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Airflow velocity estimation in Building Systems

M. Guay and N. Hariharan

Abstract— **In this paper, we propose a POD-based technique that is suitable for the design of reliable observers for the estimation of velocity field from 2D and 3D Navier-Stokes flow. POD modes are constructed using the method snapshot. Karhunen-Loeve (Galerkin) projections are used to develop a reduced-order model obtained by projecting the velocity field onto the most important POD modes. The resulting finite-dimensional dynamical system is suitable for the design of nonlinear observers. The prime application considered considered is the estimation of airflow in building systems. Two examples are provided to demonstrate the applicability of the technique.**

I. Introduction

Flow control and optimization research has been very active over the last few years. In contrast, the flow estimation problem has not been considered to such an extent. One of the major challenges of the NS equations is the fact that they are nonlinear. In most cases, a linearized version of the Navier-Stokes equation is considered. Using existing tools from linear infinite dimensional system control ([7],[8]), one can obtain a number interesting results for the control and estimation of flow systems. Kalman filter based approaches have been recently reported in [9] and [10]. Using the linearized NS equations about a velocity field of interest, the authors design an infinite dimensional Kalman filter. The infinite dimensional filter is then discretized to produce a finite dimensional filter. The main drawback of this "late-lumping" approach is that the corresponding filter can be a very large dynamical system as a result of the discretization step. Real-time implementation is thus limited to coarse discretizations or simple geometries where one can exploit symmetries. Essentially, cases where high fidelity discrete approximations can be obtained with a relatively low dimensional finite dimensional approximation. Another drawback of this technique is the requirement for a linearized version of the NS equations. Depending on the specific conditions, the assumption of linearity can be poor and some important features can be lost. Nevertheless, it provides a very effective was to estimate changes in velocity fields that are in the neighborhood of the linearization conditions.

The state estimation problem is particularly complex since the number of available sensors are generally quite limited and sensor placement is often restricted due to the lack of accessibility and extreme conditions (not suitable for sensor viability). The knowledge of 2D and 3D NS velocity field has multiple applications in very diverse areas. In building systems, the knowledge of the velocity field in building systems allows one to monitor contaminant flows inside the building and to control air quality. Velocity field estimation play an important in meteorological applications especially in cases where one is attempting to estimate contaminant flow using basic ground measurements.

POD-based techniques are generally more suitable for nonlinear system. Once a projection into a finite dimensional subspace is obtained (by the method of snapshots for example), the nonlinear NS equations can be projected onto the subspace using Galerkin projection (a.k.a. Karhunen-Loeve projection). The Galerkin projections express the distributed state variables as linear combinations of the POD modes. By setting the variables in this form and substituting to the original equations, a set of nonlinear ordinary differential equations is obtained for the coefficients. The main advantage of the Galerkin projection is that it preserves the nonlinearity of the complex system in the form of a low-dimensional set of nonlinear ordinary differential equations. In this context, the state variables of the reduced order (finite dimensional) system become the Galerkin projection coefficients. The estimation problem can be reduced to the estimation of the Galerkin coefficients. This approach does allows one to develop a suitable nonlinear filter for the estimation of the reduced order system which can then be used to estimate the state variables of the underlying complex system. As a result, the ability to accurately estimate the states of the complex system is completely dependent on the particular choice of the POD modes. In general, the POD modes should reflect the dynamical features of interest. Assuming that these features are observable from the available measurements, one can provide very accurate estimates of the state of the system. One of the major difficulty with the Galerkin approach is that the stability of the original system may not be preserved in the reduced order nonlinear system. Care must be taken to ensure that stability is preserved. Recently, Rowley [2]-[5] demonstrated that an appropriate choice of inner product can alleviate this problem. Using a simple Lyapunov argument, it was shown how one can use a suitable, energy-based, inner product to preserve the stability of the physical system. The loss stability is indicative of a more general difficulty with this approach. Although the POD modes reflect the features of the physical system, the physical attributes are lost in the Galerkin projection since their design is purely empirical in nature. Care must be taken in the application of these techniques. In addition to the sensor location, the appropriate choice of sensor is an important consideration for the development of an effective velocity field estimation algorithm. In [11], it is shown that

M. Guay is with the Department of Chemical Engineering, Queen's University, Kingston, Ontario, Canada. guaym@chee.queensu.ca
N. Hariharan is with United Technologies Research Centre, East Hartford, HarihaNS@utrc.utc.com

