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New Nonlinear Model of Macpherson Suspension System for Ride Control Applications

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Abstract—**In this paper, a new nonlinear model of Macpherson strut suspension system for ride Control applications is proposed. The model includes the vertical acceleration of the sprung mass and the motions of the unsprung mass subjected to control arm rotation. In addition, it considers physical characteristics of the spindle such as mass and inertia moment. This two degree-of-freedom (DOF) model not only provides a more accurate representation of the Macpherson suspension system for ride control applications but also facilitates evaluation of the kinematic parameters such as camber, caster and king-pin angles as well as track alterations on the ride vibrations. The performances of the nonlinear and linear models are investigated and compared. Simulation results are presented and discussed.**

I. INTRODUCTION

The Macpherson suspension was created by Earl Macpherson in 1949 for the ford company. Due to its light weight and size compatibility this kind of suspension is widely used in different vehicles. Moreover this kind of vehicle is more popular to be found in the front of the car even though it was also used as a rear suspension.

Performance requirements for a suspension system are to adequately support the vehicle weight, to provide effective ride quality which means isolation of the chassis against excitations due to road roughness, to maintain the wheels in the appropriate position so as to have a better handling and to keep tire contact with the ground. However it is well known that these requirements are conflicting, for instance to achieve better isolation of the vehicle chassis from road Irregularities, a larger suspension deflection is required with soft damping, while a large damping yields better stability at the expense of comfort. Thus, the idea of incorporating of active or semi-active suspensions can be considered so as to reach these specifications more than those passive one.

Based on a simplified two DOF quarter car model, many semi-active and active control algorithms [1-5] have been developed to handle these conflicting performance requirements. The simplified two DOF quarter car model, so-called conventional model in this paper, represents two lumped masses (sprung mass and unsprung mass) of a quarter car system. Although the conventional model of the suspension has been widely used in suspension control designs, it is not convenient for the evaluation of the suspension kinematic parameters which significantly affect handling performance of the vehicle. Hence, most of the current control algorithms focus on the enhancement of ride quality without considering structural effects. Note that, without considering the effect of the suspension kinematics, the simple model may not be considered effective. Thus the study about the impacts of the suspension kinematics on the dynamical behavior of the system is necessary. Therefore, the need for an accurate model for the Macpherson suspension system becomes increasingly important for ride control design applications.

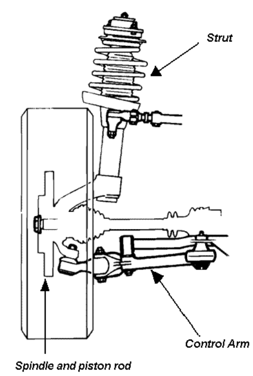

Fig. 1. Schematic of Macpherson Strut Suspension

Based on three nonlinear models of the Macpherson suspension, Stensson, *et al* [6] analyzed the dynamical behavior of this system. A spatial model of the Macpherson suspension to study its kinematic and dynamic performances was formulated by Fallah [7] and Suh [8]. Using a three-dimensional model of a Macpherson suspension, Chen and Beale [9] estimated the dynamic parameters of the mechanism. Although these models are useful in analyzing the structure, they are not suitable for ride control design. Moreover, a three-dimensional model of the Macpherson suspension was employed by Ro and Kim [10] for parameter identification and also for ride control, however, this model, as the previous models, was not applicable for observation of the kinematic parameters. Sohn, *et al* [11-12] proposed a new model of the Macpherson suspension for ride control purposes. Nevertheless, in that model the structure and properties of the spindle have not been taken into consideration.

In this paper, a comprehensive model of the Macpherson strut wheel suspension system with spindle properties is proposed for ride control applications. The model considers the kinematic properties, the vertical acceleration of the

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sprung mass and the motions of the unsprung mass subjected to control arm rotation. In addition, it includes physical characteristics of the spindle such as mass and inertia moment. With this model, it is convenient to observe the suspension kinematic parameters subjected to control actuation force, designed to improve the ride quality.

