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Arbitrarily small damping allows global output feedback tracking of a class of Euler-Lagrange systems.

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Abstract— **This paper proposes a new analysis technique called "ISS regulator approach" to show that a simple causal PD controller plus feedforward using only position measurements solves the global output feedback tracking control problem of robot manipulators with arbitrarily small damping. To this end, we first show that a causal PD regulator leads to a global input-to-state stable system with respect to a bounded input disturbance. Then, using this fact we prove that the addition of a feedforward compensation renders the overall error system uniformly globally asymptotically stable. In addition, we present a possible extension of the proposed method to other classes of Euler-Lagrange systems.**

I. INTRODUCTION

The design of a global output feedback tracking controller for robot manipulators has attracted the attention of the robotics community for many years. The pioneering works [1], [2], [3] have shown that global regulation can be guaranteed without using joint velocities. Since then, several authors have tried to derive similar output feedback controllers for the tracking problem. Unfortunately, most of them have been limited to local or semi-global results (see [4] for a literature review).

In [5], Loria developed a model-based controller that renders the one degree-of-freedom (DOF) Euler-Lagrange (EL) systems uniformly globally asymptotically stable. Unfortunately, this approach could not be extended to the general n-DOF case. To address this issue Zhang et al. proposed in [6] an output feedback adaptive controller composed by a feedforward term plus a nonlinear feedback term coupled to a dynamic nonlinear filter. This controller produces global (in the tracking initial errors) asymptotic link position tracking.

Recently, closer results to global stability were achieved in [7], [8], [9], where the initial conditions of the dynamic extensions must belong to a constrained set. In [7], using a new dynamic-kinematic model for EL-systems, which is linear in the unmeasurable velocities, a model-based controller was proposed. In [8], exploiting a separation result related with some stabilizability by state feedback and some detectability property, a model-based dynamic controller was proposed for EL-systems. In [9], a robust controller, which resembles the one presented in [6], was proposed.

On the other hand, exploiting the robot natural damping, global stability of output feedback tracking controllers were proven in [10], [11]. However, the results are guaranteed only if large enough viscous friction is present in the robot joints.

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In this paper, we show that the well known causal PD controller with feedforward compensation can provide global tracking, under the only requirement of the existence of the robot natural damping, no matter how small, which seems to be a quite realistic assumption. To this end, we propose a new method called "ISS Regulator Approach" which consists in first proving that the robot controlled by a causal PD regulator is globally input-to-state stable (ISS) [12] with respect to a bounded input disturbance and then showing that such causal PD controller plus a feedforward compensation renders the overall error system uniformly globally asymptotically stable. In addition, we suggest extensions of the proposed analysis technique to deal with uncertain robot manipulators and to consider a broader class of nonlinear systems that encompasses other classes of EL systems.

II. PRELIMINARES

A. Notation and Basic Concepts

In what follows, all κ's denote positive constants. $|\cdot|$ stands for the Euclidean norm for vectors, or the induced matrix norm for matrices. $\lambda_M(\cdot)$ ($\lambda_m(\cdot)$) denotes the largest (smallest) eigenvalue of a matrix. For any measurable function $u : [t_0, \infty) \to \mathbb{R}^m$, $\|u\|$ denotes ess $\sup\{|u(t)|, t \geq t_0\}$. Classes $\mathcal{K}$, $\mathcal{K}_\infty$, $\mathcal{KL}$ functions are defined as usual [13].

B. Basic Definitions

Definition 1: The system $\dot{x} = f(t, x)$ is said to be uniformly globally asymptotically practically stable (UGApS), if there exist $\beta \in \mathcal{KL}$ and a nonnegative constant R, such that for all $t_0 \geq 0$, $x(t_0)$ and $t \geq t_0$

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0) + R \tag{1}$$

When (1) is satisfied with $R = 0$, system $\dot{x} = f(t, x)$ is said to be uniformly globally asymptotically stable (UGAS).

Definition 2: The system $\dot{x} = f(x, u)$ is said to be input-to-state stable (ISS), if there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that for all $x(t_0)$, $u \in \mathcal{L}_\infty$ and $t \geq t_0 \geq 0$

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0) + \gamma(\|u\|) \tag{2}$$

Definition 3: A continuous function $V : \mathbb{R}^n \to \mathbb{R}$ is a storage function if there exist $\underline{\alpha}$, $\overline{\alpha} \in \mathcal{K}_\infty$ such that $\underline{\alpha}(|x|) \leq V(x) \leq \overline{\alpha}(|x|)$, $\forall x \in \mathbb{R}^n$ (V is positive definite and proper).

Definition 4: A smooth storage function $V : \mathbb{R}^n \to \mathbb{R}$ is an ISS-Lyapunov function [13] for system $\dot{x} = f(x, u)$, if there exist $\alpha \in \mathcal{K}_\infty$ and $\sigma \in \mathcal{K}_\infty$, such that for all x, u

$$\dot{V}(x) \leq -\alpha(|x|) + \sigma(|u|) \tag{3}$$

The existence of an ISS-Lyapunov function is an equivalent condition for ISS [13].

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III. Dynamic Model

The dynamic model for an n-DOF rigid robot with revolute joints can be described by [14]:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+\zeta F_v\dot{q}+g(q)=\tau \tag{4}$$

where $q(t),\ \dot{q}(t),\ \ddot{q}(t)\in\mathbb{R}^n$ denote the joint position, velocity and acceleration, respectively; $M(q)\in\mathbb{R}^{n\times n}$ is the symmetric positive definite inertia matrix; $C(q,\dot{q})\dot{q}\in\mathbb{R}^n$ is the vector of Coriolis and centrifugal torques; ζ is a generic positive constant, and $\zeta F_v\in\mathbb{R}^{n\times n}$ denotes the constant, diagonal and positive definite matrix of viscous friction; $g(q)\in\mathbb{R}^n$ is the vector of gravitational torques; and $\tau\in\mathbb{R}^n$ is the vector of torques acting at the joints. The centrifugal-Coriolis matrix is defined using the Christoffel symbols.

