

Decentralized Control of Multimachine Power Systems

Karanjit Kalsi, Jianming Lian and Stanislaw H. Żak

Abstract—A novel decentralized dynamic output feedback controller is presented to deal with the transient stability of a class of multimachine power systems. The proposed decentralized control strategy employs local sliding mode observers to estimate the states of each machine, and the feedback gain matrix of each local controller is obtained by solving two linear matrix inequalities. In addition, local sliding mode observers are capable of reconstructing unknown interconnections between machines. The effectiveness of the proposed control strategy is illustrated by simulation of a three-machine power system.

NOMENCLATURE

δ_i	rotor angle of the i -th machine, in degree
ω_i	relative speed of the i -th machine, in rad/sec
ω_o	synchronous machine speed, in rad/sec
B_{ij}	the i -th row and j -th column element of the nodal susceptance matrix at internal nodes after removing physical buses, in per unit (p.u.) system
D_i	damping coefficient of the i -th machine, in p.u.
E'_{qi}	internal transient voltage of the i -th machine, in p.u., which is assumed to be constant
E'_{qj}	internal transient voltage of the j -th machine, in p.u., which is assumed to be constant
F_{IP_i}	fraction of the turbine power generated by the intermediate pressure (IP) section
H_i	inertia constant of the i -th machine, in sec
K_{e_i}	gain of the i -th machine's speed governor
K_{m_i}	gain of the i -th machine's turbine
P_{c_i}	power control input of the i -th machine
p_{ij}	constant indicating if the i -th machine has a connection with the j -th machine; either 0 or 1
P_{m_i}	mechanical power of the i -th machine, in p.u.
R_i	regulation constant of the i -th machine, in p.u.
T_{e_i}	time constant of the i -th machine's speed governor
T_{m_i}	time constant of the i -th machine's turbine
X_{e_i}	steam valve opening of the i -th machine, in p.u.

I. INTRODUCTION

The recently proposed decentralized control strategies for multimachine power systems can be classified as decentralized turbine/governor control strategy [1], [2] and decentralized excitation control strategy [3], [4]. In [3], [5], [6], nonlinear control techniques were employed to improve the transient stability of power systems. However, these nonlinear controllers are characterized by high design complexity, which make them harder to implement than their linear counterparts proposed in [1], [2], [7]–[10]. In [8],

Karanjit Kalsi, Jianming Lian and Stanislaw H. Żak are with the School of Electrical and Computer Engineering, Purdue University, IN 47907, USA. {kkalsi, jlian, zak}@purdue.edu.

linear matrix inequalities (LMIs) were employed to develop a robust decentralized turbine/governor control strategy, while, in [10], LMIs were used to develop a decentralized excitation control strategy. In [7], the feedback gain matrix for the decentralized turbine/governor controller was obtained by solving an algebraic Riccati equation based on the bounds of the machine parameters. The main drawback of all the above discussed strategies is that they require the availability of the subsystem's states for the controller implementation. However, this cannot be guaranteed for the multimachine power system. To relax this restriction, Jiang, Wu and Wen [11] proposed high-gain observer based decentralized output feedback controller for excitation control. For turbine/governor control, Jain and Khorrami [12] proposed a decentralized output feedback based nonlinear controller. To the best of our knowledge, there has been no decentralized output feedback based linear controller developed for turbine/governor control.

In this paper, we propose a novel decentralized dynamic output feedback linear controller for turbine/governor control to stabilize the multimachine power system against faults and disturbances. We obtain a feedback gain matrix for each local controller by solving two LMIs. Local sliding mode observers are used to estimate the subsystems' states for the controller implementation. The main advantage of the sliding mode observer over the Luenberger observer and the high-gain observer is its capability of reconstructing the unknown interconnections between the subsystems.

II. MULTIMACHINE POWER SYSTEM MODELING

We consider a class of multimachine power systems consisting of N interconnected machines under turbine/governor control. The dynamics of various components of this N -machine power system can be found in [5], [7], [8], [13], [14]. Let $\mathbf{x}_i = [\delta_i \ \omega_i \ P_{m_i} \ X_{e_i}]^T$ denote the state vector of each machine. Then the dynamics of the i -th machine, $i = 1, \dots, N$, can be represented as

$$\dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i + \mathbf{B}_{i1} u_{i1} + \mathbf{B}_{i2} u_{i2}(\mathbf{x}), \quad (1)$$

$$\mathbf{y}_i = \mathbf{C}_i \mathbf{x}_i, \quad (2)$$

where

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{D_i}{2H_i} & \frac{\omega_o}{2H_i}(1 - F_{IP_i}) & \frac{\omega_o}{2H_i} F_{IP_i} \\ 0 & 0 & -\frac{1}{T_{m_i}} & \frac{K_{m_i}}{T_{m_i}} \\ 0 & -\frac{K_{e_i}}{T_{e_i} R_i \omega_o} & 0 & \frac{1}{T_{e_i}} \end{bmatrix},$$

$$\mathbf{B}_{i1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{e_i}} \end{bmatrix}, \quad \mathbf{B}_{i2} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{C}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad u_{i1} = P_{c_i},$$

and

$$u_{i2}(\mathbf{x}) = \sum_{j=1, j \neq i}^N p_{ij} \frac{\omega_o E'_{q_i} E'_{q_j} B_{ij}}{2H_i} \sin(\delta_i - \delta_j).$$

It is easy to verify that $|u_{i2}(\mathbf{x})| \leq \rho_i$ for some $\rho_i > 0$. Consider the following decentralized state feedback controller

$$u_{i1} = \mathbf{k}_i (\mathbf{x}_i - \mathbf{x}_i^d), \quad (3)$$

where $\mathbf{x}_i^d = [\delta_i^d \ \omega_i^d \ P_{m_i}^d \ X_{e_i}^d]^\top$ is an operating point and \mathbf{k}_i is the feedback gain matrix. Let $\mathbf{x}_i^e = [\delta_i^e \ \omega_i^e \ P_{m_i}^e \ X_{e_i}^e]^\top$ denote the equilibrium state of (1) corresponding to the equilibrium input u_{i1}^e . It follows from (3) that the equilibrium state \mathbf{x}_i^e satisfies the following algebraic equation,

$$\mathbf{0} = \mathbf{A}\mathbf{x}_i^e + \mathbf{B}_{i1}u_{i1}^e + \mathbf{B}_{i2}u_{i2}(\mathbf{x}^e),$$

