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Formation Control of Car-Like Mobile Robots: A Lyapunov Function Based Approach

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Abstract—**In literature leader – follower strategy has been used extensively for formation control of car-like mobile robots with the control law being derived from the kinematics.** ***This paper takes it a step further and a nonlinear control law is derived using Lyapunov analysis for formation control of car-like mobile robots using robot dynamics.*** **Controller is split into two parts. The first part is the development of a velocity controller for the follower from the error kinematics (linear and angular). The second part involves the use of the dynamics of the robot in the development of a torque controller for both the drive and the steering system of the car-like mobile robot. Unknown quantities like friction, desired accelerations (unmeasured) are computed using an online neural network. Simulations results prove the ability of the controller to effectively stabilize the formation while maintaining the desired relative distance and bearing.**

I. Introduction

THE use of dynamics coupled with kinematics for the control of autonomous mobile robots has been gaining increasing popularity in recent years. The majority of control algorithms available in literature for autonomous mobile robots use only the kinematic model [2]. The kinematic model has its own advantages. It helps in keeping the steering and velocity of the vehicle completely decoupled but in the process, the dynamics of the vehicle is not taken into account and hence remains ignored. The autonomous mobile robot considered in this paper is a front steer, rear drive car-like mobile robot. The velocity of the car-like robot is very dependant upon the dynamics of the steering system. Hence, the dynamics of the vehicle as well as dynamics of the steering must be taken into account.

Automating car-like robot has many advantages like operating in hazardous environments like mines, data collection and reconnaissance etc. These controllers can be put to use in autonomous armored vehicles (note not tanks) for patrolling the streets to detect improvised explosive devices (IED's). In most of these scenarios, employing a team of mobile robots helps in increasing the efficiency with which the task is completed. The use of a team helps in faster search of the entire search space and the operation can be carried out in a very systematic and effective way. It is extremely valuable in time critical operations. Hence the focus of research has shifted to the control of a swarm or team of mobile robots in the recent years There many references available for control of single nonholonomic mobile robots [1], [6-11].

The focus of this paper is on the formation control of a team of car-like mobile robots. There are various techniques available in literature for formation control of mobile robots. A few of the most commonly used techniques are: leader-follower [2-4] [15] [18], virtual structure based [16] [17] and behavior based approaches [11-13]. In [2] Shao *et al* use the concept of a virtual vehicle and the kinematics to derive the error system for control of multiple Pioneer 3DX vehicles. Li *et al* [3] present a kinematics model for the leader following based formation control of tricycle mobile robots and a back stepping based stabilizing controller is derived under the conditions of perfect velocity tracking and no disturbances. Dierks *et al* in [4] control a differentially steered robot by backstepping kinematics into dynamics. Desai *et al* in [5] use the kinematic model and graph theory to design a controller for multiple mobile robot formations.

Unlike other papers, the dynamics of both the drive and the steering system are considered in this study. Single leader single follower scenario is considered in this paper but the same can easily be extended to multiple follower scenarios and is proven. The asymptotic stability of the system is also guaranteed and it is proved that the position tracking errors and the velocity tracking errors go to zero asymptotically.

In Section II the mathematical model of a car-like mobile robot is derived. Both the kinematic as well as the dynamic model are derived. They are used in Section III for the derivation of velocity and torque control inputs. In Section IV weight tuning law for an online neural network is derived. Section V the stability of the formation for multiple followers is proved. Numerical results are presented in Section VI.

II. Mathematical Model

A. Kinematic Model

The kinematic model of the system will be derived taking the nonholonomic constraints into account. Nonholonomic constraints for mobile robots are non-integrable and are related to its velocity [1]. A four wheeled, front-steer, rear drive mobile robot can be modeled as a bicycle for very small angles of steering. Consider Fig.1. Let (x, y) denote

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the center of gravity (G) of the robot. The distance from G to the rear and front wheels be a and b respectively. Let θ denote the heading angle of the robot i.e. the orientation of the robot with respect to the x-axis and ϕ denotes the steering angle between the front wheel and the body axis.

