2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Design of Delay-Range-Dependent State Estimators for Discrete-Time Recurrent Neural Networks with Interval Time-Varying Delay

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Abstract—**This paper performs a global stability analysis of a particular class of recurrent neural networks (RNN) with time-varying delay. Both Lipschitz continuous activation functions and monotone nondecreasing activation functions are considered. Globally delay-dependent robust stability criteria are derived in the form of linear matrix inequalities (LMI) through the use of Leibniz-Newton formula and relaxation matrices. Finally, two numerical examples are given to illustrate the effectiveness of the given criterion.**

Index Terms—**Delay-range-dependent, state estimator, interval time-varying delay, linear matrix inequality.**

I. Introduction

Over the past few years, a great deal of interest has been devoted to the study of recurrent neural networks (RNNs) in various areas including signal processing, model identification, optimization, pattern recognition and associative memory. Many applications heavily have been presented on the dynamical behaviors for recurrent neural network. Additionally, time delays are frequently encountered in many practical areas, and it is now well known that time delays are one of the main cause of instability and oscillations in systems. Therefore, time delay is variant with time due to the finite switching speed of amplifiers. The existence of time delay could make delayed RNNs be instable or have poor performance. So, many research interests have been attracted to the stability analysis for delayed RNNs. A great deal of results related to this issue have been reported in this literature; see, e.g., [1]-[12].

State estimation is a subject of great practical and theoretical importance which has received much attention in recent years. Since the neuron states are not often fully available in the network outputs in many applications, the neuron state estimation problem is also important for many applications to utilize the estimated neuron state. The problem addressed is to estimate the neuron states through available output measurements such that the dynamics of the estimation error is globally exponentially stable. Recently, the state estimation problem for recurrent neural networks with time-varying delays was studied in [13], where an effective linear matrix inequality (LMI) approach was developed to solve the problem [14]. When the number of summands in a system equation is increased and the differences between neighboring argument values are decreased, systems with distributed delays will arise. Recently, the state estimation problem for such recurrent neural networks with mixed time delays has been dealt with in [15], where sufficient conditions for the existence of estimator have been obtained in terms of LMIs. However, it should be pointed out that the aforementioned results for both the discrete delay case and distributed delay case are delay-independent, that is, they do not include any information on the size of delays. It is known that delay-dependent conditions are generally less conservative than delay-independent ones, especially when the size of the delay is small. Therefore, delay-dependent results on the state estimation problem for RNNs with time-varying delays were proposed in [16], where the proposed method was applicable to the case that the derivative of a time-varying delay could take any value. However, it should be pointed out that the aforementioned results are continuous delayed RNNs. Recently, the dynamics analysis problem for discrete-time recurrent neural networks with or without time delays has received considerable research interest; see, e. g, [17]-[24]. Although delay-range-dependent results on the globally robust stability problem for discrete-time RNNs with interval time-varying delay were presented in [25], no delay-range dependent state estimation results on discrete-time recurrent neural networks with interval time-varying delay are available in the literature, and remain essentially open. The objective of this paper is to address this unsolved problem.

This paper deals with the problem of state estimation for discrete-time recurrent neural networks with interval time-varying delay. The interval time-varying delay includes both lower and upper bounds of delay. A delay-range-dependent condition for the existence of estimators is proposed and an LMI approach is developed. A general full order estimator is sought to guarantee that the resulting error system is globally asymptotically stable. Desired estimators can be obtained by the solution to certain LMIs, which can be solved numerically and efficiently by resorting to standard numerical algorithms [14]. Finally, an illustrative example is provided to demonstrate the effectiveness of the proposed method.

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978-1-4244-2079-7/08/$25.00 ©2008 AACC.

II. PROBLEM FORMULATION

Consider the following discrete-time recurrent neural network with interval time-varying delay

$$x(k+1) = Ax(k) + W_0 g(x(k)) + W_1 g(x(k-\tau(k))) + I(k), \quad (1)$$

where $x(k) = (x_1(k), x_2(k), \cdots, x_n(k))^T$ is the state vector, $A = diag(a_1, a_2, \cdots, a_n)$ with $|a_i| < 1$, $i = 1, 2, \cdots, n$, is the state feedback coefficient matrix, $W_0^{n\times n}$ and $W_1^{n\times n}$ are the interconnection matrices representing the weighting coefficients of the neurons, $g(x(k)) = [g_1(x_1(k)), \cdots, g_n(x_n(k))]^T \in R^n$ is the neuron activation function with $g(0) = 0$, $\tau(k)$ is the time-varying delay of the system satisfying

$$\tau_1 \le \tau(k) \le \tau_2, \; k \in \mathrm{N}, \quad (2)$$

where $0 \le \tau_1 \le \tau_2$ are known integers. $I(k)$ is the input vector.

