2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Feedforward Control to Attenuate Tension Error in Time-varying Tape Systems

Hua Zhong and Lucy Y. Pao

Abstract— **This paper presents two feedforward control algorithms to attenuate the tension error in digital tape systems. Tape systems are time-varying because the reel radii change as a result of both tape winding and reel eccentricities. The first algorithm addresses the time-varying nature of the reel radii due to tape winding by introducing a time-varying feedforward filter for the reference input. The second algorithm is designed to attenuate the tension error caused by periodic reel eccentricities. We investigate the error in previous periods and calculate the compensation input that should have canceled it. This input is then fed forward to the system for future periods. Simulation results show that the time-varying reference input achieved from the first feedforward filter leads to zero tension error if the variations in the radius are known. The second feedforward compensation input reduces the tension error caused by reel eccentricities by about three orders of magnitude.**

I. Introduction

Digital tapes were the major data storage devices when data centers were first established in the 1960s in large research centers. While hard disk drives have become the dominant storage media in the last two decades, digital tapes are still widely used in data centers for historical and economical reasons. Techniques have been developed to increase the tape winding speed and decrease the thickness of the tape in order to achieve higher data transfer rates and larger storage capacities. Thinner tape and faster rolling speed require more advanced control techniques to regulate the tape tension.

While some work has been conducted on tension control in web-winding systems including tape systems, most of that work assumes: (A) the plant is a linear time-invariant (LTI) system and (B) the reels of the web-winding system are perfectly circular [1][5][6]. Adaptive control has been investigated to reject quasi-periodic tension disturbances in web-winding systems [7][8], where the disturbances are considered as either an input or output disturbance to a LTI system. In [8], adaptive control algorithms are developed to synthesize an additional input whose system response is the additive inverse of the effects from the time-varying input or output disturbance to the tension. The synthesized input is calculated based on the frequency response of the LTI system at corresponding disturbance frequencies.

Tape systems are time-varying as the radii of both reels vary when the tape winds from the source reel to the take-up reel. The magnitude and phase of the time-varying plant at the corresponding disturbance frequencies are also time-varying. Thus using a LTI system to represent the tape system is insufficient. Moreover, reel eccentricities directly change the reel radii to aggravate the time-varying nature of the entire system. Little research has been conducted on attenuating tension errors from the reel eccentricities in time-varying tape systems.

The authors are with the Electrical and Computer Engineering Department, University of Colorado at Boulder, Boulder, CO 80309. zhongh@colorado.edu, pao@colorado.edu. This work has been partially supported by the US National Science Foundation (Grant CMMI-0700877).

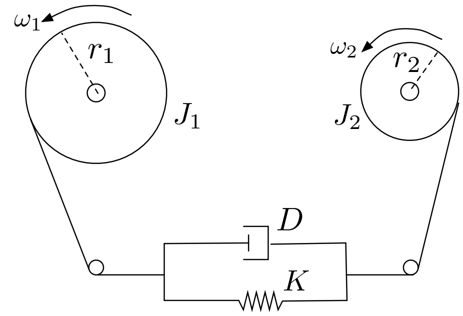

Fig. 1. Lumped-parameter model of a tape system.

In this paper, we propose two feedforward control schemes to regulate the tension in the time-varying tape system. First, we use a time-varying input reference to address the known variations in reel radii due to tape winding. Second, we use feedforward compensation to attenuate the quasi-periodic tension error caused by the unknown reel eccentricities.

This paper is organized as follows. Section II reviews a classic tape model and introduces a decoupled tape tension loop that we use in this study. Section III discusses the characteristics of the variations in the reel radii and the resulting tension errors in tape systems. Section IV outlines how the time-varying reference input filter is constructed and derives the feedforward control algorithm to attenuate tension errors caused by reel eccentricities. These control algorithms are then applied to the tension loop and simulation results are presented in Section V. Finally, conclusions and a discussion of future work are given in Section VI.

II. Tape Model and Tension Loop

The schematic in Fig. 1 illustrates a popular lumped-parameter model of a tape system [1][6]. The tape winds from the source reel (the left one) to the take-up reel. $J_i(t)$, $r_i(t)$, and $\omega_i(t)$ $(i = 1, 2)$ are the rotating inertia, radius, and angular velocity of each reel, respectively. The unsupported tape between the two tangential points on the reels is modeled by a parallel dashpot and spring with damping coefficient D and spring constant K, respectively. The tape thickness is denoted as ϵ.

978-1-4244-2079-7/08/$25.00 ©2008 AACC.

