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Control and Cohesion of Energetic Swarms

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Abstract- **An M-Member swarm system with energetic behavior is studied in this paper. A new type of swarm controller is developed such that a swarm can follow a desired trajectory with different swarm temperatures and potential energy values. The temperature allows the internal kinetic energy of the swarm to be modulated. As the temperature increases the motion of the swarm becomes more energetic and areas are covered by the swarm in less time. The potential energy controls the size of the swarm and also provides new guarantees of energetic swarm cohesion. Simulation is used to validate the results and to demonstrate the new approach**

I. Introduction

A collective pattern/behavior emerges from local action of possibly simple agents could be potentially useful in many complex engineering applications such as distributed mobile sensing [1-2], distributed robotic surveillance/rescue [3], and coverage path planning problems [4].

In the majority of works swarm model developed such that all agents are desired to move with a common velocity, while keeping a certain desired internal group formation see for example [5][6]. These models can be used to handle different control objectives in multi-agent systems such as capturing a moving target [7], formation control [8-9], and the coordinated movement of agents in the presence of multiple and moving obstacles [10]. However, moving all agents with a common velocity is not suitable for some application such as multi agent coverage path planning, where a group of autonomous vehicle needs to visit all points in its environment [11].

One of the main characteristic of swarm is cohesion i.e. boundedness of swarm size. It has been shown for a swarm model with a general class of attraction and repulsion functions that the individuals form a cohesive swarm in a finite time. Moreover, the explicit bound on swarm size can be obtained [8].

The stability of group behavior has been studied in literature under different conditions [5-6][8-9][12]. Although second order dynamic models i.e. double integrators are utilized in [13], and [14], most of them are based on a kinematic swarm model, i.e. single integrators. This agents' inertial effect can even cause unstable group behavior for a certain information topology [15]. Hence, the agents' inertial effect cannot be ignored for any information topology. Moreover, most of actuators can directly affect the acceleration not the velocity.

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In swarm literature, the internal energy interactions of the swarm have been considered only for the cases where an energy minimization controller used to achieve stable internal formation. In particular, the internal kinetic energy or molecular energy of the swarm is not investigated in these works. This paper is an extension of earlier work [16] to study the internal kinetic energy of the swarm. The temperature variable is modified. This modified temperature variable is a measure of internal kinetic swarm energy. This newly introduced temperature is directly related to velocity distribution of agents. Moreover, multi-output controller is developed such that swarm center can follow any desired trajectory with different corresponding temperature and potential energy. In this paper an implementation of the energetic swarm model in [17] is investigated for the case of wheeled mobile robots (WMR). A feedback linearization method is used to obtain double integrator model for WMR. WMRs are commonly used in industrial applications. Control and coordination of multiple robotic vehicles has been extensively studied for example [18-21]. An experimental setup in a case of multiple WMRs based on earlier work in [5] for implementation of swarming behavior is studied in [20].

Moving the swarm of WMRs with a variable temperature could potentially be useful in applications where a swarm is required to sweep out an area or volume as the center of the swarm moves around, see [4] and [11]. As the temperature of the swarm increases the surrounding area is covered in a more aggressive and rapid manner potentially covering the area in less time. Swarm of WMRs with higher temperature can perform task faster and more efficient than the one with lower temperature.

This paper is organized as follows. In section II an n-dimensional second order swarm model is presented and a complete discussion of the swarm energy is performed. In section III a temperature controller is developed. Moreover, multi-output controller is proposed. Potential energy regulation and its link to cohesion is studied The simulation result and implementation of the proposed method for WMR model is presented in section IV. The paper ends with conclusions and future research directions in section V.

II. Swarm Model

Consider a swarm of M members moving in an n-dimensional space. Each agent is modeled as a point mass. The motion equation of individual i is given by:

$$\dot{\mathbf{x}}^i = \mathbf{v}^i, \quad m_i \dot{\mathbf{v}}^i = \mathbf{u}^i_{ext} + \mathbf{u}^i_{in} - b_i \mathbf{v}^i \tag{1}$$

where $i = 1,2,\ldots,M$, $\mathbf{x}^i \in \Re^n$ is the position of the i th individual, $\mathbf{v}^i$ is the velocity of the i th individual, m_i is the mass of the i th individual, and $\mathbf{u}^i_{in}$ is the total force on the individual i as a result of inter-individual interaction, $-b_i\mathbf{v}^i$ represents the velocity damping term, and $\mathbf{u}^i_{ext}$ denotes the external input. The external input is used for tracking a desired trajectory and a velocity damping term is used for viscous dissipation. The term $\mathbf{u}^i_{in}$ is for the cohesion of swarm and is of the form:

$$\mathbf{u}^i_{in} = -\sum_{j=1, j\neq i}^{M} [g_a(\|\mathbf{x}^i - \mathbf{x}^j\|) - g_r(\|\mathbf{x}^i - \mathbf{x}^j\|)](\mathbf{x}^i - \mathbf{x}^j) \tag{2}$$

where $g_r : \Re^+ \to \Re^+$ and $g_a : \Re^+ \to \Re^+$ represents respectively the magnitude of repulsion force and the attraction force and the norm is the Euclidean norm. The following assumptions on attraction and repulsion functions are made by [8]:

Assumption 1. There exists corresponding functions $J_a : \Re^+ \to \Re$ and $J_r : \Re^+ \to \Re$ such that for any $\mathbf{y} \in \Re^n$

$$\nabla_{\mathbf{y}} J_a(\|\mathbf{y}\|) = \mathbf{y} g_a(\|\mathbf{y}\|), \nabla_{\mathbf{y}} J_r(\|\mathbf{y}\|) = \mathbf{y} g_r(\|\mathbf{y}\|). \tag{3}$$

Definition 1. The swarm center $\bar{\mathbf{x}} \in \Re^n$ and the velocity of the swarm center $\bar{\mathbf{v}} \in \Re^n$ is defined by

$$\bar{\mathbf{x}} = \sum_{i=1}^{M} m_i \mathbf{x}^i \Big/ \sum_{i=1}^{M} m_i, \bar{\mathbf{v}} = \sum_{i=1}^{M} m_i \mathbf{v}^i \Big/ \sum_{i=1}^{M} m_i \tag{4}$$

In this paper, the homogenous swarm is considered and the definition of homogenous swarm is as follows

Definition 2. The swarm described by (1) is homogenous when the damping coefficients and masses satisfy $b_i = b$ and $m_i = m$ for any i.

Definition 3. The swarm size $\rho \in \Re^+$ is defined by

$$\rho = \max_{i=1,2,\ldots,M} (\|\mathbf{x}^i - \bar{\mathbf{x}}\|). \tag{5}$$

The swarm size is the ultimate bound on the distance between the position $\mathbf{x}^i$ of the individual i and the swarm center $\bar{\mathbf{x}}$. This definition is necessary for analyzing the cohesion of swarm.

Let us define the position $\mathbf{x} \in \Re^{nM}$ and the velocity $\mathbf{v} \in \Re^{nM}$ of the swarm members as

$$\mathbf{x} = (\mathbf{x}^{1^T}, \ldots, \mathbf{x}^{M^T})^T, \mathbf{v} = (\mathbf{v}^{1^T}, \ldots, \mathbf{v}^{M^T})^T \tag{6}$$

There are different types of energy existed in swarms. Average internal energy of the swarm which is denoted as U is given by:

$$U(\mathbf{x}, \mathbf{v}) = 1/M[J(\mathbf{x}) + E_k(\mathbf{v}) - E_b(\bar{\mathbf{v}})]. \tag{7}$$

where $J : \Re^{nM} \to \Re$ is the potential energy, $E_b : \Re^n \to \Re^+$ is the bulk kinetic energy of the swarm, and $E_k : \Re^{nM} \to \Re^+$ is the total kinetic energy which are given respectively as:

$$J(\mathbf{x}) = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \left[J_a(\|x^i - x^j\|) - J_r(\|x^i - x^j\|) \right]. \tag{8}$$

$$E_b(\mathbf{v}) = \frac{1}{2}\left(\sum_{i=1}^{M} m_i\right)\|\bar{\mathbf{v}}\|^2, \quad E_k(\mathbf{v}) = \frac{1}{2}\sum_{i=1}^{M} m_i \|\mathbf{v}^i\|^2 \tag{9}$$

The complete energy discussion is provided in earlier work [16-17]. The swarm temperature can be viewed as an average of swarm internal kinetic energy.

Definition 4. The swarm temperature $T \in \Re$ is the average of the swarm internal kinetic energy :

$$T(\mathbf{v}) = [E_k(\mathbf{v}) - E_b(\bar{\mathbf{v}})]/M. \tag{10}$$

A modified swarm temperature variable gives a measure of swarm internal kinetic energy. The swarm temperature is directly related to velocity distribution.