In order to obtain our main results, the activation functions in (1) are assumed to be bounded and satisfy the following assumption.

Assumption 1

Fro any ς_1, $\varsigma_2 \in R$, the neuron activation functions satisfy

$$|g_i(\varsigma_1) - g_i(\varsigma_2)| \le \alpha_i |\varsigma_1 - \varsigma_2|, \; i = 1, 2, \cdots, n. \quad (3)$$

Our goal in this paper is to provide an efficient estimation algorithm in order to observe the neuron states from the available network outputs. For this reason, the network measurements are assumed to satisfy

$$y(k) = Cx(k) + q(k, x(k)), \quad (4)$$

where $y(k) \in R^m$ is the measurement output and C is a known constant matrix with appropriate dimension. $q(k, x(k))$ is the neuron-dependent nonlinear disturbances on the network outputs and satisfies the following Lipschitz condition

$$|q(k, \varsigma_1) - q(k, \varsigma_2)| \le |Q(\varsigma_2 - \varsigma_1)|, \quad (5)$$

where Q is a known real constant matrix.

For system (1) and (4), we now consider the following full-order estimator

$$\hat{x}(k+1) = A\hat{x}(k) + W_0 F(\hat{x}(k)) + W_1 F(\hat{x}(k-\tau(k))) + I(k) + L(y(k) - C\hat{x}(k) - q(k, \hat{x}(k)) + I(k), \quad (6)$$

where $\hat{x}(k)$ is the estimation of the neuron state and $L \in R^{m\times n}$ is the estimator gain matrix to be determined.

Our target is to choose a suitable L so that $\hat{x}(k)$ approaches $x(k)$ asymptotically. Let

$$e(k) = \hat{x}(k) - x(k) \quad (7)$$

be the state estimation error. Then, the error-state dynamics from the system (1) and (6) can be obtained as

$$e(k+1) = (A - Lc)e(k) + W_0 \hat{F}(e(k)) + W_1 \hat{F}(e(k-\tau(k))) - L\hat{q}(e(k)), \quad (8)$$

where

$$\hat{F}(e(k)) = F(\hat{x}(k)) - F(x(k)),$$

$$\hat{F}(e(k-\tau(k))) = F(\hat{x}(k-\tau(k))) - F(x(k-\tau(k))),$$

$$\hat{q}(e(k)) = q(k, \hat{x}(k)) - q(k, x(k)).$$

It is easy to see from Assumption 1 and the condition (5) that the solution of (1) exists for all $k \ge 0$ and is unique [26]. Moreover, there exists a unique zero equilibrium point to the error-state system (8).

The purpose of this paper is to develop delay-range-dependent conditions for the existence of estimators for the discrete-time recurrent neural network with interval time-varying delay. Specifically, for given scalars lower and upper bounds of delay, we are concerned with finding an asymptotically stable estimator in the form of (8) such that for any lower and upper bounds of delay satisfying $\tau_1 \le \tau(k) \le \tau_2$ the error-state system (8) is globally asymptotically stable.

III. MAIN RESULTS

This section explores the globally delay-range-dependent state estimation conditions given in (8). Specially, an LMI approach is employed to solve the estimator if the system (8) is globally asymptotically stable. The analysis commences by using the LMI approach to develop some results which are essential to introduce the following Lemma 1 for the proof of our main theorem in this section.