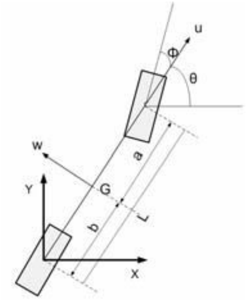

Fig.1 Bicycle model

The nonholonomic constraints for the robot given by

$$\begin{aligned} &\dot{x}\sin(\theta) - \dot{y}\cos(\theta) + b\dot{\theta} = 0 \\ &\dot{x}\sin(\theta+\phi) - \dot{y}\cos(\theta+\phi) - a\dot{\theta}\cos(\theta) = 0 \end{aligned} \tag{1}$$

Let v_u and v_w be the longitudinal and lateral velocities of the vehicle. Using the body coordinates of the vehicle i.e. along the u and w axis we can define

$$\begin{aligned} &\dot{x} = v_u\cos(\theta) - v_w\sin(\theta) \\ &\dot{y} = v_u\sin(\theta) + v_w\cos(\theta) \end{aligned} \tag{2}$$

Substituting (2) in (1) and manipulating we get

$$v_w = b\dot{\theta} \tag{3}$$

$$v_w = \frac{b\tan\phi}{L}v_u \tag{4}$$

From equations (2) through (4) the kinematic model of the robot is given by

$$\dot{x} = v_u\cos(\theta) - v_u\frac{b\tan\phi}{L}\sin(\theta) \tag{5}$$

$$\dot{y} = v_u\sin(\theta) + v_u\frac{b\tan\phi}{L}\cos(\theta) \tag{6}$$

$$\dot{\theta} = \frac{\tan\phi}{L}v_u \tag{7}$$

B. Dynamic Model

The dynamic model is derived with the following assumptions: (i) there is no slip at the wheel, (ii) the rear wheels cannot be steered and are always in the same direction as the orientation of the vehicle, (iii) and the drive force and drive torque are assumed to act at the center of the rear wheels [1] [6] [7].The forces acting on the robot are as shown in Fig. (2). F_u, F_w, F_d denote the frictional force, the force acting perpendicular to each wheel as a result of the slippage assumption made and the drive force respectively. Also m, I denote the mass of the vehicle and the moment of inertia of the vehicle. Balancing the forces (Fig.2) acting along the u and w direction we have

$$m(\dot{v}_u - v_w\dot{\theta}) = -F_{ur} + F_{dr} - F_{uf}\cos\phi - F_{wf}\sin\phi \tag{8}$$

$$m(\dot{v}_w + v_u\dot{\theta}) = F_{wr} - F_{uf}\sin\phi + F_{wf}\cos\phi \tag{9}$$

where $F_{wir} + F_{wor} = F_{wr}$; $F_{uir} + F_{uor} = F_{ur}$;
$F_{wif} + F_{wof} = F_{wf}$; $F_{uif} + F_{uof} = F_{uf}$ and $F_{dir} + F_{dor} = F_{dr}$

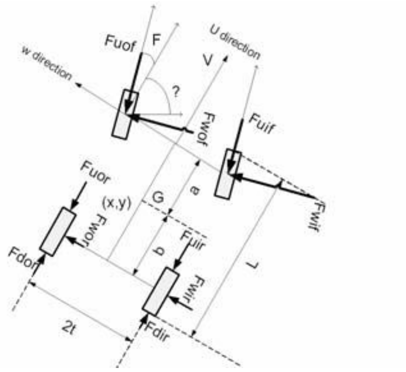

Fig.2 Free Body Diagram

Also steering system dynamics of the robot can be modeled by a first order linear system [19] represented by the differential equation

$$\dot{\phi} = \frac{1}{\tau_s}(u - \phi) \tag{10}$$

where τ_s, u denote the time constant and steering control respectively.

III. Formation Control

There are various approaches available for formation control. The most common approaches being, the leader follower, virtual structure and behavior based approach. In this paper the formation control of the robot is achieved using the leader follower approach. The separation-bearing ($l-\psi$) technique is made use of instead of the separation-separation strategy. The objective is to find a velocity control input for the follower that will drive the relative distance and relative bearing between the leader and follower to the desired value. It is assumed that the leader's motion is known i.e. there exists a control law that drives the leader independently to its desired trajectory. Most formation control techniques for car-like robots in the literature involve the kinematics and do not incorporate the dynamics [2] [3] [5]. This issue has been addressed in this paper. The dynamics of the leader and the follower are used to derive specific torque control inputs required to achieve the desired velocity profile derived earlier. Imperfect velocity tracking condition is considered. Consider the single leader single follower scenario as shown in Fig. (3).