A. State-space Tape Model

Each reel is driven by a DC motor whose motor friction viscosity coefficient is denoted as β_i and the torque constant is denoted as K_{ti}. The current applied to each motor is $u_i(t)$. Define $T(t)$ as the tape tension and $V_i(t)$ as the tangential velocity of the tape at each reel. Let the state $X = [T(t), V_1(t), V_2(t)]^\top$, the input $U = [u_1(t), u_2(t)]^\top$, and $\eta(t) = \frac{r_1^2(t)}{J_1(t)} + \frac{r_2^2(t)}{J_2(t)}$. A state-space equation of the tape system is then [1]

$$\dot{X}(t) = \mathbf{A}(t)X(t) + \mathbf{B}(t)U(t),$$

$$\mathbf{A}(t) = \begin{bmatrix} -D\eta(t) & -K+D\frac{\beta_1}{J_1(t)} & K-D\frac{\beta_2}{J_2(t)} \\ \frac{r_1^2(t)}{J_1(t)} & -\frac{\beta_1}{J_1(t)} & 0 \\ -\frac{r_2^2(t)}{J_2(t)} & 0 & -\frac{\beta_2}{J_2(t)} \end{bmatrix},$$

$$\mathbf{B}(t) = \begin{bmatrix} -D\frac{r_1(t)K_{t_1}}{J_1(t)} & D\frac{r_2(t)K_{t_2}}{J_2(t)} \\ \frac{r_1(t)K_{t_1}}{J_1(t)} & 0 \\ 0 & \frac{r_2(t)K_{t_2}}{J_2(t)} \end{bmatrix}.$$

When the tape winds, both r_i and J_i change. D and K might also vary [1]. However, this research aims at investigating the tension error caused by the time-varying reel radii in steady state and we only account for the time-varying dynamics from r_i directly. Hence other parameters are considered as constants.

The radius of the source reel decreases and that of the take-up reel increases when the tape winds. We call the time-varying reel radius without accounting for reel eccentricities the nominal radius $r_{ni}(t)$. In the tape industry, controllers are usually designed based on $r_{ni}(t)$ to regulate the tangential velocity and the tension of the tape. However, the reel eccentricities, which we denote as $r_{ri}(t)$, are often unknown and not addressed. Unavoidably, they cause errors in both velocity and tension. The velocity loop is much less sensitive to the variations in reel radii than the tension loop [4]. This study focuses on the tension loop and assumes the tape velocity is regulated to be constant.

B. Tape Tension Loop

The tension loop can be perfectly decoupled from the velocity loop at mid-pack where both reel radii are equal [4]. The transfer function of the decoupled tension loop from either input current to the output tension at mid-pack is

$$G_T = \frac{2K_t D r_m (s + \frac{K}{D})}{Js^2 + (\beta + 2Dr_m^2)s + 2Kr_m^2}, \quad (1)$$

where r_m is the reel radius of both reels at mid-pack. To simulate the time-varying tension loop in *Matlab*, the second-order system G_T in Equation (1) is converted to the observability canonical state-space form in which tension T is the first state. The other state is a function of the difference between the tangential velocities of the two reels. With the current to the motor as the input, the system model is

$$\begin{cases} \dot{X}_T(t) = A_T(t)X_T(t) + B_T u(t) \\ Y(t) = C_T X_T(t) \end{cases}, \quad (2)$$

$$A_T(t) = \begin{bmatrix} -\frac{\beta+2Dr(t)^2}{J} & 1 \\ -\frac{2Kr(t)^2}{J} & 0 \end{bmatrix},$$

$$B_T(t) = \begin{bmatrix} \frac{2K_t Dr(t)}{J} \\ \frac{2K_t Kr(t)}{J} \end{bmatrix},$$

$$C_T = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The system matrices $A_T(t)$ and $B_T(t)$ are functions of $r(t)$.

A full-state feedback K_f that stabilizes the tension loop when $r(t)$ changes from a full reel to an empty reel is applied, as shown in Fig. 2. T_d is the desired tension value, $R(t)$ is the time-varying filter for the reference input that takes into account the time-varying nominal radius, and Y is the output tension. The feedforward input $\tilde{u}$ is the compensation input that addresses reel eccentricities.

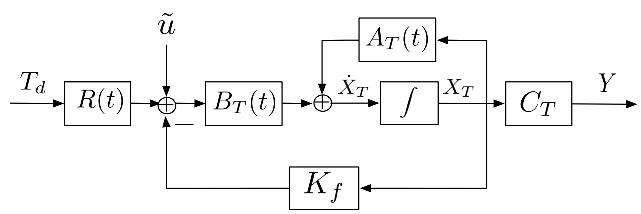

Fig. 2. The full-state feedback K_f stabilizes the tension loop. Time-varying $R(t)$ takes into account the time-varying nominal radius and $\tilde{u}$ is the feedforward input to address reel eccentricities.