where $u_{i1}^e = \mathbf{k}_i(\mathbf{x}_i^e - \mathbf{x}_i^d)$. To study the stability of the power system (1) and (2) driven by the controller (3), we consider the perturbed system about the equilibrium state. Let $\Delta\mathbf{x} = [\Delta\mathbf{x}_1^\top \ \cdots \ \Delta\mathbf{x}_N^\top]^\top$ with $\Delta\mathbf{x}_i = [\Delta x_{i1} \ \Delta x_{i2} \ \Delta x_{i3} \ \Delta x_{i4}]^\top$ denoting the deviations of δ_i , ω_i , P_{m_i} and X_{e_i} , respectively, from their equilibrium values, that is,

$$\Delta\mathbf{x}_i = [\delta_i - \delta_i^e \ \omega_i - \omega_i^e \ P_{m_i} - P_{m_i}^e \ X_{e_i} - X_{e_i}^e]^\top,$$

where $\omega_i^e = 0$. Then the dynamics of the i -th perturbed system can be represented as

$$\Delta\dot{\mathbf{x}}_i = \mathbf{A}_i\Delta\mathbf{x}_i + \mathbf{B}_{i1}\Delta u_{i1} + \mathbf{z}_i(\Delta\mathbf{x}), \quad (4)$$

$$\Delta\mathbf{y}_i = \mathbf{C}_i\Delta\mathbf{x}_i, \quad (5)$$

where $\Delta u_{i1} = u_{i1} - u_{i1}^e = \mathbf{k}_i\Delta\mathbf{x}_i$ and $\mathbf{z}_i(\Delta\mathbf{x}) = \mathbf{B}_{i2}(u_{i2}(\mathbf{x}) - u_{i2}(\mathbf{x}^e))$. In the following, we show that $\mathbf{z}_i(\Delta\mathbf{x})$ satisfies the following quadratic constraint

$$\mathbf{z}_i^\top(\Delta\mathbf{x})\mathbf{z}_i(\Delta\mathbf{x}) \leq v_i^2 \Delta\mathbf{x}^\top \mathbf{Z}_i^\top \mathbf{Z}_i \Delta\mathbf{x}, \quad (6)$$

where v_i is a known positive constant and \mathbf{Z}_i is a known interconnection matrix. In our derivation of (6), we use some ideas from [8]. Let $\alpha_{ij} = \omega_o E'_{q_i} E'_{q_j} B_{ij} / 2H_i$ and $p_{ij} = 1$ for interconnected machines. Applying standard trigonometric identities, we can represent $\mathbf{z}_i(\Delta\mathbf{x})$ as

$$\mathbf{z}_i(\Delta\mathbf{x}) = \mathbf{B}_{i2} \sum_{j=1, j \neq i}^N \alpha_{ij} \gamma_{ij} \sin w_{ij}, \quad (7)$$

where $\gamma_{ij} = 2 \cos((\delta_i - \delta_j + \delta_i^e - \delta_j^e) / 2)$ and

$$w_{ij} = \frac{1}{2} [(\delta_i - \delta_i^e) - (\delta_j - \delta_j^e)]. \quad (8)$$

Recall that $\Delta x_{i1} = \delta_i - \delta_i^e$ and $\Delta x_{j1} = \delta_j - \delta_j^e$. We can therefore represent (8) as

$$w_{ij} = \frac{1}{2} (\Delta x_{i1} - \Delta x_{j1}), \quad j \neq i.$$

To proceed, let $\boldsymbol{\gamma}_i = [\gamma_{i1} \ \cdots \ \gamma_{i(i-1)} \ \gamma_{i(i+1)} \ \cdots \ \gamma_{iN}]^\top$, $\mathbf{U}_i = \text{diag}[\alpha_{i1} \ \cdots \ \alpha_{i(i-1)} \ \alpha_{i(i+1)} \ \cdots \ \alpha_{iN}]$ and $\mathbf{h}_i = [\sin w_{i1} \ \cdots \ \sin w_{i(i-1)} \ \sin w_{i(i+1)} \ \cdots \ \sin w_{iN}]^\top$. Then we rewrite (7) as $\mathbf{z}_i(\Delta\mathbf{x}) = \mathbf{B}_{i2} \boldsymbol{\gamma}_i^\top \mathbf{U}_i \mathbf{h}_i$. Thus, we obtain

$$\mathbf{z}_i^\top(\Delta\mathbf{x})\mathbf{z}_i(\Delta\mathbf{x}) = \mathbf{h}_i^\top \boldsymbol{\Psi}_i \mathbf{h}_i, \quad (9)$$

where $\boldsymbol{\Psi}_i = \mathbf{U}_i \boldsymbol{\gamma}_i \boldsymbol{\gamma}_i^\top \mathbf{U}_i$. It follows from [5] that $E'_{q_i} E'_{q_j} B_{ij}$ is bounded. Thus, we have $|\alpha_{ij}| \leq \alpha_{ij\max}$ for some $\alpha_{ij\max} > 0$. Then it can be easily verified that each element of $\boldsymbol{\Psi}_i$, ψ_{kj} , satisfies that $|\psi_{kj}| \leq 4\alpha_{ik\max} \alpha_{ij\max}$. Applying the inequality

$$|\sin w_{ij}| |\sin w_{ik}| \leq \frac{w_{ij}^2 + w_{ik}^2}{2},$$

we rewrite (9) as

$$\mathbf{z}_i^\top(\Delta\mathbf{x})\mathbf{z}_i(\Delta\mathbf{x}) \leq \mathbf{w}_i^\top \mathbf{D}_i \mathbf{w}_i, \quad (10)$$

where $\mathbf{w}_i = [w_{i1} \ \cdots \ w_{i(i-1)} \ w_{i(i+1)} \ \cdots \ w_{iN}]^\top$ and $\mathbf{D}_i = \text{diag}[d_{i1} \ \cdots \ d_{i(i-1)} \ d_{i(i+1)} \ \cdots \ d_{iN}]$ with

$$d_{ik} = 4\alpha_{ik\max} \sum_{j=1, j \neq i}^N \alpha_{ij\max} > 0. \quad (11)$$