The dynamic system (4) exhibits the following properties (see e.g. [4], [14], [15]):

(P1) $\lambda_m(M)\left|x\right|^2\le x^TM(q)x\le\lambda_M(M)\left|x\right|^2,\forall x\in\mathbb{R}^n$, where $\lambda_m(M):=\min_{q\in\mathbb{R}^n}\lambda_m(M(q))$ and $\lambda_M(M):=\max_{q\in\mathbb{R}^n}\lambda_M(M(q))$;
(P2) $\left|M(x)z-M(y)z\right|\le c_M\left|x-y\right|\left|z\right|,\ \forall x,y,z\in\mathbb{R}^n$;
(P3) $\dot{M}(q)=C(q,\dot{q})+C^T(q,\dot{q}),\ \forall q,\ \dot{q}\in\mathbb{R}^n$;
(P4) $x^T\left(\frac{1}{2}\dot{M}(q)-C(q,\dot{q})\right)x=0,\ \forall x\in\mathbb{R}^n$;
(P5) $\left|C(x,z)w-C(y,v)w\right|\le c_1\left|z-v\right|\left|w\right|+c_2\left|z\right|\left|x-y\right|\left|w\right|$, $\forall x,y,z,v,w\in\mathbb{R}^n$
(P6) $Y(q,\dot{q},\ddot{q})\theta=M(q)\ddot{q}+C(q,\dot{q})\dot{q}+\zeta F_v\dot{q}=\bar{\tau}$, where $Y(q,\dot{q},\ddot{q})\in\mathbb{R}^{n\times l}$ is the regression matrix, $\theta\in\mathbb{R}^l$ is a constant vector of parameters, and $\bar{\tau}=\tau-g(q)$;
(P7) $\left|C(q,\dot{q})\right|\le c_1\left|\dot{q}\right|,\left|g(q)\right|\le c_3,\left|\theta\right|\le c_4$.

The constants c_M, c_1, c_2 are defined in [15].

IV. Revisiting the regulation problem using only position measurements

In this section, we consider the problem of global output regulation to a desired constant set point q_r, using only position measurements. The objective is to show that the robot controlled by a causal PD with gravity compensation is ISS with respect to a bounded input disturbance and to ensure that, in the absence of the input disturbance, the regulation error $\tilde{q}:=q-q_r$ tends asymptotically to zero.

Since it is assumed that only joint position measurements are available, the joint velocities could be estimated by means of a lead filter described by:

$$\dot{\vartheta}=-\frac{1}{\mu}\vartheta-\frac{1}{\mu^2}q,\quad \hat{\nu}=\vartheta+\frac{1}{\mu}q \tag{5}$$

where μ is a generic positive constant. However, when enhanced precision is required μ should be made small enough (c.f. Section V).

Considering that system (4) is perturbed with a bounded input disturbance $d(t)$, the causal PD regulator with gravity compensation is given by:

$$\tau=-K_p\tilde{q}-K_d\hat{\nu}+g(q)+d(t) \tag{6}$$

where K_p and K_d are symmetric positive definite matrices.

A. Stability Analysis

In order to take into account the possibility of a small natural damping, we consider that ζ may be an arbitrarily small parameter. Defining the state of the closed-loop system (4)(5)(6) as $x^T:=\left[\tilde{q}^T\ \dot{q}^T\ \hat{\nu}^T\right]$, one has:

$$\frac{d}{dt}\begin{bmatrix}\tilde{q}\\ \dot{q}\\ \hat{\nu}\end{bmatrix}=\begin{bmatrix}\dot{q}\\ M(q)^{-1}[d-C(q,\dot{q})\dot{q}-\zeta F_v\dot{q}-K_p\tilde{q}-K_d\hat{\nu}]\\ -\frac{\hat{\nu}}{\mu}+\frac{\dot{q}}{\mu}\end{bmatrix} \tag{7}$$

Consider the following ISS-Lyapunov function candidate:

$$V_1(x)=\frac{1}{2}\dot{q}^TM(q)\dot{q}+\frac{1}{2}\tilde{q}^TK_p\tilde{q}+\frac{1}{2}\mu\hat{\nu}^TK_d\hat{\nu}+\varepsilon_1\frac{\tilde{q}^TM(q)\dot{q}}{\sqrt{1+\tilde{q}^T\tilde{q}}} \tag{8}$$

Proposition 1: If ε_1 satisfies (12), then $V_1(x)$ is a smooth storage function and can be upper and lower bounded by:

$$\underline{\alpha}_1(|x|)=\kappa_1\left|x\right|^2\le V_1(x)\le\kappa_2\left|x\right|^2=\overline{\alpha}_1(|x|) \tag{9}$$

$$\kappa_1\le\frac{1}{4}\min\{\lambda_m(M),\lambda_m(K_p),2\mu\lambda_m(K_d)\}; \tag{10}$$

$$\kappa_2\ge\frac{\lambda_M(P_1)}{2},\ P_1=\begin{bmatrix}\lambda_M(M) & \varepsilon_1\lambda_M(M) & 0\\ \varepsilon_1\lambda_M(M) & \lambda_M(K_p) & 0\\ 0 & 0 & \mu\lambda_M(K_d)\end{bmatrix} \tag{11}$$

$$\varepsilon_1\le\frac{\sqrt{\lambda_m(M)\lambda_m(K_p)}}{2\lambda_M(M)} \tag{12}$$