Next step is to study the rate of change in temperature. The time differentiation of temperature is given by:

$$\dot{T}(\mathbf{v}) = \phi + \sigma + \psi \tag{11}$$

where $\phi, \sigma,$ and ψ are:

$$\phi = \frac{1}{M}\left\{\sum_{i=1}^{M}\left[(\mathbf{u}^i_{ext})^T \mathbf{v}^i\right] - \bar{\mathbf{v}}^T\left(\sum_{i=1}^{M}\mathbf{u}^i_{ext}\right)\right\} \tag{12}$$

$$\sigma = \frac{1}{M}\left\{\sum_{i=1}^{M}\left[(\mathbf{u}^i_{in})^T \mathbf{v}^i\right]\right\} \tag{13}$$

$$\psi = \frac{1}{\mathrm{M}}\left\{\bar{\mathbf{v}}^T\left(\sum_{i=1}^{M} b_i \mathbf{v}^{\mathbf{i}}\right) - \sum_{i=1}^{M}\left[(\mathbf{v}^i)^T b_i \mathbf{v}^i\right]\right\} \tag{14}$$

The swarm temperature derivative has three separate components. The first term ϕ affects the temperature due to external inputs and the second term σ and the third term ψ affects the temperature respectively due to artificial gradient force inputs and damping effect. The temperature can be computed in more distributed manner only by using relative velocity information between each pair of member.

Lemma 1. In a homogenous case, the temperature can be defined as:

$$T(\mathbf{v}) = \frac{m}{2M^2}\left\{\sum_{\mathrm{i}=1}^{M-1}\sum_{j=i+1}^{M}\|\mathbf{v}^i - \mathbf{v}^j\|^2\right\}. \tag{15}$$

Proof. See [17].

As a final step in this section, the rate of change in artificial potential function is studied. The artificial potential function $J(\mathbf{x})$ is linked to swarm cohesion. By study its behavior, more insight to cohesion problem can be achieved. First time derivative of potential function is expressed as:

$$\dot{J}(\mathbf{x}) = \sum_{i=1}^{M}\left(\sum_{j=1, j\neq i}^{M}\left[g_a(z_{ij}) - g_r(z_{ij})\right](\mathbf{x}^i - \mathbf{x}^j)^T\right)^T \mathbf{v}^i \tag{16}$$

where z_{ij} is the inter individual distance $z_{ij} = \|\mathbf{x}^i - \mathbf{x}^j\|$. To analyze the effect of inputs on $J(\mathbf{x})$, it is necessary to differentiate potential function for the second time. The second derivative has three separate components as:

$$\ddot{J} = \ddot{J}_\alpha + \ddot{J}_\beta + \ddot{J}_\gamma \tag{17}$$

where $\ddot{J}_\alpha$, $\ddot{J}_\beta$ and $\ddot{J}_\gamma$ are as follows:

$$\ddot{J}_\alpha = \sum_{i=1}^{M}\left\{\sum_{j=1, j\neq i}^{M}\left[\frac{d}{dz_{ij}}\left[g_a\left(z_{ij}\right)-g_r\left(z_{ij}\right)\right]\times \frac{(\mathbf{x}^i-\mathbf{x}^j)^T(\mathbf{v}^i-\mathbf{v}^j)}{\left\|\mathbf{x}^i-\mathbf{x}^j\right\|}\left(\mathbf{x}^i-\mathbf{x}^j\right)\right]\right\}^T \mathbf{v}^i \quad (18)$$

$$\ddot{J}_\beta = \sum_{i=1}^{M}\left\{\sum_{j=1, j\neq i}^{M}\left[\left\{g_a\left(z_{ij}\right)-g_r\left(z_{ij}\right)\right\}\left(\mathbf{v}^i-\mathbf{v}^j\right)\right]\right\}^T \mathbf{v}^i \quad (19)$$

$$\ddot{J}_\gamma = -\sum_{i=1}^{M}\left(\mathbf{u}_{in}^i\right)^T \frac{1}{m_i}\left(\mathbf{u}_{in}^i+\mathbf{u}_{ext}^i\right) \quad (20)$$

Note that the relative degree of artificial potential function $J(\mathbf{x})$ is two. The control inputs are present in the third component. The time derivatives of temperature and potential energy are used to calculate a control feedback in next section.