The subscripts l and f denote the leader and follower respectively. The relative distance L_{LF} is the distance between the rear of the leader (point B) to the front of the follower (point A) and the relative bearing ψ_{LF} is the defined as the angle measured from the leader (i.e. the direction of orientation of the leader) to the straight line joining the points A and B. The relative distance L_{LF} can be expressed in terms of the x and y coordinates of L_{LF} as

$$L_{LF}{}^{2} = L_{LFx}{}^{2} + L_{LFy}{}^{2} \tag{11}$$

where

$$L_{LFx} = x_L - x_F - d(\cos(\theta_L) + \cos(\theta_F)) \tag{12}$$

$$L_{LFy} = y_L - y_F - d(\sin(\theta_L) + \sin(\theta_F)) \tag{13}$$

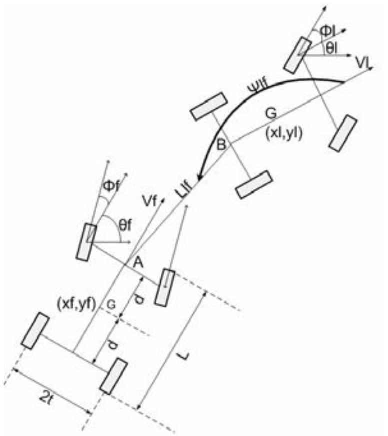

Fig.3 Formation Structure

Also from Fig (3) we can see that the relative bearing can be expressed in terms of the leader's heading angle and the x and y coordinates of the relative distance as

$$\psi_{LF} = \arctan\left(\frac{L_{LFy}}{L_{LFx}}\right) - \theta_L + \pi \tag{14}$$

Differentiating equations (12) and (13) we have

$$\dot{L}_{LFx} = \dot{x}_L - \dot{x}_F + d\dot{\theta}_L \sin(\theta_L) + d\dot{\theta}_F \sin(\theta_F) \tag{15}$$

$$\dot{L}_{LFy} = \dot{y}_L - \dot{y}_F - d\dot{\theta}_L \cos(\theta_L) - d\dot{\theta}_F \cos(\theta_F) \tag{16}$$

The kinematics of the leader and follower can be obtained from equations (5) through (7) . Let,

$$w_L = \frac{v_L \tan(\phi_L)}{L} \; ; \; w_F = \frac{v_F \tan(\phi_F)}{L} \tag{17}$$

Substituting (5) through (7), (17)in (15), (16) and taking $L = 2d$ we have

$$\begin{aligned} \dot{L}_{LFx} &= v_L \cos(\theta_L) - v_F \cos(\theta_F) + v_F \tan(\phi_F)\sin(\theta_F) \\ \dot{L}_{LFy} &= v_L \sin(\theta_L) - v_F \sin(\theta_F) - v_F \tan(\phi_F)\cos(\theta_F) \end{aligned} \tag{18}$$

From Fig (3) we can see that

$$\frac{L_{LFx}}{L_{LF}} = \cos(\psi_{LF} + \theta_L - \pi); \frac{L_{LFy}}{L_{LF}} = \sin(\psi_{LF} + \theta_L - \pi) \tag{19}$$

Define $\gamma_F = \psi_{LF} + \theta_L - \theta_F$ .Differentiating(11) and (14) , substituting(18), (19) and using trigonometric identities

$$\dot{L}_{LF} = -v_L \cos(\psi_{LF}) + v_F \tan(\phi_F)\sin(\gamma_F) + v_F \cos(\gamma_F) \tag{20}$$

$$\dot{\psi}_{LF} = \begin{pmatrix} \frac{1}{L_{LF}}\left(v_L \sin(\psi_{LF}) - v_F \sin(\gamma_F)\right) \\ +\frac{1}{L_{LF}}\left(v_F \tan(\phi_F)\cos(\gamma_F)\right) - \frac{v_L \tan(\phi_L)}{L} \end{pmatrix} \tag{21}$$