II. New model of Macpherson suspension for active control applications

The schematic of a Macpherson strut suspension is shown in Fig.1. To model a Macpherson suspension system for control application, one should take into account both the kinematics and dynamics of the system subjected to the actuation force and road disturbances.

A. Kinematics

Consider a Macpherson suspension system excited by road disturbance (z_r) as shown in Fig. 2. It comprises a quarter-car body, a spindle and a tire, a helical spring, control arm, load disturbance (f_d) and an actuation force (f_a). The structure has two degrees of freedom including vertical displacement of the sprung mass and rotational motion of the control arm when the mass of the strut is ignored and the bushing at point D is assumed to be a pin joint. In this research, we focus on building a two DOF model of a Macpherson suspension system.

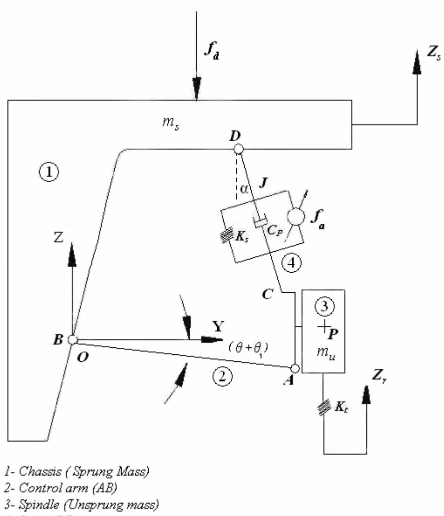

Fig. 2. Model of Macpherson suspension

The detailed assumptions in this modeling are made as follows: The sprung mass has only vertical displacement while movements in other directions are ignored. The unsprung mass (spindle and tire) is connected to the car body through the damper and spring as well as the control arm. The values of z_s, vertical displacement of the sprung mass, and θ, rotational displacement of the control arm, are measured from the static equilibrium position and are considered as generalized coordinates. It is assumed that, in the equilibrium condition, the camber angle is zero. Compared to the other links, the mass and stiffness of the strut are neglected. The spring and tire deflections and the damping force are assumed to be in the linear regions of their operation ranges.

In Fig. 2, link AB represents the control arm which is modeled as a rod, while line CD shows the strut of the mechanism. The revolute joint, located between the control arm and the chassis, is modeled as a rotational joint at point *B*. In addition, let assume that the origin of the coordinate system, O, is on point B and (y_A, z_A), (y_B, z_B), (y_J, z_J), (y_p, z_p), (y_C, z_C) and (y_D, z_D) denote the coordinates of the points A, B, J, P, C and D, respectively. Under road disturbances, the position of the key points on the sprung mass change as the following:

$$y_B = 0, \quad z_B = z_s, \quad y_D = y_{D_1}, \quad z_D = z_{D_1} + z_s \tag{1}$$

In addition, the displacements of the main points on the spindle are introduced as:

$$\begin{bmatrix} y_C & y_J & y_P \\ z_C & z_J & z_P \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & y_A - (a_{11} y_{A_1} + a_{12} z_{A_1}) \\ a_{21} & a_{22} & z_A - (a_{21} y_{A_1} + a_{22} z_{A_1}) \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} y_{C_1} & y_{J_1} & y_{P_1} \\ z_{C_1} & z_{J_1} & z_{P_1} \\ 1 & 1 & 1 \end{bmatrix} \tag{2}$$

where (y_{A1}, z_{A1}), (y_{J1}, z_{J1}) , (y_{P1}, z_{P1}), (y_{C1}, z_{C1}) are the coordinates of the points A, J, P and C at equilibrium position. Further,

$$\begin{aligned} a_{11} &= a_{22} = \cos\varphi \\ a_{12} &= -a_{21} = \sin\varphi \end{aligned} \tag{3}$$

where φ is the rotation angle of the wheel. System of equations (2), is made of six equations containing nine unknown parameters which are (y_A, z_A), (y_J, z_J), (y_P, z_P), (y_C, z_C) and φ.

