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Tuning clamping regulators for positive SISO unknown LTI systems: Industrial Hydraulic system

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***Abstract*— In this paper, the authors consider reference tracking and disturbance rejection via tuning clamping regulators for positive systems. In particular, we illustrate experimentally, for an industrial hydraulic system [2], the robustness properties of the latter control law via its application to single-input-single-output (SISO) linear time-invariant (LTI) positive systems under unmeasurable disturbances.**

I. INTRODUCTION

This paper illustrates, via an experimental study of an industrial hydraulic system [2], the effectiveness, ease and robustness of a new controller called *the tuning clamping regulator* for the tracking problem of nonnegative constant reference signals of unknown stable SISO positive LTI systems with *unmeasurable* nonnegative constant disturbances.

Positive systems, which carry the well known property that confines all state and output variables of a system to the nonnegative orthant, appear in numerous applications and in nature. For example, positive systems are visible in biology where they are used to describe the transportation, accumulation, and drainage processes of elements and compounds like hormones, glucose, insulin, metals, etc. In fact, one can simply look at the current state of interest in biological systems [3], and observe that large classes of such systems are positive. However, positive systems do not only radiate within biology, as stocking and industrial systems which involve chemical reactions, heat exchangers, and distillation columns [4] are further examples of where such systems have planted their roots.

For numerous citations on positive systems the reader can refer to [4], [5], [3] and references therein.

This paper is motivated by the study of the servomechanism problem of disturbance rejection, reference tracking, and robustness of positive LTI systems as outlined in [1], where *the mathematical model of the system is unknown which is often the case in practice*. The study of the servomechanism problem for positive systems has also appeared in [5], [6], [7], where the authors considered numerous theoretical aspects of the servomechanism problem for positive LTI systems.

The main purpose of our study is to show that with no mathematical model of a system, the tuning clamping regulator can *robustly* solve the problem of tracking a positive system under disturbances and system changes, i.e. for large perturbations[1] of the nominal plant model, the tuning clamping regulator can achieve asymptotic reference tracking regulation; more importantly, we illustrate the latter via experimental results.

The paper is organized as follows. Terminology is discussed first. The problem of interest and its solution via tuning clamping regulators are presented in Section III. The main experimental results, simulations, and discussions of the paper are described in Section IV, while all concluding remarks complete the paper.

II. BACKGROUND AND PRELIMINARIES

Let the set $\mathbb{R}_+ := \{x \in \mathbb{R} \mid x \geq 0\}$, the set $\mathbb{R}^n_+ := \{x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n \mid x_i \in \mathbb{R}_+,\ \forall i = 1, ..., n\}$. A matrix $\mathcal{A} \in \mathbb{R}^{n\times n}$ is Hurwitz or stable when all the eigenvalues (λ) of $\mathcal{A}$ are in the open left half plane of the complex plane $\mathbb{C}$, i.e. the real part of all eigenvalues is negative. A *nonnegative* matrix $\mathcal{A}$ has all of its entries greater or equal to 0, i.e. $a_{ij} \in \mathbb{R}_+$. A *Metzler* matrix $\mathcal{A}$ is a matrix for which all off-diagonal elements of $\mathcal{A}$ are nonnegative, i.e. $a_{ij} \in \mathbb{R}_+$ for all $i \neq j$. A *compartmental* matrix $\mathcal{A}$ is a matrix that is Metzler, where the sum of the components within a column is less than or equal to zero, i.e. $\sum_{i=1}^{n} a_{ij} \leq 0$ for all $j = 1, 2, ..., n$.

A positive linear system in the traditional sense [4] is defined next.

Definition 2.1: A linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{r\times n}$, and $D \in \mathbb{R}^{r\times m}$ is considered to be a *positive linear system* if for every nonnegative initial state and for every nonnegative input the state of the system and the output remain nonnegative.

Notice that Definition 2.1 states that the input to the system must be nonnegative, a restriction that we will abide to throughout this paper and illustrate via experimentation.

It turns out that Definition 2.1 has a very nice interpretation in terms of the matrix quadruple (A, B, C, D).

[1] that do not destabilize the system

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Theorem 2.1 ([4]): A linear system (1) is positive if and only if the matrix A is a Metzler matrix, and B, C, and D are nonnegative matrices.