Lemma 1: Let D, S and P be real matrices of appropriate dimensions with $P > 0$. Then, for vectors x, $y \in R^n$

$$2x^T DSy \le x^T DPD^T x + y^T S^T P^{-1} S y. \quad (9)$$

For any matrices E_i, S_i and T_i $(i = 1, 2, \cdots, 7)$ of appropriate dimensions, it can be shown that

$$\begin{aligned}\Phi_1 = 2[&e^T(k)E_1 + e^T(k-\tau(k))E_2 + e^T(k-\tau_2)E_3 + e^T(k-\tau_1)E_4 + \hat{F}^T(e(k))E_5 \\ &+ \hat{F}^T(e(k-\tau(k)))E_6 + e^T(k+1)E_7] \\ &\times [e(k) - e(k-\tau(k)) - \sum_{j=k-\tau(k)+1}^{k}(e(j) - e(j-1))] = 0\end{aligned}, \quad (10)$$

$$\begin{aligned}\Phi_2 = 2[&e^T(k)S_1 + e^T(k-\tau(k))S_2 + e^T(k-\tau_2)S_3 + e^T(k-\tau_1)S_4 + \hat{F}^T(e(k))S_5 \\ &+ \hat{F}^T(e(k-\tau(k)))S_6 + e^T(k+1)S_7] \\ &\times [e(k-\tau(k)) - e(k-\tau_2) - \sum_{j=k-\tau_2+1}^{k-\tau(k)}(e(j) - e(j-1))] = 0\end{aligned}, \quad (11)$$

$$\begin{aligned}\Phi_3 = -2[&e^T(k)T_1 + e^T(k-\tau(k))T_2 + e^T(k-\tau_2)T_3 + e^T(k-\tau_1)T_4 + \hat{F}^T(e(k))T_5 \\ &+ \hat{F}^T(e(k-\tau(k)))T_6 + e^T(k+1)T_7] \\ &\times [e(k-\tau_1) - e(k-\tau(k)) - \sum_{j=k-\tau(k)+1}^{k-\tau_1}(e(j) - e(j-1))] = 0\end{aligned}, \quad (12)$$

$$\begin{aligned}\Phi_4 = -2[&e^T(k)H_1 + e^T(k-\tau(k))\cdot 0 + e^T(k-\tau_2)\cdot 0 + e^T(k-\tau_1)\cdot 0 + \hat{F}^T(e(k))\cdot 0 \\ &+ \hat{F}^T(e(k-\tau(k)))\cdot 0 + e^T(k+1)\cdot 0] \\ &\times [e(k+1) - (A-LC)e(k) - W_0\hat{F}(e(k)) \\ &- W_1\hat{F}(e(k-\tau(k))) + L\hat{q}(e(k))]\end{aligned}, \quad (13)$$

$$\begin{aligned}\Phi_5 = &2\hat{F}^T(e(k))R_1\hat{F}(e(k)) - 2\hat{F}^T(e(k))R_1\hat{F}(e(k)) \\ &+ 2\hat{F}^T(e(k-\tau(k)))R_2 e(k-\tau(k)) - 2\hat{F}^T(e(k-\tau(k)))R_2 e(k-\tau(k)) = 0\end{aligned}. \quad (14)$$

The following theorem is essential for solving the state estimation problem formulated in the previous section.

Theorem 1: Under Assumption 1, given scalars $0 \le \tau_1 < \tau_2$, the error-state dynamics in (8) with interval time-varying delay $\tau(k)$ satisfying (2) is globally asymptotically stable, if there exist matrices $P>0$, $Q_1>0$, $Q_2>0$, $Z_1>0$, $Z_2>0$, a nonsingular H_1, diagonal matrices $R_1>0$, $R_2>0$ and matrices Y_1, E_i, S_i and T_i $(i=1,2,\cdots,7)$ of appropriate dimensions such that the following LMI holds

$$\begin{bmatrix} \Omega & Y & \tau_2 E & \tau_{21} S & \tau_{21} T \\ Y^T & -I & 0 & 0 & 0 \\ \tau_2 E^T & 0 & -\tau_2 Z_1 & 0 & 0 \\ \tau_{21} S^T & 0 & 0 & -\tau_{21}(Z_1+Z_2) & 0 \\ \tau_{21} T^T & 0 & 0 & 0 & -\tau_{21} Z_2 \end{bmatrix} < 0 \quad (15)$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} \\ \Omega_{12}^T & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & \Omega_{27} \\ \Omega_{13}^T & \Omega_{23}^T & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} \\ \Omega_{14}^T & \Omega_{24}^T & \Omega_{34}^T & \Omega_{44} & \Omega_{45} & \Omega_{46} & \Omega_{47} \\ \Omega_{15}^T & \Omega_{25}^T & \Omega_{35}^T & \Omega_{45}^T & \Omega_{55} & \Omega_{56} & \Omega_{57} \\ \Omega_{16}^T & \Omega_{26}^T & \Omega_{36}^T & \Omega_{46}^T & \Omega_{56}^T & \Omega_{66} & \Omega_{67} \\ \Omega_{17}^T & \Omega_{27}^T & \Omega_{37}^T & \Omega_{47}^T & \Omega_{57}^T & \Omega_{67}^T & \Omega_{77} \end{bmatrix},$$