Proof: see Appendix A ■

Deriving V_1 with respect to time, it follows that

$$\begin{aligned}\dot{V}_1 \;=\; & \dot{q}^Td-\zeta\dot{q}^TF_v\dot{q}-\hat{\nu}^TK_d\hat{\nu}+\varepsilon_1\frac{\dot{q}^TM(q)\dot{q}}{\sqrt{1+\tilde{q}^T\tilde{q}}}\\ & +\varepsilon_1\left(\frac{\tilde{q}^T\dot{M}(q)\dot{q}}{\sqrt{1+\tilde{q}^T\tilde{q}}}+\frac{\tilde{q}^Td}{\sqrt{1+\tilde{q}^T\tilde{q}}}-\frac{\tilde{q}^T\left(C(q,\dot{q})+\zeta F_v\right)\dot{q}}{\sqrt{1+\tilde{q}^T\tilde{q}}}\right)\\ & -\varepsilon_1\left(\frac{\tilde{q}^TK_p\tilde{q}}{\sqrt{1+\tilde{q}^T\tilde{q}}}+\frac{\tilde{q}^TK_d\hat{\nu}}{\sqrt{1+\tilde{q}^T\tilde{q}}}+\frac{(\tilde{q}^TM(q)\dot{q})(\tilde{q}^T\dot{q})}{(1+\tilde{q}^T\tilde{q})^{3/2}}\right)\end{aligned}$$

where Property **P4** was used. From Properties **P1**, **P3** and **P7**, one can conclude that:

$$\begin{aligned}\dot{V}_1 \;\le\; & -\tfrac{1}{2}\zeta\dot{q}^TF_v\dot{q}-\tfrac{1}{2}\hat{\nu}^TK_d\hat{\nu}-\varepsilon_1\tfrac{\tilde{q}^TK_p\tilde{q}}{\sqrt{1+\tilde{q}^T\tilde{q}}}+\varepsilon_1\tfrac{\left|\tilde{q}\right|}{\sqrt{1+\tilde{q}^T\tilde{q}}}\left|d\right|\\ & +\varepsilon_1\left(\tfrac{\lambda_M(M)+c_1\left|\tilde{q}\right|}{\sqrt{1+\tilde{q}^T\tilde{q}}}+\tfrac{\lambda_M(M)\left|\tilde{q}\right|^2}{(1+\tilde{q}^T\tilde{q})^{3/2}}\right)\left|\dot{q}\right|^2\\ & -\left[\tfrac{1}{4}\zeta\lambda_m(F_v)\left|\dot{q}\right|^2-\varepsilon_1\tfrac{\zeta\lambda_M(F_v)\left|\tilde{q}\right|}{\sqrt{1+\tilde{q}^T\tilde{q}}}\left|\dot{q}\right|\right]\\ & -\left[\tfrac{1}{2}\lambda_m(K_d)\left|\hat{\nu}\right|^2-\varepsilon_1\tfrac{\lambda_M(K_d)\left|\tilde{q}\right|}{\sqrt{1+\tilde{q}^T\tilde{q}}}\left|\hat{\nu}\right|\right]\\ & -\left[\tfrac{1}{4}\zeta\lambda_m(F_v)\left|\dot{q}\right|^2-\left|d\right|\left|\dot{q}\right|\right]\end{aligned}$$

After completing the squares on the bracketed terms and since $\left|\tilde{q}\right|/\sqrt{1+\left|\tilde{q}\right|^2}\le 1,\ \left|\tilde{q}\right|^2/\left(1+\left|\tilde{q}\right|^2\right)^{3/2}\le 1,\forall\tilde{q}\in\mathbb{R}^n$, it can be verified that for a sufficiently small ε_1, one has:

$$\dot{V}_1\le-\frac{1}{4}\zeta\dot{q}^TF_v\dot{q}-\frac{1}{2}\hat{\nu}^TK_d\hat{\nu}-\frac{1}{2}\varepsilon_1\frac{\tilde{q}^TK_p\tilde{q}}{\sqrt{1+\tilde{q}^T\tilde{q}}}+\kappa_3\left(\left|d\right|+\left|d\right|^2\right)$$

where: $$\kappa_3 = \max\left\{\varepsilon_1, \frac{1}{\zeta\lambda_m(F_v)}\right\}; \quad (13)$$

$$\varepsilon_1 < \min\left\{\begin{array}{c}\dfrac{\lambda_m(K_p)\lambda_m(F_v)\lambda_m(K_d)}{2\zeta\lambda_M^2(F_v)\lambda_m(K_d)+\lambda_M^2(K_d)\lambda_m(F_v)},\\ \dfrac{\zeta\lambda_m(F_v)}{4(2\lambda_M(M)+c_1)}\end{array}\right\} \quad (14)$$

The function $\dot{V}_1$ can be further upper bounded as follows:

$$\dot{V}_1(x) \leq -\alpha_1(|x|) + \sigma_1(|d|) \quad (15)$$

where $\alpha_1(r) = \kappa_4 r^2/\sqrt{1+r^2} \in \mathcal{K}_\infty$, with

$$\kappa_4 \leq \frac{1}{2}\min\left\{\frac{1}{2}\zeta\lambda_m(F_v), \lambda_m(K_d), \varepsilon_1\lambda_m(K_p)\right\} \quad (16)$$

and $\sigma_1(r) = \kappa_3(r+r^2) \in \mathcal{K}_\infty$. Thus, if ε_1 is chosen such (12) and (14) hold, then, according to Definition 4, $V_1(x)$ is an ISS-Lyapunov function for system (4)(5)(6), which implies that the closed-loop system is ISS with respect to $d(t)$.

We summarize the results in the following Theorem.

Theorem 1: Consider the robot system described by (4). If the control law is defined as in (5)(6), then the closed-loop system with state $x = \left[\tilde{q}^T \ \dot{q}^T \ \hat{\nu}^T\right]^T$ is globally ISS with respect to a bounded input disturbance $d(t)$. Moreover, if $d(t) \equiv 0$, then x tend asymptotically to zero. ■

Remark 1: Since $g(q)$ is bounded by a constant, the gravity compensation term is not relevant to conclude that a robot controlled by a causal PD is ISS.

