2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

ACTIVE DISK FOR AUTOMATIC BALANCING OF ROTOR-BEARING SYTEMS

A. Blanco-Ortega[1], F. Beltrán-Carbajal[1], A. Favela-Contreras[2] and G. Silva-Navarro[3]

Abstract

This paper leads with the active cancellation problem of mechanical vibrations in rotor-bearing systems. The use of an active disk is proposed for actively balancing a rotor by means of locating a balancing mass at a suitable position. Two nonlinear controllers with integral compensation are proposed to put the balancing mass at a specific position. Algebraic identification is used for on-line eccentricity estimation, because of the implementation of this active disk is based on knowledge of the eccentricity. An important property of this algebraic identification is that the eccentricity identification is not asymptotic but algebraic, in contrast to most of the traditional identification methods, which generally suffer of poor speed performance. In addition, a velocity control is designed to take the rotor velocity to a desired operating point over the first critical speed. The controllers are developed in the context of an off-line prespecified reference trajectory tracking problem. Some numerical simulations are included to illustrate the dynamic performance of the closed loop system and the active vibration cancellation.
Keywords: Active vibration control, Algebraic Identification, Eccentricity identification, Rotordynamics.

1 INTRODUCTION

Vibration caused by mass imbalance is a common problem in rotating machinery. Rotor imbalance occurs when the principal axis of inertia of the rotor does not coincide with its geometrical axis and leads to synchronous vibrations and significant undesirable forces transmitted to the mechanical elements and supports. Many methods have been developed to reduce the unbalance-induced vibration by using different devices such as electromagnetic bearings, active squeeze film dampers, lateral force actuators, pressurized bearings and movable bearings (see, e.g., Zhou and Shi [1]; Sheu *et al.* [2]; Guozhi *et al.* [3]; Blanco-Ortega *et al.*[4]). These active balancing control schemes require information of the eccentricity of rotating machinery. On the other hand, there exists a vast literature on identification and estimation methods, which are essentially asymptotic, recursive or complex, which generally suffer of poor speed performance (see, e.g., Ljung [5], Soderstrom [6], and Sagara and Zhao [7, 8]).

This paper leads with the active cancellation problem of mechanical vibrations in rotor-bearing systems. The use of an active disk is proposed for actively balancing a rotor by means of locating a balancing mass at a suitable position. Two nonlinear controllers with integral compensation are proposed to put the balancing mass at a specific position. Algebraic identification is used for on-line eccentricity estimation, because of the implementation of this active disk is based on knowledge of the eccentricity. An important property of this algebraic identification is that the eccentricity identification is not asymptotic but algebraic, in contrast to most of the traditional identification methods, which generally suffer of poor speed performance. In addition, a velocity control is designed to take the rotor velocity to a desired operating point over the first critical speed.

The proposed results are strongly based on the algebraic approach to parameter identification in linear systems reported by Fliess and Sira-Ramírez [9], which employs differential algebra, module theory and operational calculus. Algebraic identification has already been employed for parameter and signal estimation in nonlinear and linear vibrating mechanical systems by Beltrán-Carbajal *et al.* [10, 11]. Here numerical and experimental results show that the algebraic identification provides high robustness against parameter uncertainty, frequency variations, small measurement errors and noise.

2 VIBRATING MECHANICAL SYSTEM

2.1 Mathematical model

The rotor-bearing system consists of a planar and rigid disk of mass M mounted on a flexible shaft of negligible mass and stiffness k at the mid-span between two

[1]A. Blanco-Ortega and F. Beltrán-Carbajal are with the ITESM, Campus Guadalajara, Escuela de Ingeniería y Arquitectura, Zapopan, Jalisco, México (e-mail: andres.blanco, francisco.beltran@itesm.mx)

[2]A. Favela-Contreras is with the ITESM, Campus Monterrey, Escuela de Ingeniería, Monterrey, Nuevo León, México (e-mail: antonio.favela@itesm.mx)

[3]G. Silva-Navarro is with the CINVESTAV-IPN, Departamento de Ingeniería Eléctrica, Sección de Mecatrónica, México D.F., México (e-mail: gsilva@cinvestav.mx)

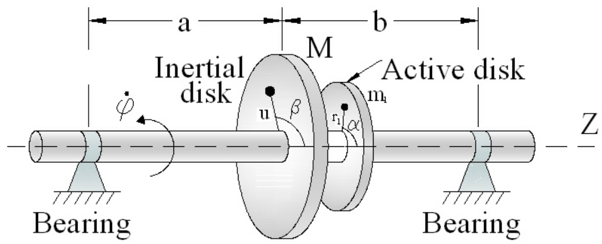

Figure 1: Rotor bearing system with active disk.