$\Omega_{11} = -P + (\tau_{21}+1)Q_1 + \tau_2 Z_1 + \tau_{21} Z_2 + E_1 + E_1^T + Q^T Q + Q_2$,

$\Omega_{12} = E_2 - E_1 + S_1 + T_1$, $\Omega_{13} = E_3^T - S_1$, $\Omega_{14} = E_4^T - T_1$, $\Omega_{15} = E_5^T + \Gamma^T R_1^T$,

$\Omega_{16} = E_6^T$, $\Omega_{17} = E_7^T - \tau_2 Z_1 - \tau_{21} Z_2 - C^T Y_1^T + A^T H_1^T$,

$\Omega_{22} = -Q_1 - E_2 - E_2^T + S_2 + S_2^T + T_2 + T_2^T$, $\Omega_{23} = -E_3^T - S_2 + S_3^T + T_3^T$,

$\Omega_{24} = -E_4^T - T_2 + S_4^T + T_4$, $\Omega_{25} = -E_5^T + S_5^T + T_5^T$, $\Omega_{26} = -E_6^T + S_6^T + T_6^T$,

$\Omega_{27} = -E_7^T + S_7^T + T_7^T$, $\Omega_{33} = -Q_2 - S_3 - S_3^T$, $\Omega_{34} = -S_4^T - T_3$,

$\Omega_{35} = -S_5^T$, $\Omega_{36} = -S_6^T$, $\Omega_{37} = -S_7^T$, $\Omega_{44} = -T_4 - T_4^T$, $\Omega_{45} = -T_5^T$,

$\Omega_{46} = -T_6^T$, $\Omega_{47} = -T_7^T$, $\Omega_{55} = -R_1 - R_1^T$, $\Omega_{56} = 0$, $\Omega_{57} = W_0^T H_1^T$,

$\Omega_{66} = -R_2\Gamma^{-1} - (R_2\Gamma^{-1})^T$, $\Omega_{67} = W_1^T H_1^T$,

$\Omega_{77} = -H_1 - H_1^T + P + \tau_2 Z_1 + \tau_{21} Z_2$,

$H = [H_1^T \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$,

$Y = HL = [Y_1^T \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$,

$E = [E_1^T \ \ E_2^T \ \ E_3^T \ \ E_4^T \ \ E_5^T \ \ E_6^T \ \ E_7^T]^T$,

$S = [S_1^T \ \ S_2^T \ \ S_3^T \ \ S_4^T \ \ S_5^T \ \ S_6^T \ \ S_7^T]^T$,

$T = [T_1^T \ \ T_2^T \ \ T_3^T \ \ T_4^T \ \ T_5^T \ \ T_6^T \ \ T_7^T]^T$, in which $\Gamma = diag(\alpha_1, \alpha_2, \cdots, \alpha_7)$, $\tau_{21} = \tau_2 - \tau_1$. In this case, a desired the estimator gain matrix L is given as $L = H_1^{-1} Y_1$.

Proof: Choose the Lyapunov-Krasovskii functional candidate for the error-state system in (8) as

$$\begin{aligned} V(k) &= V_1(k) + V_2(k) + V_3(k) + V_4(k) \\ &= e^T(k)Pe(k) + \sum_{i=-\tau_2}^{-1} \sum_{j=k+i+1}^{k} (e(j)-e(j-1))^T Z_1 (e(j)-e(j-1)) + \sum_{j=k-\tau(k)}^{k-1} e^T(j) Q_1 e(j) \\ &\quad + \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+1}^{k} e^T(j) Q_1 e(j) + \sum_{j=k-\tau_2}^{k-1} e^T(j) Q_2 e(j) \\ &\quad + \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+1}^{k} (e(j)-e(j-1))^T Z_2 (e(j)-e(j-1)) \end{aligned} \quad (16)$$