V. Global output tracking using only position measurements

In this section, the global output tracking problem of robot manipulators with dynamic model described by (4) is considered. It is assumed that only position measurements are available. The tracking error $e(t) \in \mathbb{R}^n$ is defined as:

$$e(t) = q(t) - q_d(t) \quad (17)$$

where $q_d(t)$ is the desired trajectory. The signals q_d, $\dot{q}_d$, $\ddot{q}_d$ are assumed to be continuous and bounded by $|q_d|_M$, $|\dot{q}_d|_M$ and $|\ddot{q}_d|_M$, respectively. The objective is to design a control law such that the tracking error tends asymptotically to zero. To simplify the controller design and analysis the following assumption is made:

Assumption 1: The robot dynamic model (4) is assumed as being known, which means that the constant parameter vector θ, presented in Property **P6**, is known.

To solve the tracking problem the following feedforward compensation is added to the control signal:

$$Y(q_d, \dot{q}_d, \ddot{q}_d)\theta = M(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + \zeta F_v \dot{q}_d$$

As in [16], [15], [9], the regression matrix $Y_d = Y(q_d, \dot{q}_d, \ddot{q}_d)$ is a function of the desired trajectory signals.

The signal $\dot{e}$ can be estimated by the following lead filter:

$$\dot{\vartheta} = -\frac{1}{\mu}\vartheta - \frac{1}{\mu^2}q - \frac{1}{\mu}\dot{q}_d, \quad \hat{\nu}_e = \vartheta + \frac{1}{\mu}q \quad (18)$$

The control law is designed as follows:

$$\tau = -K_p e - K_d\hat{\nu}_e + g(q) + Y_d\theta \quad (19)$$

A. Stability Analysis

From (18), it is possible to conclude that:

$$\hat{\nu}_e = \hat{\nu} + \hat{\nu}_d \quad (20)$$

where $\hat{\nu}$ is defined in (5) and $\hat{\nu}_d$ corresponds to the output of (18) with $q \equiv 0$. Thus, the control law defined in (19) can be rewritten as:

$$\tau = -K_p q - K_d\hat{\nu} + g(q) + K_p q_d - K_d\hat{\nu}_d + Y_d\theta \quad (21)$$

Note that (21) is equivalent to (6) with $\tilde{q} = q$ ($q_r = 0$) and $d = K_p q_d - K_d\hat{\nu}_d + Y_d\theta$. Since $\|\hat{\nu}_d(t)\| \leq Ke^{-at} + \|\dot{q}_d(t)\|$, for some positive scalars a, K and $\forall t$, the following upper bound for $d(t)$ that is independent of μ can be derived

$$\begin{aligned}|d(t)| \leq{}& \lambda_M(K_p)|q_d|_M + \lambda_M(K_d)(|\dot{q}_d|_M + K) + c_1|\dot{q}_d|_M^2 \\ &+ \lambda_M(M)|\ddot{q}_d|_M + \zeta\lambda_M(F_v)|\dot{q}_d|_M \leq C_d\end{aligned} \quad (22)$$

According to Theorem 1, the system (4)(18)(19) with state $x = \left[q^T \ \dot{q}^T \ \hat{\nu}^T\right]^T$ is UGApS. From (15) and (22), it follows that $\dot{V}_1(x) < 0$, if $|x| > \alpha_1^{-1} \circ \sigma_1(C_d)$. Therefore, $\dot{V}_1(x)$ is negative outside a ball of radius $R := \alpha_1^{-1} \circ \sigma_1(C_d)$. Thus, selecting a Lyapunov surface $V_1(x) = \overline{\alpha}_1 \circ \alpha_1^{-1} \circ \sigma_1(C_d) := C_R$ such that $B_R := \left\{x \in \mathbb{R}^{3n}; |x| \leq R\right\}$ is in the interior of the set $D_R := \left\{x \in \mathbb{R}^{3n}; V_1(x) \leq C_R\right\}$, one can conclude that the state x globally converges to the compact and invariant set D_R in a finite time T_D.

Remark 2: From (11), (13), (16), (22) and since μ is a sufficiently small parameter the constant C_R is independent of μ. Actually, the value of this constant is determined by the robot parameters and the desired trajectory signals, being, $O(1/\zeta^2)$, where ζ may be a small parameter related to the robot natural damping.

From (8), it is possible to show that

$$V_1(x) \geq \frac{1}{4}\lambda_m(K_p)|q|^2 + \frac{1}{4}\lambda_m(M)|\dot{q}|^2 + \frac{1}{2}\mu\lambda_m(K_d)|\hat{\nu}|^2$$

Within D_R the following upper bounds can be established:

$$|q(t)| \leq \sqrt{\frac{4C_R}{\lambda_m(K_p)}}, \quad |\dot{q}(t)| \leq \sqrt{\frac{4C_R}{\lambda_m(M)}}, \forall t \geq T_D \quad (23)$$

$$|\hat{\nu}(t)| \leq \sqrt{\frac{2C_R}{\mu\lambda_m(K_d)}}, \forall t \geq T_D \quad (24)$$

In order to improve the tracking performance μ should be chosen sufficiently small. However, as can be seen in (24), this leads to the, generally called, *peaking phenomena*, which consists of large peak amplitudes in the estimation variable $\hat{\nu}$ during the initial transient. Fortunately, this phenomena has a short duration allowing us to find an upper bound for $\hat{\nu}$ that is independent of μ, after some short finite time interval.

Indeed, from (7) the following upper bound for $\hat{\nu}$ (independent of μ) valid for all $t \geq T_D + T_p$ can be derived:

$$|\hat{\nu}(t)| \leq \sqrt{\frac{2C_R}{\lambda_m(K_d)}} + \sqrt{\frac{4C_R}{\lambda_m(M)}} \quad (25)$$

where $T_p = -\mu\ln(\sqrt{\mu})$. The results obtained in this section are formally stated in the following Theorem.

