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Semiglobal Robust Output Regulation with Generalized Immersion

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***Abstract*— The semiglobal robust output regulation problem is solved in this paper for a class of nonlinear systems that do not satisfy the standard conditions for the existence of a linear internal model, but admit a so-called "generalized immersion." It is shown how the obstacle given by the presence of the exosystem dynamics in the generalized immersion mapping can be overcome by resorting to a recently developed framework for time-varying internal model design.**

I. Problem formulation

In this paper, we consider a prototypical robust output regulation problem for systems of the form

$$\begin{aligned} \dot{w} &= Sw \\ \dot{z} &= f(z, e, w, \mu) \\ \dot{e} &= h(z, e, w, \mu) + b(\mu)[u - c(w, \mu)] \end{aligned} \tag{1}$$

with exosystem state $w \in \mathbb{R}^p$, plant state in the error-coordinates $x = (z, e) \in \mathbb{R}^{n-1} \times \mathbb{R}$, control input $u \in \mathbb{R}$, regulated error $e \in \mathbb{R}$, and unknown parameters $\mu \in \mathcal{P}$, where $\mathcal{P}$ is a given compact set in $\mathbb{R}^\rho$. The vector fields $f(x, e, w, \mu)$ and $h(x, e, w, \mu)$ are smooth and satisfy $f(0, 0, w, \mu) = 0$, $h(0, 0, w, \mu) = 0$ for all $w \in \mathbb{R}^p$ and all $\mu \in \mathcal{P}$. Moreover, $b(\mu) \geq b_0 > 0$ for all $\mu \in \mathcal{P}$. The eigenvalues of the known matrix S are all simple and lie on the imaginary axis. The *semiglobal robust regulation problem* is stated as follows:

Problem 1.1: Given arbitrary compact sets $\mathcal{K}_x \subset \mathbb{R}^n$, $\mathcal{K}_w \subset \mathbb{R}^p$, determine a dynamic error-feedback controller

$$\dot{\xi} = F(\xi, e), \quad u = H(\xi, e) \tag{2}$$

with state $\xi \in \mathbb{R}^\nu$, and a compact set $\mathcal{K}_\xi \subset \mathbb{R}^\nu$ such that all the trajectories of the closed-loop system (1)-(2) originating from any initial conditions $(w_0, x_0, \xi_0) \in \mathcal{K}_w \times \mathcal{K}_x \times \mathcal{K}_\xi$ are bounded and satisfy $\lim_{t\to\infty} e(t) = 0$ for all $\mu \in \mathcal{P}$. □

Without loss of generality, we henceforth assume that $\mathcal{K}_w$ is an invariant set for $\dot{w} = Sw$.

Assumption 1.1: There exists a smooth, positive definite function $V_0(z, w, \mu)$ such that

$$\begin{aligned} &\underline{\alpha}_0(\|z\|) \leq V_0(z, w, \mu) \leq \overline{\alpha}_0(\|z\|) \\ &\frac{\partial V_0}{\partial z} f(z, 0, w, \mu) + \frac{\partial V_0}{\partial w} Sw \leq -\alpha_0(\|z\|), \end{aligned}$$

for all $z \in \mathbb{R}^{n-1}$, $w \in \mathcal{K}_w$, and $\mu \in \mathcal{P}$, where $\underline{\alpha}_0$, $\overline{\alpha}_0$, α_0 are class-$\mathcal{K}_\infty$ functions satisfying $\underline{\alpha}_0(s) \geq \underline{a}_0 s^2$, $\overline{\alpha}_0(s) \leq \overline{a}_0 s^2$, $\alpha_0(s) \geq a_0 s^2$ for all $s \in [0, r_0]$, and for some positive numbers $\underline{a}_0$, $\overline{a}_0$, a_0, and r_0. □

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In the classical internal-model based approach (that is, in the spirit of [1]), the solvability of Problem 1.1 relies upon the possibility of embedding in the controller an internal model of the exosystem with output

$$\dot{\mu} = 0, , \quad \dot{w} = Sw, \quad y_w = c(w, \mu) \tag{3}$$

which can be accomplished if the system (3) can be immersed into a detectable nonlinear system of the form

$$\dot{\eta} = \Phi(\eta), \quad y_\eta = \Gamma(\eta). \tag{4}$$

In the literature, the nonlinear function $c(w, \mu)$ is usually assumed to satisfy the following property, which ensures the existence of an immersion into an LTI system:

Property 1.1: There exists an integer q and real numbers $a_0, \cdots, a_{q-1}$ such that $a_0 c(w, \mu) + a_1 L_S c(w, \mu) + \cdots + a_{q-1} L_S^{q-1} c(w, \mu) + L_S^q c(w, \mu) = 0$ for all $\mu \in \mathcal{P}$.