To solve this system, it is necessary to employ constraint equations as follows:

$$\begin{aligned} z_C &= \alpha(y_C - y_D) + z_D \\ z_J &= \alpha(y_J - y_D) + z_D \\ y_A &= L_A \cos(\theta + \theta_1) + y_B \\ z_A &= L_A \sin(\theta + \theta_1) + z_B \end{aligned} \tag{4}$$

where α is the slope of the strut, L_A is the length of the control arm and θ_1 is the initial angle of the control arm resulting from the static deflection and structure design.

Considering equations (2) and (4), results in ten equations including ten unknown parameters, namely, (y_A, z_A), (y_J, z_J), (y_P, z_P), (y_C, z_C), α and φ. Thus, the following equations of displacements can be established:

$$y_C = (y_{C_1} - y_{A_1})a_{11} + (z_{A_1} - z_{C_1})a_{21} + y_A$$
$$z_C = (y_{C_1} - y_{A_1})a_{21} + (z_{C_1} - z_{A_1})a_{11} + z_A$$
$$y_J = (y_{J_1} - y_{A_1})a_{11} + (z_{A_1} - z_{J_1})a_{21} + y_A$$
$$z_J = (y_{J_1} - y_{A_1})a_{21} + (z_{J_1} - z_{A_1})a_{11} + z_A$$
$$y_P = (y_{P_1} - y_{A_1})a_{11} + (z_{A_1} - z_{P_1})a_{21} + y_A$$
$$z_P = (y_{P_1} - y_{A_1})a_{21} + (z_{P_1} - z_{A_1})a_{11} + z_A$$
$$z_C = \alpha(y_C - y_D) + z_D$$
$$z_J = \alpha(y_J - y_D) + z_D$$
$$y_A = L_A \cos(\theta + \theta_1)$$
$$z_A = L_A \sin(\theta + \theta_1) + z_s \tag{5}$$

When solving the above system of equations, one determines parameter φ as a function of generalized coordinates θ and z_s. Subsequently, the other unknown parameters including (y_A, z_A), (y_J, z_J), (y_P, z_P), (y_C, z_C) and α can be specified. Hence, the displacements of all key points are determined as functions of independent variables θ and z_s. The next step is to find the velocities of the key points. By taking the derivative of (5), one can obtain the velocity components of the main points. When solving the equations of velocities, the value of $\dot{\varphi}$ is determined as following:

$$\dot{\varphi} = \frac{(\dot{z}_A - a\dot{y}_A - \dot{z}_D)(y_C - y_J)}{h} \tag{6}$$

where

$$h = (y_C - y_A + \alpha z_C - \alpha z_A)(y_J - y_D) - (y_J - y_A + \alpha z_J - \alpha z_A)(y_C - y_D)$$

B. Equations of Motion

Lagrange's method is used to obtain the equations of motion of the new model. The kinetic energy, T, is given by

$$T = \frac{1}{2}(m_s + m_{ca})(\dot{z}_s)^2 + \frac{1}{2}m_u(\dot{y}_P^2 + \dot{z}_P^2) + \frac{1}{2}I_u\dot{\varphi}^2 + \frac{1}{2}I_{ca}^B\dot{\theta}^2 \tag{7}$$

where m_s, m_u and m_{ca}, are the car body, wheel and control arm masses, respectively. I_u and I_{ca}^B represent, in turn, the inertia moments of the wheel and the control arm where the latter is around point B. The potential energy, V, is defined as

$$V = \frac{1}{2}K_s(\Delta L)^2 + \frac{1}{2}K_t(\Delta z)^2 \tag{8}$$

where K_s and K_t are the stiffness coefficients of the sprung and unsprung masses, respectively. Moreover, the deflection of the spring ΔL, and the deflection of the tire Δz are:

$$\Delta L = [(y_C - y_D)^2 + (z_C - z_D)^2]^{(1/2)} - [(y_{C_1} - y_{D_1})^2 + (z_{C_1} - z_{D_1})]^{(1/2)} \tag{9}$$

$$\Delta z = z_P - z_r = (y_{A_1} - y_{P_1})\varphi + (z_{P_1} - z_{A_1}) + L_A \sin(\theta + \theta_1) + z_s - z_r \tag{10}$$

