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Decentralized Adaptive Approximation Based Control of a Class of Large-Scale Systems

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Abstract— **This paper considers the design of a decentralized adaptive approximation based control scheme for a class of interconnected nonlinear systems. Linearly parameterized neural networks are used to adaptively approximate the unknown dynamics of each subsystem and the unknown interconnections. The feedback control and adaptation laws are based only on local measurements of the state. A dead-zone modification is used to address the issues of stability and robustness in the presence of residual approximation errors. A simulation example is used to illustrate the proposed control design methodology.**

I. INTRODUCTION

As engineering systems increase in size and complexity, the design of a single, centralized, controller is becoming an extremely difficult, if not impossible, task. This motivates the design of decentralized controllers, using only local information while guaranteeing stability of the entire system. In recent years, there has been an increased interest in the area of decentralized adaptive control and a variety of decentralized adaptive techniques have been developed.

The problem of decentralized adaptive linear control was introduced in [6], where weakly interconnected subsystems with relative degree one or two were studied, and sufficient conditions on the strength of the interconnections were derived such that global boundedness was guaranteed. In [5] and [12] it was demonstrated that the stability of the decentralized system is ensured if there exists a positive definite M-matrix, which is related to the bound of the interconnections. However, all these approaches were focused on linear subsystems with possibly nonlinear interconnections. An alternative decentralized adaptive control method using the high gain approach was developed in [3], where a standard strict matching condition is assumed on the disturbances. A methodology for handling higher-order interconnections in a decentralized adaptive control framework was developed in [14].

A recent approach is based on the use of neural networks to approximate the unknown interconnections. In [15], the authors develop a decentralized control design scheme for systems with interconnections that are bounded by first-order polynomials. In [4], the authors employ a composite Lyapunov function for handling both unknown nonlinear model dynamics and interconnections. The interconnections are assumed to be bounded by unknown smooth functions, which are indirectly approximated by neural networks. In [11], [10] and [9] it is assumed that the decentralized controllers share prior information about their reference models. Based on this assumption, it is then shown that the subsystems are able to asymptotically track their desired outputs.

In this paper we consider a system composed of nonlinear subsystems coupled by unknown nonlinear interconnections. We develop a decentralized adaptive approximation based control law [2] and derive stability results for the closed-loop system under certain assumptions. The presence of residual approximation errors and other uncertainties are addressed with the use of a dead-zone modification.

The paper is organized as follows. In Section II, we formulate the problem, while in Section III we design a feedback control law by, first, assuming that the subsystems are completely decoupled and later in the presence of unknown interconnections. In Section IV, we examine the case where the residual approximation errors are nonzero and make a dead-zone modification in the adaptive laws to account for these errors. In Section V, we illustrate by simulation of a simple interconnected system the proposed decentralized control design, while Section VI contains some concluding remarks.

II. PROBLEM FORMULATION

We consider a system comprised of n interconnected subsystems. The i-th subsystem, where $i = 1, 2, \dots, n$, is described by

$$\begin{aligned} \dot{x}_{ij} &= x_{i(j+1)}, \qquad\qquad j = 1, 2, \dots, (\rho_i - 1) \\ \dot{x}_{i\rho_i} &= f_i(x_i) + g_i(x_i)u_i + \Delta_i(x_1, x_2, ..., x_n) \\ y_i &= x_{i1}, \end{aligned}$$

where $x_i = [x_{i1}, x_{i2}, \dots, x_{i\rho_i}]^\top \in \Re^{\rho_i}$ is the state vector of the i-th subsystem, $f_i : \Re^{\rho_i} \mapsto \Re$ and $g_i : \Re^{\rho_i} \mapsto \Re$ are unknown smooth functions, $\Delta_i : \Re^{\rho} \mapsto \Re$ (where $\rho = \sum_{i=1}^{n} \rho_i$) represents the interconnection effect, $u_i \in \Re$ is the input and $y_i \in \Re$ is the output of the i-th subsystem. Our objective is to synthesize decentralized adaptive approximation based control laws u_i such that y_i tracks a smooth bounded reference trajectory y_{d_i} in the presence of the interconnections Δ_i, using only local measurements.

It is assumed that the input gain function, $g_i(x_i)$, is bounded from below by $0 < g_{i0} \le g_i(x_i)$, where g_{i0} is a known constant. This assumption is required in order to guarantee the controllability of the feeback control scheme [2]. In general, each $g_i(x_i)$ is required to be either positive or negative for all x_i in a domain of interest $\mathcal{D}_i \subset \Re^{\rho_i}$. For notational simplicity and without any loss of generality,

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here we assume that all $g_i(x_i)$ are positive. Furthermore, the desired trajectory vector $Y_{d_i} = [y_{d_i}, \dot{y}_{d_i}, \ldots, y_{d_i}^{(\rho_i)}]^\top$ of the i-th subsystem is assumed to be available and bounded.