Then, the difference of $V(k)$ along the solution of (8) is given by

$$\Delta V_1(k) = e^T(k+1)Pe(k+1) - e^T(k)Pe(k),$$

$$\begin{aligned} \Delta V_2(k) &= \sum_{i=-\tau_2}^{-1} \sum_{j=k+i+2}^{k+1} (e(j)-e(j-1))^T Z_1 (e(j)-e(j-1)) \\ &\quad - \sum_{i=-\tau_2}^{-1} \sum_{j=k+i+1}^{k} (e(j)-e(j-1))^T Z_1 (e(j)-e(j-1)) \\ &\le \tau_2 (e(k+1)-e(k))^T Z_1 (e(k+1)-e(k)) \\ &\quad - \sum_{j=k-\tau_2+1}^{k-\tau(k)} (e(j)-e(j-1))^T Z_1 (e(j)-e(j-1)) \\ &\quad - \sum_{j=k-\tau(k)+1}^{k} (e(j)-e(j-1))^T Z_1 (e(j)-e(j-1)), \end{aligned}$$

$$\begin{aligned} \Delta V_3(k) &\le e^T(k)[(\tau_{21}+1)Q_1 + Q_2]e(k) - e^T(k-\tau(k))Q_1 e(k-\tau(k)) \\ &\quad - e^T(k-\tau_2) Q_2 e(k-\tau_2) \end{aligned}$$

$$\begin{aligned} \Delta V_4(k) &= \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+2}^{k+1} (e(j)-e(j-1))^T Z_2 (e(j)-e(j-1)) \\ &\quad - \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+1}^{k} (e(j)-e(j-1))^T Z_2 (e(j)-e(j-1)) \\ &\le (\tau_2-\tau_1)(e(k+1)-e(k))^T Z_2 (e(k+1)-e(k)) \\ &\quad - \sum_{j=k-\tau_2+1}^{k-\tau(k)} (e(j)-e(j-1))^T Z_2 (e(j)-e(j-1)) \\ &\quad - \sum_{j=k-\tau(k)+1}^{k-\tau_1} (e(j)-e(j-1))^T Z_2 (e(j)-e(j-1)), \end{aligned}$$

Defining the following new variables

$\eta(k) = [e^T(k) \ e^T(k-\tau(k)) \ e^T(k-\tau_2) \ e^T(k-\tau_1) \ \hat{F}^T(e(k)) \ \hat{F}^T(e(k-\tau(k))) \ e^T(k+1)]^T$,

$E = [E_1^T \ \ E_2^T \ \ E_3^T \ \ E_4^T \ \ E_5^T \ \ E_6^T \ \ E_7^T]^T$,

$S = [S_1^T \ \ S_2^T \ \ S_3^T \ \ S_4^T \ \ S_5^T \ \ S_6^T \ \ S_7^T]^T$,

$T = [T_1^T \ \ T_2^T \ \ T_3^T \ \ T_4^T \ \ T_5^T \ \ T_6^T \ \ T_7^T]^T$,

$H = [H_1^T \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$.