The damping function, D, is given by

$$D = \frac{1}{2}C_p(\Delta\dot{L})^2 \tag{11}$$

where C_p is the damping coefficient and the relative velocity of damper $\Delta\dot{L}$ is:

$$\Delta\dot{L} = [\dot{y}_C(y_C - y_D) + (\dot{z}_C - \dot{z}_D)(z_C - z_D)] \times [(y_C - y_D)^2 + (z_C - z_D)^2]^{(-1/2)} \tag{12}$$

substituting the values of $\dot{y}_p$ and $\dot{z}_p$, obtained from derivative of (5), and $\dot{\varphi}$, attained from (6), into (7) as well as using Lagrange's equations along with the generalized coordinates z_s and θ, one can obtain the accelerations of the generalized coordinates as the following:

$$(m_s + m_u + m_{ca})\ddot{z}_s + m_u L_A\Big[\cos(\theta+\theta_1) + \Big[\cos(\theta+\theta_1) + \alpha\sin(\theta+\theta_1)\Big]\frac{(y_C - y_J)(y_P - y_A)}{h}\Big]\ddot{\theta} = f_1\ddot{z}_s + f_2\ddot{\theta} \tag{13}$$

and

$$m_u\left[L_A\cos(\theta+\theta_1) + (y_P - z_A)\frac{\partial\dot{\varphi}}{\partial\dot{\theta}}\right]\ddot{z}_s + \left[m_u L_A\begin{bmatrix}\{(\cos(\theta+\theta_1) + \alpha\sin(\theta+\theta_1)\}\frac{(y_C - y_J)(z_A - z_P)}{h} \\ -\sin(\theta+\theta_1)\end{bmatrix}\times\right.$$
$$\left[-L_A\sin(\theta+\theta_1) + (z_A - z_P)\frac{\partial\dot{\varphi}}{\partial\dot{\theta}}\right] + m_u L_A\left[L_A\cos(\theta+\theta_1) + (y_P - z_A)\frac{\partial\dot{\varphi}}{\partial\dot{\theta}}\right]\times$$
$$\begin{bmatrix}\{(\cos(\theta+\theta_1) + \alpha\sin(\theta+\theta_1)\}\frac{(y_C - y_J)(y_P - y_A)}{h} \\ +\cos(\theta+\theta_1)\end{bmatrix} +$$
$$\left. I_u\ddot{\varphi}\frac{\partial\dot{\varphi}}{\partial\dot{\theta}}\right]\ddot{\theta} = f_3\ddot{z}_s + f_4\ddot{\theta} \tag{14}$$

Since the equations are highly nonlinear and too complicated, the higher order nonlinearities in Eqs. 13 and 14 are ignored to simplify the equations. Let us denote

$$H_1 = -\frac{\partial T}{\partial z_s}, \qquad H_2 = \frac{\partial V}{\partial z_s}, \qquad H_3 = -\frac{\partial D}{\partial \dot{z}_s}$$

$$H_4 = -\frac{\partial T}{\partial \theta}, \qquad H_5 = \frac{\partial V}{\partial \theta}, \qquad H_6 = -\frac{\partial D}{\partial \dot{\theta}}$$

Hence, one has

$$f_1\ddot{z}_s + f_2\ddot{\theta}_s = \frac{\partial \vec{r}_C}{\partial z_s}.f_a + f_d + H_3 - H_1 - H_2 = F_1$$

and

$$f_3\ddot{z}_s + f_4\ddot{\theta} = -\frac{\partial \vec{r}_C}{\partial \theta}.f_a + H_6 - H_4 - H_5 = F_2\,.$$