However, the only case where the above property is guaranteed to hold is that the nonlinear function $c(w, \mu)$ is a polynomial in w, with μ-dependent coefficients. In the next section, we show how Assumption 1.1 can be relaxed by defining linear time-varying immersions.

II. Generalized Immersion

It was shown in [2] that for systems which do not admit an immersion of the form (4), it is still possible to obtain a so-called *generalized immersion* if the function $c(w, \mu)$ satisfies the following property:

Property 2.1: There exists an integer $q \in \mathbb{N}$ and smooth functions $a_0(w), \cdots, a_{q-1}(w)$ such that, for all $w \in \mathbb{R}^p$ and $\mu \in \mathcal{P}$, $a_0(w) c(w, \mu) + a_1(w) L_S c(w, \mu) + \cdots + a_{q-1}(w) L_S^{q-1} c(w, \mu) + L_S^q c(w, \mu) = 0$.

Property 2.1 is found to hold for more general classes of nonlinear functions, including sinusoidal, exponential or rational terms, (see [2]). Property 2.1 implies that the exosystem admits a generalized immersion into the system

$$\dot{w} = Sw, \quad \dot{\eta} = \Phi_{\rm p}(w)\eta, \quad y_\eta = \Gamma_{\rm p}\eta, \tag{5}$$

where the pair $(\Phi_{\rm p}(w), \Gamma_{\rm p})$ is in phase-variable form with w-dependent coefficient $a_i(w)$, $i = 1, \cdots, q$.

While the above system is dependent on w, and thus not implementable as such, it suffices to notice that, since $w(t) = {\rm e}^{St} w_0$, one can rewrite the η-dynamics as $\dot{\eta} = \Phi_{\rm p}(t, \sigma)\eta$, with $\Phi_{\rm p}(t, \sigma) := \Phi_{\rm p}({\rm e}^{St} w_0)$ and $\sigma = w_0$ plays the role of a vector of unknown parameters. Therefore, the generalized immersion is trivially transformed into a time-varying parameter-dependent immersion similar to the one considered in [3] for periodic systems. Moreover, one can always rewrite system (4) in observer canonical form $\dot{\tau}(w,\mu) = \Phi_{\rm o}(t,\sigma)\tau(w,\mu)$, $c(w,\mu) = \Gamma_{\rm o}\tau(w,\mu)$.

Assumption 2.1: There exists a re-parametrization $\sigma \mapsto \theta \in \mathbb{R}^{\varrho}$ such that the coefficients of the matrix $\Phi_{\rm p}(t,\sigma)$ depends linearly on θ. □

III. Regulator Design

The robust regulator consists of the parallel connection $u = u_{\rm st} + u_{\rm im}$ of a high-gain stabilizer $u_{\rm st} = -ke$, $k > 0$, and a parameterized internal model unit of the form (see [3])

$$\dot{\xi} = (F + G(t)H(\hat{\theta}))\xi - kG(t)e\,, \quad u_{\rm im} = H(\hat{\theta})\xi\,.$$

Note the explicit dependence on time occurs through the known matrix exponential e^{St}. It can be shown that there exists a parameterized family of almost-periodic smooth pairs $(\Sigma(t,w,\mu), H(\theta))$, satisfying

$$\begin{aligned} \frac{\partial\Sigma}{\partial t} + \frac{\partial\Sigma}{\partial w}Sw &= [F + G(t)H(\theta)]\Sigma(t,w,\theta) \\ c(w,\mu) &= H(\theta)\Sigma(t,w,\theta)\,, \end{aligned}$$

for all $\mu \in \mathcal{P}$ and all $t \geq t_0 \geq 0$. The proof then follows along the same lines of [3, Lemma 1.2]. The main result can be easily proved by combining the results of [4, Proposition 5.1] and applying LaSalle's invariance principle:

Theorem 3.1: There exists $k^\star > 0$ such that for all $k \geq k^\star$ and all $\gamma > 0$ the adaptive controller

$$\begin{aligned} \dot{\xi} &= (F + G(t)H(\hat{\theta}))\xi - kG(t)e \\ \dot{\hat{\theta}} &= \gamma\xi_1 e \\ u &= H(\hat{\theta})\xi - ke \end{aligned}$$

solves the semiglobal robust output regulation problem for the class of systems under consideration. □

Example: Consider the error system

$$\begin{aligned} \dot{w} &= Sw \\ \dot{z} &= -z^3 + r_1 e w_1 \\ \dot{e} &= a\sin(e) - z^2 + b[u - c(w,\mu)]\,, \end{aligned} \qquad (6)$$

where $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, and $c(w,\mu) = r_1 w_1 \cos(w_2)$, with r_1 a nonzero unknown constant. The exosystem with output (3) admits a generalized immersion in observer form

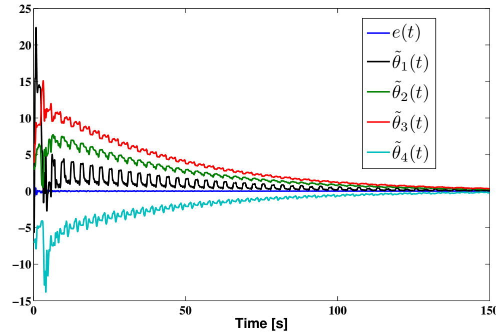
$$\Phi_{\rm o}(t,\sigma) = \begin{pmatrix} -\alpha_3(t,\sigma) & 1 & 0 & 0 \\ \vdots & & \ddots & \\ -\alpha_0(t,\sigma) & 0 & 0 & 0 \end{pmatrix},$$

with coefficients $\alpha_3(t,\sigma) = 0$,

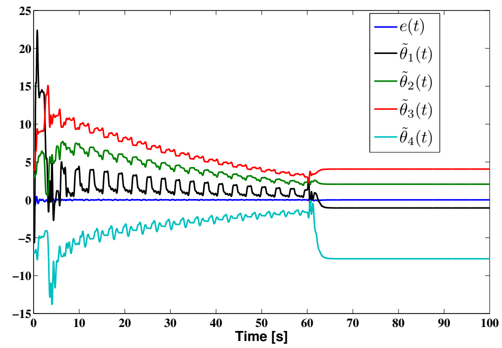
$$\begin{aligned} \alpha_2(t,\sigma) &= (5 + w_1^2(0)) + (-w_1^2(0) + w_2^2(0))\sin^2(t) + \\ &\quad + (w_1(0)w_2(0))\sin(2t) \\ \alpha_1(t,\sigma) &= 3(w_1(0)w_2(0)) - 6(w_1(0)w_2(0))\sin^2(t) \\ &\quad + 1.5(-w_1^2(0) + w_2^2(0))\sin(2t) \\ \alpha_0(t,\sigma) &= (4 + w_1^2(0) + 3w_2^2(0)) - 2(-w_1^2(0) + \\ &\quad + w_2^2(0))\sin^2(t) - 2(w_1(0)w_2(0))\sin(2t)\,. \end{aligned}$$

The proposed solution applies to the given system with

$$\theta = \begin{pmatrix} w_1^2(0) & -w_1^2(0)+w_2^2(0) & 2w_1(0)w_2(0) w_1^2(0)+3w_2^2(0) \end{pmatrix}.$$



(a) Simulation results with persistence of excitation.



(b) Simulation results without persistence of excitation.

Fig. 1. Simulation results

The controller gains and adaptation gain have been chosen as $L_0 = (15, 35, 50, 24)'$, $k = 5$ and $\gamma = 1200$, respectively (see [3]). The initial conditions of the simulations are $z(0) = 0.5$, $\xi(0) = 0_{16\times 1}$, $w(0) = (-2,\ 1)'$, and $\hat{\theta}(0) = 0_{4\times 1}$. Figure 1(a) shows asymptotic error regulation and parameter convergence. Figure 1(b) shows the results of a simulation where the exosystem is turned off at $t = 60$ s. As the method does not require persistence of excitation, the regulated error still converge to zero, while the estimation error does not.

IV. Conclusions

In this paper, we have provided an adaptive time-varying internal model design for semiglobal nonlinear robust output regulation. By combining the concept of generalized immersion [2] with the parameter-dependent time-varying immersion framework of [3] it is possible to extend the class of systems for which the problem admits a solution.

References

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