III. Time-varying Reel Radii in Tape Systems

The variations in the reel radius $r_i(t)$ consist of two components: the nominal time-varying radius $r_{ni}(t)$ due to tape winding and the reel eccentricities $r_{ri}(t)$. The changing rate of $r_{ni}(t)$ is a function of the angular velocity $\omega_i(t)$ and the thickness of the tape ϵ. With these notations, the radius of the source reel $r_1(t)$ is

$$\begin{aligned} r_1(t) &= r_{n1}(t) + r_{r1}(t) \\ &= r_1(t_0) - \int_{t_0}^{t} \frac{\epsilon\omega_1(\tau)}{2\pi} d\tau + r_{r1}(t) \end{aligned}$$

and the radius of the take-up reel $r_2(t)$ is

$$r_2(t) = r_2(t_0) + \int_{t_0}^{t} \frac{\epsilon\omega_2(\tau)}{2\pi} d\tau + r_{r2}(t).$$

Since the tape is very thin, the magnitude of the nominal radii changing rate $\frac{\epsilon\omega_i(t)}{2\pi}$ is small enough so that a) the nominal radii can be considered as invariant in a certain time period and b) in one revolution, the radii variation due to reel eccentricities is more significant than the nominal change.

Reel eccentricities, also known as reel runout, are usually unknown and not addressed in the basic controllers in tape systems. They introduce tension ripples that can lead to lateral shifts in the tape, possibly causing permanent damage to the edges of the tape, which ultimately results in data loss. If the reel rotates at a constant angular velocity, the frequencies of the tension error caused by reel eccentricities are integer multiples of the reel rotating frequency. Moreover,

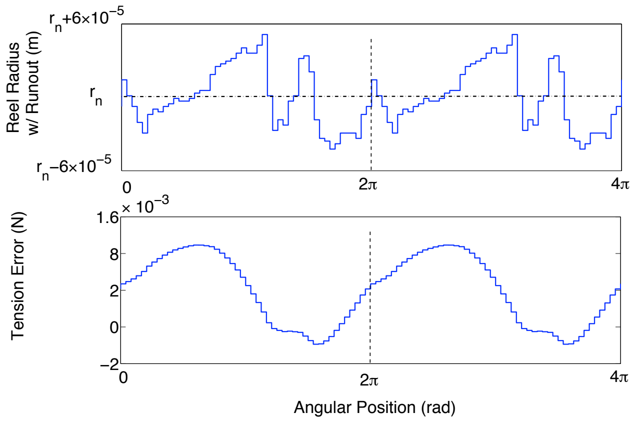

Fig. 3. Periodic reel runout and the tension error due to the runout for two revolutions.

if the nominal radius of the reel is invariant, the same reel eccentricity behavior repeats in every revolution.

Fig. 3 illustrates a periodic reel runout data set and the simulated tension error caused by the runout. The plots show the runout of a particular reel and the simulated tension error for two revolutions. The angular position is the angle the reel has rotated since the initial time. The top plot depicts the reel runout, where the horizontal dashed line is the constant nominal reel radius and the solid line is the reel radius with runout at different angular positions. Data from a tape manufacturer states that the nominal radius of a reel is on the order of 10^{-2} m and the reel runout is usually on the order of 10^{-5} m. With the current controllers applied in the industry, the 10^{-5} m reel runout causes a tension error on the order of 10^{-3} N. The simulated tension error illustrated in the bottom plot shows that the periodic reel runout leads to periodic tension error at the same frequency.

IV. Feedforward Control

A. Feedforward Controller for the Reference Input

We can derive a method [2] to introduce the reference input to a LTI system with full-state feedback so that the system perfectly tracks any reference input in steady state. For a linear time-varying system, a similar strategy can be applied to compute a time-varying gain for the reference input. Suppose the full-state feedback for the time-varying tension loop $\{A_T(t), B_T(t), C_T\}$ is K_f. When the compensation input $\tilde{u}$ is zero, the input to the plant is (Fig. 2):

$$u(t) = R(t)T_d - K_f X_T(t). \tag{3}$$

Define $\{A_T(t_i), B_T(t_i), C_T\}$ as the state matrices at a specific time t_i. Let the desired final value of the state and the control input for this particular system be $X_{Tss}(t_i)$ and $u_{ss}(t_i)$, respectively. The desired output in steady state is T_d. Then at the steady state of the system $\{A_T(t_i), B_T(t_i), C_T\}$, we have:

$$\begin{bmatrix} \mathbf{0} \\ T_d \end{bmatrix} = \begin{bmatrix} A_T(t_i) & B_T(t_i) \\ C_T & 0 \end{bmatrix} \begin{bmatrix} X_{Tss}(t_i) \\ u_{ss}(t_i) \end{bmatrix}.$$