The nonlinear equations of motion are obtained as below

$$\begin{aligned} &f_1(z_s,\dot{z}_s,\theta,\dot{\theta})\ddot{z}_s + f_2(z_s,\dot{z}_s,\theta,\dot{\theta})\ddot{\theta} = F_1(z_s,\dot{z}_s,\theta,\dot{\theta}) \\ &f_3(z_s,\dot{z}_s,\theta,\dot{\theta})\ddot{z}_s + f_4(z_s,\dot{z}_s,\theta,\dot{\theta})\ddot{\theta} = F_2(z_s,\dot{z}_s,\theta,\dot{\theta}) \end{aligned} \tag{15}$$

Solving the above system, the acceleration of the generalized coordinates are obtained as follows:

$$\begin{aligned} \ddot{z}_s &= \frac{f_4F_1 - f_2F_2}{f_4f_1 - f_2f_3} = g_1(z_s,\dot{z}_s,\theta,\dot{\theta}) \\ \ddot{\theta} &= \frac{f_1F_2 - f_3F_1}{f_4f_1 - f_2f_3} = g_2(z_s,\dot{z}_s,\theta,\dot{\theta}) \end{aligned} \tag{16}$$

At this point let us introduce the state variables as $[x_1, x_2, x_3, x_4]^T = [z_s, \dot{z}_s, \theta, \dot{\theta}]^T$, then (16) can be written in the state space format as follows.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g_1(z_s,\dot{z}_s,\theta,\dot{\theta},f_a,f_d,z_r) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= g_2(z_s,\dot{z}_s,\theta,\dot{\theta},f_a,f_d,z_r) \end{aligned} \tag{17}$$

Since the equations are nonlinear and working with them is non-trivial task and employing a complex nonlinear controller is essential, all of equations are linearized at the equilibrium state where, $(x_{1e}, x_{2e}, x_{3e}, x_{4e}) = (0, 0, 0, 0)$. The resulting equations are:

$$\begin{aligned} \dot{x} &= Ax(t) + B_1f_a(t) + B_2z_r(t) + B_3f_d(t) \\ x(0) &= x_e \end{aligned} \tag{18}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} & \frac{\partial g_1}{\partial x_4} \\ 0 & 0 & 0 & 1 \\ \frac{\partial g_1}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} & \frac{\partial g_2}{\partial x_4} \end{bmatrix}_{x_e}$$

$$B_1 = \begin{bmatrix} 0 & \frac{\partial g_1}{\partial f_a} & 0 & \frac{\partial g_2}{\partial f_a} \end{bmatrix}_{f_a=0}$$

$$B_2 = \begin{bmatrix} 0 & \frac{\partial g_1}{\partial z_r} & 0 & \frac{\partial g_2}{\partial z_r} \end{bmatrix}_{z_r=0}$$

$$B_3 = \begin{bmatrix} 0 & \frac{\partial g_1}{\partial f_d} & 0 & \frac{\partial g_2}{\partial f_d} \end{bmatrix}_{f_d=0}$$

III. Simulation and Verification of Model

A. Comparison of the conventional, linear and nonlinear models

In order to compare the models, the following values have been taken from [12] and ADAMS software default:

$m_s = 453$ (Kg), $m_u = 71$ (Kg), $K_s = 17658$ (N/m)

$K_t = 183887$ (N/m), $I_u = 0.021$ (Kg.m^2)

$C_p = 1950$ (N.sec/m)

The positions for the key points on the Macpherson suspension are considered as the below (all dimensions are in mm):

$A_1 = (206.5,\ 249,\ -60.8)$, $C_1 = (222, 152.6, 236.2)$

$J_1 = (229.2, 134.5, 374.8)$, $P_1 = (211.1, 292.1, 27.5)$

$D_1 = (240, 107.4, 582.5)$

The output variables of the conventional model are the vertical displacements of the sprung mass z_s and the unsprung mass z_u whereas in the new model the output vector consists of the displacement of the sprung mass z_s and the angular displacement of the control arm θ. Thus, the displacement of the sprung mass, z_s, is considered as the output variable in order to compare the two models . The frequency response of the two models is shown in Fig. 3. As can be seen, the first resonance frequency is smaller than that of the conventional model and the second resonant frequency is larger than that of the conventional model.