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the velocity field of the flow between two infinite plates can be reconstructed exactly by measuring the skin frictions and the pressure at the wall. Using this result, a reduced-rank Kalman filter approach is used for the estimation. While the results obtained are good at the wall, significant deviations were observed at a distance from the wall. Thus, despite the knowledge of sensors that allows one to reconstruct exactly the velocity field in the vicinity of the wall, the problem of estimation remains unresolved. In this study, the POD-based approach is developed for the estimation of velocity field in 2D and 3D Navier-Stokes flow. The effectiveness of the technique is demonstrated for a 2D flow problem. The application to a challenging 3D flow problem arising in building systems flow estimation is considered.

II. BACKGROUND

In this paper, we consider the estimation of velocity fields in air flow in building systems. For the purpose of this study, we assume that the airflow velocity field dynamics are governed by the incompressible Navier-Stokes equation given by:

$$\begin{aligned} \mathrm{div}(u) &= 0 \\ \frac{\partial u}{\partial t} &= -(u \cdot \nabla)u + \nu \nabla^2 u - \nabla p \end{aligned} \tag{1}$$

where $u : \Omega \times \mathbb{R} \rightarrow \mathbb{R}^3$ represents the velocity field taking values over a spatial domain Ω, p is the pressure field, $\nu = 1/\mathrm{Re}$, Re is the Reynolds number. Here it is assumed that the velocity and pressure fields are defined on a closed-subset of $\mathbb{R}^3$ and take values on a normed space $\mathcal{V}$. This equation constitutes a scaled formulation of the Navier-Stokes equation where the velocities are scaled by a factor U, time by U/L, the viscosity by ρUL and the pressure by ρU^2 where ρ is the density, U and L are nominal velocities and length.

The incompressible flow assumption is justifiable for the set of conditions considered for the modeling of airflow in commercial buildings. We note that the approach described below can be directly applied to the compressible flow assumption (as demonstrated in Rowley et al.).

III. POD BASED MODEL REDUCTION

In POD based model reduction, the velocity field $u(t,x)$ is expressed as an expansion in the POD modes $\phi(x)$ defined on the spatial domain Ω. (Note that depending on the application, temporal models $\psi(t)$ may be more appropriate). The expansion is given as:

$$u(x,t) = \sum_{j=1}^{n} a_j(t)\phi_j(x) \tag{2}$$

In general, the decomposition is taken over a Hilbert space H, the space of smooth divergence-free vector-valued functions on Ω. The choice of inner product becomes a crucial aspect of the decomposition. In the incompressible flow approach however, the standard inner product

$$\langle u\ ,\ v\rangle = \int_{\Omega} u(x) \cdot v(x) dV \tag{3}$$

where $u(x) \cdot v(x)$ represents the standard dot product between vectors $u(x)$ and $v(x)$ in Euclidean space, dV is a volume element.

The basis of the technique described here is to restate the Navier-Stokes equation in terms of the modal decomposition (2). Assuming that $\mathrm{div}(\phi_i(x) = 0$ $(i = 1, \ldots, n)$, substitution of (2) in (1) yields

$$\begin{aligned} \sum_{i=1}^{n} \dot{a}_i(t)\phi_i(x) &= -(\sum_{j=1}^{n} a_j(t)\phi_j(x) \cdot \nabla) \sum_{k=1}^{n} a_k(t)\phi_k(x) \\ &\quad + \nu \sum_{i=1}^{n} a_i(t)\nabla^2\phi_i(x) - \nabla p \end{aligned} \tag{4}$$

Projecting onto the space of POD modes leads to,

$$\begin{aligned} &\langle \sum_{k=1}^{n} \dot{a}_k(t)\phi_k(x), \phi_i(x)\rangle = \\ &-\langle \left(\sum_{j=1}^{n} a_j(t)\phi_j(x) \cdot \nabla\right) \sum_{k=1}^{n} a_k(t)\phi_k(x), \phi_i(x)\rangle \\ &+\nu\langle \sum_{k=1}^{n} a_k(t)\nabla^2\phi_k(x), \phi_i(x)\rangle - \langle \nabla p, \phi_i(x)\rangle. \end{aligned} \tag{5}$$