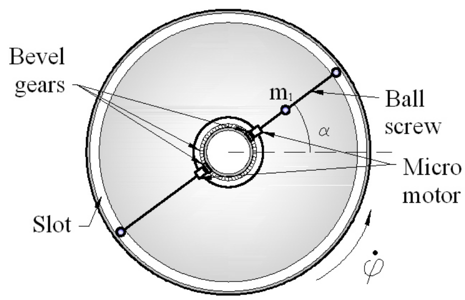

Figure 2: Configuration of the Active disk.

symmetric bearing supports (see Fig. 1 when $a = b$). Due to rotor imbalance the mass center is not located at the geometric center of the disk S but at the point G (center of mass of the unbalanced disk), the distance u between these points is known as disk eccentricity or static unbalance (see Vance [12]; Dimarogonas [13]).

In our analysis the rotor-bearing system has an active disk mounted on the shaft and near the main disk (see Fig. 1). The active disk is designed in order to move a mass m_1 in all angular and radial positions inside the disk, which are given by α and r_1, respectively. In fact, these movements can be got with some mechanical elements such as bevel gears and ball screw (see Fig. 2). The mass m_1 and the radial distance r_1 are designed in order to compensate the residual unbalance of the rotor bearing system.

An end view of the whirling rotor is also shown in Fig. 3, with coordinates that describe its motion. The coordinate system (η, ξ, ψ) of this figure is fixed to the active disk, and the coordinate system $(X, Y, Z\ )$ is an inertial frame with Z the nominal axis of rotation.

The mathematical model of the five degree-of-freedom rotor-bearing system with active disk was obtained using Euler-Lagrange equations, which is given by

$$\begin{aligned} (M + m_1)\,\ddot{x} + c\dot{x} + kx &= p_x\,(t) \\ (M + m_1)\,\ddot{y} + c\dot{y} + ky &= p_y\,(t) \\ J_e\ddot{\varphi} + c_\varphi\dot{\varphi} &= \tau_1 + p_\varphi\,(t) \\ m_1 r_1^2\ddot{\alpha} + 2m_1 r_1\dot{r}_1\dot{\alpha} + m_1 g r_1\cos\alpha &= \tau_2 \\ m_1\ddot{r}_1 - m_1 r_1\dot{\alpha}^2 + m_1 g\sin\alpha &= F \end{aligned} \tag{1}$$

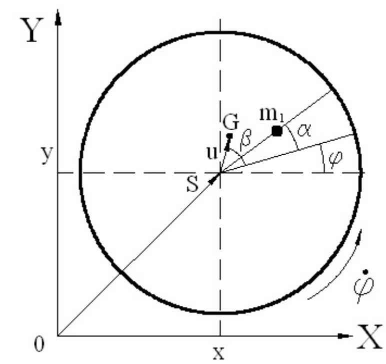

Figure 3: Inertial disk considering the balancing mass of the active disk.

with

$$\begin{aligned} p_x\,(t) &= Mu\left[\ddot{\varphi}\sin(\varphi+\beta) + \dot{\varphi}^2\cos(\varphi+\beta)\right] \\ &\quad + m_1 r_1\left[\ddot{\varphi}\sin(\varphi+\alpha) + \dot{\varphi}^2\cos(\varphi+\alpha)\right] \\ p_y\,(t) &= Mu\left[\dot{\varphi}^2\sin(\varphi+\beta) - \ddot{\varphi}\cos(\varphi+\beta)\right] \\ &\quad + m_1 r_1\left[\dot{\varphi}^2\sin(\varphi+\alpha) - \ddot{\varphi}\cos(\varphi+\alpha)\right] \\ p_\varphi\,(t) &= -M\ddot{y}u\cos(\varphi+\beta) - m_1\ddot{y}r_1\cos(\varphi+\alpha) \\ &\quad + M\ddot{x}u\sin(\varphi+\beta) + m_1\ddot{x}r_1\sin(\varphi+\alpha) \end{aligned}$$

Here J and c_φ are the inertia polar moment and the viscous damping of the totor, $\tau_1(t)$ is the applied torque (control input) for rotor speed regulation, x and y are the orthogonal coordinates that describe the disk position, r_1 and α denote the radial and angular position of the balancing mass, which are controlled by means of the control force $F(t)$ and control torque $\tau_2\,(t)$ (servo-mechanism).