III. Decentralized Adaptive Control

For the sake of clarity, we first consider the special case of completely decoupled subsystems (i.e., $\Delta_i = 0$, $i = 1, 2, \ldots, n$), and then proceed to make design modifications in the feedback control and adaptive laws to account for the presence of the interconnections. Towards this direction, we define $u_i = u_i^* + u_{l_i}$, where u_i^* is the nominal local controller for the decoupled case and u_{l_i} is an augmented control law designed, later on, to account for the interconnections Δ_i. In both cases, we use linearly parametrized neural network models (such as Radial Basis Function networks) to approximate the unknown functions.

Following the universal approximation results of neural networks (see, e.g. [13]), given any continuous function $f(x)$ where $f : \Re^q \mapsto \Re$ is defined on a compact set $\mathcal{D} \subset \Re^q$, and an arbitrary $\varepsilon^* > 0$, there exists a set of bounded constant weights $\theta_f \in \Re^p$ and a set of basis functions $\phi_f(x)$, where $\phi_f : \Re^q \mapsto \Re^p$ is such that the following representation holds $\forall x \in \mathcal{D}$,

$$f(x) = \phi_f(x)^\top \theta_f + \varepsilon(x), \quad \|\varepsilon(x)\|_{\mathcal{D}} < \varepsilon^*. \tag{1}$$

In the above representation, $\varepsilon(x)$ denotes the Minimum Functional Approximation Error (MFAE) which is the minimum possible deviation between the unknown function $f(x)$ and the appromixation of it, $\phi_f(x)^\top \theta_f$, in the ∞-norm sense over the compact set $\mathcal{D}$.

In this section, we consider the ideal case where the unknown functions f_i, g_i and Δ_i are assumed to be approximated exactly. Later, in Section IV, we consider the robustness issues and modify the adaptive laws so as to handle nonzero approximation errors.

A. Decoupled Subsystems

To design the local controller we consider the tracking error dynamics of a decoupled subsystem

$$\begin{aligned}
\dot{\tilde{x}}_{ij} &= \tilde{x}_{i(j+1)}, \qquad\qquad j = 1, 2, \ldots, (\rho_i - 1) \\
\dot{\tilde{x}}_{i\rho_i} &= f_i(x_i) + g_i(x_i) u_i^* - y_{di}^{(n)},
\end{aligned}$$

where $\tilde{x}_{ij} = x_{ij} - y_{d_i}^{(j-1)}$. The tracking error dynamics can be written in matrix state-space form as

$$\dot{\tilde{x}} = A\tilde{x}_i + B(f_i(x_i) + g_i(x_i)u_i^* - y_{di}^{(n)}), \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

We consider the ideal case where the unknown functions f_i and g_i can be approximated exactly as follows:

$$f_i(x_i) = \phi_{f_i}(x_i)^\top \theta_{f_i} \tag{3}$$

$$g_i(x_i) = \phi_{g_i}(x_i)^\top \theta_{g_i}. \tag{4}$$

Let $\hat{\theta}_{f_i}$ and $\hat{\theta}_{g_i}$ be the estimated weights of the approximators of f_i and g_i respectively, and define $\tilde{\theta}_{f_i} = \hat{\theta}_{f_i} - \theta_{f_i}$, $\tilde{\theta}_{g_i} = \hat{\theta}_{g_i} - \theta_{g_i}$ as the corresponding parameter estimation errors. The feedback linearizing approximation based control law of the i-th decoupled subsystem is defined as

$$u_i^* = \frac{-K_i^\top \tilde{x}_i + y_{di}^{(n)} - \phi_{f_i}(x_i)^\top \hat{\theta}_{f_i}}{\phi_{g_i}(x_i)^\top \hat{\theta}_{g_i}}, \tag{5}$$

where $K_i = [k_{i1}, k_{i2}, \cdots, k_{i\rho_i}]^\top \in \Re^{\rho_i}$ is chosen such that $A - BK_i^\top$ is a Hurwitz matrix.

Since $A - BK_i^\top$ is Hurwitz, for any $Q_i > 0$ there exists positive definite matrix P_i satisfying the Lyapunov equation

$$P_i(A - BK_i^\top) + (A - BK_i^\top)^\top P_i = -Q_i.$$

Define the scalar training error $e_i = B^\top P_i \tilde{x}_i$. The parameters $\hat{\theta}_{f_i}$ and $\hat{\theta}_{g_i}$ are updated according to the following adaptive laws:

$$\dot{\hat{\theta}}_{f_i} = \Gamma_{f_i} \phi_{f_i}(x_i) e_i \tag{6}$$

$$\dot{\hat{\theta}}_{g_i} = \mathcal{P}_s\{\Gamma_{g_i} \phi_{g_i}(x_i) e_i u_i^*\}, \tag{7}$$

where $\Gamma_{f_i} > 0$, $\Gamma_{g_i} > 0$ are positive definite matrices characterizing the adaptive gain of the parameter estimates and $\mathcal{P}_s$ is a projection operator that is used to ensure that the term $\phi_{g_i}(x_i)^\top \hat{\theta}_{g_i}$ stays away from zero [2].