In the literature, the acceleration of the sprung mass is considered as a criterion for assessment of the effect of a suspension on the ride comfort, specially, in the high frequency ranges. Fig. 4 compares the acceleration transmissibility of three models for frequencies between 0-20 Hz. As shown in Fig. 4, the linear model represents a good performance of the nonlinear model for the frequencies between 0-5 Hz. However, the conventional model shows the performance of the Macpherson suspension systems with some discrepancies.

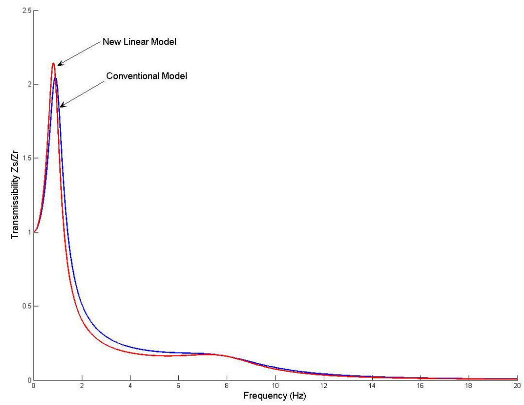

Fig. 3 Frequency response of new model and conventional model

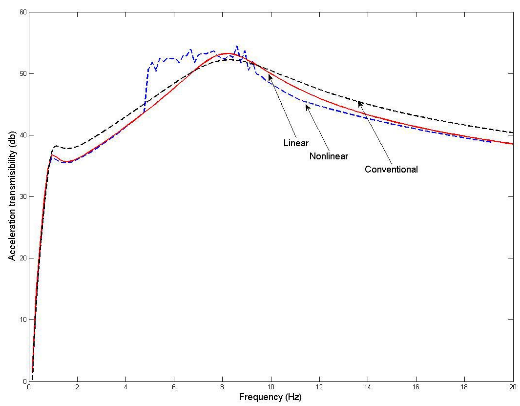

Fig. 4 Acceleration transmissibility of the nonlinear, linear and conventional models

B. *Evaluation of the kinematic parameters*

Some of the main kinematical parameters which are important in chassis design and affect handling and stability of the vehicle are 1) camber angle; 2) kingpin angle 3) caster angle 4) track. Camber angle alterations are due to rubbing of tires and produce lateral forces acting on the wheel and cause the vehicle to steer to one side. Alterations of kingpin and caster angles affect the self aligning torques and consequently affect the stability and handling of the vehicle when wheels bounce or rebound. When the wheels travel on a bump and rebound, the track changes cause the rolling tire to slip and, also produce lateral forces [13]. In the following simulations, we set the step input for road disturbance z_r equal to 100 *mm* and time step equal to 0.0001 (s).

The camber angle, represented by φ in (3), is the angle between the wheel center plane and a vertical line to the road [13]. Fig. 5 shows this parameter variation for both the linear and the nonlinear models. In definition, the steering axis is the line passing through the point *D* and *A* in the three-dimensional case and the kingpin angle is the angle between the projection of the steering axis on *y-z* plane and the vertical line to the road. In Fig. 6, King-Pin angle variations are plotted for both the linear and nonlinear models. The angle between the projection of the steering axis on the *x-z* plane and the vertical line to the road is defined as caster angle. The performance of this parameter is illustrated in Fig. 7. Track is the lateral distance between the centers of the front wheels. Fig. 8 shows the alteration of track for the linear and nonlinear models. It is obvious that, unlike the previous parameters, the linearization has a large impact on the track. As a result, linear model is not sufficiently accurate for studying the track behavior.