Define the vector $\gamma_\mathbf{1}(t_i)$ and the scalar $\gamma_2(t_i)$ as

$$\begin{bmatrix} \gamma_\mathbf{1}(t_i) \\ \gamma_2(t_i) \end{bmatrix} = \begin{bmatrix} A_T(t_i) & B_T(t_i) \\ C_T & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix},$$

then

$$\begin{bmatrix} X_{Tss}(t_i) \\ u_{ss}(t_i) \end{bmatrix} = T_d \begin{bmatrix} \gamma_\mathbf{1}(t_i) \\ \gamma_2(t_i) \end{bmatrix}.$$

If the input filter in (3) at t_i is

$$R(t_i) = K_f \gamma_\mathbf{1}(t_i) + \gamma_2(t_i),$$

the steady-state tension error should be zero. In the discrete-time domain, one reference input filter can be computed at each sampling step if the state matrices are known. In a tape system, the nominal radii of the reels are usually known and hence the input reference filter can address variations in nominal radii.

B. Control to Address Reel Eccentricities

Consider the discrete-time state-space system:

$$\begin{cases} X_{k+1} &= A_k X_k + B_k u_k \\ Y_k &= C X_k \end{cases},$$

where A_k and B_k are the discrete-time state matrices at step k and C is constant [3]. Suppose the initial state is X_1, then the output series from Y_1 to Y_k is

$$\begin{aligned} Y_1 &= CX_1 \\ Y_2 &= CX_2 = CA_1X_1 + CB_1u_1 \\ &\vdots \\ Y_k &= C\prod_{i=1}^{k-1} A_i X_1 + C\sum_{j=1}^{k-2}\prod_{i=j+1}^{k-1} A_i B_j u_j + CB_{k-1}u_{k-1}. \end{aligned}$$

Defining $\vec{Y} = [Y_2, Y_3, \cdots, Y_k]^\top$ and $\vec{u} = [u_1, u_2, \cdots, u_{k-1}]^\top$, we now have

$$\vec{Y} = \Phi X_1 + \Gamma \vec{u},$$

where

$$\Phi = C\left[A_1, A_2A_1, \cdots, \prod_{i=1}^{k-1} A_i\right]^\top \quad \text{and}$$

$$\Gamma = C\begin{bmatrix} B_1 & 0 & \cdots & \cdots & 0 \\ A_2B_1 & B_2 & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \prod_{i=2}^{j=k-1} A_iB_1 & \prod_{i=3}^{j=k-1} A_iB_2 & \cdots & \cdots & B_{k-1} \end{bmatrix}.$$

Over a pre-determined period of time, the lower block triangular matrix Γ is a constant matrix. If it is invertible and the matrices A_i and B_i are known, we can solve for the input series $\vec{u}$ from $\vec{Y}$ and X_1:

$$\vec{u} = \Gamma^{-1}\left(\vec{Y} - \Phi X_1\right).$$

Denoting the desired output as y_d, the output error in Y_k as $\tilde{Y}_k = y_d - Y_k$, and the compensation input to cancel $\tilde{Y}_k$ as $\tilde{u}_k$, the new input with the compensation input at the k-th step $\hat{u}_k$ then is

$$\hat{u}_k = u_k + \tilde{u}_k.$$

The state-space equation for the new system becomes

$$\begin{cases} \hat{X}_{k+1} &= A_k \hat{X}_k + B_k \hat{u}_k \\ \hat{Y}_k &= C\hat{X}_k \end{cases}.$$

If the initial state is X_1 as in the original system, the new output at the k-th step is

$$\begin{aligned} \hat{Y}_k &= C\prod_{i=1}^{k-1} A_i X_1 + C\sum_{j=1}^{k-2}\prod_{i=j+1}^{k-1} A_i B_j \hat{u}_j + CB_{k-1}\hat{u}_{k-1} \\ &= Y_k + C\sum_{j=1}^{k-2}\prod_{i=j+1}^{k-1} A_i B_j \tilde{u}_j + CB_{k-1}\tilde{u}_{k-1}. \end{aligned}$$