Defining the error system given by

$$e_1 = x_{FD} - x_F ; e_2 = y_{FD} - y_F ; e_3 = \theta_{FD} - \theta_F \tag{22}$$

From the Fig(3), it can be seen that, the actual and desired coordinates of point A can be expressed in terms of coordinates of point B, L_{LF}, L_{LFD}, ψ_{LF} and ψ_{LFD} .Using these equations and transforming from inertial to body coordinates, for better intuitive sense, results in

$$e_{F1} = [L_{LFD} \cos(\psi_{LFD} + e_{F3}) - L_{LF} \cos(\psi_{LF} + e_{F3}) - d\cos(e_{F3}) + d] \tag{23}$$

$$e_{F2} = [L_{LFD} \sin(\psi_{LFD} + e_{F3}) - L_{LF} \sin(\psi_{LF} + e_{F3}) - d\sin(e_{F3})] \tag{24}$$

$$e_{F3} = \theta_L - \theta_F \tag{25}$$

Differentiating (23) through(25), substituting(20), (21) and simplifying

$$\dot{e}_{F3} = \frac{v_L \tan(\phi_L)}{L} - \frac{v_F \tan(\phi_F)}{L} \tag{26}$$

$$\dot{e}_{F1} = \left(v_L \cos(e_{F3}) - v_F + w_L\left[-L_{LF} \sin(\gamma_F) - e_{F2}\right] + w_F e_{F2}\right) \tag{27}$$

$$\dot{e}_{F2} = \begin{pmatrix} v_L \sin(e_{F3}) - Lw_F + w_L\left[e_{F1} - d\right] - w_F\left[e_{F1} - d\right] \\ +w_L L_{LF} \cos(\gamma_F) \end{pmatrix} \tag{28}$$

To stabilize the kinematic system and maintain the desired relative bearing and distance, velocity control inputs for the follower robot can be designed using Lyapunov analysis. Choosing the Lyapunov function candidate as

$$V = \frac{K_1 e_{F1}{}^{2} + K_2 e_{F2}{}^{2}}{2} + K_3(1 - \cos(e_{F3})) \tag{29}$$

Differentiating (29)

$$\dot{V} = K_1 e_{F1}\dot{e}_{F1} + K_2 e_{F2}\dot{e}_{F2} + K_3 \sin(e_{F3})\dot{e}_{F3} \tag{30}$$

The control inputs v_F and w_F that make $\dot{V} < 0$ and the system asymptotically stable i.e. $e_F \to 0$ as $t \to \infty$ are given by

$$v_F = k_{VF}\, e_{F1} + v_L \cos(e_{F3}) - w_L L_{LF} \sin(\gamma_F) \tag{31}$$

$$\begin{aligned} w_F = {} & -w_L + \frac{(k_v + v_L)\sin(e_{F3})}{d} + \frac{w_L L_{LF} \cos(\gamma_F)}{d} + e_{F2} \\ & + \frac{-2K_3 w_L - \frac{K_3 w_L L_{LF}}{d} - K_3\left|e_{F2}\right| - Kk_V\left|e_{F2}\right|}{Kd\left|e_{F2}\right| + K_3} \end{aligned} \tag{32}$$

Substituting (26) through(28),(31),(32) in (30) and simplifying ,

$$\dot{V} < -Kk_{VF} e_{F1}{}^{2} - Kde_{F2}{}^{2} - \frac{K_3(k_V + v_L)\sin^2(e_{F3})}{d} \tag{33}$$

Since $v_L \geq 0$,with $K_1 = K_2 = K$ and $K, K_3, k_V > 0$, $\dot{V} < 0$. In order to track the velocity and the angular velocity derived using Lyapunov analysis, the follower robot dynamics needs to be considered. The torque control inputs for the drive and steering system which will produce the desired velocity profile need to be obtained. Define a velocity tracking error given by

$$\mathbf{e}_{FD} = \mathbf{Z}_{FD} - \mathbf{Z}_F \tag{34}$$

where

$$\mathbf{Z}_{FD} = \left[v_{FD} \quad \phi_{FD}\right]^T \text{ and } \mathbf{Z}_F = \left[v_F \quad \phi_F\right]^T \tag{35}$$