Defining the state variables as $z_1 = x$, $z_2 = \dot{x}$, $z_3 = y$, $z_4 = \dot{y}$, $z_5 = \varphi$, $z_6 = \dot{\varphi}$, $z_7 = r_1$, $z_8 = \dot{r}_1$, $z_9 = \alpha$ and $z_{10} = \dot{\alpha}$, one obtains the following state space description

$$\begin{aligned} &\dot{z}_1 = z_2 \\ &\dot{z}_2 = \tfrac{1}{\Delta}\left(\tfrac{1}{M_e}\left(b^2 + J_e M_e\right)f_1 + \tfrac{ab}{M_e}f_2 + a\left(\tau_1 - z_6 c_\varphi\right)\right) \\ &\dot{z}_3 = z_4 \\ &\dot{z}_4 = \tfrac{1}{\Delta}\left(\tfrac{ab}{M_e}f_1 + \tfrac{1}{M_e}\left(J_e M_e - a^2\right)f_2 + b\left(\tau_1 - z_6 c_\varphi\right)\right) \\ &\dot{z}_5 = z_6 \\ &\dot{z}_6 = \tfrac{1}{\Delta}\left(-af_1 - bf_2 - M_e\left(\tau_1 - z_6 c_\varphi\right)\right) \\ &\dot{z}_7 = z_8 \\ &\dot{z}_8 = \tfrac{1}{m_1}\left(F - gm_1\sin z_9 + mz_7 z_{10}^2\right) \\ &\dot{z}_9 = z_{10} \\ &\dot{z}_{10} = \tfrac{1}{m_1 r_1^2}\left(\tau_2 - gm_1 z_7\cos z_9 - 2m_1 z_7 z_8 z_{10}\right) \\ &\ \ y = z_1^2 + z_3^2 \end{aligned} \tag{2}$$

with $f_1 = c_\varphi z_2 + kz_1 - Mz_6^2 u_y - m_1 r_y z_6^2$, $f_2 = cz_4 + kz_3 - Mz_6^2 u_x - m_1 r_x z_6^2$, $a = -Mu_x - m_1 r_x$, $b = Mu_y +$

$m_1 r_y, J_e = J + Mu^2 + m_1 r_1^2, M_e = M + m_1$ and $\Delta = a^2 + b^2 - J_e M_e$.

The rotor-bearing system with active disk is then described by the five degree-of-freedom, highly nonlinear and coupled model (2). The proposed control objective consists of reduce as much as possible the rotor vibration amplitude, denoted in adimensional units by

$$R = \frac{\sqrt{z_1^2 + z_3^2}}{u} \tag{3}$$

for run-up, coast-down or steady state operation of the rotor system, even in presence of small exogenous or endogenous perturbations.

In the following table are given the rotor system parameters employed troughtout the paper:

Table 1: Rotor system parameters		
$M = 1.2\,\text{kg}$	$m_1 = 0.003\,\text{kg}$	$a = b = 0.3\,\text{m}$
$\beta = \pi/6$ rad	$\alpha = 0$ rad	$r_{disk} = 0.04\,\text{m}$
$u = 100\,\mu\text{m}$	$c_\varphi = 1.5 \times 10^{-3} \frac{\text{N}}{\text{m/s}}$	$D = 0.01\,\text{m}$

3 ACTIVE VIBRATION CONTROL

3.1 Active disk control

We are proposing to use an active disk for actively balancing of the rotor (see Fig. 1 and Fig. 3). We can see that if the mass m_1 is located at the position $\left(\bar{r} = \frac{Mu}{m_1},\ \bar{\alpha} = \beta + \pi\right)$ the unbalance can be cancelled.