Theorem 2: Consider system (4). If the control law is defined as in (18)(19), then the closed-loop system with state $x = \left[q^T\ \dot{q}^T\ \hat{\nu}^T\right]^T$ is globally uniformly asymptotically practically stable. Moreover, after a finite time an upper bound for x that is independent of μ can be obtained. ■

B. Convergence Analysis

In this section, we provide an analysis of the convergence properties guaranteed by the controller (19).

Defining the lead filter estimation error $\epsilon_e \in \mathbb{R}^n$ as:

$$\epsilon_e(t) = \hat{\nu}_e(t) - \dot{e}(t), \tag{26}$$

the estimation error dynamics can be described by:

$$\dot{\epsilon}_e = -\frac{1}{\mu}\epsilon_e - \ddot{e} \tag{27}$$

Using (26) the control law (19) can be rewritten as:

$$\tau = -K_p e - K_d\dot{e} - K_d\epsilon_e + g(q) + Y_d\theta \tag{28}$$

Substituting the control law (28) into (4), one has:

$$\ddot{e} = M^{-1}(q)[-C(q,\dot{q})\dot{e} - \zeta F_v\dot{e} - K_p e - K_d\dot{e} - K_d\epsilon_e - h(e,\dot{e})] \tag{29}$$

where $h(e,\dot{e}) = [M(q) - M(q_d)]\,\ddot{q}_d + [C(q,\dot{q}) - C(q_d,\dot{q}_d)]\,\dot{q}_d$ can be upper bounded by: (see [15])

$$|h(e,\dot{e})| \leq c_1 |\dot{q}_d|_M |\dot{e}| + c_h \mathrm{sat}\left(\frac{|e|}{\Delta_h}\right) \tag{30}$$

with $\Delta_h = 2(\lambda_M(M)\,|\ddot{q}_d|_M + c_1 |\dot{q}_d|_M^2)/c_h$ and $c_h = c_M |\ddot{q}_d|_M + c_2 |\dot{q}_d|_M^2$ Now, defining the error state as:

$$x_e = \left[z_e^T\ \epsilon_e^T\right]^T, \quad z_e = \left[e^T\ \dot{e}^T\right]^T, \tag{31}$$

the error system dynamics can be described by (27) and (29). From the results obtained in Section V-A the error system is UGApS and the system trajectories are globally driven to the compact set D_R.

In order to analyze the convergence properties of the error system we first show that the z_e-subsystem defined in (29) is ISS with respect to the input ϵ_e. To this end, we consider the following ISS-Lyapunov function candidate:

$$V_2(z_e) = \frac{1}{2}\dot{e}^T M(q)\dot{e} + \frac{1}{2}e^T K_p e + \varepsilon_2 f^T(e)M(q)\dot{e} \tag{32}$$

where ε_2 is a sufficiently small positive constant and the function $f(e) : \mathbb{R}^n \to \mathbb{R}^n$ is defined by

$$f(e) = c_h\sqrt{\frac{1}{\eta^2} + \Delta_h^2}\,\frac{\eta e}{\sqrt{1 + \eta^2 e^T e}} \tag{33}$$

where $0 < \eta \leq 1$ is a suitable chosen small positive constant.

Proposition 2: If ε_2 satisfies (37), then $V_2(z_e)$ is a smooth storage function and can be upper and lower bounded by:

$$\underline{\alpha}_2(|z_e|) = \kappa_5 |z_e|^2 \leq V_2(z_e) \leq \kappa_6 |z_e|^2 = \overline{\alpha}_2(|z_e|) \tag{34}$$

$$\kappa_5 \leq \frac{1}{4}\min\{\lambda_m(M), \lambda_m(K_p)\}; \tag{35}$$

$$\kappa_6 \geq \frac{1}{2}\lambda_M(P_2), \quad P_2 = \begin{bmatrix} \lambda_M(K_p) & \chi_1 \\ \chi_1 & \lambda_M(M) \end{bmatrix} \tag{36}$$

$$\chi_1 = \varepsilon_2\lambda_M(M)c_h\sqrt{1 + \eta^2\Delta_h^2}$$

$$\varepsilon_2 \leq \frac{1}{2}\frac{\sqrt{\lambda_m(M)\lambda_m(K_p)}}{\lambda_M(M)c_h\sqrt{1 + \eta^2\Delta_h^2}} \tag{37}$$

Proof: the proof follows the same steps presented in Appendix A. ■

The following Lemma shows that the z_e-subsystem is ISS with respect to ϵ_e, if K_p and K_d are properly chosen.

Lemma 1: If the control gains K_p and K_d are selected such that

$$\lambda_m(K_d) \geq \chi_2 \tag{38}$$

$$\lambda_m(K_p) \geq 2c_h\sqrt{1 + \eta^2\Delta_h^2}\left[1 + \frac{\chi_3^2}{4\varepsilon_2(\lambda_m(K_d) - \chi_2)}\right] \tag{39}$$

with χ_2 and χ_3 defined as:

$$\chi_2 := c_1 |\dot{q}_d|_M + 2\varepsilon_2\lambda_M(M)c_h\sqrt{1 + \eta^2\Delta_h^2} + \varepsilon_2 c_1 c_h\sqrt{\frac{1}{\eta^2} + \Delta_h^2}$$

$$\chi_3 := 1 + \varepsilon_2\left(\zeta\lambda_M(F_v) + \lambda_M(K_d) + 2c_1 |\dot{q}_d|_M\right);$$

then, the z_e-subsystem defined in (29) is ISS w.r.t ϵ_e.

$$|z_e(t)| \leq \beta_z(|z_e(t_0)|, t - t_0) + \gamma_z(\|\epsilon_e\|) \tag{40}$$

where $\beta_z \in \mathcal{KL}$ and $\gamma_z \in \mathcal{K}_\infty$. Moreover, within D_R the ISS gain $\gamma_z(r) = \kappa_z r$, where κ_z is independent of μ.