An interesting subset of positive systems is that of compartmental systems. The main mathematical distinction, for LTI systems, between a positive system and a compartmental system is that a positive system's A matrix is Metzler, while a compartmental system's A matrix is compartmental. The inclusion of compartmental systems is made because in general, compartmental systems are stable, a property of great significance throughout the paper and in our experimental setup. For a more complete study and interesting results on compartmental systems see [8] and references therein.

III. The Tuning Clamping Regulator

In this section, we consider the problem of [1], i.e. the servomechanism problem for positive LTI systems under nonnegative control. Here we consider the same problem; however, unlike in [1] we also consider saturating signals from above - a case which we encounter in our experimental setup in the sequel. The goal of this section is to provide the gist of the servomechanism problem for positive LTI systems and its solution, while improving on the clamping controller as presented in [1].

Throughout this paper we consider the following LTI SISO plant:

$$\begin{array}{rcl} \dot{x} & = & Ax + bu + e_\omega \omega \\ y & = & cx + du + f\omega \\ e & := & y_{ref} - y \end{array} \tag{2}$$

where A is an $n \times n$ Metzler Hurwitz matrix, $b \in \mathbb{R}_+^n$, $c \in \mathbb{R}_+^{1\times n}$, $d \in \mathbb{R}_+$, $e_\omega \omega \in \Omega_1 \subset \mathbb{R}_+^n$, $f\omega \in \Omega_2 \subset \mathbb{R}_+$, $y_{ref} \in Y_{ref} \subset \mathbb{R}_+$.

Next, we provide an important assumption, which is needed in order to ensure that the steady state values of the closed loop system be nonnegative, under the choice of the reference signals and the unmeasurable disturbances of the plant. If this assumption was not true, then clearly we cannot attempt to satisfy any sort of nonnegativity of the states.

Assumption 3.1 ([1]): Given (2) assume that the *existence* condition $rank(d - cA^{-1}b) = 1$ holds and that the sets Ω_1, Ω_2, and Y_{ref} are chosen such that the steady state values of the input is nonnegative, i.e. $u_{ss} \in \mathbb{R}_+$.

Notice that if $u_{ss} \in \mathbb{R}_+$, then the steady-state of x and y will be nonnegative as desired.

Before we present the problem of interest, we would like to point out that one can easily check if the *existence* condition $rank(d - cA^{-1}b) = 1$ holds true from steady-state experiments performed on the plant, see [1].

Now with the above plant and assumption given, we outline the main problem of interest. The following problem differs from that of [1] in that it considers saturations from above and below.

Problem 3.1: Consider the plant (2) under unmeasurable disturbances, with initial condition $x_0 \in \mathbb{R}_+^n$. Find a nonnegative controller $0 \le u \le \overline{u}$, with $\overline{u} > 0$, under Assumption 3.1 with $u_{ss} \in (0, \overline{u})$ that

(a) guarantees closed loop stability;
(b) ensures the plant (2) is nonnegative, i.e. the states x and the output y are nonnegative for all time; and
(c) ensures tracking of the reference signals, i.e. $e = y - y_{ref} \to 0$, as $t \to \infty$, $\forall y_{ref} \in Y_{ref}$ and $\forall \omega \in \Omega$. In addition,
(d) assume that a controller has been found so that conditions (a), (b), (c) are satisfied; then for all perturbations of the nominal plant model which maintain properties (a) and (b) under Assumption 3.1, it is desired that the controller can still achieve asymptotic tracking and regulation, i.e. property (c) still holds.

The solution to the above problem is summarized in a Theorem below.

Theorem 3.1: Consider system (2). Then there exists an ϵ^* such that for all $\epsilon \in (0, \epsilon^*]$ the tuning clamping regulator:

$$\begin{array}{ll} \dot{\eta} = \epsilon(y_{ref} - y) & \begin{array}{l}\text{if } ((0 < u < \overline{u}) \text{ or} \\ (u = 0 \text{ and } e > 0) \text{ or} \\ (u = \overline{u} \text{ and } e < 0))\end{array} \\ & \\ \dot{\eta} = 0 & \text{else,} \end{array} \tag{3}$$

with

$$u = k\eta, \;\; \eta_0 = 0$$

where

$$k = \begin{cases} 0 & \eta \le 0 \\ 1 & 0 < \eta < \overline{u} \\ \overline{u}/\eta & \eta \ge \overline{u} \end{cases}$$

solves Problem 3.1.