In order to design the position controllers for the balancing mass m_1, consider its associated dynamics:

$$\begin{aligned}
&\dot{z}_7 = z_8 \\
&\dot{z}_8 = \tfrac{1}{m_1}\left(F - gm_1 \sin z_9 + m_1 z_7 z_{10}^2\right) \\
&\dot{z}_9 = z_{10} \\
&\dot{z}_{10} = \tfrac{1}{m_1 z_7^2}\left(\tau_2 - gm_1 z_7 \cos z_9 - 2m_1 z_7 z_8 z_{10}\right) \\
&y_2 = z_7 \\
&y_3 = z_8
\end{aligned}$$

From these equations, one can get the following nonlinear controllers with integral compensation to take the balancing mass to the equilibrium position ($\bar{y}_2 = \bar{r} = \frac{Mu}{m_1}$, $y_3 = \bar{\alpha} = \beta + \pi$):

$$\begin{aligned}
F &= m_1 v_2 + gm_1 \sin z_9 - m_1 z_7 z_{10}^2 && (4) \\
\tau_2 &= m_1 z_7^2 v_3 + gm_1 z_7 \cos z_9 + 2m_1 z_7 z_8 z_{10} && (5)
\end{aligned}$$

with

$$\begin{aligned}
v_2 &= \ddot{y}_2^*(t) - \gamma_{21}\left[\dot{y}_2 - \dot{y}^*(t)\right] - \gamma_{21}\left[y_2 - y_2^*(t)\right] \\
&\quad -\gamma_{20}\int_0^t \left[y_2 - y_2^*(\sigma)\right] d\sigma \\
v_3 &= \ddot{y}_3^*(t) - \gamma_{32}\left[\dot{y}_3 - \dot{y}_3^*(t)\right] - \gamma_{31}\left[y_3 - y_3^*(t)\right] \\
&\quad -\gamma_{30}\int_0^t \left[y_3 - y_3^*(\sigma)\right] d\sigma
\end{aligned}$$

where $y_2^*(t)$ and $y_3^*(t)$ are desired trajectories for the outputs y_2 and y_3. Thus the tracking errors $e_2 = y_2 - y_2^*(t)$, $e_3 = y_3 - y_3^*(t)$, obey the following set of linear, decoupled, homogeneous differential equations:

$$\begin{aligned}
e_2^{(3)} + \gamma_{22}\ddot{e}_2 + \gamma_{21}\dot{e}_2 + \gamma_{20}e_2 &= 0 \\
e_3^{(3)} + \gamma_{32}\ddot{e}_3 + \gamma_{31}\dot{e}_3 + \gamma_{30}e_3 &= 0
\end{aligned}$$

which can be made to have the point: $(e_2, e_3) = (0, 0)$, as an exponentially asymptotically stable equilibrium point by selecting the design parameters $\{\gamma_{20}, \gamma_{21}, \gamma_{22}, \gamma_{30}, \gamma_{31}, \gamma_{32}\}$ such that the characteristic polynomials

$$\begin{aligned}
p_2(s) &= s^3 + \gamma_{22}s^2 + \gamma_{21}s + \gamma_{20} \\
p_3(s) &= s^3 + \gamma_{32}s^2 + \gamma_{31}s + \gamma_{30}
\end{aligned}$$

are Hurwitz polynomials.

It is evident, however, that the controllers (4) and (5) require information of the disk eccentricity (u, β). In what follows we will apply the algebraic identification method to estimate the disk eccentricity (u, β).

3.2 A certainty equivalence angular velocity controller

In order to control the speed of the rotor, consider its associated dynamics, under the assumption that effect of the unbalance was cancelled by the active disk and that the disk eccentricity (u, β) is perfectly known:

$$\begin{aligned}
\left[J + \left(Mu^2 + m_1 r_1^2\right)\right]\dot{z}_6 + c_\varphi z_6 &= \tau_1 \\
y_1 &= z_6
\end{aligned} \tag{6}$$

From this equation, one can get the following PI controller to asymptotically track a desired reference trajectory $y_1^*(t)$:

$$\begin{aligned}
\tau &= \left[J + \left(Mu^2 + m_1 r_1^2\right)\right] v_1 + c_\varphi z_6 \\
v_1 &= \dot{y}_1^*(t) - \gamma_{11}\left[y_1 - y_1^*(t)\right] \\
&\quad -\gamma_{10}\int_0^t \left[y_1 - y_1^*(\sigma)\right] d\sigma
\end{aligned} \tag{7}$$

The use of this controller yields the following closed-loop dynamics for the trajectory tracking error $e_1 = y_1 - y_1^*(t)$ as follows

$$\ddot{e}_1 + \gamma_{11}\dot{e}_1 + \gamma_{10}e_1 = 0 \tag{8}$$

Therefore, selecting the design parameters $\{\gamma_{10}, \gamma_{11}\}$ such that the associated characteristic polynomial for (8) be *Hurwitz,* one guarantees that the error dynamics be globally asymptotically stable.