Lemma 1: Given the tracking error dynamics of the i-th subsystem (2), the control law (5) with the adaptation laws (6) and (7) ensure that the tracking errors $\tilde{x}_{ij}(t)$ converge asymptotically to zero; i.e., $\lim_{t\to\infty} \tilde{x}_{ij}(t) = 0$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, \rho_i$.

Proof: By incorporating the control law (5) in the tracking error dynamics (2), we obtain

$$\dot{\tilde{x}}_i = (A - BK_i^\top)\tilde{x}_i - B\phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} - B\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i.$$

Consider the Lyapunov function candicate of the i-th subsystem

$$V_i = \tilde{x}_i^T P_i \tilde{x}_i + \tilde{\theta}_{f_i}^T \Gamma_{f_i}^{-1} \tilde{\theta}_{f_i} + \tilde{\theta}_{g_i}^T \Gamma_{g_i}^{-1} \tilde{\theta}_{g_i}.$$

After some algebraic manipulation it can be shown that the time derivative of the Lyapunov function satisfies

$$\begin{aligned}
\dot{V}_i &= -\tilde{x}_i^\top Q_i \tilde{x}_i + 2\tilde{\theta}_{f_i}^\top \Gamma_{f_i}^{-1}(\dot{\hat{\theta}}_{f_i} - \Gamma_{f_i}\phi_{f_i} e_i) \\
&\quad + 2\tilde{\theta}_{g_i}^\top \Gamma_{g_i}^{-1}(\dot{\hat{\theta}}_{g_i} - \Gamma_{g_i}\phi_{g_i} e_i u_i).
\end{aligned}$$

With the adaptive laws (6) and (7), the Lyapunov time derivative satisfies $\dot{V}_i \le -\tilde{x}_i^\top Q_i \tilde{x}_i$, which is negative semidefinite. Therefore, using Barbalat's Lemma [7], [8], and by the use of the projection modification in (7) to ensure the controllability of the system during the adaptation, we obtain that $\tilde{x}_i(t) \to 0$ as $t \to \infty$. ■

It is important to note that the above result for decoupled subsystems is obtained under two key assumptions: (i) the unknown functions f_i and g_i can be represented exactly by the approximating neural network model, as given in (3), (4); i.e., the MFAE is zero. (ii) The exact approximation property holds in a compact set $\mathcal{D}$, therefore the trajectories $x_i(t)$ should remain in $\mathcal{D}$ for all $t \geq 0$. If the trajectory leaves the set $\mathcal{D}$, then the zero approximation property does not hold, hence the convergence of the tracking error to zero is not guaranteed. In Section IV, we will address the case of non-zero MFAE .

B. Interconnected Subsystems

In the case of interconnected subsystems the tracking error dynamics of the i-th subsystem are given by

$$\begin{aligned} \dot{\tilde{x}}_i &= A\tilde{x}_i + B\left(f_i(x_i) + g_i(x_i)u_i + \Delta_i(x_1, x_2, \ldots, x_n)\right) \\ &\quad - By_{d_i}^{(n)}. \end{aligned} \tag{8}$$

We impose the following assumption on the interconnection terms Δ_i.

Assumption 1: The interconnections Δ_i are bounded by

$$|\Delta_i(x_1, x_2, ..., x_n)| \leq \sum_{j=1}^{n} \gamma_{ij}(|e_j|),$$

where $\gamma_{ij} : \Re \mapsto \Re^+$ are unknown analytic functions.

According to Assumption 1, the magnitude of the interconnections is allowed to be significantly large and also unknown. As we will see later on, a surrogate of the unknown bounding functions γ_{ij} will be adaptively approximated for use in the feedback control law. The above assumption is similar to the corresponding assumption used in [4].

To address the presence of unknown (or uncertain) interconnection terms Δ_i satisfying Assumption 1, an augmented control term u_{l_i} is added to the nominal control law u_i^*, defined in (5), such that $u_i = u_i^* + u_{l_i}$. Due to the unknown interconnection terms Δ_i, the augmented control law u_{l_i} is a dynamic controller designed in an adaptive approximation framework as follows:

$$u_{l_i} = -\frac{\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i}}{\phi_{g_i}(x_i)^\top \hat{\theta}_{g_i}} \tag{9}$$

$$\dot{\hat{\theta}}_{\Delta_i} = \Gamma_{\Delta_i} \phi_{\Delta_i}(e_i) e_i, \tag{10}$$

where $\Gamma_{\Delta_i} > 0$ is a positive definite adaptive gain matrix and $\phi_{\Delta_i}(|e_i|)$ is a vector of basis functions with the corresponding adaptable weights $\hat{\theta}_{\Delta_i}$. Therefore the overall decentralized control law for the i-th subsystem is given by

$$u_i = \frac{-K_i^\top \tilde{x}_i + y_{di}^{(n)} - \phi_{f_i}(x_i)^\top \hat{\theta}_{f_i} - \phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i}}{\phi_{g_i}(x_i)^\top \hat{\theta}_{g_i}}. \tag{11}$$

It is important to note that the feedback control law described by (11) is decentralized, since each local control law u_i does not use the states x_j, $j = 1, 2, \ldots, n$, $j \neq i$, of the other subsystems.