Proof: The proof follows similar guidelines as the proof of Theorem 7 in [1]; thus, we omit most detail. The only remaining link, which needs proof is the need to show that the tuning clamping regulator will not saturate from above forever, i.e. $k \neq \frac{\overline{u}}{\eta}$ for all time, under the given assumptions. This is shown below via contradiction. Assume that there exists a time $t_1 \ge 0$ such that for all $t > t_1$ $u = \overline{u}$. Therefore, the closed loop system becomes

$$\begin{array}{rcl} \dot{x} & = & Ax + b\overline{u} + e_\omega \omega \\ \dot{\eta} & = & \epsilon(y_{ref} - cx - f\omega) \end{array}$$

and since A is stable

$$x \to -A^{-1}b\overline{u} - A^{-1}e_\omega\omega = \overline{x}_{ss}, \quad t \to \infty.$$

Recall,

$$\begin{array}{rcl} 0 & = & Ax_{ss} + bu_{ss} + e_\omega\omega \\ -e_\omega\omega & = & Ax_{ss} + bu_{ss} \\ -A^{-1}e_\omega\omega & = & x_{ss} + A^{-1}bu_{ss} \\ \overline{x}_{ss} & = & x_{ss} + A^{-1}bu_{ss} - A^{-1}b\overline{u} \\ & = & x_{ss} - A^{-1}b(\overline{u} - u_{ss}) \end{array}$$

i.e. if $u = \overline{u}$ for all time $t > 0$, then the system tends toward $\overline{x}_{ss}$ as $t \rightarrow \infty$, but this implies that

$$\begin{aligned}
\dot{\eta} &\rightarrow \epsilon(y_{ref} - c\overline{x}_{ss} - d(\overline{u}) - f\omega) \\
&\rightarrow \epsilon(y_{ref} - c(x_{ss} - A^{-1}b(\overline{u} - u_{ss})) \\
&\qquad\qquad - d(\overline{u}) + du_{ss} - du_{ss} - f\omega) \\
&\rightarrow \epsilon(y_{ref} - cx_{ss} - du_{ss} - f\omega) \\
&\qquad\qquad - \epsilon(d - cA^{-1}b)(\overline{u} - u_{ss}) \\
&\rightarrow 0 - \epsilon(d - cA^{-1}b)(\overline{u} - u_{ss}) \\
&< 0,
\end{aligned}$$

since $\epsilon > 0$ by assumption, $(d - cA^{-1}b) > 0$ by [5], $(\overline{u} - u_{ss}) > 0$ by assumption, and by continuity there exists a time $t_2 \geq t_1$ such that $\dot{\eta}(t_2) < 0$, $u(t_2) = \overline{u}$, and hence there exists a t_3 such that $u(t_3) \neq \overline{u}$, a contradiction to the assumption made that $u = \overline{u}$ for all time. The reminder of the proof follows that of [1], and we omit details. ■

The improvement of the tuning clamping regulator, over the one presented in [1], comes from the fact that both upper and lower saturations are considered; moreover, we incorporate reset windup type behavior into (3) something that was not done in [1], i.e. reset windup is present through the addition of the extra constraint of $\dot{\eta} = 0$ and the *if-else* statement of (3).

Remark 3.1: We note that the tuning clamping regulator can be easily combined with feedforward inputs, i.e.

$$u = u_{ff} + u_{tcr}, \tag{4}$$

where u_{ff} is a feedforward term (e.g. $u_{ff} = \beta y_{ref}$, β a constant) and u_{tcr} is the tuning clamping regulator (3).

This latter regulator controls a system with an unknown model under unmeasurable disturbances. Thus we will introduce an algorithm 3.1 that uses the tuning clamping regulator to solve the servomechanism problem if the property of the steady-state existence conditions (Assumption 3.1 with $0 < u_{ss} < \overline{u}$) are satisfied; otherwise, if the controller (3) does not solve the servomechanism problem defined by Problem 3.1, then *no other control law* will provide a solution, i.e. this is the best which any controller can do, given the limited information which we have.

Algorithm 3.1: Apply the tuning clamping regulator (3) to the unknown plant, by using "on-line tuning" [1], [9]. In this case,

1) The tuning clamping regulator either solves Problem 3.1, or
2) the servomechanism problem is not solvable under any control law.

Note that the controller does not take the system model into account, and moreover it is extremely simple to use and *it shuts itself off if the disturbances of the system are too large*. We are now ready to show experimentally the advantages of the tuning clamping regulator.

