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Operator based Robust Control for Nonlinear Systems with Uncertain Non-symmetric Backlash

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Abstract— **In this paper, a method of operator based robust control for nonlinear systems with uncertain non-symmetric backlash is proposed. In details, using robust right coprime factorization condition, the operator based controller is designed for stabilizing the nonlinear systems.**

I. INTRODUCTION

In recent papers, a great deal of effort has been made for analysis and design of nonlinear output feedback control systems (for example, [1,-,5]). In particular, the problem of designing the nonlinear control systems by using operator based robust right coprime factorization approach is a problem of considerable practical importance [1,2].

The purpose of the paper is to discuss the design problem of nonlinear control systems with uncertain non-symmetric backlash by using operator based robust right coprime factorization approach. That is, by proposing a new robust condition such that the robust stability of the control system with uncertain non-symmetric backlash can be guaranteed. So far, operator based robust conditions have been established for ensuring robust stability of nonlinear uncertain systems [1,2]. However, it is difficult to apply the above conditions to plants with uncertain non-symmetric backlash. In this paper, a robust condition for plants with uncertain non-symmetric backlash is given. Under the existence of the uncertain non-symmetric backlash, a robust nonlinear control system design method based on the proposed condition is studied.

II. PROBLEM STATEMENT

Consider a nonlinear unstable system $P : U \to Y$, where U and Y are the input and output spaces respectively. It's described by the following right coprime factorization:

$$P = ND^{-1} \tag{1}$$

where $D : W \to U$ and $N : W \to Y$ are stable operators from the quasi-state space W to the input and output spaces. A feedback control system is said to be well-posed if every signal in the control system is uniquely determined for any input signal in U. For the above nonlinear system (1), under the condition of well-posedness, N and D are said to be right coprime factorization if there exist two stable operators $A : Y \to U$ and $B : U \to U$ satisfying the following Bezout identity

$$AN + BD = M, \text{ for some } M \in \mathcal{S}(W,U) \tag{2}$$

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where $\mathcal{S}(W,U)$ is the set of unimodular operator. Then controller A and B could stabilize the unstable system (1). Let the overall plant with perturbation ΔP be $\tilde{P}$, shown as: $\tilde{P} = P + \Delta P$, where $\tilde{P}$ and P are nonlinear and unstable operators. As above mentioned, the right coprime factorization of $\tilde{P}$ is $\tilde{P} = P + \Delta P = N(D + \Delta D)^{-1}$. We assume that ΔD is unknown but the upper and lower bounds of it are known. According to (2), we can obtain

$$AN + B(D + \Delta D) = M, \text{ for some } M \in \mathcal{S}(W,U) \tag{3}$$

However, in some case, (3) is not satisfied since ΔD is unknown. The stability of the nonlinear feedback system with perturbation is guaranteed by Theorem 1 of [2].

Our considering system is either nonlinear or preceded by a non-symmetric backlash which is defined as follows [3,5].

$$\begin{aligned} u(t) &= B_a(v(t)) \qquad (4) \\ &= \begin{cases} m_r(v(t)-h), & \text{if } \dot{v}(t) > 0 \text{ and } u(t) = m_r(v(t)-h) \\ m_l(v(t)+h), & \text{if } \dot{v}(t) < 0 \text{ and } u(t) = m_l(v(t)+h) \\ u(t_-), & \text{otherwise} \end{cases} \end{aligned}$$

where the parameters m_r and m_l stand for the right and left slope of the backlash, $h > 0$ is the backlash distance.

Assumption 1: The coefficients m_r, m_l are strictly positive and unknown.

Assumption 2: The maximum and the minimum values of the slopes of the backlash are known, $\max\{m_l, m_r\} = \overline{m}$, $\min\{m_l, m_r\} = \underline{m}$.

III. PROPOSED DESIGN SCHEME

The original nonlinear system is preceded by the non-symmetric backlash. Namely, the output of the backlash is the input of the original nonlinear system. Based on (3), the original nonlinear system could be stabilized by two stable controllers A and B. However, since the existence of backlash, the stabilization will be affected. So the non-symmetric backlash will considered as one part of the original nonlinear system for being stabilized by new controllers based on Bezout identity.