Lemma 2: Given the tracking error dynamics (8), the decentralized control law (11) with adaptation laws (6), (7) and (10) ensures that the tracking errors $\tilde{x}_{ij}$ converge asymptotically to zero; i.e., $\lim_{t\to\infty} \tilde{x}_{ij}(t) = 0$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, \rho_i$.

Proof: The proof of this Lemma follows the same procedure as in [4]. Let the Lyapunov equation of the i-th subsystem be given by $V_i = V_{i1} + V_{i2}$, where

$$\begin{aligned} V_{i1} &= \tilde{x}_i^\top P \tilde{x}_i, \\ V_{i2} &= \tilde{\theta}_{f_i}^\top \Gamma_{f_i}^{-1} \tilde{\theta}_{f_i} + \tilde{\theta}_{g_i}^\top \Gamma_{g_i}^{-1} \tilde{\theta}_{g_i} + \tilde{\theta}_{\Delta_i}^\top \Gamma_{\Delta_i}^{-1} \tilde{\theta}_{\Delta_i}. \end{aligned}$$

By substituting the control law (11) into the tracking error dynamics (8), we obtain the following expression for the closed-loop tracking error dynamics

$$\begin{aligned} \dot{\tilde{x}}_i &= \left(A - BK_i^\top\right)\tilde{x}_i - B\phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} \\ &\quad - B\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i + B\left(-\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} + \Delta_i\right). \end{aligned}$$

The time derivative of V_{i1} satisfies

$$\begin{aligned} \dot{V}_{i1} &= -\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i\phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} - 2e_i\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i \\ &\quad + 2e_i\left(-\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} + \Delta_i\right) \\ &\leq -\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i\phi_{f_i}^\top(x_i)\tilde{\theta}_{f_i} - 2e_i\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i \\ &\quad -2e_i\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} + 2|e_i| \cdot |\Delta_i|. \end{aligned}$$

Based on Assumption 1,

$$2|e_i| \cdot |\Delta_i| \leq 2|e_i| \sum_{j=1}^{n} \gamma_{ij}(|e_j|).$$

Since γ_{ij} are analytic functions, using Taylor's Theorem (see, for example, [1]), there exist smooth functions ξ_{ij} such that

$$\gamma_{ij}(|e_j|) = \gamma_{i0} + |e_j|\xi_{ij}(|e_j|),$$

where $\gamma_{i0} = \gamma_{ij}(0)$ is a constant. Therefore, using the inequality $2\alpha\beta \leq \alpha^2 + \beta^2$ for $\alpha, \beta \in \Re$, we obtain

$$\begin{aligned} \dot{V}_{i1} &\leq -\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i\phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} - 2e_i\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i \\ &\quad - 2e_i\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} + 2\gamma_{i0}|e_i| \\ &\quad + 2|e_i| \sum_{j=1}^{n} |e_j|\xi_{ij}(|e_j|) \\ &\leq -\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i\phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} - 2e_i\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i \\ &\quad - 2e_i\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} + 2\gamma_{i0}|e_i| \\ &\quad + ne_i^2 + \sum_{j=1}^{n} e_j^2 \xi_{ij}^2(|e_j|). \end{aligned}$$

Hence, after some re-ordering of terms,

$$\begin{aligned} \sum_{i=1}^{n} \dot{V}_{i1} &\leq \sum_{i=1}^{n} \Big[-\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i\phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} \\ &\quad - 2e_i\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i - 2\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} e_i \\ &\quad + ne_i^2 + 2\gamma_{i0}|e_i| + e_i^2 \sum_{j=1}^{n} \xi_{ji}^2(|e_i|)\Big]. \end{aligned}$$

Let

$$d_i(e_i) = \frac{1}{2}\left[2\gamma_{i0} sgn(e_i) + ne_i + e_i \sum_{j=1}^{n} \xi_{ji}^2(|e_i|)\right]. \quad (12)$$

Since d_i is a smooth function and e_i is in a compact set, the following representation holds:

$$d_i(e_i) = \phi_{\Delta_i}(e_i)^\top \theta_{\Delta_i} + \mu_{\Delta_i}.$$

In this section we consider the ideal case where all the approximation errors μ_{Δ_i} are zero. Thus, we have

$$\begin{aligned}\sum_{i=1}^{n} \dot{V}_{i1} \quad \leq \quad & \sum_{i=1}^{n} \Big[-\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i \phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} \\ & - 2e_i \phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i - 2e_i \phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} \\ & + 2e_i \phi_{\Delta_i}(e_i)^\top \theta_{\Delta_i} \Big].\end{aligned}$$