III. Control of Energetic Swarm

A. Temperature Controller

In this section, the objective is to track the desired trajectory of the swarm center with variable temperatures. Let $\bar{\mathbf{x}}_d(t)$ denote the desired trajectory. Equations of motion for the swarm center can be shown as:

$$(\sum_{i=1}^{M} m_i)\ddot{\bar{\mathbf{x}}} = \sum_{i=1}^{M}\mathbf{u}_{ext}^i - \sum_{i=1}^{M} b_i \mathbf{v}^i \quad (21)$$

It is assumed that there is no velocity damping term; that means $b_i = 0$, for any i. In other words, this assumption implies that the environment is non-viscous.

Design procedures include two different steps. The first step is to design the controller such that the swarm center tracks the desired trajectory. The second step is to control the swarm temperature by modifying the control input designed in the first step. In the first stage, the tracking controller is developed as:

$$\sum_{i=1}^{M}\mathbf{u}_{ext}^i = -k_P(\bar{\mathbf{x}}-\bar{\mathbf{x}}_d) - k_D(\bar{\mathbf{v}}-\dot{\bar{\mathbf{x}}}_d) + \left(\sum_{i=1}^{M} m_i\right)\ddot{\bar{\mathbf{x}}}_d \quad (22)$$

where k_p and k_D are positive constants. The above controller is a simple PD controller. Any combination of individual external input $\mathbf{u}_{ext}^i$ that satisfies (22) guarantees the tracking for the swarm center. However, it does not guarantee the cohesion of swarm.

Suppose that individuals are initially in a cohesive configuration, a simple way not to disturb the cohesion of swarm is to move all individuals with almost the same velocity. This implies a homogenous distribution of the total external control input among swarm members; that introduce a controller such as

$$\mathbf{u}_{ext}^i = -1/M\left(k_P(\bar{\mathbf{x}}-\bar{\mathbf{x}}_d)+k_D(\bar{\mathbf{v}}-\dot{\bar{\mathbf{x}}}_d)\right)+m\ddot{\bar{\mathbf{x}}}_d \quad (23)$$

where $i = 1,2,...,M$. Note that the above distribution is designed for the homogenous swarm.

The second step is to introduce a separate extra control input as $\mathbf{u}_T^i$ for temperature control. Unlike the tracking controller, a nonlinear controller is required for temperature control.

Proposition 1. Consider the following controller for homogenous swarm $m_i = m$ in non viscous environment e.g., $b_i = 0$ for any i.

$$\mathbf{u}_{ext}^i = -\frac{1}{M}\left[k_P(\bar{\mathbf{x}}-\bar{\mathbf{x}}_d)+k_D(\bar{\mathbf{v}}-\dot{\bar{\mathbf{x}}}_d)\right]+m\ddot{\bar{\mathbf{x}}}_d+\mathbf{u}_T^i \quad (24)$$

where the extra control is given by:

$$\mathbf{u}_T^i = -\sum_{j=1, j\neq i}^{M}\alpha_{ij}\left(\mathbf{v}^i-\mathbf{v}^j\right) \quad (25)$$

The α_{ij} is the control parameter and developed as:

$$\alpha_{ij} = -\frac{(\mathbf{x}^i-\mathbf{x}^j)^T\beta_{ij}(\mathbf{v}^i-\mathbf{v}^j)}{\left\|\mathbf{v}^i-\mathbf{v}^j\right\|^2}+\lambda\frac{\left(\left\|\mathbf{v}^i-\mathbf{v}^j\right\|^2-\gamma\right)}{\left\|\mathbf{v}^i-\mathbf{v}^j\right\|^2} \quad (26)$$

where λ is a positive constant and β_{ij} is given as:

$$\beta_{ij} = g_a\left(\left\|\mathbf{x}^i-\mathbf{x}^j\right\|\right)-g_r\left(\left\|\mathbf{x}^i-\mathbf{x}^j\right\|\right) \quad (27)$$

It can be shown that a swarm is tracking a desired trajectory $\bar{\mathbf{x}}_{des}(t)$ with a temperature $T_{des} = (M-1)\gamma/\left(4mM\right)$ for any $\mathbf{x}(t_0)$, $\mathbf{v}(t_0)$ and $T(t_0)$.