4 ON-LINE ALGEBRAIC IDENTIFICATION OF ECCENTRICITY

Consider the first two equations in (1), where measurements the position coordinates of the disk (z_1, z_3) are available to be used in the on-line eccentricity identification scheme.

$$\begin{aligned}
&(M+m_1)\,\dot z_2 + c_\varphi z_2 + k z_1 = \\
&\quad M\left[\dot z_6 u \sin(z_5+\beta) + z_6^2 u \cos(z_5+\beta)\right] \\
&\quad + m_1\left[\dot z_6 r_1 \sin(z_5+\alpha) + z_6^2 r_1 \cos(z_5+\alpha)\right] \\
&(M+m_1)\,\dot z_4 + c_\varphi z_4 + k z_3 = \\
&\quad M\left[z_6^2 u \sin(z_5+\beta) - \dot z_6 u \cos(z_5+\beta)\right] \\
&\quad + m_1\left[z_6^2 r_1 \sin(z_5+\alpha) - \dot z_6 r_1 \cos(z_5+\alpha)\right]
\end{aligned} \tag{9}$$

Multiplying (9) by the quantity t^2 and integrating the result twice with respect to time t, one gets

$$\begin{aligned}
&\int^{(2)}\left((M+m_1)\,t^2\tfrac{dz_2}{dt} + c_\varphi t^2 z_2 + k t^2 z_1\right) = \\
&\quad \int^{(2)}\left(M u t^2 \tfrac{d}{dt}\left[z_6 \sin(z_5+\beta)\right]\right) \\
&\quad + \int^{(2)}\left(m_1 r_1 t^2 \tfrac{d}{dt}\left[z_6 \sin(z_5+\alpha)\right]\right) \\
&\int^{(2)}\left((M+m_1)\,t^2\tfrac{dz_4}{dt} + c_\varphi t^2 z_4 + k t^2 z_3\right) = \\
&\quad -\int^{(2)}\left(M u t^2 \tfrac{d}{dt}\left[z_6 \cos(z_5+\beta)\right]\right) \\
&\quad -\int^{(2)}\left(m_1 r_1 t^2 \tfrac{d}{dt}\left[z_6 \cos(z_5+\alpha)\right]\right)
\end{aligned} \tag{10}$$

where $\int^{(n)} \varphi(t)$ are iterated integrals of the form $\int_0^t \int_0^{\sigma_1} \cdots \int_0^{\sigma_{n-1}} \varphi(\sigma_n)\, d\sigma_n \cdots d\sigma_1$, with $\int \varphi(t) = \int_{t_0}^t \varphi(\sigma)\, d\sigma$ and n a positive integer.
Using integration by parts, one gets

$$\begin{aligned}
&(M+m_1)\left[t^2 z_1 - 4\int t z_1 + 2\int^{(2)} z_1\right] \\
&+c_\varphi\left[\int t^2 z_1 - 2\int^{(2)} t z_1\right] + k\int^{(2)} t^2 z_1 = \\
&\quad Mu\left[\int t^2 z_6 \sin(z_5+\beta) - 2\int^{(2)} t z_6 \sin(z_5+\beta)\right] \\
&\quad + mr\left[\int t^2 z_6 \sin(z_5+\alpha) - 2\int^{(2)} t z_6 \sin(z_5+\alpha)\right] \\
&(M+m_1)\left[t^2 z_3 - 4\int t z_3 + 2\int^{(2)} z_3\right] \\
&+c_\varphi\left[\int t^2 z_3 - 2\int^{(2)} t z_3\right] + k\int^{(2)} t^2 z_3 = \\
&\quad - Mu\left[\int t^2 z_6 \cos(z_5+\beta) - 2\int^{(2)} t z_6 \cos(z_5+\beta)\right] \\
&\quad - mr\left[\int t^2 z_6 \cos(z_5+\alpha) - 2\int^{(2)} t z_6 \cos(z_5+\alpha)\right]
\end{aligned} \tag{11}$$