Let $\boldsymbol{\Theta}_{i1} \in \mathbb{R}^{(N-1) \times (N-1)}$ and $\boldsymbol{\Theta}_{i2} \in \mathbb{R}^{N \times N}$ be defined as

$$\boldsymbol{\Theta}_{i1} = \begin{bmatrix} 0 & \cdots & 0 & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

and

$$\boldsymbol{\Theta}_{i2} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Then we can express \mathbf{w}_i as $\mathbf{w}_i = \frac{1}{2} \mathbf{T}_{(i)} \Delta\mathbf{x}_{(1)}$, where $\Delta\mathbf{x}_{(1)} = [\Delta x_{11} \ \cdots \ \Delta x_{N1}]^\top$ and $\mathbf{T}_{(i)} \in \mathbb{R}^{(N-1) \times N}$ is determined by $\mathbf{T}_{(i)} = \boldsymbol{\Theta}_{i1} \mathbf{T}_{(i-1)} \boldsymbol{\Theta}_{i2}$ for $i = 2, \dots, N$ with

$$\mathbf{T}_{(1)} = \begin{bmatrix} 1 & | & -1 & 0 & 0 & \cdots & 0 \\ 1 & | & 0 & -1 & 0 & \cdots & 0 \\ \vdots & | & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & | & 0 & \cdots & 0 & -1 & 0 \\ 1 & | & 0 & \cdots & 0 & 0 & -1 \end{bmatrix}. \quad (12)$$

Therefore, we can represent (10) as

$$\mathbf{z}_i^\top(\Delta\mathbf{x})\mathbf{z}_i(\Delta\mathbf{x}) \leq \frac{1}{4} \Delta\mathbf{x}_{(1)}^\top \mathbf{T}_{(i)}^\top \mathbf{D}_i \mathbf{T}_{(i)} \Delta\mathbf{x}_{(1)}. \quad (13)$$

Let $\mathbf{Z}_{(i)} = \frac{1}{2} \mathbf{D}_i^{\frac{1}{2}} \mathbf{T}_{(i)}$, where

$$\mathbf{Z}_{(i)} = \begin{bmatrix} Z_{i11} & \cdots & Z_{i1N} \\ \vdots & \ddots & \vdots \\ Z_{i(N-1)1} & \cdots & Z_{i(N-1)N} \end{bmatrix}, \quad (14)$$

and let

$$\mathbf{Z}_i = \begin{bmatrix} \mathbf{Z}_{i11} & \mathbf{0}_3^\top & \cdots & \mathbf{Z}_{i1N} & \mathbf{0}_3^\top \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{Z}_{i(N-1)1} & \mathbf{0}_3^\top & \cdots & \mathbf{Z}_{i(N-1)N} & \mathbf{0}_3^\top \end{bmatrix}. \quad (15)$$

Thus, we have $\mathbf{z}_i^\top(\Delta\mathbf{x})\mathbf{z}_i(\Delta\mathbf{x}) \leq \Delta\mathbf{x}^\top \mathbf{Z}_i^\top \mathbf{Z}_i \Delta\mathbf{x}$ following from (13)–(15), which satisfies (6) with $v_i = 1$.

In this paper, the control objective is to develop a decentralized dynamic output feedback controller of the form

$$\mathbf{u}_{i1} = \mathbf{k}_i(\hat{\mathbf{x}}_i - \mathbf{x}_i^d)$$

instead of the decentralized state feedback controller (3) to stabilize the multimachine power system against faults and disturbances, where $\hat{\mathbf{x}}_i$ is the estimate of the state vector \mathbf{x}_i .

III. LOCAL SLIDING MODE OBSERVER DESIGN

In this section, we consider the design of local observers for the following generalized versions of the systems modeled by (1) and (2),

$$\dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i + \mathbf{B}_{i1} \mathbf{u}_{i1} + \mathbf{B}_{i2} \mathbf{u}_{i2}(\mathbf{x}), \quad (16)$$

$$\mathbf{y}_i = \mathbf{C}_i \mathbf{x}_i. \quad (17)$$

where $\mathbf{x}_i \in \mathbb{R}^{n_i}$, $\mathbf{y}_i \in \mathbb{R}^{p_i}$, $\mathbf{u}_{i1} \in \mathbb{R}^{m_{i1}}$ and $\mathbf{u}_{i1} = \mathbf{K}_i(\hat{\mathbf{x}}_i - \mathbf{x}_i^d)$. The unknown input $\mathbf{u}_{i2} \in \mathbb{R}^{m_{i2}}$ satisfies $\|\mathbf{u}_{i2}(\mathbf{x})\| \leq \rho_i$ for some $\rho_i > 0$, where $\|\cdot\|$ is the standard Euclidean norm. It follows from [15] that local sliding mode observers can be constructed for the system described by (16) and (17), if

$$\text{rank } \mathbf{B}_{i2} = \text{rank}(\mathbf{C}_i \mathbf{B}_{i2}) = r_i, \quad (18)$$

where $r_i \leq m_{i2} \leq p_i$, and the system zeros of the system model given by the triple $(\mathbf{A}_i, \mathbf{B}_{i2}, \mathbf{C}_i)$ are located in the open left-hand complex plane, that is,

$$\text{rank} \begin{bmatrix} s\mathbf{I}_{n_i} - \mathbf{A}_i & \mathbf{B}_{i2} \\ \mathbf{C}_i & \mathbf{O} \end{bmatrix} = n_i + r_i \quad (19)$$

for all s such that $\Re(s) \geq 0$. It is easy to verify that the multimachine power system modeled by (1) and (2) satisfies the above two conditions.

The constructed local sliding mode observer for the i -th subsystem has the form,

$$\dot{\hat{\mathbf{x}}}_i = (\mathbf{A}_i - \mathbf{L}_i \mathbf{C}_i) \hat{\mathbf{x}}_i + \mathbf{L}_i \mathbf{y}_i + \mathbf{B}_{i1} \mathbf{u}_{i1} - \mathbf{B}_{i2} \mathbf{E}_i(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i), \quad (20)$$

with

$$\mathbf{E}_i(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i) = \begin{cases} \eta_i \frac{\mathbf{F}_i(\hat{\mathbf{y}}_i - \mathbf{y}_i)}{\|\mathbf{F}_i(\hat{\mathbf{y}}_i - \mathbf{y}_i)\|} & \text{if } \mathbf{F}_i(\hat{\mathbf{y}}_i - \mathbf{y}_i) \neq \mathbf{0} \\ \mathbf{0} & \text{if } \mathbf{F}_i(\hat{\mathbf{y}}_i - \mathbf{y}_i) = \mathbf{0}, \end{cases}$$