Proof. First, a proof for trajectory tracking is given. For tracking, it is required to investigate the total external input. From (24) the total external input is calculated as:

$$\sum_{i=1}^{M}\mathbf{u}_{ext}^i = -k_P(\bar{\mathbf{x}}-\bar{\mathbf{x}}_d)-k_D(\bar{\mathbf{v}}-\dot{\bar{\mathbf{x}}}_d)+mM\ddot{\bar{\mathbf{x}}}_d+\sum_{i=1}^{M}\mathbf{u}_T^i \quad (28)$$

To keep the tracking characteristic of the above control, the extra input $\mathbf{u}_T^i$ should satisfy the following condition $\sum\mathbf{u}_T^i = 0$. This condition holds since control parameters α_{ij} satisfies the symmetry, e.g., $\alpha_{ij} = \alpha_{ji}$ for any i, j. Hence, the summation of $\mathbf{u}_T^i$ is given by:

$$\sum_{i=1}^{M}\mathbf{u}_T^i = -\sum_{i=1}^{M-1}\sum_{\mathbf{j}=i+1}^{M}\left[\alpha_{\mathrm{ij}}\left(\mathbf{v}^i-\mathbf{v}^j\right)+\alpha_{\mathrm{ji}}\left(\mathbf{v}^j-\mathbf{v}^i\right)\right]=0 \quad (29)$$

This means that the extra control input $\mathbf{u}_T^i$ can be viewed as an internal input. The first part of proof is complete. It is required to calculate the temperature derivative. Three different terms of the temperature derivative is calculated as:

$$\phi = 0,\quad \sigma = \frac{1}{M}\left\{\sum_{i=1}^{M}\left[(\mathbf{u}_{in}^i+\mathbf{u}_T^i)^T\mathbf{v}^i\right]\right\},\quad \psi = 0 \quad (30)$$

Since $\mathbf{u}_T^i$ can be viewed as a part of internal controller, it can be concluded that the external input is distributed homogenously. From homogenous distribution of external control and (12) it can be concluded that the first term ϕ is zero for homogenous swarm. The third term ψ is zero due to non-viscous environment; i.e. $b_i = b = 0$. Combining (2),(25), and (30):

$$\sigma = \frac{1}{M}\left\{\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}(\mathbf{x}^i - \mathbf{x}^j)^T \beta_{ij}(\mathbf{v}^i - \mathbf{v}^j)\right\} - \frac{1}{M}\left\{\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}(\mathbf{v}^i - \mathbf{v}^j)^T \alpha_{ij}(\mathbf{v}^i - \mathbf{v}^j)\right\} \tag{31}$$

Substituting (26) in (31):

$$\dot{T} = \sigma = -\frac{1}{M}\left\{\sum_{i=1}^{M-1}\sum_{j=1}^{M}\lambda\left[\left\|\mathbf{v}^i - \mathbf{v}^j\right\|^2 - \gamma\right]\right\} \tag{32}$$

Form lemma 1 and above equation, it can be concluded that:

$$\left(\frac{m}{2M}\right)\dot{T} + \lambda\left(T - \frac{(M-1)}{4mM}\gamma\right) = 0 \tag{33}$$

The above first order differential equation is stable since $\lambda > 0$. This implies that $T(t) \to T_{des}$. This completes the proof. □

B. Multi-output Controller

In this section, the multi-output controller is desired. It is assumed that the swarm is homogenous and the environment is non-viscous. The control objectives are as follows: trajectory tracking for swarm center; temperature and potential energy regulation.

Let $\overline{\mathbf{x}}_d(t)$ denote the desired trajectory, T_{des} is desired temperature and J_d is a desired artificial potential energy. The design procedure is like before so the tracking controller is developed as (23) and will be modified by extra control input $\mathbf{u}_i^T$ as (24).Then inputs are modified for temperature and potential energy control. The design is based on feedback linearization method. Therefore, the control input should be designed in such a way that it satisfies the following condition:

$$\dot{T} + \lambda(T - T_{des}) = 0 \tag{34}$$

$$\ddot{J} + 2\tau(\dot{J}) + \tau^2(J - J_{des}) = 0 \tag{35}$$

where λ and τ are positive constant. Like before, to keep the tracking characteristic of the above control, the extra input should satisfy the following condition $\sum \mathbf{u}_T^i = 0$. In consequence, temperature derivative can be calculated similarly by (30). Combining (30) and (34):

$$\sum_{i=1}^{M}(\mathbf{u}_T^i)^T \mathbf{v}^i = -\lambda M(T - T_{des}) - \sum_{i=1}^{M}(\mathbf{u}_{in}^i)^T \mathbf{v}^i \tag{36}$$