The above integral-type equations (11), after some algebraic manipulations, leads to the following *linear* system of equations

$$A(t)\theta = b(t) \tag{12}$$

where $\theta = \left[u_\eta = u\cos\beta, u_\xi = u \sin\beta\right]^T$ denotes the eccentricity parameter vector to be identified and $A(t)$, $b(t)$ are 2×2 and 2×1 matrices, respectively, which are described by

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) \\ -a_{12}(t) & a_{11}(t) \end{bmatrix}, \quad b(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}$$

whose components are time functions specified as

$$\begin{aligned}
a_{11} &= M\left[\int t^2 z_6 \sin z_5 - 2\int^{(2)} t z_6 \sin z_5\right] \\
a_{12} &= M\left[\int t^2 z_6 \cos z_5 - 2\int^{(2)} t z_6 \cos z_5\right] \\
b_1 &= (M+m_1)\,t^2 z_1 \\
&\quad + \int\left(-4(M+m_1)\,t z_1 + c t^2 z_1\right) \\
&\quad - \int\left(-m_1 z_6 z_7 t^2 \sin(z_5+\alpha)\right) \\
&\quad + \int^{(2)}\left(2(M+m_1)\,z_1 - 2ctz_1 + kt^2 z_1\right) \\
&\quad + \int^{(2)}\left(2 m_1 z_6 z_7 t \sin(z_5+\alpha)\right) \\
b_2 &= (M+m_1)\,t^2 z_3 \\
&\quad + \int\left(-(M+m_1)\,4tz_3 + ct^2 z_3\right) \\
&\quad + \int\left(m_1 z_6 z_7 t^2 \cos(z_5+\alpha)\right) \\
&\quad + \int^{(2)}\left(2(M+m_1)\,z_3 - 2ctz_3 + kt^2 z_3\right) \\
&\quad - \int^{(2)}\left(2 m_1 t z_6 z_7 \cos(z_5+\alpha)\right)
\end{aligned}$$

From the equation (12) can be concluded that the parameter vector θ is *algebraically identifiable* if, and only if, the trajectory of the dynamical system is *persistent* in the sense established by Fliess and Sira-Ramírez [9], that is, the trajectories or dynamic behavior of the system satisfy the condition

$$\det A(t) \neq 0$$

In general, this condition holds at least in a small time interval $(t_0, t_0+\delta]$, where δ_0 is a positive and sufficiently small value.
By solving the equations (12) it is obtained the following algebraic identifier for the unknown eccentricity parameters

$$\left.\begin{aligned}
u_{\eta e} &= \frac{b_1 a_{11} - b_2 a_{12}}{a_{11}^2 + a_{12}^2} \\
u_{\xi e} &= \frac{b_1 a_{12} + b_2 a_{11}}{a_{11}^2 + a_{12}^2} \\
u_e &= \sqrt{u_{\eta e}^2 + u_{\xi e}^2} \\
\beta_e &= \cos^{-1}\left(\tfrac{u_{\eta e}}{u_e}\right)
\end{aligned}\right\} \forall t \in (t_0, t_0+\delta_0] \tag{13}$$

5 SIMULATION RESULTS

In Fig. 4 is depicted the identification process of the eccentricity. We can observe a good and fast estimation ($t << 0.1\,\text{s}$).
Fig. 5 shows the dynamic behavior of the adaptive-like control scheme (7), which starts using the nominal value $u = 0$ μm. We can observe the asymptotic output tracking of a desired reference trajectory. Here, we selected a desired *Hurwitz* polynomial $r(s)$ given as $r(s) = \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)$ with $\zeta = 0.7071$, $\omega_n = 12$.
The planned trajectory for the output $y_1 = z_6$ is given by

$$y_1^*(t) = \begin{cases} 0 & \text{for } 0 \le t < T_1 \\ \psi(t, T_1, T_2)\,\bar y_1 & \text{for } T_1 \le t \le T_2 \\ \bar y_1 & \text{for } t > T_2 \end{cases}$$

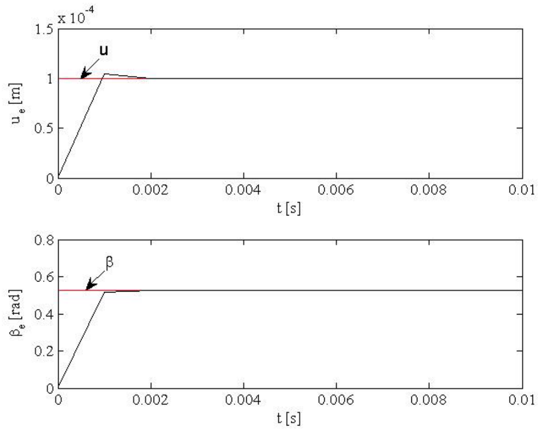

Figure 4: Excentricty (u) and angular position (β) identification.