Ideally, $\hat{Y}_k$ is equal to y_d and hence

$$\begin{aligned} \tilde{Y}_k &= y_d - Y_k = \hat{Y}_k - Y_k \\ &= C\sum_{j=1}^{k-2}\prod_{i=j+1}^{k-1} A_i B_j \tilde{u}_j + CB_{k-1}\tilde{u}_{k-1}. \end{aligned}$$

If CB_{k-1} is non-singular, we can solve for $\tilde{u}$:

$$\tilde{u}_{k-1} = \frac{1}{CB_{k-1}}\left(\tilde{Y}_k - C\sum_{j=1}^{k-2}\prod_{i=j+1}^{k-1} A_i B_j \tilde{u}_j\right).$$

If the system is periodic with a period N, i.e., $A_{k+N} = A_k$, $B_{k+N} = B_k$, and $Y_{k+N} = Y_k$, suppose the initial state in period μ is $X_{(1,\mu)}$, we can calculate the compensation input $\tilde{u}_{(i,\mu)}$ $(i = 1, 2, \cdots, N)$ that should have canceled the tension error in period μ. Then $\tilde{u}_{(i,\mu)}$ can be applied to the next period $\mu+1$ whose initial state $X_{(1,\mu+1)}$ equals $X_{(1,\mu)}$ and the system behaves the same as in period μ due to the periodicity of the system. Thus the output of period $\mu+1$ with the compensation input $\tilde{u}_{(i,\mu)}$ should be the desired output y_d. However, in the following period $\mu+2$, the initial state $X_{(1,\mu+2)}$ is not the same as $X_{(1,\mu)}$ because the compensation input $\tilde{u}$ has been applied for one period. Hence, $\tilde{u}_{(i,\mu)}$ will not cancel the error for period $\mu+2$. Thus we need to investigate at least two periods of the output error signal to construct the compensation input series $\tilde{u}$.

In the tape tension loop, if the nominal radius of the reel and the tape velocity are constant, the reel runout is periodic and so are the system matrices of the tape tension loop. The period N is determined by the sample rate and the rotational frequency. Hence the above feedforward control scheme can then be applied to attenuate tension error. In the simulations in Sections V-B and V-C, two periods of tension error signal are used to compute an input series $\tilde{u}_0(i)(i = 1, 2, \cdots, 2N)$. The complete compensation input signal $\tilde{u}$ then is constructed as illustrated in Fig. 4:

$$\begin{cases} \tilde{u}(k) &= \tilde{u}_0(k-2N), \quad & 2N < k \leq 3N \\ \tilde{u}(k) &= \tilde{u}_0\left((\bmod \frac{k}{N}) + N\right), \quad & k > 3N \end{cases}.$$

V. Simulation Results

In the simulations, the tension loop is implemented as in Equation (2) with the parameter values listed in Table I. The magnitude and phase of G_T as a function of the time-varying reel radius evaluated at the corresponding reel rotating frequency are illustrated in Fig. 5. The response varies from full reel ($r = 0.028$ m) to empty reel ($r = 0.014$ m). The time-varying matrices of the tension loop are

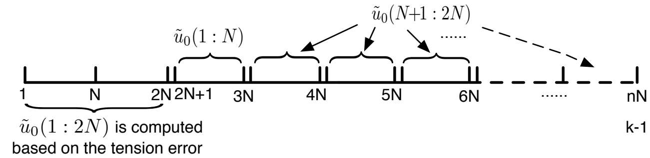

Fig. 4. Two periods of tension error signal are used to compute the compensation input series.

$$\begin{aligned} A_T(t) &= \begin{bmatrix} -1.95 + 5.87\times 10^4 r(t)^2 & 1 \\ -3.9\times 10^7 r(t)^2 & 0 \end{bmatrix} \\ B_T(t) &= \begin{bmatrix} 1.11\times 10^3 r(t) \\ 7.39\times 10^5 r(t) \end{bmatrix}. \end{aligned}$$

TABLE I

Tape system parameters for simulation

Parameter	Definition	Value
ϵ	Thickness of the tape	$10e$-5 m
K_t	Motor torque constant	0.0189 N m/Amp
D	Dashpot constant	0.9 N sec/m
$r(t)$	Radius of the reel	0.014 m-0.028 m
r_m	Mid-pack radius of the reel	0.02389 m
J	Inertia of the reel	$3.05e$-5 kg m^2
K	Spring constant	600 N/m
β	Motor viscosity coefficient	$5.9828e$-5 N m sec/rad