IV. Experiment: Industrial Hydraulic System

The purpose of this section is to validate and justify, experimentally, the results presented in the prequel via the use of an experimental industrial test bed. Our interest will not be focused on the technicalities of the experimental setup, model identification, or even linearization of the model[2], but rather we concentrate on how well the tuning clamping regulator (3) performs and abides to robustness issues. The control law is definitely tested within the setup, as the experiment incorporates nonlinear effects of the plant, actuator valve dynamics, time-delays, as well as sizing effects due to actuator valve constraints [2].

This section is organized as follows. The details of the setup and industrial components used within the experiment are initially provided. Thereafter, we concentrate on the experimental results and comparisons of various perturbations of the model to the performance of the tuning clamping regulator (3). We also show an example of how our current control law compares to that of [1].

A. Experimental apparatus

First, we describe the industrial components associated with the experiment used within this paper (see Figure 1). A more in-depth summary is given in [2].

The entire apparatus has been assembled from *industrial components*; this includes the actuators, sensors, valves, piping, and all digital communication. We note that the actuators (valves) are controlled by compressed air, and all signal communication between the actuators/sensors to the digital computer are obtained by commercial current variation ($4ma$ to $20ma$) techniques [2], and controlled within the loop by voltages ($0V$ to $10V$). Although all components used are industrial we have chosen to incorporate a standard personal computer running MATLAB Version 7.2.0.232 (R2006a) to carry out all real-time control. Below is a full list of components used within the experiment.

- 1 personal computer with an AMD Athlon(tm) 64 Processor 3200+ and 896 RAM
- 2 PCI-DAS6014 Analog and Digital I/O Boards
- 2 Foxboro Model V4A 1/2 inch Body H Needle Diaphram Control valve C/W I/P Transducer Model E69-BIIQ-R-S
  The I/P transducer is of the equal % type (3psi = 4 ma and 15 psi s 20 ma)
- 2 Taylor Model B 3401T 1/2 inch Differential Pressure Transmitter
- 1 Foxboro (Canada) Model E13-DL-I KAL2 Differential Transmitter and Model IFO-F2-S1 Integral Orifice Manifold Assembly in-lin type. This flow meter has a range of O to 2 (US) gallons/ rein
- 2 ASCOsolenoid valves. These solenoids are 100 V(AC) on/off 1/4 inch size and are activated by a

[2] both model identification and linearization were not performed during experimentation

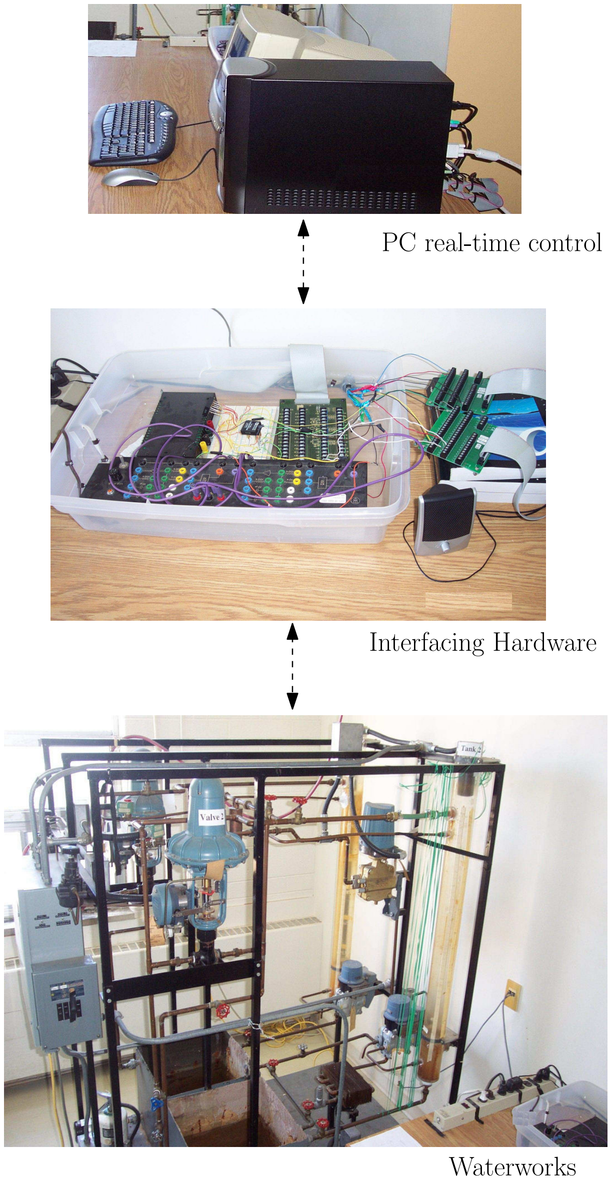

Fig. 1. Experimental setup.