As (4) shown, according to the conditions, the non-symmetric backlash is regarded as two cases: one is the case of linear relationship between input $v(t)$ and output $u(t)$; the other is the case of output $u(t)$ holding on.

For the first case, we define an operator D_b to describe the slopes of the backlash.

$$u(t) = D_b(v(t) \pm h) = m_0(v(t) \pm h) \tag{5}$$

where m_0 is the designed slope. For the backlash distance h, we regard it as disturbance. So operator D_b could be fused in

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D to be $\tilde{D} = D_b^{-1}D$. Based on the above presentation, two controllers A and B are designed to satisfy $AN + B\tilde{D} = M$, where M is the unimodular operator. And the output of system is obtained, $y(t) = N(AN + B\tilde{D})^{-1}(r^*(t) \pm B(h))$.

Since the coefficients m_r and m_l are unknown and $m_r \neq m_l$, we have the following lemma to guarantee the stability of the nonlinear feedback control system with uncertain non-symmetric backlash based on Theorem 1 of [2]. Before the lemma, some notations should be introduced. As (4) shown, $D_b(v(t)) = m_0 v(t)$, so $D_b^{-1} = 1/m_0$. $\tilde{D}$ can be represented by $\tilde{D_0}$, $\tilde{D_1}$ and $\tilde{D_2}$ when $m = m_0, \overline{m}, \underline{m}$, respectively.

Lemma 1: Let U^e and Y^e be two extended linear spaces, which are associated respectively with two given Banach spaces U_B and Y_B. Let D^e be a linear subspace of U^e and let $(B\tilde{D_1} - B\tilde{D_0})M^{-1} \in Lip(D^e)$, $(B\tilde{D_2} - B\tilde{D_0})M^{-1} \in Lip(D^e)$. Let the Bezout identity of the nominal plant be $AN + B\tilde{D_0} = M \in S(W,U)$, $AN + B\tilde{D_1} = \tilde{M_1}$ or $AN + B\tilde{D_2} = \tilde{M_2}$, when slope of backlash is $\overline{m}$ or $\underline{m}$. Under the condition of controller A to satisfying (3), if

$$\|[B\tilde{D_1} - B\tilde{D_0}]M^{-1}\| < 1, \ \|[B\tilde{D_2} - B\tilde{D_0}]M^{-1}\| < 1 \quad (6)$$

the system is stable, where $\|\cdot\|$ is defined as

$$\|F\| := \sup_{T\in[0,\infty)} \sup_{\substack{x,\tilde{x}\in D^e \\ x_T \neq \tilde{x_T}}} \frac{\|[Fx]_T - [F\tilde{x}]_T\|_{Y_B}}{\|x_T - \tilde{x_T}\|_{U_B}} \quad (7)$$

Proof: M is unimodular operator, then M is invertible. From $AN + B\tilde{D_0} = M$, $AN + B\tilde{D_1} = \tilde{M_1}$, we have

$$\tilde{M_1} = M + [B\tilde{D_1} - B\tilde{D_0}]. \quad (8)$$

Since $\tilde{M_1} = M + [B\tilde{D_1} - B\tilde{D_0}] = [I + (B\tilde{D_1} - B\tilde{D_0})M^{-1}]M$ and $(B\tilde{D_1} - B\tilde{D_0})M^{-1} \in Lip(D^e)$, $I + (B\tilde{D_1} - B\tilde{D_0})M^{-1}$ is invertible based on the result in [2], where I is the identity operator. Consequently, we have $\tilde{M_1}^{-1} = M^{-1}[I + (B\tilde{D_1} - B\tilde{D_0})M^{-1}]^{-1}$. Meanwhile, since $\tilde{M_1} = M + [B\tilde{D_1} - B\tilde{D_0}]$, $(B\tilde{D_1} - B\tilde{D_0})M^{-1} \in Lip(D^e)$, and $M \in \mathcal{U}(W,U)$, we have $\tilde{M_1} \in \mathcal{U}(W,U)$. For $AN + B\tilde{D_2} = \tilde{M_2}$, it can also be proofed by the same method.