In (35) v_{FD} and ϕ_{FD} are the desired linear velocity and steering angle profiles derived from the Lyapunov analysis, while v_F and ϕ_F denote the actual values. Substituting $\tau_F = rF_{dr}$ where τ_F, r denote the drive torque and wheel radius of the follower and taking $v_u = v_F$

$$\dot{v}_F = \left(-\frac{F_{uf}\cos\phi_F}{m} - \frac{F_{wf}\sin\phi_F}{m} + \frac{bv_F^2\tan^2\phi_F}{L^2} - \frac{F_{ur}}{m} + \frac{\tau_F}{rm} \right) \quad (36)$$

From (36) and (10) we have

$$\dot{\mathbf{Z}}_F = -\mathbf{A}\mathbf{Z}_F - \mathbf{B} + \mathbf{E}\mathrm{T} \quad (37)$$

where

$$\mathbf{A}_{11} = \frac{-dv_F\tan^2\phi_F}{L^2} \qquad \mathbf{A}_{12} = 0 \quad (38)$$

$$A_{21} = 0 \qquad \mathbf{A}_{22} = \frac{1}{\tau_s} \quad (39)$$

$$\mathbf{B}_{11} = +\frac{F_{ur}}{m} + \frac{F_{uf}\cos\phi_F}{m} + \frac{F_w\sin\phi_F}{m} \quad \mathbf{B}_{21} = 0 \quad (40)$$

$$\mathbf{E} = diag\left(\frac{1}{rm}, \frac{1}{\tau_s}\right) \quad \text{and} \quad \mathrm{T} = \begin{bmatrix} \tau_F \\ u \end{bmatrix} \quad (41)$$

Adding and subtracting $\dot{\mathbf{Z}}_{FD}, \mathbf{A}\mathbf{Z}_{FD}$ in (37) and simplifying we have

$$\dot{\mathbf{e}}_{FD} = -\mathbf{A}\mathbf{e}_{FD} + \mathbf{A}\mathbf{Z}_{FD} + \dot{\mathbf{Z}}_{FD} + \mathbf{B} - \mathbf{E}\mathrm{T} \quad (42)$$

Define $\mathbf{f}(\mathbf{x}_{Fnew}) = \mathbf{A}\mathbf{Z}_{FD} + \dot{\mathbf{Z}}_{FD} + \mathbf{B}$. Note that $\mathbf{f}(\mathbf{x}_{Fnew})$ involves friction terms and desired acceleration terms that cannot be computed in a real life accurately. Hence, neural network will be used to estimate $\mathbf{f}(\mathbf{x}_{Fnew})$.The error dynamics can now be written as

$$\dot{\mathbf{e}}_{FD} = -\mathbf{A}\mathbf{e}_{FD} + \mathbf{f}(\mathbf{x}_{Fnew}) - \mathbf{E}\mathrm{T} \quad (43)$$

where $\mathbf{x}_{Fnew} = \begin{bmatrix} e_{F1} & e_{F2} & e_{F3} \end{bmatrix}^T$. A torque control

$$\mathrm{T} = \mathbf{E}^{-1}\left[\mathbf{K}_1\mathbf{e}_{FD} + \mathbf{f}(\mathbf{x}_{Fnew})\right] \quad (44)$$

is designed. Using (44),

$$\dot{\mathbf{e}}_{FD} = -(\mathbf{A} + \mathbf{K}_1)\mathbf{e}_{FD} \quad (45)$$

An appropriate choice of $\mathbf{K}_1$ will result in the system in (45) being asymptotically stable and the velocity tracking error will go to zero. Now consider a new Lyapunov candidate function obtained by appending the one in (29)

$$V_{new} = V_{old} + \frac{1}{2}\mathbf{e}^T{}_{FD}\mathbf{e}_{FD} \quad (46)$$

Differentiating (46) and substituting (45)

$$\dot{V}_{new} = \dot{V}_{old} - \mathbf{e}^T{}_{FD}(\mathbf{A} + \mathbf{K}_1)\mathbf{e}_{FD} \quad (47)$$