It follows from (16), (10)-(14) that

$$\begin{aligned} &\Delta V(k) \\ &= \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) + \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 \\ &\le e^T(k+1)Pe(k+1) - e^T(k)Pe(k) + \tau_2 (e(k+1)-e(k))^T Z_1 (e(k+1)-e(k)) \\ &- \sum_{j=k-\tau_2+1}^{k-\tau(k)} (e(j)-e(j-1))^T Z_1 (e(j)-e(j-1)) \\ &- \sum_{j=k-\tau(k)+1}^{k} (e(j)-e(j-1))^T Z_1 (e(j)-e(j-1)) \\ &+ e^T(k)((\tau_2-\tau_1)+1)Q_1 e(k) - e^T(k-\tau(k))Q_1 e(k-\tau(k)) \\ &+ (\tau_2-\tau_1)(e(k+1)-e(k))^T Z_2 (e(k+1)-e(k)) \\ &- \sum_{j=k-\tau_2+1}^{k-\tau(k)} (e(j)-e(j-1))^T Z_2 (e(j)-e(j-1)) \\ &- \sum_{j=k-\tau(k)+1}^{k-\tau_1} (e(j)-e(j-1))^T Z_2 (e(j)-e(j-1)) \\ &+ e^T(k) Q_2 e(k) - e^T(k-\tau_2) Q_2 e(k-\tau_2) \\ &+ 2\eta^T(k) E [e(k) - e(k-\tau(k)) - \sum_{j=k-\tau(k)+1}^{k} (e(j)-e(j-1))] \\ &+ 2\eta^T(k) S [e(k-\tau(k)) - e(k-\tau_2) - \sum_{j=k-\tau_2+1}^{k-\tau(k)} (e(j)-e(j-1))] \\ &- 2\eta^T(k) T [e(k-\tau_1) - e(k-\tau(k)) - \sum_{j=k-\tau(k)+1}^{k-\tau_1} (e(j)-e(j-1))] \\ &- 2\eta^T(k) H [e(k+1) - (A-LC)e(k) - W_0 \hat{F}(e(k)) \\ &\qquad - W_1 \hat{F}(e(k-\tau(k))) + L\hat{q}(e(k))] \\ &+ 2\hat{F}^T(e(k)) R_1 \hat{F}(e(k)) - 2\hat{F}^T(e(k)) R_1 \hat{F}(e(k)) \\ &+ 2\hat{F}^T(e(k-\tau(k))) R_2 e(k-\tau(k)) - 2\hat{F}^T(e(k-\tau(k))) R_2 e(k-\tau(k)). \end{aligned} \quad (17)$$

Moreover,

$$-2\eta^T(k)E\sum_{j=k-\tau(k)+1}^{k}(e(j)-e(j-1))$$
$$\le \tau_2\eta^T(k)EZ_1^{-1}E^T\eta(k)+\sum_{j=k-\tau(k)+1}^{k}(e(j)-e(j-1))^T Z_1(e(j)-e(j-1)), \quad (18)$$

$$-2\eta^T(k)S\sum_{j=k-\tau_2+1}^{k-\tau(k)}(e(j)-e(j-1))$$
$$\le(\tau_2-\tau_1)\eta^T(k)S(Z_1+Z_2)^{-1}S^T\eta(k)$$
$$+\sum_{j=k-\tau_2+1}^{k-\tau(k)}(e(j)-e(j-1))^T Z_2(e(j)-e(j-1)), \quad (19)$$

$$-2\eta^T(k)T\sum_{j=k-\tau(k)+1}^{k-\tau_1}(e(j)-e(j-1))$$
$$\le(\tau_2-\tau_1)\eta^T(k)TZ_2^{-1}T^T\eta(k)+\sum_{j=k-\tau(k)+1}^{k-\tau_1}(e(j)-e(j-1))^T Z_2(e(j)-e(j-1)) \quad (20)$$

Using Assumption 1 and noting that $R_1>0$ and $R_2>0$ are diagonal matrices, one has

$$2\hat{F}^T(e(k))R_1\hat{F}(e(k))\le 2\hat{F}^T(e(k))R_1\Gamma e(k), \quad (21)$$
$$-2\hat{F}^T(e(k-\tau(k)))R_2e(k-\tau(k))\le -2\hat{F}^T(e(k-\tau(k)))R_2\Gamma^{-1}\hat{F}(e(k-\tau(k))), \quad (22)$$

where $\Gamma=diag(\alpha_1,\alpha_2,\cdots,\alpha_7)$.

Substituting (18)-(22) into (17), it is not difficult to deduce that

$$\Delta V(k)\le\eta^T(k)[\Omega+\tau_2EZ_1^{-1}E^T+\tau_{21}SZ_2^{-1}S^T+\tau_{21}TZ_3^{-1}T^T+YY^T]\eta(k), \quad (23)$$

From condition (15) and Schur complement, it can be concluded that

$$\Omega+\tau_2EZ_1^{-1}E^T+\tau_{21}SZ_2^{-1}S^T+\tau_{21}TZ_3^{-1}T^T+YY^T<0 \quad . \quad (24)$$

It follows from (23) that the estimation error-state system (8) is asymptotically stable for interval time-varying delay $\tau(k)$ satisfying (2). This completes the proof of Theorem 1. □

Remark 1: Theorem 1 provides a sufficient condition for the globally stability of the discrete-time recurrent neural network with interval time-varying delay given in (1) and proposes a delay-range-dependent criterion. Even for $\tau_1=0$, the result in Theorem 1 may lead to the delay-dependent stability criteria. In fact, if $Z_2=\varepsilon_1 I$, with $\varepsilon_1>0$, being sufficient small scalars, $T_i=0$, $i=1,2,\cdots,7$, $E_4=0$, $S_4=0$, Theorem 1 yields the following delay-dependent criterion.