On the other hand, combining (35) and (16)-(20):

$$\sum_{i=1}^{M}\left(\mathbf{u}_T^i\right)^T \mathbf{u}_{in}^i = m\left(\tau^2(J - J_d) + 2\tau\dot{J} + \ddot{J}_\alpha + \ddot{J}_\beta\right) - \sum_{i=1}^{M}\left(\mathbf{u}_{in}^i\right)^T \mathbf{u}_{ext}^i - \sum_{i=1}^{M}\left\|\mathbf{u}_{in}^i\right\|^2 \tag{37}$$

Any choice of $\mathbf{u}_i^T$ which satisfies conditions (36), (37) and $\sum \mathbf{u}_T^i = 0$ ensure output regulation and trajectory tracking of swarm center. The extra control input is developed as:

$$\mathbf{u}_T^i = -\sum_{j=1, j\neq i}^{M} c_{ij}\left(\mathbf{v}^i - \mathbf{v}^j\right) - \sum_{j=1, j\neq i}^{M}\kappa_{ij}\left(\mathbf{x}^i - \mathbf{x}^j\right) \tag{38}$$

where c_{ij} and κ_{ij} are the control parameter. The symmetry of control parameters ensures the condition $\sum \mathbf{u}_T^i = 0$. Symmetry of control parameter means that $c_{ij} = c_{ji}$ and $\kappa_{ij} = \kappa_{ji}$ for any i, j. One symmetric choice can be the case that $c_{ij} = c$ and $\kappa_{ij} = \kappa$ for any i, j.

To calculate the control parameters in this case, control input (38) is substituted into right hand side of (36) and (37). Then, Solving (36) and (37) for κ and c as

$$\begin{bmatrix}\kappa \\ c\end{bmatrix} = \mathbf{A}^{-1}\mathbf{B} \tag{39}$$

where $\mathbf{A}$ matrix and $\mathbf{B}$ vector are expressed as

$$\begin{aligned} b_1 &= \lambda M(T - T_{des}) + \sum_{i=1}^{M}\left(\mathbf{u}_{in}^i\right)^T \mathbf{v}^i \\ b_2 &= m\left(\tau^2(J - J_d) + 2\tau\dot{J} + \ddot{J}_\alpha + \ddot{J}_\beta\right) - \sum_{i=1}^{M}\left(\mathbf{u}_{in}^i\right)^T \mathbf{u}_{ext}^i - \sum_{i=1}^{M}\left\|\mathbf{u}_{in}^i\right\|^2 \end{aligned} \tag{40}$$

$$\mathbf{A} = \begin{bmatrix} \sum_{i=1}^{M}\left\{\sum_{j\neq i}^{M}\left(\mathbf{x}^i - \mathbf{x}^j\right)\right\}^T \mathbf{v}^i & \sum_{i=1}^{M}\left\{\sum_{j\neq i}^{M}\left(\mathbf{v}^i - \mathbf{v}^j\right)\right\}^T \mathbf{v}^i \\ \sum_{i=1}^{M}\left\{\sum_{j\neq i}^{M}\left(\mathbf{x}^i - \mathbf{x}^j\right)\right\}^T \mathbf{u}_{in}^i & \sum_{i=1}^{M}\left\{\sum_{j\neq i}^{M}\left(\mathbf{v}^i - \mathbf{v}^j\right)\right\}^T \mathbf{u}_{in}^i \end{bmatrix} \tag{41}$$

The proposed method controls swarm temperature and artificial potential energy simultaneously. Control of the potential energy guarantees the cohesion of the swarm. Bounding potential energy will result in swarm cohesion. Suppose the combine attraction repulsion function $h(\cdot)$ given by:

$$h(\|\mathbf{y}\|) = J_a(\|\mathbf{y}\|) - J_r(\|\mathbf{y}\|) \tag{42}$$

It is assumed that $h(\cdot)$ is radially unbounded function and it can be rewritten as:

$$h(\|\mathbf{y}\|) = \begin{cases} h_1(\|\mathbf{y}\|), & \|\mathbf{y}\| \geq \delta \\ h_2(\|\mathbf{y}\|), & \|\mathbf{y}\| \leq \delta \end{cases} \tag{43}$$

where $h_1(\cdot)$ is absolutely increasing and $h_2(\cdot)$ is absolutely decreasing and $h(\cdot)$ has global minimum at $\|\mathbf{y}\| = \delta$. If the potential energy is bounded:

$$J(\mathbf{x}) \leq J_{\max} \tag{44}$$

Then the size of swarm is bounded

$$\max_{i=1,2,\ldots,M}\left(\left\|\mathbf{e}^i\right\|\right) \leq \rho_{\max} \tag{45}$$

where $\rho_{\max}$ can be estimated as

$$\rho_{\max} = (M - 1/M)h_1^{-1}\left(J_{\max} - 0.5(M^2 - M - 3)h(\delta)\right)$$

The formal derivation of the bound is given in [17]. The multi output control with potential regulation guarantee the cohesion of swarm.