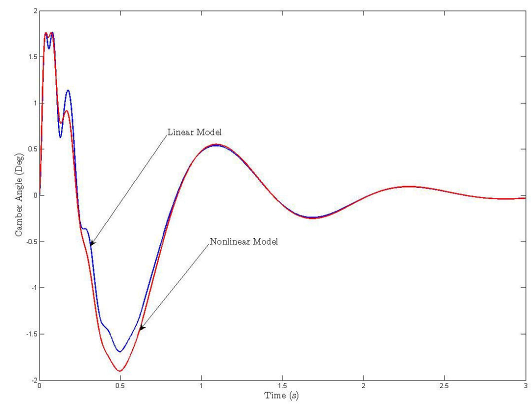

Fig. 5 Camber angle alterations

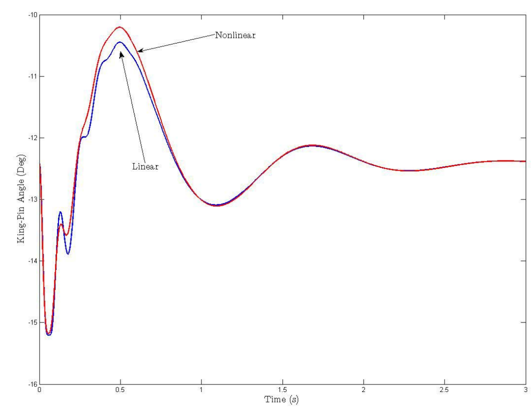

Fig. 6 King-pin angle alterations

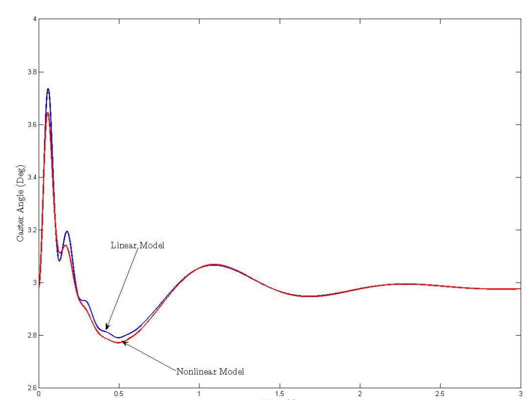

Fig. 7 Caster angle alterations

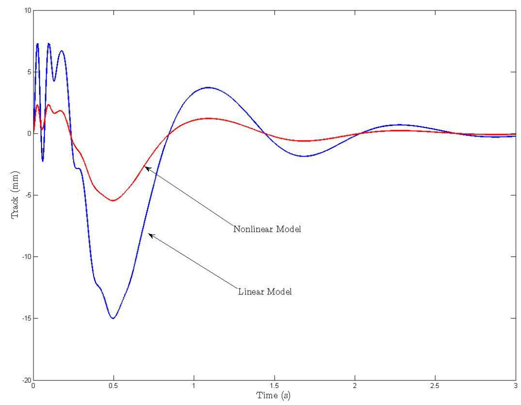

Fig. 7 Track alteration

IV. Conclusion

A new nonlinear model of Macpherson suspension is proposed and equations of motion are derived. The new model is more general than conventional model where the structural kinematics and spindle properties are taken into account. In addition, the new model allows investigation of the suspension kinematic parameters affecting on handling and stability of the vehicle while it is impossible or difficult using the other models proposed for the Macpherson suspension in the case of ride control implementation. The nonlinear and linear responses of the model are investigated and shown that the linear model is a good approximation of the nonlinear model for ride quality assessment. However, for evaluation of the kinematic parameter performances nonlinear kinematic relations are used which provide a more accurate study of handling performance and stability condition of the vehicle.

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