voltage 3 to 32 V DC to the solid state relay

- 1 Compressed Air Regulator Model 2515346 (from Canox Toronto)
- 2 Magnetic Drive Pumps Model 13-874-11 (from Fisher Scientific). The pumps are 1/12 HP, 1/2 in in/out and can deliver 32 l/rein at 10 head
- 4 Solid state relays model EOM1DE42 (5VDC) (from Electrosonic)
- 1 24V power supply model HPFSO24O1O (from Electrosonic)
- 1 Disk Drive power supply model CP206-A (from Active Components, Toronto)
- Hammond power supply HPFT 00512015 (from Electrosonic).

The apparatus (Figure 1) consists of four water tanks, interconnected via numerous piping and valves, where the water circulates between the tanks and can be controlled via two digitally controlled valves (we will only be interested in using one valve due to the SISO constraint of our theory) that provide water inflow into two upper tanks. An overview diagram of the system is provided by Figure 2. The experimental apparatus has numerous valves which can be opened/closed to increase/decrease the water flow between respective tanks during the experimentation, thus allowing for major perturbation of the system model. Moreover, unmeasurable disturbances are present within the apparatus; in particular a water inflow disturbance is available via a digital on/off control input (both are not measured during experimentation). The only measurements taken during the experimentation are that of the height of the water in Tank 1 (Figure 2) via a sensor which provides a voltage level (varying from $1V$ corresponding to near empty, to $5V$ near full, with a $1V$ increase/decrease representing approximately $2.4L$ of water rise/drop) and the valve control voltage ($0V$ corresponding to nearly closed and $10V$ corresponding to fully open) of the input into Tank 1 (Valve A in Figure 2).

Note: by inspection since the system is compartmental and stable the setup is a positive system.

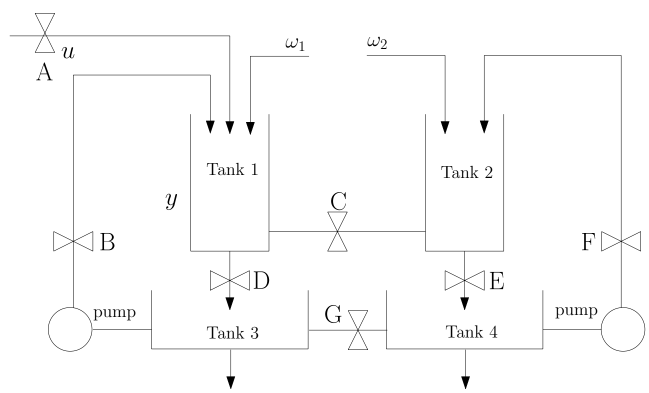

Fig. 2. Diagram of the system.

We now turn our focus to experimental results.

B. Experimental results

Throughout this subsection we refer to Figure 2 and Table I. Our goal will be to illustrate the theory behind the tuning clamping regulator via the use of various perturbations and disturbances on the nominal plant, which is represented by Case 1 from Table I. In all cases, the initial level of Tank 1 is equal to $x_0 = 4.4V$, and $\eta_0 = 0$.

C. Experiment I: Case 1 of Table I

In our first experiment we consider Case 1 (see Table I) under various initial conditions and ϵ values, i.e. various

TABLE I
EXPERIMENTAL CASES

Case	A	B	C	D	E	F	G	ω_1	ω_2
1	on	off	off	on	off	off	on	small	none
2	on	off	on	on	on	off	on	small	none
3	on	off	on	on	on	on	on	small	small
4	on	on	on	on	on	on	on	small	small
5	on	on	on	on	on	on	on	large	large

water levels of Tank 1 and for different types of tuning parameters. In particular, we consider the cases of the

(a) clamping controller used in [1], which has no reset anti-windup, with $\epsilon = 0.07$, $y_{ref} = 2V$ (Figure 3), which results in a settling time of 16.5 minutes.