IV. Simulation Results and Application

A. Simulation Result

In this section, simulation results are based on two different experiments. In the simulations, the swarm has 10 members and is moving in 2-D space for easy visualization. The attraction and repulsion functions are expressed by:

$$g_a(\|\mathbf{y}\|) = 0.2/\|\mathbf{y}\|, \quad g_r(\|\mathbf{y}\|) = 0.2/\|\mathbf{y}\|^2 \tag{46}$$

The purpose of the first experiment is to validate the proposed multi-output controller in section III. Fig. 1 shows how the swarm temperature and swarm potential energy approaches to the corresponding desired values. Fig. 2 shows the bound on the swarm size at two different level of potential energy. Note that the swarm internal energy is equal for both cases. Therefore, the one with smaller potential energy has more temperature. The size of the swarm expands due to increase of potential energy. The potential control guarantees the cohesion of the swarm since the upper bound on potential function implies the bound on swarm size.

In the second experiment the swarm center moves such that its center moves around a circle of radius 15.00 with different desired temperature. Fig. 3 shows the covered area by swarm of WMRs while the desired temperature is set at two different temperatures. The swarm covered more area when it has higher temperature. This shows that a more thorough foraging behavior can be obtained using a higher temperature.

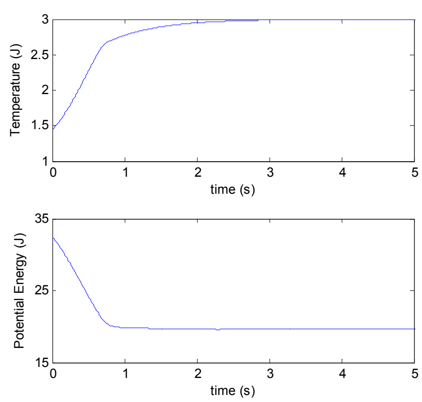

Fig.1 swarm temperature and potential energy versus time

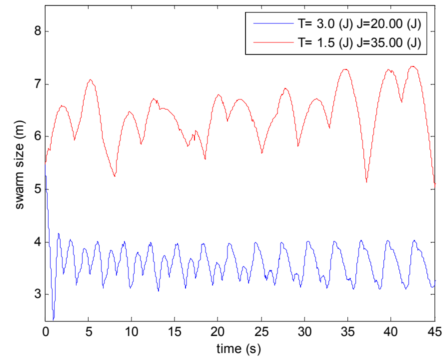

Fig.2 swarm size at two different potential energy levels

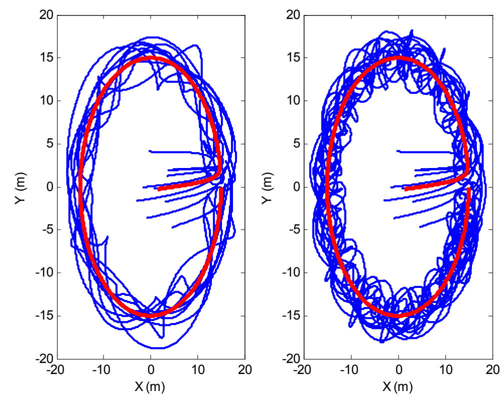

Fig.3 swept area at T=0.5 (J) vs. swept area at T=4.5 (J) while J=20.00

B. Swarm of Wheeled Mobile Robots

In this section, the feedback linearization method is used to implement the proposed temperature controller. The kinematic and dynamic equations of WMR agent as shown in Fig. 4 are given by:

$$\begin{aligned} \dot{x}_c &= v_c \cos\theta \\ \dot{y}_c &= v_c \sin\theta \\ \dot{\theta} &= \omega \\ m\dot{v}_c &= \frac{1}{\rho}(\tau_R + \tau_L) \\ J\dot{\omega}_c &= \frac{d}{2\rho}(\tau_R - \tau_L) \end{aligned} \tag{47}$$

where v_c is the surge speed of WMR. (x_c, y_c) denotes the position of the center of robot. The orientation of the robot is given by θ and ω is angular speed, m is the mass of WMR and J is its moment of inertia, τ_R and τ_l are the torques generated by the right and left wheel respectively, $d/2$ is the moment arm and ρ denotes the radius of the wheel.