Proof: see Appendix B ■

Now, considering the ISS-Lyapunov function candidate

$$V_3(\epsilon_e) = \frac{1}{2}\epsilon_e^2, \tag{41}$$

the following Lemma proves that for a sufficiently small μ the ϵ_e-subsystem (27) is ISS with respect to the input z_e.

Lemma 2: If μ is chosen such that

$$\mu \leq \frac{\lambda_m(M)}{4\lambda_M(K_d)}, \tag{42}$$

then, the ϵ_e-subsystem defined in (27) is ISS w.r.t. z_e

$$|\epsilon_e(t)| \leq \beta_\epsilon(|\epsilon_e(t_0)|, t - t_0) + \gamma_\epsilon(\mu\,\|z_e\|) \tag{43}$$

where $\beta_\epsilon \in \mathcal{KL}$ and $\gamma_\epsilon \in \mathcal{K}_\infty$. Moreover, within D_R the ISS gain $\gamma_\epsilon(\mu r) = \mu\kappa_\epsilon r$, where κ_ϵ is independent of μ.

Proof: see Appendix C ■

From Lemmas 1 and 2, it follows that within D_R the composite gain $\gamma_\epsilon \circ \gamma_z(r) = \mu\kappa_z\kappa_\epsilon r$. Thus, if μ satisfies

$$\mu \leq \frac{1}{\kappa_z\kappa_\epsilon} \tag{44}$$

then, uniform global asymptotic stability of the error system with state x_e follows from the nonlinear generalized small-gain theorem [17], [18]. The results obtained in this section are formally stated in the following Theorem.

Theorem 3: Consider the robot system described by (4). If the control law is defined as in (18)(19), then the error system (27)(29) with state $x_e = \left[e^T\ \dot{e}^T\ \epsilon_e^T\right]^T$ is uniformly globally asymptotically practically stable. Moreover, if the control gains K_d and K_p are selected such that (38) and (39) hold and, in addition, the lead filter parameter μ is chosen such that (42) and (44) are satisfied, then the closed-loop error system is uniformly globally asymptotically stable. ■

Remark 3: High gain control can be used in case the robot parameters are only known "nominally". In this case our analysis predicts that the close-loop system would still be globally stable and in addition arbitrarily small residual errors could be achieved selecting the control gains K_p and K_d sufficiently large and setting μ sufficiently small.

VI. Extensions

A. Uncertain Robot Manipulators

From the above results, a control strategy can be derived for the uncertain case achieving global exact tracking. The idea is to use the recently proposed global robust exact differentiator (GRED) [19], [20] and to add an unit vector term in the control law to cope with the unknown parameters of the feedforward compensation. Another possibility which is being investigated is to adapt the unknown parameters of the feedforward compensation to also achieve exact tracking.

B. Broader Class of Nonlinear Systems

Although, in the previous analysis a linear damping was considered, the proposed approach can deal with nonlinear damping (e.g $|\dot{q}|\,\dot{q}$). Considering this type of damping, it is easy to see that in the regulation case the system would still be global ISS with respect to a bounded input disturbance. In the tracking analysis instead of using $F_v\dot{q}_d$ in the feedforward compensation, now we would use $|\dot{q}_d|\,\dot{q}_d$. Noting that $\dot{e}^T(|\dot{q}|\,\dot{q}-|\dot{q}_d|\,\dot{q}_d)\geq 0$ and using the fact that $(|\dot{q}|\,\dot{q}-|\dot{q}_d|\,\dot{q}_d)\leq(|\dot{e}|+2\,|\dot{q}_d|)\,|\dot{e}|$, it is possible to prove global output feedback tracking for this class of systems.

Since this kind of damping can represent the hydrodynamic damping of an underwater vehicle, the proposed analysis can be extended to a broader class of nonlinear systems that encompasses other classes of EL systems.

VII. Conclusion

In this paper, a new analysis technique called "ISS Regulator Approach" was proposed in order to show that a robot controlled by the well known causal PD controller with a feedforward compensation can provide global tracking, requiring only the existence of the robot natural damping, which can be arbitrary small. The main idea was to first prove that the robot controlled by a causal PD regulator is globally input-to-state stable with respect to a bounded input disturbance and then use this result to show that such causal PD controller plus a feedforward compensation yields uniform global asymptotic stability for the general n-DOF case. We have also provided suggestions to extend the proposed approach to a broader class of nonlinear systems and to consider uncertain robot manipulators.

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Appendix

A. Proof of Proposition 1

From Property **P1**, it follows that $V_1\geq\frac{1}{2}\lambda_m(K_p)\,|\tilde{q}|^2+\frac{1}{2}\mu\lambda_m(K_d)\,|\hat{\nu}|^2+\left[\frac{1}{2}\lambda_m(M)\,|\dot{q}|^2-\frac{\varepsilon_1\lambda_M(M)|\tilde{q}||\dot{q}|}{\sqrt{1+|\tilde{q}|^2}}\right]$. By completing the squares on the bracketed term and using the fact that $|\tilde{q}|/(1+|\tilde{q}|^2)\leq|\tilde{q}|$, it can be verified that for $\varepsilon_1\leq\sqrt{\lambda_m(M)\lambda_m(K_p)}/2\lambda_M(M)$, it follows that $V_1\geq\frac{1}{4}\lambda_m(M)\,|\dot{q}|^2+\frac{1}{4}\lambda_m(K_p)\,|\tilde{q}|^2+\frac{1}{2}\mu\lambda_m(K_d)\,|\hat{\nu}|^2$. Therefore, $V_1(x)\geq\underline{\alpha}_1(|x|)$, where $\underline{\alpha}_1(r)=\kappa_1r^2\in\mathcal{K}_\infty$ and κ_1 is defined in (10). The function $V_1(x)$ can be upper bounded by $V_1\leq\bar{x}^TP_1\bar{x}$, where $\bar{x}^T=[|\tilde{q}|\ \ |\dot{q}|\ \ |\hat{\nu}|]$, P_1 is defined in (11). Thus, $V_1(x)\leq\overline{\alpha}_1(|x|)$, where $\overline{\alpha}_1(r)=\kappa_2r^2\in\mathcal{K}_\infty$ and κ_2 is defined in (11). Therefore, from Definition 3, it follows that $V_1(x)$ is a smooth storage function. ■