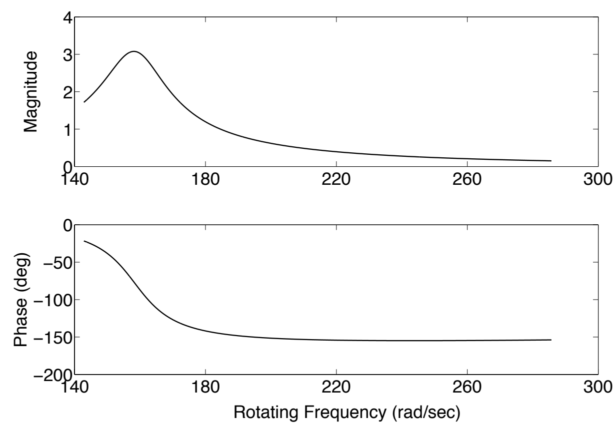

Fig. 5. Magnitude and phase of the time-varying G_T as a function of the time-varying reel radius evaluated at the corresponding rotating frequency.

We freeze the parameters of the time-varying tension loop at every time step and convert the continuous-time model to discrete-time with the sample period $T_s = 7.5 \times 10^{-4}$ seconds. The state feedback is designed as $K_f = [-0.7766, 0.0109]$ to guarantee the stability of the system over the entire web-winding process. The desired tension value T_d is 1 N. The desired tape velocity V_d is 4 m/sec and the actual tangential velocity of the reel is assumed to track V_d perfectly in steady state due to a velocity control loop such as in [1].

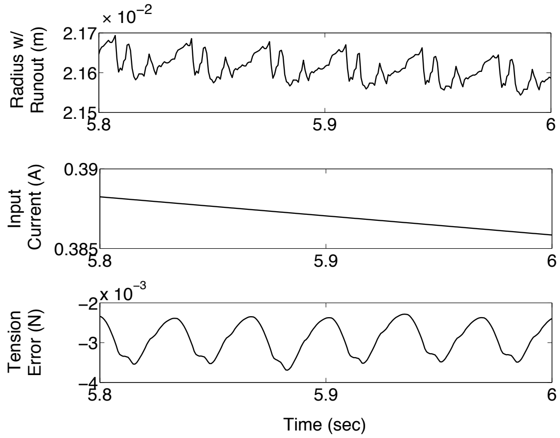

Fig. 6. The reel runout is from a tape manufacturer. The input current does not account for reel eccentricities and the tension error is on the order of 10^{-3} N.

A. Reel Runout is Known

Fig. 6 shows simulated tension error on a source reel using runout data from an actual tape industry sample. The maximum reel runout is 6×10^{-5} m. When only the compensation for the time-varying nominal reel radius is applied, the tension error caused by the reel eccentricities is on the order of 10^{-3} N.

If the reel runout data is collected during an initialization procedure when the tape cartridge is inserted into a tape drive, we can design the input reference filter as discussed in Section IV-A to address both the time-varying nominal radius and the reel eccentricities. Fig. 7 shows simulation results. As we know the nominal radius and the runout data in advance, the input current is designed to cancel the effects from the overall variations in the radius to the tension.

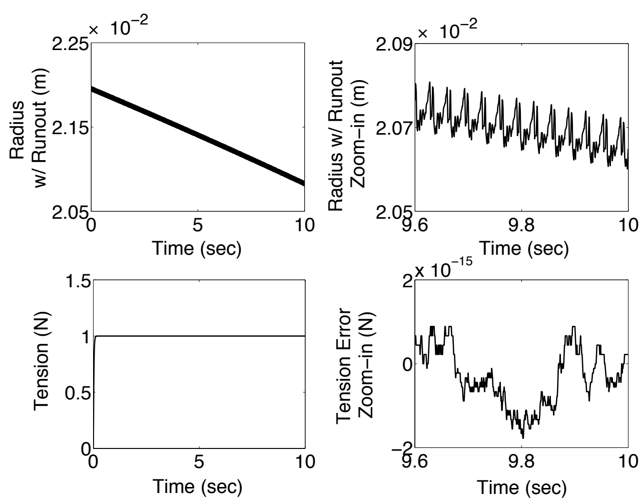

Fig. 7. Tension error is eliminated by a time-varying reference input that addresses both nominal reel radius variations and runout.