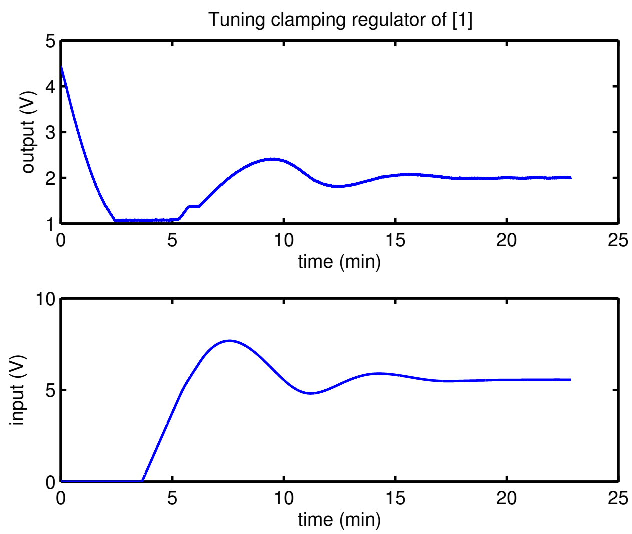

Fig. 3. Experiment 1: Clamping controller used in [1].

(b) reset anti-windup tuning clamping regulator (3) with $\overline{u} = 10$, $\epsilon = 0.07$, $y_{ref} = 2V$ (Figure 4), which results in a settling time of 15 minutes; and
(c) reset anti-windup tuning clamping regulator (3) under Remark 3.1 with $\overline{u} = 10$, $u_{ff} = 2.5y_{ref}$, $\epsilon = 0.07$, $y_{ref} = 2V$ (Figure 5), which results in a settling time of 12 minutes.

We note that in the case of initial condition $x_0 = 0$ the two cases (a) and (b) are identical, as no clamping occurs, while case (c) outperforms both (a) and (b) cases. In general, we found that for various initial conditions the tuning clamping regulator with the addition of the extra feedforward works best, i.e. has best $\%OS$ and settling times.

D. Experiment II: Case 1,2,3,4 of Table I

Next, we put the tuning clamping regulator with the addition of the extra feedforward term above to the test by considering numerous perturbations into the system, as

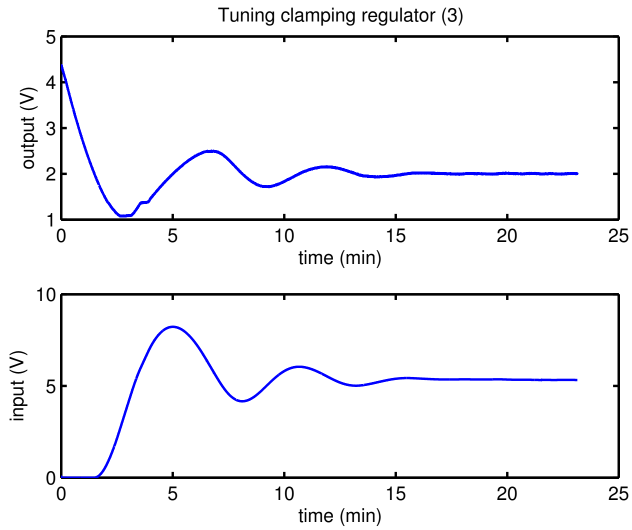

Fig. 4. Experiment 1: Tuning clamping regulator (3).

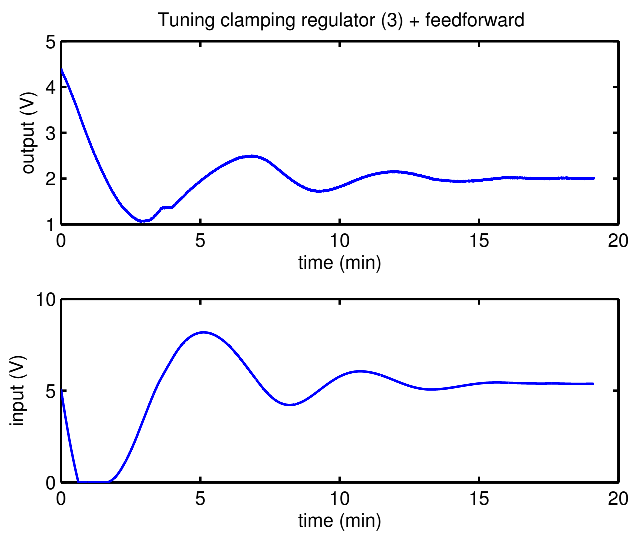

Fig. 5. Experiment 1: Tuning clamping regulator (4).

described in Table II. In particular, we consider the case of

$$\begin{aligned}\overline{u} &= 10\\ x_0 &= 3.53V\\ \epsilon &= 0.07\\ y_{ref} &= 2V,\end{aligned}$$

under the transitions of Table II. Figure 6 illustrates the results.