We know that $\dot{V}_{old} < 0$, so to make $\dot{V}_{new} < 0$ choosing,

$$\mathbf{K}_1 = diag\left[\left(k_1 + \frac{dv_F\tan^2\phi_F}{L^2}\right), \left(-\frac{1}{\tau_s} + k\right)\right] \quad (48)$$

with $k_{1,}k_4 > 0$ results in

$$\dot{V}_{new} = \dot{V}_{old} - e^2_{FD1}k_1 - e^2_{FD2}k_4 \quad (49)$$

From (49) it can be inferred that the tracking error system in (43) and the error system given by (26) through (28) are asymptotically stable. Since the function $\mathbf{f}(\mathbf{x}_{Fnew})$ is approximated by a neural network a weight update rule is needed for the neural network.

IV. WEIGHT UPDATE RULE AND PROOF OF BOUNDED-NESS OF WEIGHTS

A single layer functional link neural network (FLNN) is used for the approximation of $\mathbf{f}(\mathbf{x}_{Fnew})$. The activation function is chosen as a basis set for the universal approximation property to hold. There exists a weight W such that $\mathbf{f}(\mathbf{x}_{Fnew}) = W^T\phi(\mathbf{x}_{Fnew}) + \varepsilon$ with the estimation error $\|\varepsilon\| < \varepsilon_N$. The ideal approximating weights are unknown and nonunique. So an assumption is made that $\|W\|_F < W_B$ with the bound known. $\| \, \|_F$ denotes the Forbenius norm. Then, an estimate of $\mathbf{f}(\mathbf{x}_{Fnew})$ is given by

$$\hat{\mathbf{f}}(\mathbf{x}_{Fnew}) = \hat{W}^T\phi(\mathbf{x}_{Fnew}) \quad (50)$$

with $\hat{W}$ being neural network weights. Therefore,

$$\mathrm{T} = \mathbf{E}^{-1}\left[\mathbf{K}_1\mathbf{e}_{FD} + \hat{\mathbf{f}}(\mathbf{x}_{Fnew})\right] \quad (51)$$

Define $\tilde{\mathbf{f}}(\mathbf{x}_{Fnew})$ as

$$\tilde{\mathbf{f}}(\mathbf{x}_{Fnew}) = \mathbf{f}(\mathbf{x}_{Fnew}) - \hat{\mathbf{f}}(\mathbf{x}_{Fnew}) \quad (52)$$

An online weight update rule is now developed to guarantee stable tracking and yet guarantee boundedness of weights. The weight estimation error is defined as

$$\tilde{W} = W - \hat{W} \quad (53)$$

Now substituting (51) in (43) and using (53)

$$\dot{\mathbf{e}}_{FD} = -(\mathbf{A} + \mathbf{K}_1)\mathbf{e}_{FD} + \tilde{W}^T\phi(\mathbf{x}_{Fnew}) \quad (54)$$

Lyapunov candidate function as given below is chosen.

$$V_W = \frac{1}{2}\mathbf{e}^T_{FD}\mathbf{e}_{FD} + \frac{1}{2}tr\{\tilde{W}^T F^{-1}\tilde{W}\} \quad (55)$$

where F is a user defined tuning matrix. Differentiating (55) and substituting (54) we have

$$\dot{V} = -\mathbf{e}^T_{FD}(\mathbf{A} + \mathbf{K}_1)\mathbf{e}_{FD} + tr\left\{\tilde{W}^T(F^{-1}\dot{\tilde{W}} + \phi\mathbf{e}^T{}_{FD})\right\} \quad (56)$$

Selecting $\dot{\hat{W}} = F\phi\mathbf{e}^T_{FD} - kF\|\mathbf{e}_{FD}\|\hat{W}$ as the weight tuning law, we can show that

$$\dot{V} \le -\|\mathbf{e}_{FD}\|\left[(\mathbf{A} + \mathbf{K}_1)_{\min}\|\mathbf{e}_{FD}\| + k\|\tilde{W}\|_F(\|\tilde{W}\|_F - W_B)\right] \quad (57)$$

where $(.)_{\min}$ denotes the minimum singular value. When,

$$\|\mathbf{e}_{FD}\| \ge \frac{kW_B^2}{4(\mathbf{A} + \mathbf{K}_1)_{\min}} \equiv b_{e_{FD}} or \|\tilde{W}\|_F > \frac{W_B}{2} + \sqrt{\frac{kW_B^2}{4}} \equiv b_W \quad (58)$$