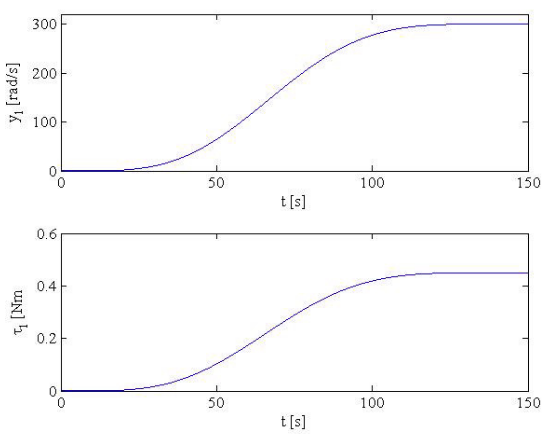

Figure 5: Close loop system response using an adaptive-like control scheme.

where $\bar{y}_1 = \bar{z}_6 = 300\,\mathrm{rad/s}$, $T_1 = 0$ s, $T_2 = 50$ s and $\psi(t, T_1, T_2)$ is a Bézier polynomial, with $\psi(T_1, T_1, T_2) = 0$ and $\psi(T_2, T_1, T_2) = 1$, described by

$$\begin{aligned}\psi(t) = \left(\tfrac{t-T_1}{T_2-T_1}\right)^5 & [r_1 - r_2\left(\tfrac{t-T_1}{T_2-T_1}\right) \\ & + r_3\left(\tfrac{t-T_1}{T_2-T_1}\right)^2 - \ldots - r_6\left(\tfrac{t-T_1}{T_2-T_1}\right)^5]\end{aligned}$$

with $r_1 = 252$, $r_2 = 1050$, $r_3 = 1800$, $r_4 = 1575$, $r_5 = 700$, $r_6 = 126$.

Fig. 6 shows the dynamic behavior of the active disk controllers to take the balancing mass to the equilibrium position $\left(\bar{r} = \frac{M}{m_1}u_e,\ \bar{\alpha} = \beta_e + \pi\right)$. In this position the active disk cancels the unbalance, as it is shown in Fig. 7. The controllers were implemented when the eccentricity has been estimated. The gains of both controllers were selected to have a third order characteristic polynomial $r(s) = (s+p)\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)$, with $\zeta = 0.7071$, $\omega_n = 12$, $p = 10$.

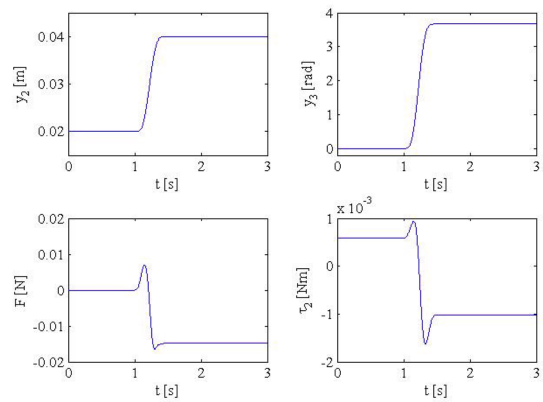

Figure 6: Dynamic response of the balancing mass: radial position (z_7), angular position (z_9), control force (F) and control torque (τ_2).

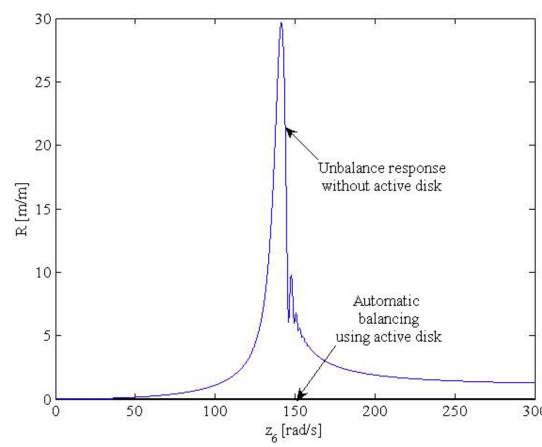

Figure 7: Active balancing response using the active disk.