Then we define $Z = \max\{\frac{1}{m_0} - \frac{1}{\overline{m}}, \frac{1}{\underline{m}} - \frac{1}{m_0}\}$. Let the exact plant be $AN + B\tilde{D} = \tilde{M}$, because $m \in [\underline{m}, \overline{m}]$, we obtain

$$\|[B\tilde{D} - B\tilde{D_0}]M^{-1}\| \leq \|[BD \cdot Z]M^{-1}\| < 1 \quad (9)$$

So $\tilde{M} \in \mathcal{U}(W,U)$. Refer to the proof of theorem in [2], the nonlinear system is overall stable. ■

For the second case, we regard $u(t_-)$ as disturbance and D_b needs not be considered. So we can design another two controllers A^* and B^* which are satisfying (2) to stabilize the nonlinear system, and the output of the system is obtained, $y(t) = N(A^*N + B^*D)^{-1}(r^*(t) + B(u(t_-)))$.

IV. NUMERICAL EXAMPLE

In this section, a numerical example is given to show the effectiveness of the proposed condition of robust stability.

Let the given plant operator $P = ND^{-1}$ be defined by $P(u^*(t)) = \int_0^t u^{*1/3}(\tau)\mathrm{d}\tau + e^{t/3}u^{*1/3}(t)$, $N(w(t)) = \int_0^t e^{\tau/3}w^{1/3}(\tau)\mathrm{d}\tau + w^{1/3}(t)$, $D(w(t)) = e^{-t}w(t)$, where $u^* \in U^*$, $P(u^*) \in Y$, we choose space $W = U$. Obviously, operator D^{-1} is unstable.

As above section mentioned, the non-symmetric backlash can be regarded as two cases. Since the second case is not related to the proposed condition, we just consider the first case here. Let the input r^* of the considering system be bounded, and bounds of the slopes be $m_r \in [1, 1.2]$, $m_l \in [0.92, 1.1]$. That is $\overline{m} = \max(m_r) = 1.2$, $\underline{m} = \min(m_l) = 0.92$. According to the designed framework, we design the controllers A and B as follows:

$$A(y(t)) = (e^t - 1)(g(t))^3, \ B(u(t)) = m_0 u(t) \quad (10)$$

where $g(t) = e^{-t/3}w^{1/3}(t)$, $m_0 = 1.05$.

Then the output of considering system is obtained as

$$y(t) = N(AN + B\tilde{D})^{-1}(r^*(t) \pm B(h)) \quad (11)$$

From (4), we obtain that h is bounded, D_b and D_b^{-1} are stable operators. For proofing the operators D and N stable [1], we pick any $x \in W$. There is a constant k such that $\|x\|_\infty < k$. Then for all $t \in [0,\infty)$, $| D(x(t)) | = e^{-t} | x(t) | < k$ and $| \int_0^t e^{-\tau/3}x^{1/3}(\tau)\mathrm{d}\tau | < k^{1/3} | \int_0^t e^{-\tau/3}\mathrm{d}\tau | \leq 3k^{1/3}$, so that $|N(x(t))| < 4k^{1/3}$. Thus both D and N are stable. Since the input r^* is bounded, the output $y(t)$ is bounded if $(AN + B\tilde{D})^{-1}$ is stable.

According to known parameters,

$$\|[B\tilde{D_1} - B\tilde{D_0}]M^{-1}\| = \|\left(\frac{m_0}{\overline{m}} - \frac{m_0}{m_0}\right)e^{-t}M^{-1}\| < 1$$

is obtained. We can also get $\|[B\tilde{D_2} - B\tilde{D_0}]M^{-1}\| < 1$, the robust condition in Lemma 1 is satisfied.

Let $AN + B\tilde{D} = \tilde{M}$, according to Lemma 1, we obtain $\tilde{M} \in S(W,U)$, because $m \in [\underline{m}, \overline{m}]$. Since $S(W,U)$ is the set of unimodular operator, $(AN + B\tilde{D})^{-1}$ is stable. Thus, the output $y(t)$ is bounded. The unstable nonlinear system with uncertain non-symmetric backlash is stabilized depending on the proposed robust condition.

V. CONCLUSION

Operator based robust control for nonlinear systems with uncertain non-symmetric backlash is studied. Using the proposed robust condition, the operator based controller is designed for nonlinear systems with uncertain non-symmetric backlash.

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