By orthogonality of the modes

$$\langle \phi_i(x), \phi_j(x)\rangle = \left\{ \begin{array}{ll} 1 & i = j \\ 0 & i \neq j \end{array} \right. , \tag{6}$$

it follows that (5) reduces to the following set of ordinary differential equations:

$$\begin{aligned} &\dot{a}_i(t)\langle \phi_i(x), \phi_k(x)\rangle = \\ &-\sum_{i=1}^{n}\sum_{j=1}^{n} a_i(t)a_j(t)\langle (\phi_j(x) \cdot \nabla)\phi_i(x), \phi_k(x)\rangle \\ &+\nu \sum_{i=1}^{n} a_i(t)\langle \nabla^2\phi_i(x), \phi_k(x)\rangle - \langle \nabla p, \phi_k(x)\rangle \end{aligned} \tag{7}$$

which reduces to

$$\begin{aligned} \dot{a}_k(t) &= -\sum_{i=1}^{n}\sum_{j=1}^{n} a_i(t)a_j(t)\langle (\phi_j(x) \cdot \nabla)\phi_i(x), \phi_k(x)\rangle \\ &\quad + \nu \sum_{i=1}^{n} a_i(t)\langle \nabla^2\phi_i(x), \phi_k(x)\rangle - \langle \nabla p, \phi_k(x)\rangle. \end{aligned} \tag{8}$$

Equation (8) provides the basis for the design of the observer. Given a set of representative POD modes, the reduced-order system of ordinary differential equations constitutes a description of the fluid flow dynamics. Thus by building an observer for system (8), an (indirect) observer of the fluid flow velocity field $u(t,x)$ is obtained.

IV. POD MODES

The ability to reconstruct the velocity field $u(t,x)$ using estimates of the time-varying coefficients $a_i(t)$ depends completely on the choice of POD modes obtained.

Let $\mathcal{H}$ be a Hilbert space with inner product $\langle\ ,\ \rangle$. It is assumed that a data ensemble is given, $\{u_k \in \mathcal{H} | k = 1,\ldots,m\}$, which provides a representative sample of the system dynamics. In general, the ensemble $\{u_k\}$ is composed of a number of experiments designed to highlight various aspects of the process dynamics. These experiments are typically formed a set of snapshots of the velocity field u_k taken at specific times t_k.

These snapshots are usually obtained using CFD simulation of the process and the times t_k of the snapshots are designed to highlight various aspects of the system dynamics. The snapshots can also be obtained through experiments by placing sensors at predefined locations.

For the purpose of this study, the Hilbert space $\mathcal{H}$ is the set of functions defined on the spatial domain Ω where the fluid flows. (Here the spatial domain is the geometry of a room in a building system). Let $\mathcal{S}$ be a subspace of $\mathcal{H}$ of fixed dimension $n < m$. The projection of any element u_k of $\mathcal{H}$ is given by $P_{\mathcal{S}}u_k$ where $P_{\mathcal{S}}$ is the orthogonal projection operator. The objective of proper orthogonal decomposition is to find a subspace $\mathcal{S}$ of fixed dimension $n < m$ such that error $E(\|u_k - P_{\mathcal{S}}u_k\|)$ is minimized, where $\|\cdot\|$ is the induced norm on $\mathcal{H}$ and $E(\cdot)$ is the expectation operator.

The solution of the optimization problem leads to the eigenvalue problem (see [1] for a detailed development),

$$R\phi = \lambda\phi \tag{9}$$

where $R : \mathcal{H} \to \mathcal{H}$ is the linear operator given by

$$R = E(u_k \otimes u_k^*) \tag{10}$$

where $u^* \in \mathcal{H}^*$ is the adjoint (dual) of a $u \in \mathcal{H}$ and $\mathcal{H}^*$ is the space if functionals $u^*(\cdot) = \langle \cdot, u\rangle$. The operation $\otimes$ represents the standard tensor product. Thus for any u, v and w in $\mathcal{H}$, $(u \otimes v^*)(w) = u\langle w, v\rangle$.

In practice, the snapshots u_k are a sampled value of the nvelocity field $u(x,t_k)$ at time t_k evaluated at a finite number, N, of location x_i ($i = 1,\ldots,N$) over the spatial domain Ω. Since, in general, the number of spatial locations N is large, the corresponding spectral decomposition problem can become prohibitively complex.