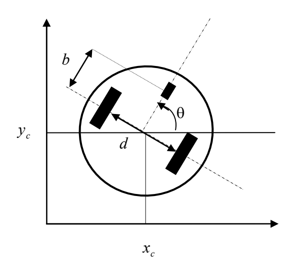

Fig.4 wheeled mobile robot

The following system can be transformed via feedback to simple integrators. The input-output linearization transformation matrix is not unique. One simple choice for output can be expressed as:

$$\begin{aligned} y_1 &= x_c + b\cos\theta \\ y_2 &= y_c + b\sin\theta \end{aligned} \tag{48}$$

(y_1, y_2) denotes the Cartesian coordinate of the tip of the robot. As shown in Fig. 4, b is the drift distance along the main axis of the unicycle. Second times differentiation of y_1 and y_2 are given as:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \mathbf{A}_w \begin{bmatrix} \tau_R \\ \tau_L \end{bmatrix} + \mathbf{B}_w \tag{49}$$

$$\mathbf{A}_w = \begin{bmatrix} \frac{1}{\rho m}\cos\theta - \frac{bd}{2\rho J}\sin\theta & \frac{1}{\rho m}\cos\theta + \frac{bd}{2\rho J}\sin\theta \\ \frac{1}{\rho m}\sin\theta + \frac{bd}{2\rho J}\cos\theta & \frac{1}{\rho m}\sin\theta - \frac{bd}{2\rho J}\cos\theta \end{bmatrix} \tag{50}$$
$$\mathbf{B}_\mathrm{w} = \begin{bmatrix} -b\omega^2\cos\theta - \omega v_c \sin\theta \\ -b\omega^2\sin\theta + \omega v_c\cos\theta \end{bmatrix}$$

Note that $\mathbf{A}_w$ is invertible so the system is transformed into input-output linear form by using the following input transformation:

$$\begin{bmatrix} \tau_R \\ \tau_L \end{bmatrix} = \mathbf{A}_w^{-1}\left(-\mathbf{B}_w + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \right) \tag{51}$$

(u_1, u_2) is a linear feedback controller. Consequently, the input-output equation is as follows:

$$\begin{aligned} \ddot{y}_1 &= u_1 \\ \ddot{y}_2 &= u_2 \end{aligned} \tag{52}$$

The defined controller will make the tip of the unicycle follow any desired trajectory. Moreover, it can be showed that the internal state is stable. For more details, see [21] and the reference in there.

After feedback linearization, the model is turned to simple double integrator model. Therefore, the swarm energy controller can be implemented for a case of swarm of unicycle. A control input for the i^{th} WMR can be shown as:

$$\begin{aligned} u_1^i &= (\mathbf{u}_{in}^i + \mathbf{u}_{ext}^i + \mathbf{u}_T^i)_x \\ u_2^i &= (\mathbf{u}_{in}^i + \mathbf{u}_{ext}^i + \mathbf{u}_T^i)_y \end{aligned} \tag{53}$$

where $\mathbf{u}_{in}^i$, $\mathbf{u}_{ext}^i$ and $\mathbf{u}_T^i$ are respectively proposed by (2), (23) and (38) . Also note that the choice of $\mathbf{x}^i$ and $\mathbf{v}^i$ are as follows: $\mathbf{x}^i = \begin{pmatrix} y_1^i & y_2^i \end{pmatrix}^T$ and $\mathbf{v}^i = \begin{pmatrix} \dot{y}_1^i & \dot{y}_2^i \end{pmatrix}^T$ for a case of groups of unicycle.

V. Conclusion and Future Works

In this paper, an M-member continuous-time energetic swarm problem is studied. A new swarm temperature variable is introduced to investigate the internal kinetic energy properties of the swarm. A controller is developed such that the swarm center can follow any desired trajectory with different corresponding swarm temperatures. Moreover, multi-output controller is established to control swarm temperature and its potential energy simultaneously. The swarm size can be varied by changing the potential energy. Finally, it is shown that feedback linearization method can be used to implement the proposed method for a group of wheeled mobile robot. Simulation is used to validate the results and to demonstrate the proposed method can be used effectively for multi-agent coverage path planning problem. Future work includes a control method to move a swarm with a predefined velocity distribution.

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