The Lyapunov equation of the overall system is given by

$$V = \sum_{i=1}^{n} V_{i1} + V_{i2}.$$

The time derivative of the Lyapunov function of the overall system is given by

$$\begin{aligned}\dot{V} \quad = \quad & \sum_{i=1}^{n} \dot{V}_{i1} + \dot{V}_{i2} \\ \leq \quad & \sum_{i=1}^{n} \Big[-\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i \phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} \\ & - 2e_i \phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i - 2e_i \phi_{\Delta_i}(e_i)^\top \tilde{\theta}_{\Delta_i}\Big] \\ & + \sum_{i=1}^{n} \Big[2\tilde{\theta}_{f_i}^\top \Gamma_{f_i}^{-1} \dot{\hat{\theta}}_{f_i} + 2\tilde{\theta}_{g_i}^\top \Gamma_{g_i}^{-1} \dot{\hat{\theta}}_{g_i} + 2\tilde{\theta}_{\Delta_i}^\top \Gamma_{\Delta_i}^{-1} \dot{\hat{\theta}}_{\Delta_i}\Big].\end{aligned}$$

By grouping terms we obtain

$$\begin{aligned}\dot{V} \quad \leq \quad & \sum_{i=1}^{n} \Big[-\tilde{x}_i^\top Q_i \tilde{x}_i + 2\tilde{\theta}_{f_i}^\top \Gamma_{f_i}^{-1} \left(\dot{\hat{\theta}}_{f_i} - \Gamma_{f_i} \phi_{f_i}(x_i) e_i\right) \\ & + 2\tilde{\theta}_{g_i}^\top \Gamma_{g_i}^{-1} \left(\dot{\hat{\theta}}_{g_i} - \Gamma_{f_i} \phi_{g_i}(x_i) e_i u_i\right) \\ & + 2\tilde{\theta}_{\Delta_i}^\top \Gamma_{\Delta_i}^{-1} \left(\dot{\hat{\theta}}_{\Delta_i} - \Gamma_{\Delta_i} \phi_{\Delta_i}(e_i) e_i\right)\Big].\end{aligned}$$

By substituting the adaptive laws (6), (7) and (10), the Lyapunov function derivative satisfies

$$\dot{V} \leq -\sum_{i=1}^{n} \tilde{x}_i^\top Q_i \tilde{x}_i,$$

which is negative semidefinite. Therefore, using Barbalat's Lemma, it can be shown that all the tracking errors $\tilde{x}_i(t)$ go to zero as $t \to \infty$. ■

IV. Dead-Zone Compensation

In the previous section, it was assumed that the Minimum Functional Approximation Error (MFAE) of each adaptive approximator was zero; in other words, it was possible to match exactly the unknown functions f_i, g_i and d_i by the use of the corresponding adaptive approximators $\phi_{f_i}(x_i)^\top \hat{\theta}_{f_i}$, $\phi_{g_i}(x_i)^\top \hat{\theta}_{g_i}$ and $\phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i}$, within a certain approximation region $\mathcal{D}_i$. In most practical situations there will be nonzero approximation error, which is an issue that needs to be addressed in the control design. Let μ_{f_i}, μ_{g_i} and μ_{Δ_i} be the corresponding MFAE of each approximator. In this case, based on (8) with the decentralized control law (11), the equation of the tracking errors dynamics becomes

$$\begin{aligned}\dot{\tilde{x}}_i \quad = \quad & (A - BK_i^\top)\tilde{x}_i - B\phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} + B\mu_{f_i} + Bu_i\mu_{g_i} \\ & - B\phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i + B\left(\Delta_i - \phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i}\right).\end{aligned}$$

Thus, following a similar procedure as in the proof of Lemma 2, we obtain

$$\begin{aligned}\sum_{i=1}^{n} \dot{V}_{i1} \quad \leq \quad & \sum_{i=1}^{n} \Big[-\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i \phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} + 2e_i \mu_{f_i} \\ & - 2e_i \phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i + 2e_i \mu_{g_i} u_i \\ & -2e_i \phi_{\Delta_i}(e_i)^\top \hat{\theta}_{\Delta_i} + ne_i^2 + 2\gamma_{i0}|e_i| \\ & + e_i^2 \sum_{j=1}^{n} \xi_{ji}^2(|e_i|)\Big].\end{aligned}$$

Using the adaptive approximation of $d_i(e_i)$, where d_i is defined in (12), and this time including the corresponding MFAE, μ_{Δ_i}, the time derivative of the Lyapunov function can be expressed as

$$\begin{aligned}\sum_{i=1}^{n} \dot{V}_{i1} \quad \leq \quad & \sum_{i=1}^{n} \Big[-\tilde{x}_i^\top Q_i \tilde{x}_i - 2e_i \phi_{f_i}(x_i)^\top \tilde{\theta}_{f_i} + 2e_i \mu_{f_i} \\ & - 2e_i \phi_{g_i}(x_i)^\top \tilde{\theta}_{g_i} u_i + 2e_i \mu_{g_i} u_i \\ & - 2e_i \phi_{\Delta_i}(e_i)^\top \tilde{\theta}_{\Delta_i} + 2e_i \mu_{\Delta_i}\Big].\end{aligned}$$