where η_i is a positive design parameter, and $\mathbf{L}_i \in \mathbb{R}^{n_i \times p_i}$ and $\mathbf{F}_i \in \mathbb{R}^{m_{i2} \times p_i}$ are matrices satisfying

$$(\mathbf{A}_i - \mathbf{L}_i \mathbf{C}_i)^\top \mathbf{P}_i^o + \mathbf{P}_i^o (\mathbf{A}_i - \mathbf{L}_i \mathbf{C}_i) = -\mathbf{Q}_i^o \quad (21)$$

and

$$\mathbf{F}_i \mathbf{C}_i = \mathbf{B}_{i2}^\top \mathbf{P}_i^o \quad (22)$$

for some symmetric positive definite $\mathbf{P}_i^o \in \mathbb{R}^{n_i \times n_i}$ and $\mathbf{Q}_i^o \in \mathbb{R}^{n_i \times n_i}$. It follows from [16] that the $\hat{\mathbf{x}}_i$ converges

asymptotically to \mathbf{x}_i for $\eta_i \geq \rho_i$. Moreover, it is shown in [15] that if \mathbf{B}_{i2} is of full rank, then we can obtain $\mathbf{u}_{i2}(\mathbf{x}) = -\mathbf{E}_i^{eq}(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i)$ as $t \rightarrow \infty$, where $\mathbf{E}_i^{eq}(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i)$ is the so-called ‘‘equivalent injection term’’ [17]. Thus, we can reconstruct the unknown interconnections using local sliding mode observers.

The design procedures for the matrices \mathbf{L}_i , \mathbf{F}_i and \mathbf{P}_i^o , which satisfy (21) and (22), is given in [15]. For the subsequent stability analysis of the closed-loop system, we need the following lemma, whose proof is given in [15].

Lemma: Let $\mathbf{Q}_i = (\mathbf{B}_{i1} \mathbf{K}_i)^\top (\mathbf{B}_{i1} \mathbf{K}_i)$. Then we have $\lambda_{\min}(\mathbf{Q}_i^o) > \lambda_{\max}(\mathbf{Q}_i)$.

IV. DECENTRALIZED DYNAMIC OUTPUT FEEDBACK CONTROLLER CONSTRUCTION

In this section, we design a decentralized dynamic output feedback controller in the form of $\mathbf{u}_{i1} = \mathbf{K}_i(\hat{\mathbf{x}}_i - \mathbf{x}_i^d)$ for the generalized system (16) and (17). The feedback gain matrix \mathbf{K}_i is derived by solving Linear Matrix Inequalities (LMIs). The stability of the closed-loop system driven by the decentralized dynamic output feedback controller is then analyzed. To proceed, consider the following generalized version of the perturbed system modeled by (4) and (5),

$$\Delta \dot{\mathbf{x}}_i = \mathbf{A}_i \Delta \mathbf{x}_i + \mathbf{B}_{i1} \Delta \mathbf{u}_{i1} + \mathbf{z}_i(\Delta \mathbf{x}), \quad (23)$$

$$\Delta \mathbf{y}_i = \mathbf{C}_i \Delta \mathbf{x}_i. \quad (24)$$

Then we can represent the overall system consisting of (23) and (24), for $i = 1, \dots, N$, in the compact form

$$\Delta \dot{\mathbf{x}} = \mathbf{A}_D \Delta \mathbf{x} + \mathbf{B}_{1D} \Delta \mathbf{u}_1 + \mathbf{z}(\Delta \mathbf{x}), \quad (25)$$

$$\Delta \mathbf{y} = \mathbf{C}_D \Delta \mathbf{x}, \quad (26)$$

where \mathbf{A}_D , \mathbf{B}_{1D} , and \mathbf{C}_D are block-diagonal matrices, $\Delta \mathbf{u}_1 = [\Delta \mathbf{u}_{11}^\top \cdots \Delta \mathbf{u}_{N1}^\top]^\top$, $\Delta \mathbf{y} = [\Delta \mathbf{y}_1^\top \cdots \Delta \mathbf{y}_N^\top]^\top$ and $\mathbf{z}(\Delta \mathbf{x}) = [\mathbf{z}_1^\top(\Delta \mathbf{x}) \cdots \mathbf{z}_N^\top(\Delta \mathbf{x})]^\top$ satisfies the following quadratic constraint

$$\mathbf{z}^\top(\Delta \mathbf{x}) \mathbf{z}(\Delta \mathbf{x}) \leq \Delta \mathbf{x}^\top \left(\sum_{i=1}^N v_i^2 \mathbf{Z}_i^\top \mathbf{Z}_i \right) \Delta \mathbf{x}. \quad (27)$$

Let $\hat{\mathbf{x}} = [\hat{\mathbf{x}}_1^\top \cdots \hat{\mathbf{x}}_N^\top]^\top$ and $\mathbf{K}_D = \text{diag}[\mathbf{K}_1 \cdots \mathbf{K}_N]$. The controller $\Delta \mathbf{u}_1$ has the form

$$\Delta \mathbf{u}_1 = \mathbf{K}_D (\hat{\mathbf{x}} - \mathbf{x}^e). \quad (28)$$

Let $\mathbf{P}_D^c = \text{diag}[\mathbf{P}_1^c \cdots \mathbf{P}_N^c]$ and $\mathbf{P}_D^o = \text{diag}[\mathbf{P}_1^o \cdots \mathbf{P}_N^o]$, where \mathbf{P}_i^c is symmetric positive definite and \mathbf{P}_i^o is defined in Section III. Consider the Lyapunov function candidate

$$V = \Delta \mathbf{x}^\top \mathbf{P}_D^c \Delta \mathbf{x} + \mathbf{e}^\top \mathbf{P}_D^o \mathbf{e},$$

where $\mathbf{e} = [\mathbf{e}_1^\top \cdots \mathbf{e}_N^\top]^\top$ with $\mathbf{e}_i = \hat{\mathbf{x}}_i - \mathbf{x}_i$. Evaluating the time derivative of V on the solutions to (25), we obtain

$$\begin{aligned} \dot{V} &= 2\Delta \mathbf{x}^\top \mathbf{P}_D^c \Delta \dot{\mathbf{x}} + 2\mathbf{e}^\top \mathbf{P}_D^o \dot{\mathbf{e}} \\ &= 2\Delta \mathbf{x}^\top \mathbf{P}_D^c (\mathbf{A}_D + \mathbf{B}_{1D} \mathbf{K}_D) \Delta \mathbf{x} + 2\mathbf{e}^\top \mathbf{P}_D^o \dot{\mathbf{e}} \\ &\quad + 2\Delta \mathbf{x}^\top \mathbf{P}_D^c (\mathbf{B}_{1D} \mathbf{K}_D) \mathbf{e} + 2\Delta \mathbf{x}^\top \mathbf{P}_D^c \mathbf{z}(\Delta \mathbf{x}). \end{aligned}$$