6 CONCLUSIONS

The active vibration control of rotor-bearing systems using an active disk is addressed. This approach consists in locating a balancing mass at a suitable position. Since this active control scheme requires information of the eccentricity, a novel algebraic identification approach is proposed for the on-line estimation of the eccentricity parameters. This approach is quite promising, in the sense that from a theoretical point of view, the algebraic identification is practically instantaneous and robust with respect to parameter uncertainty, frequency variations, small measurement errors and noise. Thus the algebraic identification is combined with two control schemes to put the balancing mass at the correct position to cancel the unbalance of the rotor. A velocity control is designed to take the rotor velocity to a desired operating point over the first critical speed in order to show the vibration cancellation. The controllers were developed in the context of an off-line prespecified reference trajectory tracking problem. Numerical simulations were included to illustrate the high dynamic performance of the active vibration control scheme proposed. Some experiments are being implemented on a real rotor system in order to validate the results.

References

[1] S. Zhou and J. Shi, "Active Balancing and Vibration Control of Rotating Machinery: a survey", *The Shock and Vibration Digest, 33(4)*, 2001, pp. 361-371.

[2] G. J. Sheu, S. M. Yang, and C. D. Yang, "Design of Experiments for the Controller of Rotor Systems With a Magnetic Bearing", *Journal of Vibration and Acoustics*, *119(2)*, 1997, pp. 200-207.

[3] Y. Guozhi, Y. F. Fah, C. Guang, M. Guang, F. Tong and Q. Yang, "Electro-Rheological Multi-layer Squeeze Film Damper and its Application to Vibration Control of Rotor System", *Journal of Vibration and Acoustics, 122(1)*, 2000, pp. 7-11.

[4] A. Blanco-Ortega, F. Beltrán-Carbajal, "On-Line Algebraic Identification of Eccentricity in Active Vibration Control of Rotor Bearing Systems", *Proceedings of 4th International Conference on Electrical and Electronics Engineering*, México, D.F., 2007.

[5] L. Ljung, *Systems Identification: Theory for the User*. Englewood Cliffs, NJ: Prentice-Hall, 1987.

[6] T. Soderstrom and P. Stoica, *System Identification.* New York: Prentice-Hall, 1989.

[7] S. Sagara and Z. Y. Zhao, "Numerical integration approach to on-line identification of continuous systems", *Automatica, 26*(1), 1990, pp. 63-74.

[8] S. Sagara and Z. Y. Zhao, "Recursive identification of transfer function matrix in continuous systems via linear integral filter", *International Journal of Control, 50*(2), 1989, pp. 457-477.

[9] M. Fliess and H. Sira-Ramírez, "An algebraic framework for linear identification", *ESAIM: Control, Optimization and Calculus of Variations,* 9, 2003, pp. 151-168.

[10] F. Beltrán-Carbajal, H. Sira-Ramírez and G. Silva-Navarro, "Adaptive-like Active Vibration Supression for a Nonlinear Mechanical System Using On-Line Algebraic Identification", *Proceedings of the Thirteenth International Congress on Sound and Vibration*, Vienna, Austria, 2006.

[11] F. Beltrán-Carbajal, G. Silva-Navarro, H. Sira-Ramírez, J. Quezada-Andrade, "Active vibration control using on-line algebraic identification of harmonic vibrations", *Proceedings of American Control Conference*, Portland, Oregon, June 2005.

[12] J. M. Vance, *Rotordynamics of Turbomachinery.* New York: John Wiley and Sons Inc, 1988.

[13] Dimarogonas, A., *Vibration for Engineers.* New Jersey: Prentice Hall, 1996.

[14] Sandler, B.Z., *Robotics: designing the mechanism for automated machinery.* San Diego, CA: Academic Press, 1999.

[15] K. T. Millsaps and G. L. Reed, "Reducing Lateral Vibrations of a Rotor Passing Through Critical Speeds by Acceleration Scheduling", *Journal of Engineering for Gas Turbines and Power, 120(3)*, 1998, pp. 615-620.

[16] M. Fliess, R. Marquez, E. Delaleau and H. Sira-Ramírez, "Correcteurs Proportionnels-Integraux Généralisés", *ESAIM Control, Optimisation and Calculus of Variations, 7(2)*, 2002, 23-41.