Substituting the adaptive laws into the time derivative of the Lyapunov function, V, yields

$$\dot{V} \leq \sum_{i=1}^{n} -\tilde{x}_i^\top Q_i \tilde{x}_i + 2e_i \mu_{f_i} + 2e_i \mu_{g_i} u_i + 2e_i \mu_{\Delta_i}.$$

We define δ_i as

$$\delta_i = \mu_{f_i} + \mu_{g_i} u_i + \mu_{\Delta_i}$$

and rewrite the inequality of $\dot{V}$ as follows:

$$\dot{V} \leq \sum_{i=1}^{n} -\tilde{x}_i^\top Q_i \tilde{x}_i + 2e_i \delta_i. \quad (13)$$

To avoid instabilities that may occur due to parameter drift and to enhance the robustness properties of the adaptive scheme, we introduce a dead-zone modification in the update laws. For notational simplicity, we assume that $Q_i = I$ and $K_i = K$ for all $i = 1, 2, \ldots, n$, and let P be the solution of the Lyapunov equation with $Q_i = I$ and $K_i = K$.

The parameter estimates are adjusted according to

$$\begin{aligned}\dot{\hat{\theta}}_{f_i} \quad &= \quad \Gamma_{f_i} \phi_{f_i}(x_i) q_i(e_i, \tilde{x}_i, \epsilon_i) \quad (14) \\ \dot{\hat{\theta}}_{g_i} \quad &= \quad \Gamma_{g_i} \phi_{g_i}(x_i) q_i(e_i, \tilde{x}_i, \epsilon_i) u_i \quad (15) \\ \dot{\hat{\theta}}_{\Delta_i} \quad &= \quad \Gamma_{\Delta_i} \phi_{\Delta_i}(e_i) q_i(e_i, \tilde{x}_i, \epsilon_i), \quad (16)\end{aligned}$$

where $q_i(e_i, \tilde{x}_i, \epsilon_i)$ is defined as

$$
\begin{aligned}
q_i(e_i, \tilde{x}_i, \epsilon_i) &= \begin{cases} 0 & \tilde{x}_i^\top P \tilde{x}_i \leq \bar{\lambda}_P \epsilon_i^2 \\ \epsilon & \tilde{x}_i^T P \tilde{x}_i > \bar{\lambda}_P \epsilon_i^2 \end{cases} \\
\epsilon_i &= 2\|PB\|\delta_{0_i} + \mu_i
\end{aligned}
$$

and δ_{0_i} is an upper bound of δ_i (i.e., $|\delta_i| < \delta_{0_i}$), μ_i is a positive constant and $\bar{\lambda}_P$ and $\underline{\lambda}_P$ are the maximum and minimum eigenvalues of P, respectively.

Lemma 3: Given the tracking error dynamics (8) with non-zero MFAEs, the decentralized control law (11) with adaptation laws (14), (15) and (16) guarantees that the following hold:

1) $\tilde{x}_i$ is small-in-the-mean-square sense, satisfying
$$\int_t^{t+T} \|\tilde{x}_i^2(\tau)\|^2 d\tau \leq 2V_i(t) + \frac{\bar{\lambda}_P}{\underline{\lambda}_P}\epsilon_i^2 T;$$
2) $\|\tilde{x}(t)\|$ is uniformly ultimately bounded by ϵ; i.e., the total time such that $\tilde{x}_i^\top P \tilde{x}_i > \bar{\lambda}_P \epsilon_i^2$ is finite.

Proof: Following a similar framework as in [2] let the condition $\tilde{x}_i^\top P \tilde{x}_i > \bar{\lambda}_P \epsilon_i^2$ be satisfied for $t \in (t_{s_k}, t_{f_k}), k = 1, 2, 3, \ldots$, where $t_{s_k} < t_{f_k} \leq t_{s_{k+1}}$ and $\tilde{x}_i^\top P \tilde{x}_i \leq \bar{\lambda}_P \epsilon_i^2$ for $t \in (t_{f_k}, t_{s_{k+1}})$. The start of the sequence t_{s_0} is assumed to be zero without loss of generality and $t_{s_{k+1}}$ may be infinity for some k. Since $\tilde{x}_i(t_{f_k})^\top P \tilde{x}_i(t_{f_k}) = \tilde{x}_i(t_{s_{k+1}})^\top P \tilde{x}_i(t_{s_{k+1}}) = \bar{\lambda}_P \epsilon_i^2$ and parameter adaptation is off for $t \in [t_{f_k}, t_{s_{k+1}}]$, we have that $V(t_{f_k}) = V(t_{s_{k+1}})$. Note that when $t \in (t_{s_k}, t_{f_k})$ for any k, the fact that $\tilde{x}_i^\top P \tilde{x}_i > \bar{\lambda}_P (2\|PB\|\delta_{0_i} + \mu_i)^2$ ensures that $\|\tilde{x}_i\|_2 > 2\|PB\|\delta_{0_i} + \mu_i$. Therefore (13) becomes,