As an alternative, one can compute the POD modes using the method of snapshots. Starting with an ensemble $\{u_k\}$ (with $k = 1,\ldots,m$ where $m << N$), the POD modes are taken as linear combinations of the elements of the ensemble. That is,

$$\phi(x) = \sum_{k=1}^{m} c_k u_k. \tag{11}$$

Note that this choice is not arbitrary since elements of the range of the linear operator R are by construction in the span of the ensemble $\{u_k\}$.

Rewriting the eigenvalue problem yields

$$Uc = \lambda c \tag{12}$$

where U is a m by m matrix with elements $U_{ij} = \frac{1}{m}\langle u_i, u_j\rangle$. (Thus the problem is reduced to an m dimensional eigenvalue problem. Note that this is true even when the original problem is infinite dimensional.)

In this study, the method of snapshots was used to extract the POD modes from a CFD simulation that yield snapshots over a detailed grid over a small number of time instants.

V. OBSERVER DESIGN

A. Reduced Order Nonlinear System

The dynamical system (8 yields a set of quadratic differential equations of the form:

$$\dot{a}_k(t) = L_k a(t) + a(t)^T Q_k a(t) \tag{13}$$

where L_k are is a row vector with elements are given by

$$L_{ik} = \langle \nabla^2\phi_i(x), \phi_k(x)\rangle \tag{14}$$

and Q_k is an n by n matrix with elements

$$Q_{ijk} = \langle(\phi_j(x)\cdot\nabla)\phi_i(x), \phi_k(x)\rangle \tag{15}$$

for $k = 1,\ldots,n$. In general, the pressure term is ignored. This can be justified as follows.

Since the POD modes are such that $\text{div}(\phi) = 0$, it follows that

$$\begin{aligned}\int_\Omega \nabla p \cdot \phi_k(x) dV &= \int_\Omega div(p\phi_k(x))dV \\ &= \int_{\partial\Omega} p\phi_k(x)\cdot \mathbf{n}_\Omega dS \end{aligned} \tag{16}$$

where $\mathbf{n}_\Omega$ represents the unit vector normal to the spatial domain Ω. Hence, the pressure term will vanish altogether over a closed domain ($\phi_k(x) = 0$ on the boundary of Ω, $\partial\Omega$).

B. Measurements

It is assumed that several measurements are available. If one assumes that p velocity field measurements are available at p predefined locations, these measurements must first be expressed in terms of the modal decomposition. For example, if one measures the average velocity, $u_avg(t,x_0) = (u(t,x_0) + v(t,x_0) + w(t,x_0))$, at a point x_o, then the corresponding measurement becomes

$$u_{\text{avg}}(t,x_0) = \sum_{i=1}^{n} a_i(t)(\phi_i^1(x) + \phi_i^2(x) + \phi_i^3(x)) \tag{17}$$

where $\phi_i^j(x)$ represents the j^{th} element of the i^{th} POD mode. Since the POD modes are time independent, the resulting output map can be written in the form

$$u_avg(t,x_0) = Ca_(t) \tag{18}$$

where C is a 1 by n matrix and $a(t)$ is the n-dimensional vector of time varying coefficients of the Galerkin approximation of $u(t, x)$. In general, the output map will be written as

$$y(t) = Ca(t)$$

which is constructed by expressing the measured quantity using the POD modes.

If pressure is measured at a point x_0, the pressure can be expressed in modal form as follows:

$$p(t, x_0) = \sum_{i=1}^{n} a_i(t) p_i(x) \tag{19}$$

where $p_i(x_0)$ is the pressure associated to mode i $\phi_i(x_0)$.

Assuming that the available measurements include velocity measurements and pressure measurements, the complete dynamical system considered in this study takes the form:

$$\begin{aligned} \dot{a}_k(t) &= L_k a(t) + a(t)^T Q_k a(t)\ ,\ k = 1, \ldots, n \\ y(t) &= Ca(t) \end{aligned} \tag{20}$$

The objective of this study is to consider the design of an observer for the system (20). Assuming that the POD modes provide an accurate description of the features of the flow field, the estimation of the Galerkin coefficients $a_i(t)$ yields an estimate of the velocity field using the expression (2). Note that if the initial conditions $a_i(0)$ are known, then the predictions of the dynamical system (20) will be in agreement with actual value $a_i(t)$ and a good estimate of the flow field should result. However, the initial conditions are generally not known and the value of $a_i(t)$ must be replaced by an estimate $\hat{a}_i(t)$. n addition, the flow system is subject to uncertainties and disturbances that must be filtered in some way.