Using the inequality, $2\mathbf{a}^\top \mathbf{b} \leq \mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b}$, where \mathbf{a} and \mathbf{b} are arbitrary vectors, we obtain

$$2\Delta \mathbf{x}^\top \mathbf{P}_D^c (\mathbf{B}_{1D} \mathbf{K}_D) \mathbf{e} \leq \Delta \mathbf{x}^\top \mathbf{P}_D^c \mathbf{P}_D^c \Delta \mathbf{x} + \mathbf{e}^\top \mathbf{Q}_D \mathbf{e},$$

where $\mathbf{Q}_D = (\mathbf{B}_{1D} \mathbf{K}_D)^\top (\mathbf{B}_{1D} \mathbf{K}_D)$. It follows that

$$\dot{V} \leq \dot{V}_c + \dot{V}_o, \quad (29)$$

where

$$\begin{aligned} \dot{V}_c &= 2\Delta \mathbf{x}^\top \mathbf{P}_D^c (\mathbf{A}_D + \mathbf{B}_{1D} \mathbf{K}_D) \Delta \mathbf{x} \\ &\quad + \Delta \mathbf{x}^\top \mathbf{P}_D^c \mathbf{P}_D^c \Delta \mathbf{x} + 2\Delta \mathbf{x}^\top \mathbf{P}_D^c \mathbf{z}(\Delta \mathbf{x}), \end{aligned} \quad (30)$$

and $\dot{V}_o = 2\mathbf{e}^\top \mathbf{P}_D^o \dot{\mathbf{e}} + \mathbf{e}^\top \mathbf{Q}_D \mathbf{e}$. If \dot{V}_c and \dot{V}_o are both negative, we have $\dot{V} < 0$, which implies the asymptotic stability of the closed-loop system. In the following subsections, we first derive the matrices \mathbf{P}_D^c and \mathbf{K}_D using LMIs similar to [8], [18] such that $\dot{V}_c < 0$. Then we analyze the stability of the closed-loop system.

A. Feedback Gain Matrix Selection

It follows from (30) that $\dot{V}_c < 0$ implies the existence of \mathbf{P}_D^c and \mathbf{K}_D such that $\mathbf{P}_D^c > 0$ and

$$\begin{aligned} &2\Delta \mathbf{x}^\top \mathbf{P}_D^c (\mathbf{A}_D + \mathbf{B}_{1D} \mathbf{K}_D) \Delta \mathbf{x} \\ &\quad + \Delta \mathbf{x}^\top \mathbf{P}_D^c \mathbf{P}_D^c \Delta \mathbf{x} + 2\Delta \mathbf{x}^\top \mathbf{P}_D^c \mathbf{z}(\Delta \mathbf{x}) < 0. \end{aligned} \quad (31)$$

On the other hand, it follows from (27) that

$$\Delta \mathbf{x}^\top \left(\sum_{i=1}^N v_i^2 \mathbf{Z}_i^\top \mathbf{Z}_i \right) \Delta \mathbf{x} - \mathbf{z}^\top(\Delta \mathbf{x}) \mathbf{z}(\Delta \mathbf{x}) \geq 0.$$

Thus, we can guarantee that (31) holds if there exists some $\tau > 0$ so that

$$\begin{aligned} &\Delta \mathbf{x}^\top \left((\mathbf{A}_D + \mathbf{B}_{1D} \mathbf{K}_D)^\top \mathbf{P}_D^c + \mathbf{P}_D^c (\mathbf{A}_D + \mathbf{B}_{1D} \mathbf{K}_D) \right. \\ &\quad \left. + \mathbf{P}_D^c \mathbf{P}_D^c + \tau \left(\sum_{i=1}^N v_i^2 \mathbf{Z}_i^\top \mathbf{Z}_i \right) \right) \Delta \mathbf{x} \\ &\quad + 2\Delta \mathbf{x}^\top \mathbf{P}_D^c \mathbf{z}(\Delta \mathbf{x}) - \tau \mathbf{z}^\top(\Delta \mathbf{x}) \mathbf{z}(\Delta \mathbf{x}) < 0. \end{aligned} \quad (32)$$

Let $\mathbf{A}_D^c = \mathbf{A}_D + \mathbf{B}_{1D} \mathbf{K}_D$. It follows from (32) that, in order to have $\dot{V}_c < 0$, it is equivalent to find \mathbf{P}_D^c and \mathbf{K}_D such that

$$\begin{bmatrix} \mathbf{A}_D^{c\top} \mathbf{P}_D^c + \mathbf{P}_D^c \mathbf{A}_D^c + \mathbf{P}_D^c \mathbf{P}_D^c & & \mathbf{P}_D^c \\ & + \tau \sum_{i=1}^N v_i^2 \mathbf{Z}_i^\top \mathbf{Z}_i & \\ & \mathbf{P}_D^c & -\tau \mathbf{I} \end{bmatrix} < 0$$

$$\mathbf{P}_D^c > 0$$

for some $\tau > 0$. Pre- and post-multiplying the above matrix by $\text{diag}[\tau \mathbf{P}_D^c^{-1} \mathbf{I}]$ and defining $\mathbf{Y}_D = \tau \mathbf{P}_D^c^{-1}$, we obtain

$$\begin{bmatrix} \mathbf{Y}_D \mathbf{A}_D^{c\top} + \mathbf{A}_D^c \mathbf{Y}_D + \tau \mathbf{I} & & \mathbf{I} \\ & + \mathbf{Y}_D \left(\sum_{i=1}^N v_i^2 \mathbf{Z}_i^\top \mathbf{Z}_i \right) \mathbf{Y}_D & \\ & \mathbf{I} & -\mathbf{I} \end{bmatrix} < 0$$