This scheme requires knowledge of both the nominal radius and runout. In practice, the nominal radius usually is available but the reel runout data is not. When the reel runout data is not available, we can take advantage of the periodicity of the reel runout to estimate an input that should

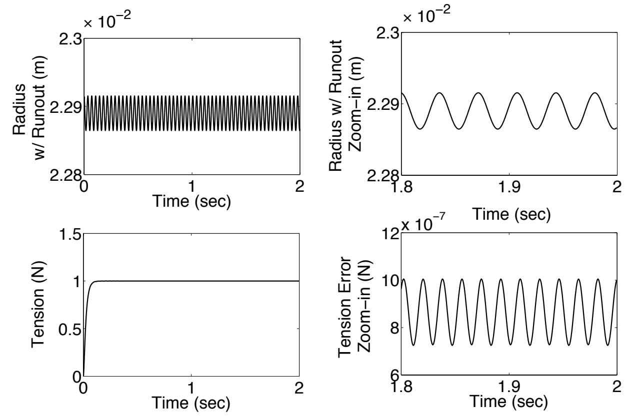

Fig. 8. Sinusoidal reel runout causes sinusoidal tension error; the compensation input is sinusoidal as well. The frequency of the residual tension error is twice that of the runout.

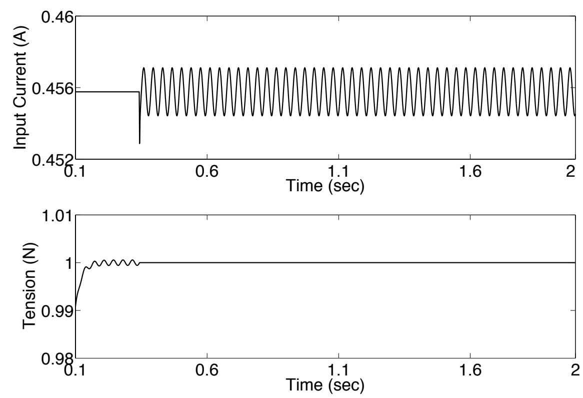

Fig. 9. Tension error is reduced after the compensation input is applied.

have canceled the effect from the runout on the tension in a previous period for future periods. In the next two simulations, the nominal radius is kept constant and unknown reel runout is considered.

B. Sinusoidal Reel Eccentricity

In this simulation, we apply a sinusoidal reel runout $r_r(t) = d_r \cos(\alpha(t))$ to the tension loop, where $d_r = 2.54 \times 10^{-5}$ m and $\alpha(t)$ is the angle that the reel has rotated at time t. Figs. 8 and 9 illustrate the tension error and the input. Two periods of tension error signal in steady state are used to compute the compensation input series. After the compensation input is applied, the steady state is reached very quickly ($\ll N$). The tension error caused by this runout is sinusoidal and the frequency is twice that of the runout. This is because both the input and the plant are sinusoidal at the same frequency as the runout. The feedforward control scheme reduces the tension error to the order of 10^{-6} N from 10^{-3} N.

C. Actual Reel Runout from a Tape Industry Sample

The reel runout data in this simulation is the same as in Section V-A but assumed to be unknown. We additionally add a bias (2.14×10^{-5} m) to the runout. Results are shown in Figs. 10 and 11. Similar to the results in Section V-B, the

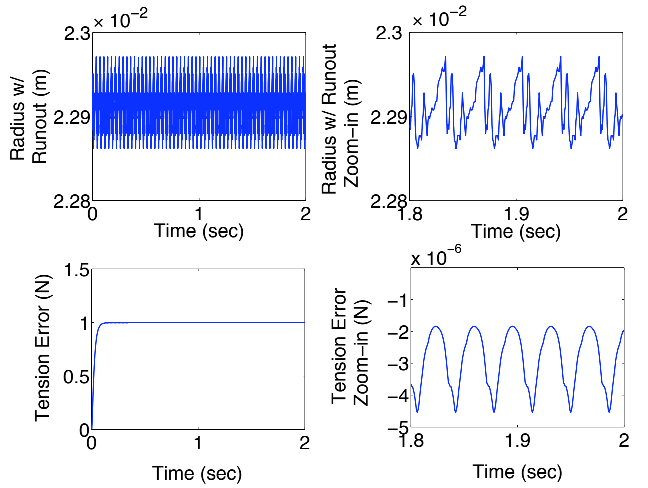

Fig. 10. Results using reel runout data from a tape manufacturer. The period of the residual tension error is the same as that of the runout.