Corollary 1: Under assumption 1, given scalars $\tau_2>0$, $\tau_1=0$, the discrete-time recurrent neural network with time-varying delay satisfying (2) is globally asymptotically stable, if there exist matrices $P>0$, $Q_1>0$, $Q_2>0$, $Z_1>0$ and diagonal matrices $R_1>0$, $R_2>0$ and matrices E_i and S_i $(i=1,2,\cdots,7)$ of appropriate dimensions among $E_4=S_4=0$ such that the following LMI holds

$$\begin{bmatrix}\Omega & Y & \tau_2E & \tau_2S\\ Y^T & -I & 0 & 0\\ \tau_2E^T & 0 & -\tau_2Z_1 & 0\\ \tau_2S^T & 0 & 0 & -\tau_2Z_1\end{bmatrix}<0, \quad (25)$$

where

$$\Omega=\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{15} & \Omega_{16} & \Omega_{17}\\ \Omega_{12}^T & \Omega_{22} & \Omega_{23} & \Omega_{25} & \Omega_{26} & \Omega_{27}\\ \Omega_{13}^T & \Omega_{23}^T & \Omega_{33} & \Omega_{35} & \Omega_{36} & \Omega_{37}\\ \Omega_{15}^T & \Omega_{25}^T & \Omega_{35}^T & \Omega_{55} & \Omega_{56} & \Omega_{57}\\ \Omega_{16}^T & \Omega_{26}^T & \Omega_{36}^T & \Omega_{56}^T & \Omega_{66} & \Omega_{67}\\ \Omega_{17}^T & \Omega_{27}^T & \Omega_{37}^T & \Omega_{57}^T & \Omega_{67}^T & \Omega_{77}\end{bmatrix},$$

$\Omega_{11}=-P+(\tau_{21}+1)Q_1+\tau_2Z_1+E_1+E_1^T+Q^TQ+Q_2$,
$\Omega_{12}=E_2-E_1+S_1$, $\Omega_{13}=E_3^T-S_1$, $\Omega_{15}=E_5^T+\Gamma^TR_1^T$, $\Omega_{16}=E_6^T$,
$\Omega_{17}=E_7^T-\tau_2Z_1-\tau_{21}(Z_2+Z_3)-C^TY_1^T+A^TH_1^T$,
$\Omega_{22}=-Q_1-E_2-E_2^T+S_2+S_2^T$, $\Omega_{23}=-E_3^T-S_2+S_3^T$,
$\Omega_{25}=-E_5^T+S_5^T$, $\Omega_{26}=-E_6^T+S_6^T$, $\Omega_{27}=-E_7^T+S_7^T$,
$\Omega_{33}=-Q_2-S_3-S_3^T$, $\Omega_{35}=-S_5^T$, $\Omega_{36}=-S_6^T$, $\Omega_{37}=-S_7^T$,
$\Omega_{55}=-R_1-R_1^T$, $\Omega_{56}=0$, $\Omega_{57}=W_0^TH_1^T$, $\Omega_{66}=-R_2\Gamma^{-1}-(R_2\Gamma^{-1})^T$,
$\Omega_{67}=W_1^TH_1^T$, $\Omega_{77}=-H_1-H_1^T+P+\tau_2Z_1$,
$H=[H_1^T\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$, $Y=HL=[Y_1^T\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$,
$E=[E_1^T\ \ E_2^T\ \ E_3^T\ \ E_5^T\ \ E_6^T\ \ E_7^T]^T$,
$S=[S_1^T\ \ S_2^T\ \ S_3^T\ \ S_5^T\ \ S_6^T\ \ S_7^T]^T$. Therefore, the estimation error-state system (8) (i.e. the lower bounds $\tau_1=0$ and the given upper bounds τ_2) approaches globally asymptotically delay-dependent stability. In this case, a desired the estimator gain matrix L is given as $L=H_1^{-1}Y_1$.