B. Proof of Lemma 1

The time derivative of V_2 is given by:

$$\begin{aligned}\dot{V}_2 \;=\; & -\zeta\dot{e}^TF_v\dot{e}-\dot{e}^TK_d\dot{e}-\dot{e}^TK_d\epsilon_e-\dot{e}^Th(e,\dot{e})\\ & +\varepsilon_2\dot{f}^T(e)M(q)\dot{e}+\varepsilon_2f^T(e)C^T(q,\dot{q})\dot{e}\\ & -\varepsilon_2\zeta f^T(e)F_v\dot{e}-\varepsilon_2f^T(e)K_pe-\varepsilon_2f^T(e)K_d\dot{e}\\ & -\varepsilon_2f^T(e)K_d\epsilon_e-\varepsilon_2f^T(e)h(e,\dot{e}) \qquad (45)\end{aligned}$$

where Property **P3** and **P4** were utilized.

The function $f(e)$ defined in (33) satisfies the following inequalities, for all $e\in\mathbb{R}^n$ **(I1)** $|f(e)|\geq c_h\mathrm{sat}\left(|e|/\Delta_h\right)$; **(I2)** $\left|\dot{f}(e)\right|\leq 2c_h\sqrt{1+\eta^2\Delta_h^2}\,|\dot{e}|$; **(I3)** $|f(e)|\leq c_h\sqrt{\frac{1}{\eta^2}+\Delta_h^2}$; **(I4)** $|f(e)|^2\leq c_h\sqrt{1+\eta^2\Delta_h^2}f^T(e)e$.

From (30) and **I1**, it is possible to show that $-\dot{e}^Th(e,\dot{e})\leq c_1\,|\dot{q}_d|_M\,|\dot{e}|^2+|\dot{e}|\,|f(e)|$. Using **I2**, one has that $\varepsilon_2\dot{f}^T(e)M(q)\dot{e}\leq 2\varepsilon_2\lambda_M(M)c_h\sqrt{1+\eta^2\Delta_h^2}\,|\dot{e}|^2$. From **I3** and Property **P7**, the following result can be obtained $\varepsilon_2f^T(e)C^T(q,\dot{q})\dot{e}\leq\varepsilon_2c_1\,|\dot{q}_d|_M\,|\dot{e}|\,|f(e)|+\varepsilon_2c_1c_h\sqrt{\frac{1}{\eta^2}+\Delta_h^2}\,|\dot{e}|^2$. Using (33) and **I4** it can be verified that $-\varepsilon_2f^T(e)K_pe\leq-\frac{1}{2}\varepsilon_2\lambda_m(K_p)c_h\sqrt{1+\eta^2\Delta_h^2}\frac{|e|^2}{\sqrt{1+\eta^2|e|^2}}-\frac{1}{2}\varepsilon_2\frac{\lambda_m(K_p)}{c_h\sqrt{1+\eta^2\Delta_h^2}}\,|f(e)|^2$. From (30) and **I1**, it follows that $-\varepsilon_2f^T(e)h(e,\dot{e})\leq\varepsilon_2c_1\,|\dot{q}_d|_M\,|\dot{e}|\,|f(e)|+\varepsilon_2\,|f(e)|^2$. Thus, the function $\dot{V}_2(z_e)$ can be upper bounded as follows:

$$\begin{aligned}\dot{V}_2 \;\leq\; & -\zeta\dot{e}^TF_v\dot{e}-\frac{1}{2}\dot{e}^TK_d\dot{e}-\left[\frac{1}{2}\dot{e}^TK_d\dot{e}-\lambda_M(K_d)\,|\dot{e}||\epsilon_e|\right]\\ & +(c_1|\dot{q}_d|_M+2\varepsilon_2\lambda_M(M)c_h\sqrt{1+\eta^2\Delta_h^2})|\dot{e}|^2+|\dot{e}||f(e)|\\ & +\varepsilon_2(\lambda_M(K_d)+2c_1|\dot{q}_d|_M+\zeta\lambda_M(F_v))\,|\dot{e}|\,|f(e)|\\ & +\varepsilon_2c_1c_h\sqrt{\frac{1}{\eta^2}+\Delta_h^2}\,|\dot{e}|^2-\frac{1}{4}\varepsilon_2\frac{\lambda_m(K_p)}{c_h\sqrt{1+\eta^2\Delta_h^2}}\,|f(e)|^2\\ & -\left[\frac{1}{4}\varepsilon_2\frac{\lambda_m(K_p)}{c_h\sqrt{1+\eta^2\Delta_h^2}}\,|f(e)|^2-\varepsilon_2\lambda_M(K_d)|f(e)||\epsilon_e|\right]\\ & -\frac{1}{2}\varepsilon_2\lambda_m(K_p)c_h\frac{\sqrt{1+\eta^2\Delta_h^2}\,|e|^2}{\sqrt{1+\eta^2\,|e|^2}}+\varepsilon_2\,|f(e)|^2 \qquad (46)\end{aligned}$$

Completing the squares on the bracketed terms, one has:

$$\dot{V}_2 \le -\frac{1}{2}\varepsilon_2\lambda_m(K_p)c_h\sqrt{1+\eta^2\Delta_h^2}\frac{|e|^2}{\sqrt{1+\eta^2|e|^2}} - [|\dot{e}|\ \ |f(e)|]\, Q \begin{bmatrix} |\dot{e}| \\ |f(e)| \end{bmatrix} + \frac{\lambda_M^2(K_d)}{2\lambda_m(K_d)}|\epsilon_e|^2 + \frac{\varepsilon_2 c_h\sqrt{1+\eta^2\Delta_h^2}\lambda_M^2(K_d)}{\lambda_m(K_p)}|\epsilon_e|^2 \tag{47}$$

where $Q = \begin{bmatrix} \frac{1}{2}\lambda_m(K_d)-\chi_2 & -\frac{\chi_3}{2} \\ -\frac{\chi_3}{2} & \varepsilon_2\left(\frac{\lambda_m(K_p)}{4c_h\sqrt{1+\eta^2\Delta_h^2}}-1\right) \end{bmatrix}$