Notice that no adjustments to the controller are made during this experimentation and no model is ever used. Note that saturation has played a key role in this set up.

E. Experiment III: Case 1,2,3,4,5 of Table I

In this experiment we repeat Experiment II of the previous subsection and at approximately 75 minutes add a large disturbance, described by the addition of Case 5 of Table

TABLE II

PURTURBATION EXPERIMENT

Time (min)	Current Case (of Table I)
0	Case 1
≈ 26	Case 2
≈ 53	Case 3
≈ 67	Case 4.

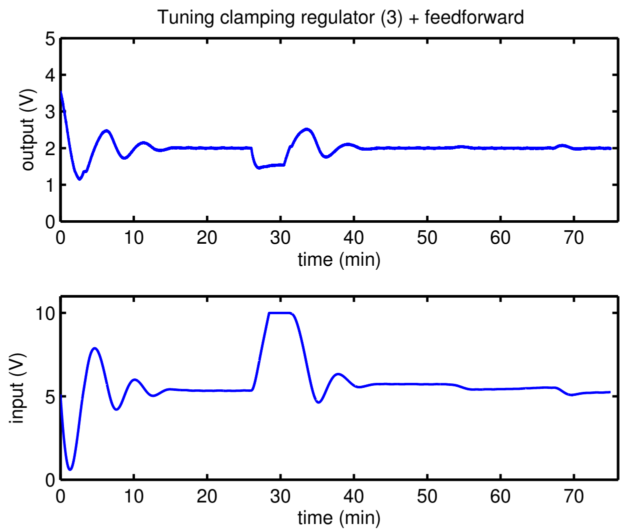

Fig. 6. Experiment 2: Tuning clamping regulator (4) under perturbations.

I to Table II, coming into both Tank 1 and Tank 2. The point of this experiment is to show that if the steady state u_{ss} does not abide to Assumption 3.1, then by Algorithm 3.1 no solution exists under any type of control law. Figure 7 illustrates the results.

We note that if the disturbance of the system is too large, then the tuning clamping regulator will shut itself off, as is done in Figure 7.

V. CONCLUSION

In this paper, we have used the tuning clamping regulator to illustrate experimentally the servomechanism problem for unknown stable positive systems, i.e. a positive system whose mathematical model is unknown. The main contribution of the tuning clamping regulator is its ability to provide a solution to the servomechanism problem for stable positive systems if and only if a solution exists. The use of the regulator is effective and very practical from the perspective of cost and validation of a solution. Currently the authors are extending the results, theoretical and experimental, to improve the transient and settling time characteristics of the clamping regulator.

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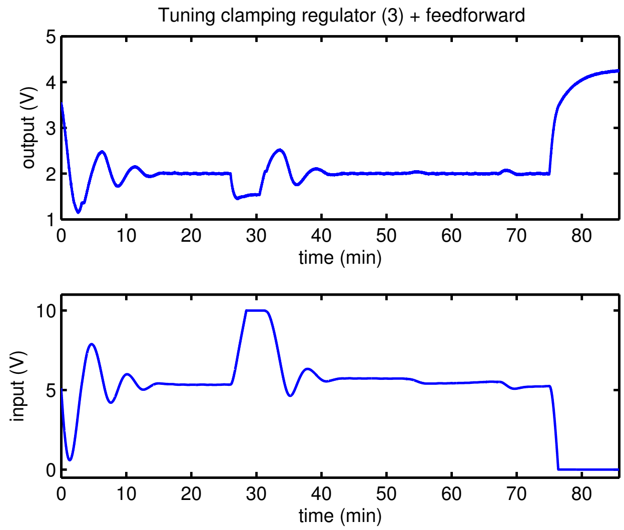

Fig. 7. Experiment 3: Tuning clamping regulator (4) under large disturbances.

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