Given the dynamical system 20 and assuming that the system is observable, one can rely on a number of potential approaches to provide estimates of the Galerkin coefficients. In addition, the flow system is subject to uncertainties and disturbances that must be filtered in some way. The use of an observer for the estimation of the Galerkin coefficients was proposed in [5] for the design of an feedback control scheme. In order to reduce the complexity of the observer design, only the linear approximation of (20) was considered. It is clear that a nonlinear observer approach would provide improvement in the performance of the observer. This aspect of the problem is treated in this study.

Alternative POD-based estimation have been proposed in the literature. One technique of potential interest is described in the next section.

C. Observer design

In this section, we discuss the design of a suitable observer for the dynamical system. The linear approximation of (20) about $a_i(0) = 0$ is given by:

$$\begin{aligned} \dot{a}_k(t) &= L_k a(t),\ k = 1, \ldots, n \\ y(t) &= Ca(t) \end{aligned} \tag{21}$$

In this case, a linear observer can be designed of the form:

$$\dot{\hat{a}}(t) = L\hat{a}(t) + K(y(t) - C\hat{a}(t)),\ k = 1, \ldots, n \tag{22}$$

In [12], the performance of different observers for varying number of POD modes. As expected, observers based that use larger numbers of modes tend to outperform simpler observers. The choice of POD modes is also shown to be important. POD modes based on the method of snapshots are shown to provide rather poor observers. POD modes based on a balanced truncation [4] are argued to provide superior performance. However, the results remain biased by the potential detrimental affect of the quadratic term on the estimation of the $a_i(t)$. The application of more suitable nonlinear filters is therefore of interest in this case.

In this study, we consider the application of the extended Kalman filter for the estimation of the Galerkin coefficients. We also consider the application of numerical observer. The subject of Kalman filtering is well known. We refer the reader to the wide literature available on the subject. The extended Kalman filter yields a design for the nonlinear system (20) which is based on a time-varying linear approximation of the nonlinear system. The performance and limitations of EKF are well documented.

A numerical observer for nonlinear systems was proposed in ([6]). In this formulation, state estimates are chosen to minimize the least-squares error between the past measured outputs and the past predicted outputs. Formally, the observer is based on the following optimization problem:

$$\begin{array}{ll} \min_{\hat{a}(t)} & J(\hat{a}_t) \int_{t-T}^{t} \|y(\tau) - Ca_p(\tau)\| d\tau \\ \text{subject to} & \\ & a^p(t) = \hat{a}(t) \\ & \dot{a}_k^p(t) = L_k a^p(t) + a^p(t)^T Q_k a^p(t) \\ & y^p(t) = Ca^p(t)\ ,\ k = 1, \ldots, n \end{array} \tag{23}$$

The estimate $\hat{a}(t)$ is given as the minimizer of this problem. A real-time optimization version of this observer, given in [6], consists of evaluating the gradient of J with respect to $\hat{a}(t)$, $\nabla_{\hat{a}}(t)$ and to use a steepest descent update to obtain the following numerical observer:

$$\dot{\hat{a}}(t) = -\Gamma \nabla_{\hat{a}}(t) J(\hat{a}(t)) \tag{24}$$

where Γ is a positive definite matrix which acts as an estimate of the Hessian of J with respect to $\hat{a}(t)$, $\nabla^2_{\hat{a}(t)} J(\hat{a}(t))$.

For the purpose of this study, we have considered the application of the EKF for the estimation of the Galerkin projection coefficients $a(t)$ based on the nonlinear system (20). The next section summarizes some results on the application of the POD based observer for the estimation of airflow velocity field in a 2D and a 3D environment.

VI. Results

A. 2D Room Case Study

In the first case study, we consider the application of the POD based approach for the estimation of the velocity in a two-dimensional room. The room is assumed to be 8 ft by 16 ft. It contains one inlet flow of ambient air and one outlet flow. A single window is also added. A far field approximation is used to simulate the effect of outdoor flow disturbances when the window is open. The diagram also depicts the mesh used in the CFD simulation. The velocity is assumed to be measured at a single measurement location at an airflow return outlet.