$\dot{V}$ is negative outside a compact set. Let the NN function approximation property hold for $\mathbf{f}(\mathbf{x}_{Fnew})$ with an accuracy of ε_N for all $\mathbf{x}_{Fnew}$ in the compact set $S_{Fnew} \equiv \{x_{Fnew} \mid \|x_{Fnew}\| < b_{x_{Fnew}}\}$ with $b_{x_{Fnew}} > Z_{FB}$ where Z_{FB} is the bound on the desired trajectory Z_{FD}.

Define $S_{e_{FD}} \equiv \{e_{FD} \mid \|e_{FD}\| < (b_{x_{Fnew}} - Z_{FB})/(c_0 + c_1)\}$. Selecting the gain $(\mathbf{A}+\mathbf{K}_1)_{\min} > \dfrac{\frac{kW_B^2}{4}(c_0+c_1)}{(b_{x_{Fnew}} - Z_{FB})}$ ensures that the compact set defined by $\|e_{FD}\| < b_{e_{FD}}$ is contained in $S_{e_{FD}}$. This guarantees that the error e_{FD} and the NN weight estimates $\hat{w}$ are uniformly ultimately bounded (UUB) with bounds given by(58). [20]

V. FORMATION STABILITY

Consider a formation of $N+1$ robots consisting of a leader "l_i" and N followers. Let there be a smooth velocity control input $[v_L \quad w_L]^T$ for the leader and let the torque control inputs $[\tau_L \quad u_L]^T$ be applied to the leader such that the leader tracks a virtual reference robot. The smooth velocity control inputs $[v_{Fi} \quad w_{Fi}]^T$ for the i^{th} follower are given by(31), (32) and torque control inputs by(51). Consider the following Lyapunov candidate

$$V_{Formation} = \sum_{1}^{N} V_{Wi} + V_{l_1} \tag{59}$$

where V_{Wi} is given by (55) and $V_{l_1} = e_{l_1}^2 + e_{l_2}^2 + e_{l_3}^2 + e_{vl}^2 + e_{\phi l}^2$
Differentiating (59) yields

$$\dot{V}_{Formation} = \sum_{1}^{N} \dot{V}_{Wi} + \dot{V}_{l_1} \tag{60}$$

In the previous subsection it has been proved that V_{Wi} for all $i = 1toN$ individually is negative outside a compact set and that the error e_{FD} and the NN weight estimates $\hat{w}$ are uniformly ultimately bounded (UUB). Hence, when $\dot{V}_{Wi} < 0$ for all $i = 1toN$, so it automatically follows that $\sum_{1}^{N} \dot{V}_{Wi} < 0$. Also, the leader torque control and velocity control inputs are designed such that the errors go to zero asymptotically and hence, $\dot{V}_{l_1}$ is negative. Therefore, $\dot{V}_{Formation} < 0$, and the entire formation is asymptotically stable.

VI. RESULTS

A single leader single follower scenario is considered and the simulations are carried out using MATLAB for the same. The leader executes a circular trajectory with radius = 60 m, linear velocity of 5 m/sec and an angular velocity ~ 0.08 rad/sec. It is desired for the follower to execute a circle of radius = 56 m being parallel to the leader at all times. So the desired relative distance to be maintained is 4.0774 m and a relative bearing angle of 78.8199 degrees. The gains used during simulation are $k_{VF} = 8$, $K_3 = 0.01$, $k_V = 0.057$, $K = 100$. The constants $k = 0.5$ and $F = 30 * eye(20)$ are used in the NN weight update rule The NN has 20 hidden neurons. Measurement noise is added in the form Gaussian noise with zero mean. The noise added is one percent of the states that are inputs to the neural network. Also the simulations were carried out with different time constants for the steering dynamics and increased friction parameters. The plots obtained are given below. From Fig. 4 it can be seen that the follower achieves the desired position and orientation, with the position and orientation errors going to zero asymptotically as shown in Fig. 5.