IV. Numerical Examples

Example 1: Consider the discrete-time recurrent neural network with the following parameters

$$A=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.2\end{bmatrix},\ W_0=\begin{bmatrix}0.1 & -0.5 & 0.4\\ -0.8 & -0.7 & 0.9\\ 0.2 & 0.3 & -0.6\end{bmatrix},$$

$W_1=\begin{bmatrix}0.5 & 0.3 & 0.1\\ 0.1 & 0.5 & 0.2\\ 0.2 & 0.3 & 0.5\end{bmatrix}$. The activation functions in this example are assumed to satisfy Assumption 1 with $\alpha_1=0.034$, $\alpha_2=0.429$, $\alpha_3=0.508$. The non-linearity $q(k,x(k))$ is assumed to satisfy (5) with

$Q=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.2 & 0\\ 0 & 0 & 0.3\end{bmatrix}$, for the network output, the parameter C is given as $C=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$. By the Matlab LMI Control Toolbox, it can be verified that Theorem 1 in this paper is feasible solution for all delays $2\le\tau(k)\le14$ (i.e. the lower bound $\tau_1=2$ and the upper bound $\tau_2=14$) as follows

$$P=\begin{bmatrix}3.1336 & -0.1112 & 0.1746\\ -0.1112 & 1.8526 & 0.0769\\ 0.1746 & 0.0769 & 3.5254\end{bmatrix},\ Q_1=\begin{bmatrix}1.2066 & -0.0397 & 0.0629\\ -0.0397 & 0.8434 & 0.0660\\ 0.0629 & 0.0660 & 1.4637\end{bmatrix}$$

$$Q_2=\begin{bmatrix}0.7478 & -0.0289 & 0.0330\\ -0.0289 & 0.2403 & -0.0524\\ 0.0330 & -0.0524 & 0.5401\end{bmatrix},\ Z_1=\begin{bmatrix}0.0270 & -0.0028 & 0.0030\\ -0.0028 & 0.0121 & 0.0009\\ 0.0030 & 0.0009 & 0.0198\end{bmatrix},$$

$$Z_2=\begin{bmatrix}0.0316 & -0.0033 & 0.0034\\ -0.0033 & 0.0142 & 0.0012\\ 0.0034 & 0.0012 & 0.0230\end{bmatrix},\ Z_3=\begin{bmatrix}0.0348 & -0.0043 & 0.0042\\ -0.0043 & 0.0124 & 0.0013\\ 0.0042 & 0.0013 & 0.0243\end{bmatrix}$$

$$R_1=\begin{bmatrix}1.9166 & 0 & 0\\ 0 & 1.6715 & 0\\ 0 & 0 & 1.8935\end{bmatrix},\ R_2=\begin{bmatrix}1.7912 & 0 & 0\\ 0 & 1.4864 & 0\\ 0 & 0 & 1.4369\end{bmatrix},$$

$$H_1=\begin{bmatrix}3.4569 & -0.1965 & 0.3201\\ 0.1234 & 2.0734 & 0.1097\\ 0.0025 & 0.0543 & 3.6305\end{bmatrix},\ Y_1=\begin{bmatrix}0.3713 & -0.0400 & 0.0373\\ 0.0828 & -0.2346 & -0.1317\\ -0.1313 & -0.3393 & 0.3519\end{bmatrix}$$

Therefore, according to Theorem 1, a desired estimator can be computed as

$$L=H_1^{-1}Y_1=\begin{bmatrix}0.1128 & -0.0092 & -0.0022\\ 0.0352 & -0.1077 & -0.0686\\ -0.0368 & -0.3393 & 0.0979\end{bmatrix}.$$

Thus, by Theorem 1, the desired estimator guarantees asymptotic stability of the error-state system in (8) (i.e. the lower bound $\tau_1=2$ and the upper bound $\tau_2=14$)

V. Conclusions

In this paper, the problem of state estimation for a discrete-time recurrent neural network with interval time-varying delay has been studied. A sufficient condition for the solvability of this problem, which takes into account the range for the time delay, has been established. The desired estimator guarantees asymptotic stability of the estimation error-state system. An illustrative example has been presented to demonstrate the effectiveness of the proposed approach.

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