9], the decentralized robust controller can be constructed. Therefore, the proposed design method is very useful in the sense that the resulting decentralized robust controller can be implemented to more practical large-scale interconnected stochastic systems.

From Theorem 2 the initial states of (1) must hold inequality $\sqrt{\lambda_{\max}(\mathbf{P})/\lambda_{\min}(\mathbf{P})}\|x_i(0)\|\le\gamma_i=2$. Thus the stability region of (1) is $\|x_i(t)\|\le 4.954702e-01$ because $\sqrt{\lambda_{\max}(\mathbf{P})/\lambda_{\min}(\mathbf{P})}=\sqrt{6.527155/4.005890e-01}=4.036570$.

VI. CONCLUSION

In this paper, the stochastic GCC problem for uncertain nonlinear large-scale interconnected systems under gain perturbations has been addressed. Particularly, the high-order interconnections are considered for a different model in [8], [9]. A concept for the reliable controllers is based on the practical case of the model of actuator failures. The decentralized robust controller which minimizes the value of the guaranteed cost for the closed-loop uncertain large-scale interconnected stochastic systems can be computed by means of a feasible LMI optimization problem. Moreover it is easy to solve the decentralized guaranteed cost controller by using software such as MATLAB's LMI control Toolbox. Finally, the proposed design synthesis is useful in that the resulting reliable decentralized linear feedback controller attains EMSS and the optimal cost bound against the uncertain large-scale interconnected stochastic systems with high-order nonlinearity and the actuator failure.

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