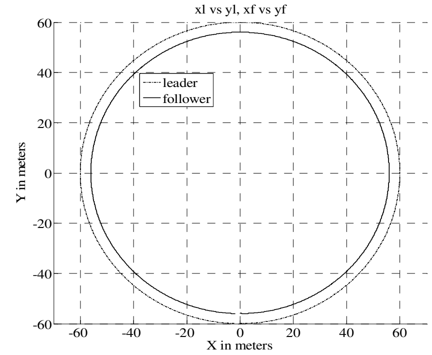

Fig. 4 Leader and follower trajectories

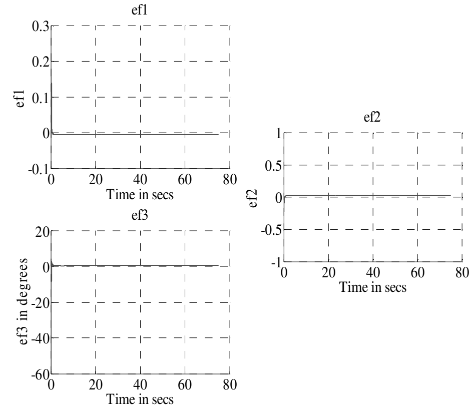

Fig. 5 Position and orientation errors

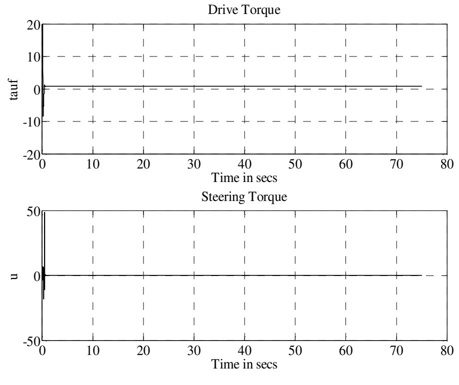

Fig.6 Drive and Steering torques

It can be seen that the follower is parallel to the leader at all times tracking a circle of radius 56 m. The torque control inputs to the drive and steering system, which achieve the velocity profile in (31) and(32) are as shown in Fig.6. From Fig. 7 it can be inferred that the velocity tracking errors also go to zero asymptotically. From Fig. 10 it can bee seen that the neural network is able to approximate $\mathbf{f}(\mathbf{x}_{Fnew})$ accurately. This work is being currently implemented on the jeep like mobile robots shown in Fig. 9.

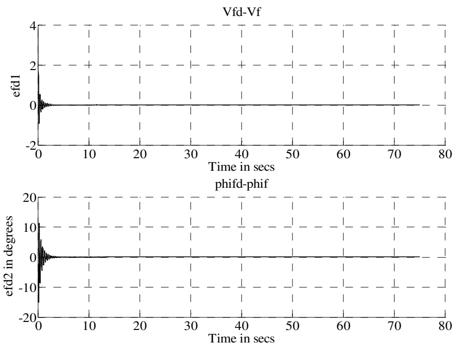

Fig.7 efd1 and efd2 plots

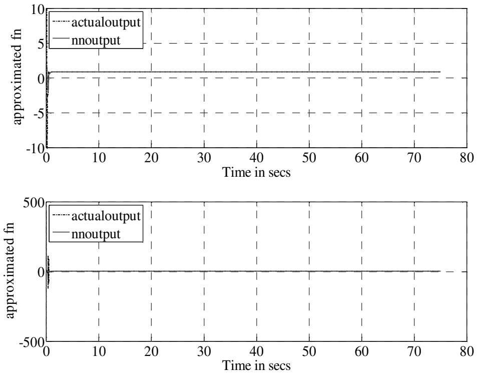

Fig. 8 Neural Network output

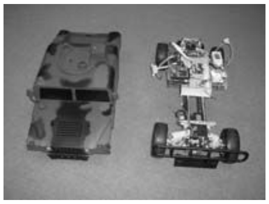

Fig. 9 Jeep Robot currently used for real time implementation

VII. Conclusion and Future Work

In this paper simplified dynamic equations are used to obtain the torque control inputs for the drive and steering system of a car-like follower mobile robot to maintain a desired relative distance and bearing angle between the leader and the follower. Imperfect velocity tracking and uncertainties in the friction forces and the steering system modeling is taken into account. In future simulations for multiple follower scenarios have to be carried out.

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