$$\mathbf{Y}_D > 0.$$

Applying the Schur complement to the above, we obtain

$$\begin{aligned} &\mathbf{Y}_D \mathbf{A}_D^{c\top} + \mathbf{A}_D^c \mathbf{Y}_D \\ &\quad + \mathbf{Y}_D \left(\sum_{i=1}^N v_i^2 \mathbf{Z}_i^\top \mathbf{Z}_i \right) \mathbf{Y}_D + \tau \mathbf{I} + \mathbf{I} < 0 \\ &\mathbf{Y}_D > 0, \end{aligned}$$

which is equivalent to

$$\begin{bmatrix} \mathbf{W}_D & \mathbf{I} & \mathbf{Y} \mathbf{Z}_1^\top & \cdots & \mathbf{Y} \mathbf{Z}_N^\top \\ \mathbf{I} & -\mathbf{I} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{Z}_1 \mathbf{Y}_D & \mathbf{O} & -\gamma_1 \mathbf{I} & \cdots & \mathbf{O} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_N \mathbf{Y}_D & \mathbf{O} & \mathbf{O} & \cdots & -\gamma_N \mathbf{I} \end{bmatrix} < 0 \quad (33)$$

$$\mathbf{Y}_D > 0$$

with $\gamma_i = 1/v_i^2$ and $\mathbf{W}_D = \mathbf{Y}_D \mathbf{A}_D^\top + \mathbf{A}_D \mathbf{Y}_D + \mathbf{B}_{1D} \mathbf{K}_D \mathbf{Y}_D + (\mathbf{B}_{1D} \mathbf{K}_D \mathbf{Y}_D)^\top + \tau \mathbf{I}$. Note that (33) is a bilinear matrix inequality. To proceed, we introduce the transformation, $\mathbf{K}_D \mathbf{Y}_D = \mathbf{M}_D$, as in [8], [18]. Applying the above transformation in (33), we obtain

$$\begin{bmatrix} \tilde{\mathbf{W}}_D & \mathbf{I} & \mathbf{Y} \mathbf{Z}_1^\top & \cdots & \mathbf{Y} \mathbf{Z}_N^\top \\ \mathbf{I} & -\mathbf{I} & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{Z}_1 \mathbf{Y}_D & \mathbf{O} & -\gamma_1 \mathbf{I} & \cdots & \mathbf{O} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z}_N \mathbf{Y}_D & \mathbf{O} & \mathbf{O} & \cdots & -\gamma_N \mathbf{I} \end{bmatrix} < 0 \quad (34)$$

$$\mathbf{Y}_D > 0 \quad (35)$$

where

$$\tilde{\mathbf{W}}_D = \mathbf{Y}_D \mathbf{A}_D^\top + \mathbf{A}_D \mathbf{Y}_D + \mathbf{B}_{1D} \mathbf{M}_D + (\mathbf{B}_{1D} \mathbf{M}_D)^\top + \tau \mathbf{I}.$$

Therefore, if the LMIs given by (34) and (35) are feasible for some $\tau > 0$, we can obtain \mathbf{P}_D^c and \mathbf{K}_D as $\mathbf{P}_D^c = \frac{1}{\tau} \mathbf{Y}_D^{-1}$ and $\mathbf{K}_D = \mathbf{M}_D \mathbf{Y}_D^{-1}$.

B. Stability Analysis

We now proceed with the stability analysis of the closed-loop system.

Theorem: If the LMIs given by (34) and (35) are feasible for some $\tau > 0$, then the closed-loop system (25) and (26) driven by the dynamic output feedback controller (28) with (20) is asymptotically stable.

Proof: Recall from (29) that $\dot{V} \leq \dot{V}_c + \dot{V}_o$. It is clear from the above that if the LMIs given by (34) and (35) are feasible for some $\tau > 0$, then $\dot{V}_c < 0$. Thus, it remains to show that $\dot{V}_o < 0$. It follows from (16) and (20) that

$$\dot{\mathbf{e}}_i = (\mathbf{A}_i - \mathbf{L}_i \mathbf{C}_i) \mathbf{e}_i - \mathbf{B}_{i2} \mathbf{u}_{i2}(\mathbf{x}) - \mathbf{B}_{i2} \mathbf{E}_i(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i). \quad (36)$$

Then it follows from (36) that

$$\begin{aligned} \dot{V}_o &= 2\mathbf{e}^\top \mathbf{P}_D^o \dot{\mathbf{e}} + \mathbf{e}^\top \mathbf{Q}_D \mathbf{e} \\ &= \sum_{i=1}^N (2\mathbf{e}_i^\top \mathbf{P}_i^o (\mathbf{A}_i - \mathbf{L}_i \mathbf{C}_i) \mathbf{e}_i - 2\mathbf{e}_i^\top \mathbf{P}_i^o (\mathbf{B}_{i2} \mathbf{u}_{i2}) \\ &\quad - 2\mathbf{e}_i^\top \mathbf{P}_i^o \mathbf{B}_{i2} \mathbf{E}_i(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i) + \mathbf{e}_i^\top \mathbf{Q}_i \mathbf{e}_i), \end{aligned} \quad (37)$$

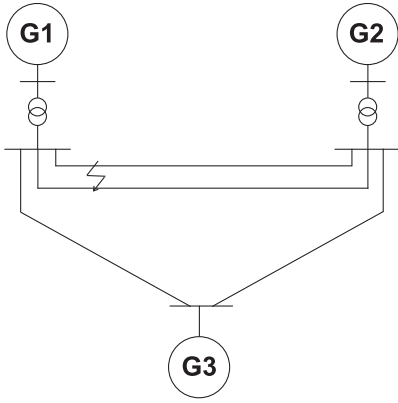


Fig. 1. Three-machine power system

where $\mathbf{Q}_i = (\mathbf{B}_{i1}\mathbf{K}_i)^\top(\mathbf{B}_{i1}\mathbf{K}_i)$. If $\mathbf{F}_i(\hat{\mathbf{y}}_i - \mathbf{y}_i) = \mathbf{0}$, it follows from (22) that $\mathbf{B}_{i2}^\top \mathbf{P}_i^\circ \mathbf{e}_i = \mathbf{0}$ and therefore

$$-2\mathbf{e}_i^\top \mathbf{P}_i^\circ \mathbf{B}_{i2} \mathbf{E}_i(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i) - 2\mathbf{e}_i^\top \mathbf{P}_i^\circ (\mathbf{B}_{i2} \mathbf{u}_{i2}) = 0. \quad (38)$$