$$
\begin{aligned}
\dot{V} &\leq \sum_{i=1}^n -\tilde{x}_i^\top \tilde{x}_i + 2e_i\delta_i \\
&\leq \sum_{i=1}^n -\|\tilde{x}_i\|_2^2 + 2\|\tilde{x}_i\|_2\|PB\|_2|\delta_i| \\
&\leq \sum_{i=1}^n -\|\tilde{x}_i\|_2 \left(\|\tilde{x}_i\|_2 - 2\|PB\|\delta_{0_i}\right) \\
&\leq \sum_{i=1}^n -\epsilon_i\mu_i.
\end{aligned}
$$

We integrate both sides over (t_{s_k}, t_{f_k}),

$$
\begin{aligned}
V(t_{f_k}) &\leq V(t_{s_k}) - (t_{f_k} - t_{s_k})\sum_{i=1}^n \epsilon_i\mu_i \\
&\leq V(t_{f_{k-1}}) - (t_{f_k} - t_{s_k})\sum_{i=1}^n \epsilon_i\mu_i \\
&\vdots \\
&\leq V(0) - \left(\sum_{m=1}^k (t_{f_m} - t_{s_m})\right)\left(\sum_{i=1}^n \epsilon_i\mu_i\right).
\end{aligned}
$$

Hence, since $V(t_{f_k}) \geq 0$,

$$\left(\sum_{m=1}^k (t_{f_m} - t_{s_m})\right) \leq \frac{V(0)}{(\sum_{i=1}^n \epsilon_i\mu_i)},$$

which shows that the total time spent with $\tilde{x}_i^\top P \tilde{x}_i > \bar{\lambda}_P \epsilon_i^2$ (i.e., outside the dead zone) is finite. In addition, $V(t_{f_k})$, $k = 1, 2, 3, \ldots$ is a positive decreasing sequence, and either this is finite sequence or $\lim_{t\to\infty} V(t_{f_k}) = V_\infty$ exists and is finite. In addition, if $t > t_{f_k}$, then $V(t) < V(t_{f_k})$.

Within the dead zone, it is obvious that $\underline{\lambda}_P\|\tilde{x}_i\|^2 \leq \tilde{x}_i^\top P \tilde{x}_i \leq \bar{\lambda}_P \epsilon^2$ implies

$$\int_t^{t+T} \|\tilde{x}_i^2(\tau)\|^2 d\tau \leq \frac{\bar{\lambda}_P}{\underline{\lambda}_P}\epsilon_i^2 T.$$

Outside the dead zone, using the inequality,

$$xy \leq \beta^2 x^2 + \frac{1}{4\beta^2}y^2, \quad \forall \beta \neq 0,$$

we have that

$$
\begin{aligned}
\dot{V}_i &\leq -\|\tilde{x}_i\|_2^2 + 2\|\tilde{x}_i\|_2\|PB\|_2\,|\delta_{0_i}| \\
&\leq -\|\tilde{x}_i\|_2^2 + \beta^2\|\tilde{x}_i\|_2^2 + \frac{\|PB\|_2}{\beta^2}\delta_{0_i}^2 \\
&= -\|\tilde{x}_i\|_2^2\left(1 - \beta^2\right) + \frac{\|PB\|_2^2}{\beta^2}\delta_{0_i}^2 \\
&= -0.5\|\tilde{x}_i\|_2^2 + 2\|PB\|_2^2\delta_{0_i}^2,
\end{aligned}
$$

for $\beta^2 = 0.5$. Integrating over $[t, t+T]$ we obtain

$$
\begin{aligned}
\int_t^{t+T} \dot{V}_i(\tau)d\tau &\leq -0.5\int_t^{t+T} \|\tilde{x}_i(\tau)\|_2^2 d\tau \\
&\quad + 2\|PB\|_2^2\delta_{0_i}^2\int_t^{t+T} d\tau \\
\int_t^{t+T} \|\tilde{x}_i(\tau)\|_2^2 d\tau &\leq 2V_i(t) + 4\|PB\|_2^2\delta_{0_i}^2 T.
\end{aligned}
$$