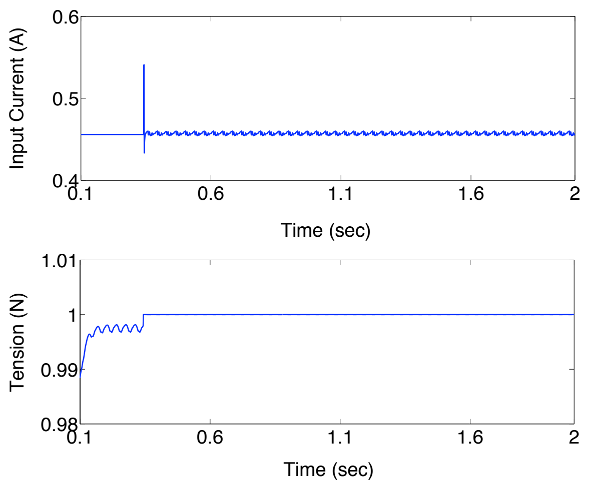

Fig. 11. The time-varying input attenuates the tension error by 3 orders of magnitude.

steady state is reached quickly after the compensation input is applied. The tension error is attenuated by about three orders of magnitude and the offset in the tension caused by the bias in the runout is compensated by the feedforward controller.

VI. Conclusions, Discussion, and Future Work

Feedforward control schemes to attenuate tension error in time-varying tape systems have been discussed. We use a feedforward time-varying controller to regulate the reference input to address the variations in the radius. Simulation results show that the output achieves zero steady-state error when all the varying components in the radius are known. This condition is usually satisfied if the variation in the radius is only due to the nominal change.

When the reel is not perfectly circular, eccentricities add variations to the reel radius and cause tension errors. To ameliorate this error, we design a feedfoward controller in the discrete-time domain based on the assumption that the time-varying tape system can be considered as periodic within a certain time span. We compute the compensation input that should have canceled the error in previous periods, and construct a complete series of the input based on periodicity for future periods. This algorithm is independent of the characteristics of the reel eccentricities as long as the system is periodic. It is first implemented on the tape tension loop with artificial sinusoidal reel eccentricities and then with real industry reel eccentricities, respectively. The tension errors caused by reel eccentricities are reduced by 3 orders of magnitude in both cases.

There are a few issues worth addressing about this latter feedforward algorithm. First, in both cases, the steady-state tension error does not go to zero. This is because we use the nominal state matrices to compute the compensation input instead of the state matrices that includes reel eccentricities in the radius. Thus the computed input is not ideally accurate. Second, this algorithm works best if the system under investigation is purely periodic. Moreover, the compensation input $\tilde{u}$ should be perfectly synchronized with the plant. A small time shift between $\tilde{u}$ and the plant deteriorates the performance of the tension loop. Since there is no feedback loop to account for tension errors caused by reel eccentricities, the error from the mismatch between the feedforward input and the plant can build up over time. Third, the sampling rate plays a significant role in the simulation as it directly impacts the periodicity of the sampled system.

We have thus far investigated the effects of time-varying reel radius in tape systems and implemented feedforward controllers to account for nominal radii changes and reel runout. Future work includes developing a feedforward control scheme that simultaneously takes into account unknown reel eccentricities and time-varying nominal radii for the entire tape winding process. Moreover, a feedback controller that accounts for potential tension error from the mismatch between the feedforward input and the system will improve overall stability and robustness. Further, adaptive feedback controllers to cancel output error caused by unknown time-varying parameters should also be investigated.

References

[1] M. Baumgart and L. Pao. "Robust control of nonlinear web-winding systems with and without tension sensors," *ASME J. Dynamic Systems, Measurement, and Control*, vol. 129, no. 1, January 2007, pp. 41-55.

[2] G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, Prentice Hall, 4th edition, 2002.

[3] G. F. Franklin, J. D. Powell, and M. Workman, *Digital Control of Dynamic Systems*, Prentice Hall, 3rd edition, 1998.

[4] Y. Lu, *Advanced Control for Tape Transport*, Ph.D. Thesis, Carnegie Mellon University 2002.

[5] Y. Lu and W. Messner, "Robust servo design for tape transport," *Proc. IEEE International Conference on Control Applications*, September 2001, pp. 1014-1019.

[6] P. Mathur and W. Messner, "Controller development for a prototype high-speed low-tension tape transport," *IEEE Trans. Control Systems Technology*, vol. 6, no. 4, July 1998, pp. 534-542.

[7] Y. Xu, M. Mathelin, and D. Knittel, "Adaptive rejection of quasi-periodic tension disturbances in the unwinding of a non-circular roll," *Proc. American Control Conference*, May 2002, pp. 4009-4014.

[8] H. Zhong, V. Kulkarni, and L. Pao, "Adaptive control for rejecting disturbances with time-varying frequencies in tape systems," *Proc. American Control Conference*, June 2005, pp. 533-538.