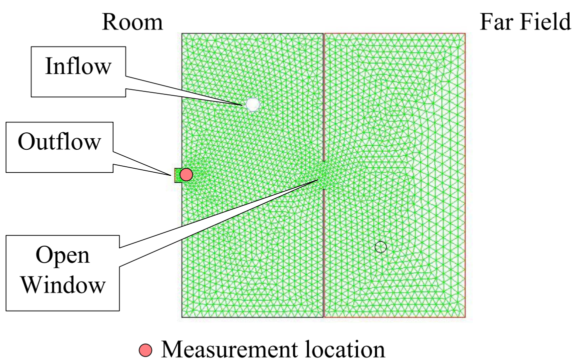

Fig. 1. Geometry of the two-dimensional room.

A nominal inlet flow of 0.1 m/sec is considered for the generation of the snapshots. The snapshots are generated to capture changes in inlet flowrate and the opening and closing of the outlet to the far field. Starting with a flowrate of 0.1 m/sec with outlet closed, the flowrate is then changed to 0.5 m/sec. The outlet is then opened and the flow is reduced from 0.5 to 09.1 m/sec. The corresponding CFD simulation are monitored at every 5 seconds until equilibrium is reached after each change. The POD modes are generated using the method of snapshots. Two modes are retained which explain 99% of the variation.

The Galerkin projection on the POD modes yields a two dimensional dynamical system. An extended Kalman filter was used as the observer in this case. The dynamical equation was first discretized using a sampling time of 5 seconds.

A transient CFD simulation is then used to demonstrate the applicability of the observer. Starting from the equilibrium is obtained for the equilibrium velocity field at an inlet flow of 0.2 m/sec, the inlet flow is decreased at time $t = 0$ to 0.6 m/sec. The outlet, initially closed is then opened halfway through the simulation. The initial values for the estimates of the Galerkin projection coefficients are $\hat{a}_1(0) = 0$ and $\hat{a}_2(0) = 0$. The estimates of $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are shown in Figure 2.

Figures 3 show a sample comparison of the velocity field from CFD calculations and the estimated velocity field. Results demonstrate the effectiveness of the technique. The performance of the estimator is summarized in Figure 4 where the total mean squared error in the estimate of the

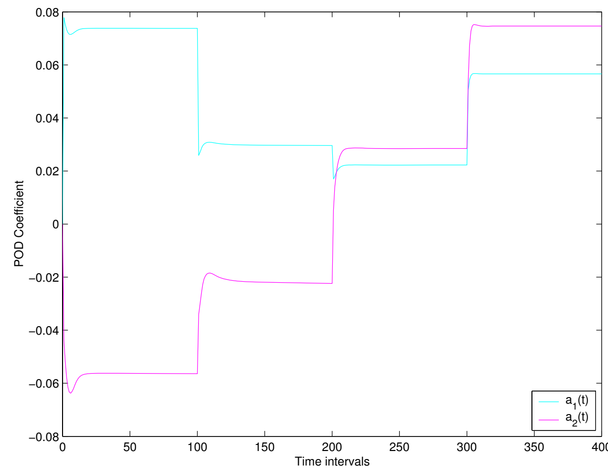

Fig. 2. Estimated Galerkin projection coefficients for the 2D Room example. The data shows the four different modes of operations resulting from opening/closing the window and changing the inlet flowrate.

velocity. The peaks in the mean squared error show the times when the conditions are changed by modifying the flowrate or closing/opening the outlet flow to the far field.

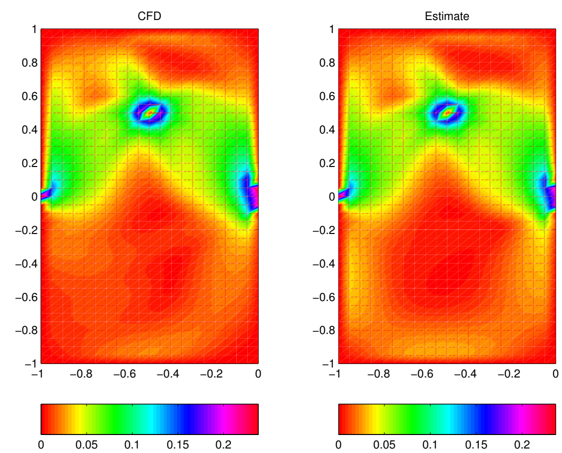

Fig. 3. Mean squared velocity field estimation results for the 2D room at t=250 sec. Thee actual velocity field is shown on the left and the estimated velocity field on the right.