If, on the other hand, $\mathbf{F}_i(\hat{\mathbf{y}}_i - \mathbf{y}_i) \neq \mathbf{0}$, it follows from (22) and $\eta_i \geq \rho_i$ that

$$\begin{aligned} & -2\mathbf{e}_i^\top \mathbf{P}_i^\circ \mathbf{B}_{i2} \mathbf{E}_i(\mathbf{y}_i, \hat{\mathbf{y}}_i, \eta_i) - 2\mathbf{e}_i^\top \mathbf{P}_i^\circ (\mathbf{B}_{i2} \mathbf{u}_{i2}) \\ &= -\frac{2\eta_i}{\|\mathbf{F}_i \mathbf{C}_i \mathbf{e}_i\|} (\mathbf{e}_i^\top \mathbf{P}_i^\circ \mathbf{B}_{i2}) (\mathbf{F}_i \mathbf{C}_i \mathbf{e}_i) - 2\mathbf{e}_i^\top \mathbf{P}_i^\circ (\mathbf{B}_{i2} \mathbf{u}_{i2}) \\ &\leq -2(\eta_i - \rho_i) \|\mathbf{e}_i^\top \mathbf{P}_i^\circ \mathbf{B}_{i2}\| \leq 0. \end{aligned} \quad (39)$$

It follows from (37)–(39) that

$$\begin{aligned} \dot{V}_o &\leq \sum_{i=1}^N (2\mathbf{e}_i^\top \mathbf{P}_i^\circ (\mathbf{A}_i - \mathbf{L}_i \mathbf{C}_i) \mathbf{e}_i + \mathbf{e}_i^\top \mathbf{Q}_i \mathbf{e}_i) \\ &\leq \sum_{i=1}^N -(\lambda_{\min}(\mathbf{Q}_i^\circ) - \lambda_{\max}(\mathbf{Q}_i)) \|\mathbf{e}_i\|^2. \end{aligned} \quad (40)$$

It follows from the lemma and (40) that $\lambda_{\min}(\mathbf{Q}_i^\circ) > \lambda_{\max}(\mathbf{Q}_i)$ and then $\dot{V}_o < 0$. Thus, we have

$$\dot{V} \leq \dot{V}_c + \dot{V}_o < 0,$$

which implies that the closed-loop system is asymptotically stable. The proof of the theorem is complete. ■

Remark: When solving the LMIs (34) and (35), we could obtain the feedback gain matrix \mathbf{K}_D of big size, which may not be practical in real applications. Effective methods of restricting the size of the feedback gain matrix \mathbf{K}_D deserves further investigation, which definitely increases the applicability of the proposed decentralized output feedback controller. This problem is left open for future research.

V. CASE STUDY

In this section, we apply the proposed decentralized dynamic output feedback controller to stabilize a three-machine power system shown in Fig. 1, where Generator 3 is assumed to be an infinite bus with the same dynamics as Generator 2 [9]. The parameter values for each machine are the same as given in [7] and are listed in Table I.

TABLE I
MACHINE PARAMETERS

Parameter	Machine 1	Machine 2
H	4	5.1
D	5	3
k_c	1	1
F_{IP}	0.3	0.3
T_m	0.35	0.35
T_e	0.1	0.1
R	0.05	0.05
K_m	1	1
K_e	1	1
ω_o	314.159	314.159

The complete set of plant dynamics used for simulations are give in [5]. It is given in [7] that $\alpha_{12_{\max}} = \alpha_{13_{\max}} = 27.49$ and $\alpha_{21_{\max}} = \alpha_{23_{\max}} = 23.10$. However, with these conservative bounds, the corresponding feedback gain matrix \mathbf{K}_D turns out to be of very big size. Therefore, in order to better illustrate the performance of our decentralized output feedback controller, we use $\alpha_{12_{\max}} = \alpha_{13_{\max}} = 1.6494$ and $\alpha_{21_{\max}} = \alpha_{23_{\max}} = 1.3860$ in our simulations. We choose the operating points, $\mathbf{x}_1^d = [49 \ 0 \ 0.57 \ 0.57]^\top$ and $\mathbf{x}_2^d = [53 \ 0 \ 0.56 \ 0.56]^\top$. Applying the definitions given in (11) and (12), we calculate $\mathbf{T}_{(1)}$, \mathbf{D}_1 , $\mathbf{T}_{(2)}$ and \mathbf{D}_2 . Then we use (14) and (15) to calculate \mathbf{Z}_1 and \mathbf{Z}_2 . Solving the LMIs given by (34) and (35) with $\tau = 1$, we obtain $\mathbf{k}_1 = [-65.0516 \ -6.1643 \ -7.6241 \ -3.2470]$ and $\mathbf{k}_2 = [-63.0106 \ -6.6794 \ -6.9836 \ -2.9745]$. The initial conditions for the first and second sliding-mode observers are chosen to be zero. We also choose $\eta_1 = 114$ and $\eta_2 = 136$. Using the design procedures given in Section III, we obtain

$$\mathbf{L}_1 = \begin{bmatrix} 0 & 2.5 \\ 434284 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{L}_2 = \begin{bmatrix} 0 & 2.5 \\ 407531 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

and $\mathbf{F}_1 = \mathbf{F}_2 = [-1 \ 0]$.

We consider a symmetrical three phase short circuit fault as in [5], [7], which is assumed to be on the transmission line between the first and the second machine. Let λ denote the fraction of the transmission line to the left of the fault. The fault sequence is as follows:

- 1) The system is in the pre-fault steady state;
- 2) At $t = 3.1$ sec, the fault occurs;
- 3) At $t = 3.25$ sec, the fault is removed by opening the breakers of the line at which the fault occurs;
- 4) At $t = 4.0$ sec, the transmission line is restored;
- 5) The system is in the post-fault steady state.

Simulation results for the first and second subsystems with $\lambda = 0.5$ are shown in Fig. 2, where we only show the plots of δ_i due to lack of space. The unknown interconnection reconstruction for each subsystem is shown in Fig. 3. It can be seen that the dynamic output feedback controller performs as expected and the unknown interconnections are reconstructed perfectly.

VI. SUMMARY

In this paper, we have proposed a novel decentralized dynamic output feedback controller to stabilize a class of multimachine power systems against faults and disturbances. The developed control strategy incorporates local sliding mode observers to estimate the subsystem's states for the controller implementation, where a feedback gain matrix is obtained by solving two LMIs. The local sliding mode observers are also able to effectively reconstruct unknown interconnections between machines. This, in fact, can be used to detect faults in multi-machine power systems. The transient stability of the closed-loop system driven by the decentralized observer-based controller is guaranteed.