If K_p and K_d are chosen such that (38) and (39) hold, then Q is positive definite. From (47) and since $\frac{|e|^2}{\sqrt{1+\eta^2|e|^2}} \ge \frac{|e|^2}{\sqrt{1+|e|^2}}$, the function $\dot{V}_2$ can be further upper bounded by:

$$\dot{V}_2(z_e) \le -\alpha_2(|z_e|) + \sigma_2(|\epsilon_e|) \tag{48}$$

where $\alpha_2(r) = \kappa_7 r^2/\sqrt{1+r^2},\ \sigma_2(r) = \kappa_8 r^2 \in \mathcal{K}_\infty$ with

$$\kappa_7 = \min\left\{\lambda_m(Q), \frac{1}{2}\varepsilon_2\lambda_m(K_p)c_h\sqrt{1+\eta^2\Delta_h^2}\right\}$$

$$\kappa_8 = \frac{\lambda_M^2(K_d)\left(\lambda_m(K_p)+2\varepsilon_2 c_h\sqrt{1+\eta^2\Delta_h^2}\lambda_m(K_d)\right)}{2\lambda_m(K_d)\lambda_m(K_p)}$$

Thus, from Definition 4, $V_2(z_e)$ is an ISS-Lyapunov function for the z_e-subsystem. Moreover, from Definition 2, follows (40), where $\gamma_z(r) = \underline{\alpha}_2^{-1} \circ \overline{\alpha}_2 \circ \alpha_2^{-1} \circ \sigma_2(r) \in \mathcal{K}_\infty$.

From (17), (23) and (31), it is possible to conclude that $\sup_{t\ge T_d}|z_e(t)| \le C_z$. Thus, $\alpha_2(|z_e|)$ in (48) can be redefined as $\alpha_2(r) = \bar{\kappa}_7 r^2$, with $\bar{\kappa}_7 = \kappa_7/\sqrt{1+C_z^2}$. Hence, within D_R, the ISS gain $\gamma_z(r) = \kappa_z r$, where $\kappa_z = \sqrt{(\kappa_6\kappa_8)/(\kappa_5\bar{\kappa}_7)}$. ■

C. Proof of Lemma 2

From (29), (27) and (41), using (30) and Properties **P1** and **P7**, the time derivative of V_3 can be upper bounded by:

$$\dot{V}_3 \le -\frac{1}{4\mu}|\epsilon_e|^2 - \left[\frac{1}{8\mu}|\epsilon_e|^2 - \frac{\lambda_M(K_p)+c_h}{\lambda_m(M)}|e||\epsilon_e|\right] - \left[\frac{1}{8\mu}|\epsilon_e|^2 - \frac{2c_1|\dot{q}_d|_M+\zeta\lambda_M(F_v)+\lambda_M(K_d)}{\lambda_m(M)}|\dot{e}||\epsilon_e|\right] - \left[\frac{1}{4\mu}|\epsilon_e|^2 - \frac{c_1|\dot{e}|^2|\epsilon_e|}{\lambda_m(M)}\right] - \left(\frac{1}{4\mu} - \frac{\lambda_M(K_d)}{\lambda_m(M)}\right)|\epsilon_e|^2$$

After completing the squares on the bracketed terms, the following result can be obtained for $\mu \le \frac{\lambda_m(M)}{4\lambda_M(K_d)}$

$$\dot{V}_3 \le -\frac{1}{4\mu}|\epsilon_e|^2 + 2\mu\frac{(\lambda_M(K_p)+c_h)^2}{\lambda_m^2(M)}|e|^2 + \mu\frac{c_1^2}{\lambda_m^2(M)}|\dot{e}|^4 + 2\mu\frac{(2c_1|\dot{q}_d|_M+\zeta\lambda_M(F_v)+\lambda_M(K_d))^2}{\lambda_m^2(M)}|\dot{e}|^2 \tag{49}$$

From (49), it can be shown that $\dot{V}_3$ can be upper bounded by the following inequality:

$$\dot{V}_3(\epsilon_e) \le -\alpha_3(|\epsilon_e|) + \sigma_3(\mu|z_e|) \tag{50}$$

where $\alpha_3(r) = r^2/4\mu,\ \sigma_3(\mu r) = \mu r^2(\kappa_9+\kappa_{10}r^2) \in \mathcal{K}_\infty$, with

$$\kappa_9 = \frac{2}{\lambda_m^2(M)}\max\left\{\begin{array}{c}(\lambda_M(K_p)+c_h)^2, \\ (2c_1|\dot{q}_d|_M+\zeta\lambda_M(F_v)+\lambda_M(K_d))^2\end{array}\right\}$$

and $\kappa_{10} = c_1^2/\lambda_m^2(M)$. Thus, from Definition 4, $V_3(\epsilon_e)$ is an ISS-Lyapunov function for the ϵ_e-subsystem. Furthermore, from Definition 2, follows (43), where $\gamma_\epsilon(\mu r) = \alpha_3^{-1} \circ \sigma_3(\mu r) \in \mathcal{K}_\infty$. Inside D_R the function $\sigma_3(z_e)$ can be redefined as follows $\alpha_3(\mu r) = \mu\kappa_{11}r^2$ with $\kappa_{11} = \kappa_9 + \kappa_{10}C_z^2$ Therefore, within D_R, $\gamma_\epsilon(\mu r) = \mu\kappa_\epsilon r$, where $\kappa_\epsilon = 2\sqrt{\kappa_{11}}$ ■

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