B. 3D Room Case Study

In order to demonstrate the applicability of the technique in a 3D flow environment, a CFD simulation of the air flow of the lobby of a two story building is considered. The room is shown in Figure 5.

The room is connected to the rest of the building through four corridors. The doors are presented as dark shaded areas. The lobby contains one entrance and one back door. For the purpose of this study, only the main entrance door is used.

A CFD simulation of the room was developed. Several snapshots were captured using two different flowrates with the main entrance door open and closed. In this case, 3 POD modes were shown to express more that 97% of the

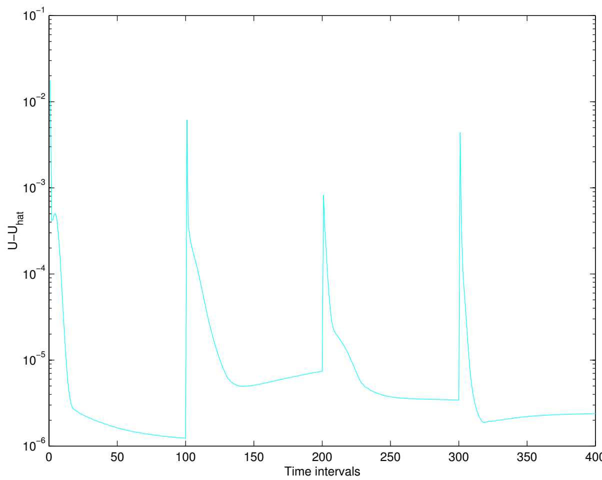

Fig. 4. Total mean squared error in the estimate of the velocity field for the 2D room example.

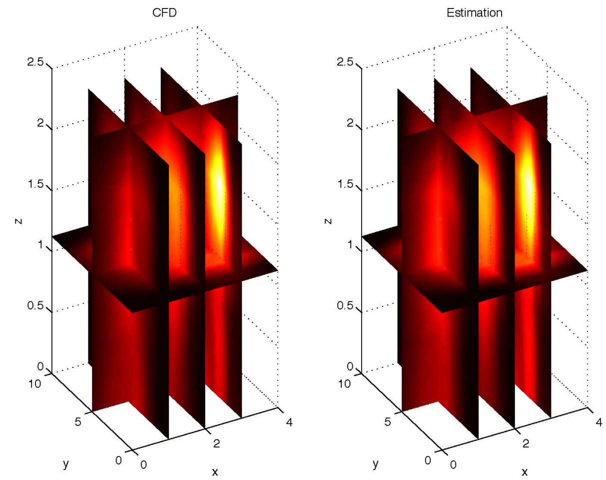

Fig. 6. Simulation results for the 3D room case study. Shown are six slices of the mean squared velocity field for the CFD and the estimated velocity field.

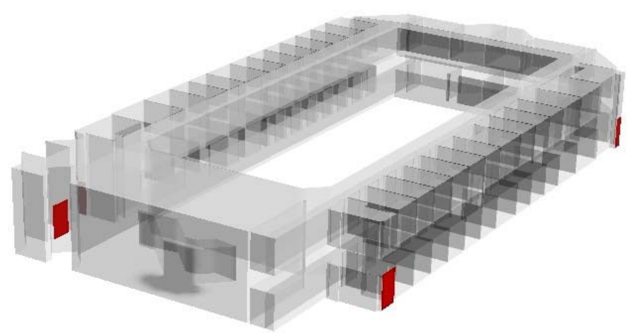

Fig. 5. Schematic of the 3D room case study. The area of interest is the large room located in the forefront of the two-story building.

variation. A simple 3 dimensional reduced-order model was developed as prescribed above. One velocity sensor located at the main entrance was assumed to be available. A second measurement located on the 1st floor near the back-door was used. A transient simulation was performed using the CFD simulation model and two different flowrates where the main entrance door was opened and closed. Figure 6 shows the comparison between the CFD simulation and the estimated velocity field using the two-dimensional reduced order model. As in the 2D case, the approach provides a very effective and reliable estimation of the entire velocity field. The results demonstrate the ability of the estimator to reconstruct the 3D velocity field from limited measurements.

VII. Conclusions

A POD-based observer design method is developed for the estimation of velocity field from the 2D and 3D Navier-Stokes flow. In building systems, the POD-based approach provides very simple low-order representation of the flow that are both accurate and reliable. Further work will be focussed on the incorporation of contamination flow estimation for large building systems.

VIII. Acknowledgements

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