REFERENCES

- [1] Z. Qu, J. F. Dorsey, J. Bond, and J. D. McCalley, "Application of robust control to sustained oscillations in power systems," *IEEE Trans. Circuits Syst. I*, vol. 39, no. 6, pp. 470–476, Jun. 1992.
- [2] H. Jiang, H. Cai, J. Dorsey, and Z. Qu, "Toward a globally robust decentralized control for large-scale power systems," *IEEE Trans. Control Syst. Technol.*, vol. 5, no. 3, pp. 309–319, Mar. 1997.
- [3] J. W. Chapman, M. D. Ilic, C. A. King, L. Eng, and H. Kaufman, "Stabilizing a multimachine power system via decentralized feedback linearizing excitation control," *IEEE Trans. Power Syst.*, vol. 8, no. 3, pp. 830–839, Aug. 1993.
- [4] C. King, J. Chapman, and M. Ilic, "Feedback linearizing excitation control on a full-scale power system model," *IEEE Trans. Power Syst.*, vol. 9, no. 2, pp. 1102–1109, May 1994.
- [5] G. Guo, Y. Wang, and D. J. Hill, "Nonlinear decentralized control of large-scale power systems," *Automatica*, vol. 36, no. 9, pp. 1275–1289, Sep. 2000.
- [6] Q. Lu, S. Mei, W. Hu, F. F. Wu, Y. Ni, and T. Shen, "Nonlinear decentralized disturbance attenuation excitation control via new recursive design for multimachine power system," *IEEE Trans. Power Syst.*, vol. 16, no. 4, pp. 729–736, Nov. 2001.
- [7] Y. Wang, D. J. Hill, and G. Guo, "Robust decentralized control for multimachine power systems," *IEEE Trans. Circuits Syst. I*, vol. 45, no. 3, pp. 271–279, Mar. 1998.
- [8] D. D. Šiljak, D. M. Stipanovic, and A. I. Zecevic, "Robust decentralized turbine/governor control using linear matrix inequalities," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 715–722, Aug. 2002.
- [9] S. Elloumi and E. B. Braiek, "Robust decentralized control for multimachine power systems—The LMI approach," in *Proc. IEEE International Conference on Systems, Man and Cybernetics*, vol. 6, Yasmine Hammamet, Tunisia, Oct. 2002.
- [10] A. I. Zecevic, G. Neskovic, and D. D. Šiljak, "Robust decentralized exciter control with linear feedback," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 1096–1103, May 2004.
- [11] L. Jiang, Q. H. Wu, and J. Y. Wen, "Decentralized nonlinear adaptive control for multimachine power systems via high-gain perturbation observer," *IEEE Trans. Circuits Syst. I*, vol. 51, no. 10, pp. 2052–2059, Oct. 2004.
- [12] S. Jain and F. Khorrami, "Robust decentralized control of power system utilizing only swing angle measurements," *Int. J. Contr.*, vol. 46, pp. 581–601, 1997.
- [13] P. Kundur, *Power system stability and control*. New York: McGraw-Hill, 1994.
- [14] P. W. Sauer and M. A. Pai, *Power systems dynamics and stability*. Englewood Cliffs, New Jersey: Prentice-Hall, 1998.
- [15] K. Kalsi, J. Lian, and S. H. Žak, "Decentralized control of nonlinear interconnected systems," in *Proc. 16th Mediterranean Conference on Control and Automation*, Corsica, France, Jun. 2008, pp. 952–957.
- [16] S. Hui and S. H. Žak, "Observer design for systems with unknown inputs," *Int. J. Appl. Math. Comput. Sci.*, vol. 15, no. 4, pp. 431–446, 2005.
- [17] C. Edwards and S. K. Spurgeon, *Sliding Mode Control: Theory and Applications*. London, UK: Taylor and Francis Group, 1998.
- [18] D. D. Šiljak and D. M. Stipanovic, "Robust stabilization of nonlinear systems: The LMI approach," *Mathematical Problems in Engineering*, vol. 6, no. 5, pp. 461–493, 2000.

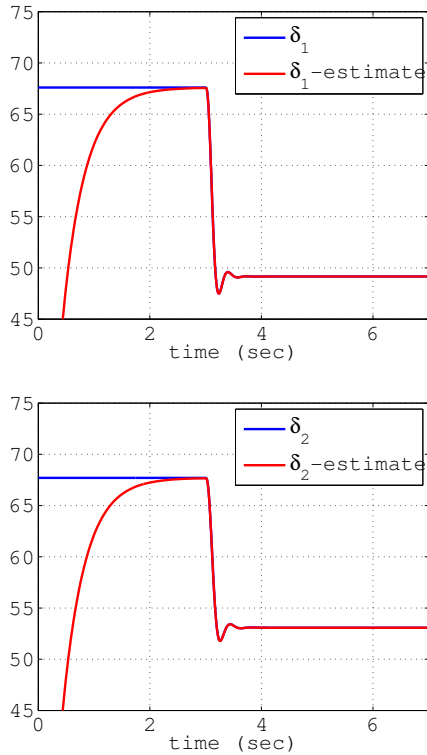


Fig. 2. Controller performance for Generator 1.

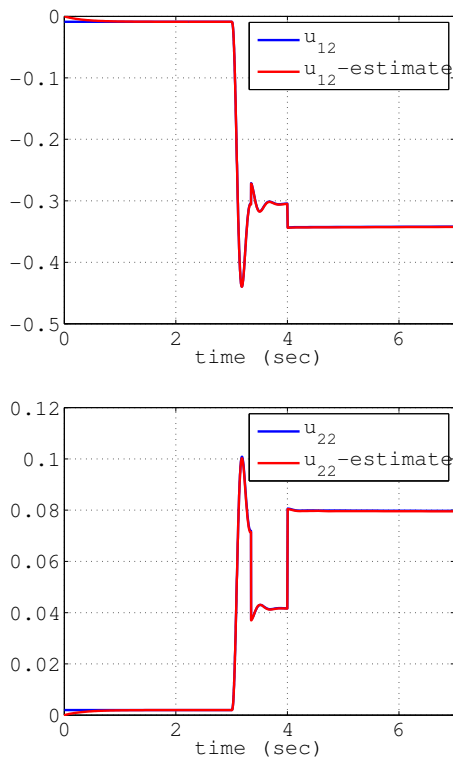


Fig. 3. Unknown interconnection reconstruction.