Therefore,

$$\int_t^{t+T} \|\tilde{x}_i(\tau)\|_2^2 d\tau \leq 2V_i(t) + \epsilon_i^2 T$$

which completes the proof. ■

V. Simulation Example

A simple simulation example is presented to illustrate the design methodology for decentralized adaptive approximation based control. We consider the following interconnected uncertain system:

$$
\begin{aligned}
\Sigma_1 \ :\ \dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= \cos\left(\frac{\pi}{1.2}R_1\right) + \Delta_1(x_1, x_2) + g_1(x_1)u_1 \\
\Sigma_2 \ :\ \dot{x}_{21} &= x_{22} \\
\dot{x}_{22} &= 0.5\cos\left(\frac{\pi}{1.2}R_2\right) + \Delta_2(x_1, x_2) + g_2(x_2)u_2
\end{aligned}
$$

where $R_i = x_{i1}^2 + x_{i2}^2$, $\Delta_1(x_1, x_2) = (0.5x_{21} + x_{22})^2$, $\Delta_2(x_1, x_2) = (x_{11} + x_{12})^2$ and $g_i(x_i) = 2 + (x_{i1} + x_{i2})^2 + 2e^{-R_i}$. The output of the i-th subsystem is $y_i = x_{i1}$ and the desired trajectories are given by $y_{d_1}(t) = 0.7\sin(4.93t)$ and $y_{d_2}(t) = 0.8\sin(4.31t)$. The matrix P satisfying the Lyapunov equation is given by

$$P = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1 \end{bmatrix},$$

where $K_1 = K_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^\top$. For the approximators of f_i and g_i, a lattice network of basis functions was designed with centers evenly spaced between $[-4, 4]$ for the x_{i1} input and $[-8, 8]$ for the x_{i2} input. The approximators of the bounding functions d_i are designed with basis functions evenly spaced in $[-10, 10]$. The dead-zone parameters ϵ_i are set to $\epsilon_1 = \epsilon_2 = 0.2$. The initial conditions are assumed to be: $x_{11}(0) = x_{12}(0) = 1$ and $x_{21}(0) = x_{22}(0) = -1$.

In Figure 1 we plot the tracking performance of each subsystem with and without adaptive approximation of the function d_i. In the case that no adaptive approximation is used, the radial basis function neural networks are turned off. As illustrated by the plot, the use of adaptive approximation results in a significantly better tracking performance. In fact, in the case of adaptive approximation, the tracking performance continues to improve after the time period shown in the plot. However, the rate of improvement is reduced as the subsystems spend more time in the dead-zone, until approximately the time $t = 80$ sec, when the scalar errors e_i stay within the dead-zone thereafter.

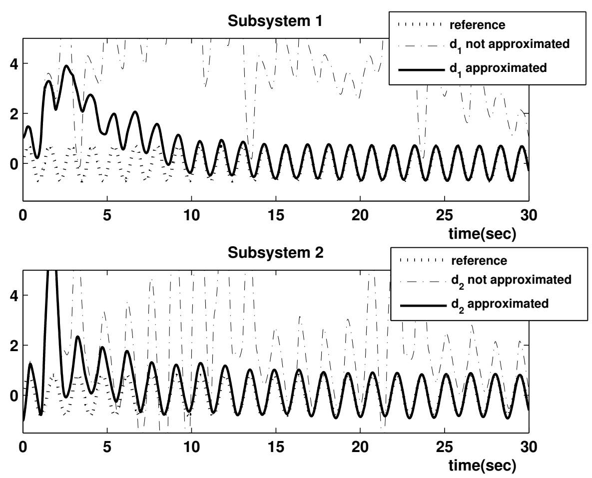

Fig. 1. Tracking performance of simulation example.

Figure 2 shows the control effort of the proposed decentralized adaptive approximation based control scheme along with the control effort of a nominal centralized control, which assumes that all the functions are known exactly and all the states are available to each subsystem. As we can see, while the control effort of the proposed scheme is quite large at the initial stages of simulation (due to large approximation errors), as time passes and the approximators keep learning the unknown functions, the decentralized control effort becomes closer to the nominal centralized control effort.

VI. Concluding Remarks

In this paper, we have presented some preliminary results regarding the design of a stable and robust decentralized adaptive approximation based control scheme. To address the presence of non-zero residual approximation errors, we used a dead-zone modification in the adaptive laws, and derived some analytical properties of the closed loop-system. One

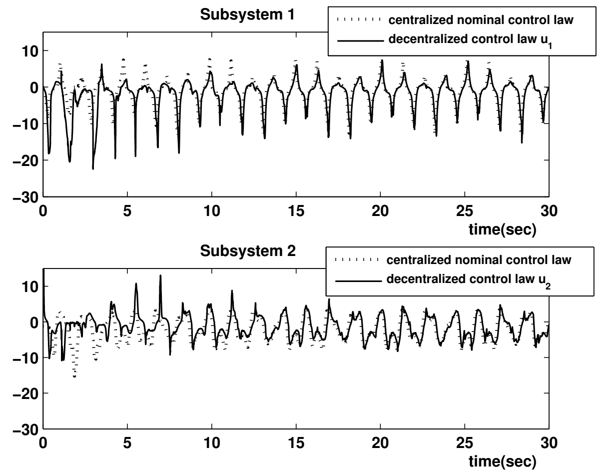

Fig. 2. Control effort of simulation example.

of the key assumptions made is that the trajectories remain within a certain approximation region. Efforts are underway for relaxing this assumption.

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